Artificial Intelligence 1
Assignment7: Propositional Logic
– Given Dec 8, Due Dec 18 –

Problem 7.1 (PL Concepts) 30 pt

Which of the following statements are true? In each case, give an informal argument why it is true or a counter-example.

1. Every satisfiable formula is valid.
2. Every valid formula is satisfiable.
3. If A is satisfiable, then $\neg A$ is unsatisfiable.
4. If $A \models B$, then $A \wedge C \models B \wedge C$.
5. Every admissible inference rule is derivable.
6. If $\vdash$ is sound for $\models$ and $\{A, B\} \vdash C$, then C is satisfiable if A and B are.

Problem 7.2 (Equivalence of CSP and SAT) 30 pt

We consider

- CSPs (V, D, C) with finite domains as before
- SAT problems (V, A) where V is a set of propositional variables and A is a propositional formula over V.

We will show that these problem classes are equivalent by reducing their instances to each other.

1. Given a SAT instance $P = (V, A)$, define a CSP instance $P' = (V', D', C')$ and two bijections
   - f mapping satisfying assignments of P to solutions of P'
   - f' the inverse of f

   We already know that binary CSPs are equivalent to higher-order CSPs. Therefore, it is sufficient to give a higher-order CSP.
2. Given a CSP instance (V, D, C), define a SAT instance (V', A') and bijections as above

Problem 7.3 (Calculi Comparison) 60 pt

Prove (or disprove) the validity of the following formulae in i) Natural Deduction ii) Tableau and iii) Resolution.

1. $(P \wedge Q) \Rightarrow (P \vee Q)$ (to be done in the tutorial, not part of grading)
2. $((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)) \Rightarrow C$
3. $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$