AI 1 2022/23
Assignment9: First-Order Logic
– Given Jan. 12, Due Jan. 22 –

Problem 9.1 (Induction) 20 pt

Use structural induction on terms and formulas to define a function C that maps every term/formula to the number of occurrences of free variables. For example, $C(\forall x.P(x,x,y,y,z)) = 3$ because the argument has 2 free occurrences of y and 1 of z.

Hint: Use an auxiliary function $C'(V,A)$ that takes the set V of bound variables and a term/formula A. Define C' by structural induction on A. Then define $C(A) = C'(\emptyset, A)$.

Problem 9.2 (First-Order Semantics) 30 pt

Let $=\in \Sigma_2^p$, $P \in \Sigma_1^p$ and $+ \in \Sigma_2^f$. We use the semantics of first-order logic without equality.

Prove or refute the following formulas semantically. That means you must show that $I_\varphi(A) = T$ for all models I and assignments φ (without using a proof calculus) or to give some I, φ such that $I_\varphi(A) = F$.

1. $P(X)$
2. $\forall X.\forall Y. = (+(X,Y), +(Y,X))$
3. $\exists X.(P(X) \Rightarrow \forall Y.P(Y))$
4. $P(Y) \Rightarrow \exists X.P(X)$

Problem 9.3 (Natural Deduction) 25 pt

Let $R \in \Sigma_2^p$, $P \in \Sigma_1^p$, $c \in \Sigma_0^f$.

Prove the following formula in Natural Deduction:

$$((\forall X.\forall Y.R(Y,X) \Rightarrow P(Y)) \;\wedge\; (\exists Y.R(c,Y))) \Rightarrow P(c)$$