Problem 7.1 PL Concepts

Answer:

1. **Every satisfiable formula is valid [False]**

   Let, model $M \coloneqq \langle U, I \rangle$, then

   A satisfiable in M, iff $I_\varphi(A) = T$ for some assignment φ

   A valid in M, iff $M \models^\varphi A$ for all assignments φ

   Therefore, if a formula is satisfiable for some assignment, that doesn't mean it will be valid for all assignments.

2. **Every valid formula is satisfiable [True]**

   Considering the previous model $M \coloneqq \langle U, I \rangle$, if a formula already valid for all the assignments, then it will certainly satisfiable.

3. **If $\boldsymbol{A}$ is satisfiable, then $\neg \boldsymbol{A}$ is unsatisfiable [True]**

   Let's say, $A = T$ (satisfiable) then $\neg A = F$ (unsatisfiable), but if $A = F$ (unsatisfiable) then $\neg A = T$ (satisfiable).

4. **If $\boldsymbol{A} \models \boldsymbol{B}$, then $\boldsymbol{A} \wedge \boldsymbol{C} \models \boldsymbol{B} \wedge \boldsymbol{C}$ [True]**

   $A \models B$ means all assignments that make A true also make B true, if that is the case then,

   if $(A \wedge C)$ is true for C then $(B \wedge C)$ will also be true for same value of C, thus the statement is true.

5. **Every admissible inference rule is derivable [True]**

   As the admissible rules are those inference rules which can be consistently employed in derivations in a given system, admissible inference rule is derivable.

6. **If $\vdash$ is sound for $\models$ and $\{\boldsymbol{A}, \boldsymbol{B}\} \vdash \boldsymbol{C}$, then $\boldsymbol{C}$ is satisfiable if $\boldsymbol{A}$ and $\boldsymbol{B}$ are [True]**

   As C can be derived from A and B, that means it is sound, therefore C is satisfiable if A and B are satisfiable.