

1.

For example, let's say we have the following SAT instance:

$$V = \{x, y, z\}$$

$$A = (x \vee \neg y) \wedge (\neg x \vee y \vee z)$$

We can reduce this to a CSP instance P' as follows:

$$V' = \{x, y, z\}$$

$$D' = \{0, 1\}$$

$$C' = \{(x, \{0, 1\}), (y, \{0, 1\}), (z, \{0, 1\}), (x \vee \neg y, \{0\}), (\neg x \vee y \vee z, \{0\})\}$$

We can then define the bijection f as follows:

$$f: \{x=1, y=0, z=1\} \rightarrow \{x=1, y=0, z=1\}$$

This bijection maps a satisfying assignment of the SAT instance (i.e., $\{x=1, y=0, z=1\}$) to a solution of the CSP instance (i.e., an assignment of truth values to the variables x , y , and z in the CSP instance).

The inverse bijection f' would then be defined as follows:

$$f': \{x=1, y=0, z=1\} \rightarrow \{x=1, y=0, z=1\}$$

This bijection maps a solution of the CSP instance (i.e., an assignment of truth values to the variables x , y , and z) back to a satisfying assignment of the SAT instance (i.e., $\{x=1, y=0, z=1\}$).

2.

consider the following CSP instance with three variables a , b , and c , each with a domain of $\{1, 2, 3\}$. The constraints are:

- $a \neq b$
- $a \neq c$
- $b \neq c$

To define a SAT instance (V', A') from this CSP instance, we can follow these steps:

1. Create Boolean variables a' , b' , and c' in V' .
2. Create Boolean variables $d1'$, $d2'$, and $d3'$ in V' .
3. Create the following conjunctions in A' :
 - $(\neg a' \vee \neg b')$
 - $(\neg a' \vee \neg c')$
 - $(\neg b' \vee \neg c')$

This results in the following SAT instance (V', A') :

- $V' = \{a', b', c', d1', d2', d3'\}$
- $A' = \{(\neg a' \vee \neg b'), (\neg a' \vee \neg c'), (\neg b' \vee \neg c')\}$

Now, let's say we have a satisfying assignment for this SAT instance that sets the variables as follows:

- $a' = \text{True}$
- $b' = \text{False}$
- $c' = \text{True}$
- $d1' = \text{True}$
- $d2' = \text{False}$
- $d3' = \text{False}$

The function f maps this satisfying assignment to a solution for the original CSP instance as follows:

- $a = 1$ (because $d1'$ is True)
- $b = 2$ (because $d2'$ is False and b' is False)
- $c = 3$ (because $d3'$ is False and c' is True)

the inverse function f' would map the solution $a = 1, b = 2, c = 3$ for the CSP instance to the following satisfying assignment for the SAT instance:

- $a' = \text{True}$ (because 1 is the value of a in the solution and $d1'$ is True)
- $b' = \text{False}$ (because 2 is the value of b in the solution and $d2'$ is False)
- $c' = \text{True}$ (because 3 is the value of c in the solution and $d3'$ is False)
- $d1' = \text{True}$ (because 1 is the value assigned to a in the solution)
- $d2' = \text{False}$ (because 2 is the value assigned to b in the solution)
- $d3' = \text{False}$ (because 3 is the value assigned to c in the solution)

This inverse function f' can be used to determine whether a given solution for the CSP instance is a valid satisfying assignment for the corresponding SAT instance.