Problem 10.3 First-Order Tableaux

Answer:

Given that, $P \in \sum_1^p$

Formula: $\exists X.\,(P(X) \Longrightarrow \forall Y.\,P(Y))$

First-Order Tableaux:

$$\exists X.\,(P(X) \Longrightarrow \forall Y.\,P(Y))^F$$

$$\forall X.\,\neg(P(X) \Longrightarrow \forall Y.\,P(Y))^T \qquad [De\ Morgan's\ Law\ for\ Existential\ Quantifer]$$

$$\forall X.\,\neg(\neg P(X) \vee \forall Y.\,P(Y))^T \qquad [p \longrightarrow q \equiv \neg p \vee q]$$

$$\forall X.\,(\neg\neg P(X) \wedge \neg\forall Y.\,P(Y))^T \qquad [De\ Morgan's\ Law]$$

$$\forall X.\,(P(X) \wedge \exists Y.\,\neg P(Y))^T \qquad [De\ Morgan's\ Law\ for\ Universal\ Quantifer]$$

$$\forall X.\,P(X)^T$$

$$\exists Y.\,\neg P(Y)^T$$

$$P(c)^T$$

$$\neg P(c)^T$$

$$P(c)^F$$

$$\perp$$