Eigen Values and Vectors

Definition Rhyme

"A transforms v but keeps its line,
Scale it by lambda, perfectly fine."

Direction Meaning Rhyme

"No twist, no turn, just stretch or squeeze —
Eigenvectors move with ease."

Eigenvalue Rhyme

"Lambda tells how big the stride,
Stretch or shrink — nothing to hide."

Lets Find Eigen Value and Vectors for following problem

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Step 1: Compute

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$$

Step 2: Determinant

$$(2-\lambda)(2-\lambda) - 1 = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda)^2 = 1$$

$$2 - \lambda = \pm 1$$

Step 3: Solve both cases

1. $2 - \lambda = 1 \rightarrow \lambda = 1$
2. $2 - \lambda = -1 \rightarrow \lambda = 3$

Step 4: Find Eigen Vecors for labbda value 1

$$(A - I)v = 0$$

$$\begin{pmatrix} 2-1 & 1 \\ 1 & 2-1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$x + y = 0$$

$$y = -x$$

$$v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Step 5: Find Eigen Vecors for labbda value 3

$$(A - 3I)v = 0$$

$$\begin{pmatrix} 2-3 & 1 \\ 1 & 2-3 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$-x + y = 0 \quad \Rightarrow \quad y = x$$

$$v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Final Answer

$$\lambda_1 = 1, \quad v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\lambda_2 = 3, \quad v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$