Problem 11.2 (ALC Semantics)

01. ALC concepts $\forall R.(C \sqcap D)$ & $(\forall R.C) \sqcap (\forall R.D)$

$$[[\forall R.(C \sqcap D)]] = \{ x \in D \mid \forall y.\ \text{if } \langle x,y \rangle \in [[R]] \text{ then } y \in [[C \sqcap D]] \}$$

$$= \{ x \in D \mid \forall y.\ \text{if } \langle x,y \rangle \in [[R]] \text{ then } (y \in [[C]] \cap y \in [[D]]) \}$$

$$= \{ x \in D \mid \forall y.\ \text{if } \langle x,y \rangle \in [[R]] \text{ then } y \in [[C]] \} \cap$$
$$\{ x \in D \mid \forall y.\ \text{if } \langle x,y \rangle \in [[R]] \text{ then } y \in [[D]] \}$$

$$= [[\forall R.C]] \sqcap [[\forall R.D]]$$

02. FOL conversion:

(i) $\forall R.(C \sqcap D) = \forall_Y R(X,Y) \rightarrow (C(Y) \wedge D(Y))$

(ii) $\forall R.C \sqcap \forall R.D = (\forall_Y R(X,Y) \rightarrow C(Y)) \wedge (\forall_Y R(X,Y) \rightarrow D(Y))$

$$\forall_Y R(X,Y) \rightarrow (C(Y) \wedge D(Y)) \leftrightarrow (\forall_Y R(X,Y) \rightarrow C(Y)) \wedge (\forall_Y R(X,Y) \rightarrow D(Y))$$

Since, we need to show that two formulas are equivalent, then Natural Deduction calculus will be best suited for this case.