Artificial Intelligence 1
Assignment5: Constraint Satisfaction
– Given Nov. 28, Due Dec. 4 –

Problem 5.1 (3 Rooks on a Small Board) 40 pt

Consider the following problem: We want to place 3 rooks (german: Turm) on a 4×7 chess-board such that no two rooks threaten each other. (Rooks can move horizontally and vertically as far as they like.)

Model the problem above as a *constraint satisfaction problem* $\langle V, D, C\rangle$. Explain your model briefly by saying how rook placements correspond to the *assignments* for the problem.

Note: Make sure you give a formally exact definition, i.e., explicitly define the sets V and all sets D_v. You can describe each *constraint* as a set of tuples or as a formula.

Grading: 2 points each for the variables, domains, and constraints, and 1 point for the argument. Deductions for mistakes.

Problem 5.2 (CSP as a Search Problem) 30 pt

We consider a binary CSP P with

- a set V of variables
- a family D of domains D_v for $v \in V$
- a family C of constraints $C_{uv} \subseteq D_u \times D_v$ for $u, v \in V$, $u \neq v$ where C_{uv} is the dual of C_{vu}

Note: We assume here that a constraint C_{uv} is given for all pairs of unequal variables — if we want to omit a constraint, we can simply assume $C_{uv} = D_u \times D_v$ or $C_{uv} =$ "`true`", i.e., all pairs are allowed and thus there is no constraint. That could be problematic in implementations, but is practical on paper.

Define the search problem (S, A, T, I, G) corresponding to P.

Note: This problem formalizes the informal statement that CSPs are a special case of search problems.

Problem 5.3 (Basic Definitions) 30 pt

Consider the following binary CSP:

- $V = \{a, b, c, d\}$
- $D_a = \texttt{bool}$, $D_b = D_c = \{0, 1, 2, 3\}$, $D_d = \{0, 1, 2, 3, 4, 5, 6\}$
- Constraints:

- if a, then $b \leq 2$
- if $c < 2$, then a
- $b + c < 4$
- $b > d$
- $d = 2c$

1. Give all solutions.
2. Give an inconsistent total assignment.
3. Give all consistent partial assignments α such that $\mathrm{dom}(\alpha) \subseteq \{a, b\}$.