Problem 10.2 First-Order Resolution

Answer:

Given that, $P \in \sum_1^p, R \in \sum_2^p, a, b \in \sum_0^f$

Formula: $\exists X. \forall Y. \exists Z. \exists W. ((\neg P(Z) \wedge \neg R(b,a)) \vee \neg R(a,b) \vee R(W,a) \vee (P(Y) \wedge R(X,b)))$

Negate:

$$\forall X. \exists Y. \forall Z. \forall W. \neg((\neg P(Z) \wedge \neg R(b,a)) \vee \neg R(a,b) \vee R(W,a) \vee (P(Y) \wedge R(X,b)))$$

$$\equiv \forall X. \exists Y. \forall Z. \forall W. (\neg(\neg P(Z) \wedge \neg R(b,a)) \wedge \neg\neg R(a,b) \wedge \neg R(W,a) \wedge \neg(P(Y) \wedge R(X,b)))$$

$$\equiv \forall X. \exists Y. \forall Z. \forall W. ((P(Z) \vee R(b,a)) \wedge R(a,b) \wedge \neg R(W,a) \wedge (\neg P(Y) \vee \neg R(X,b)))$$ [CNF]

Substituting bound variables:

$$(P(Z) \vee R(b,a)) \wedge R(a,b) \wedge \neg R(W,a) \wedge (\neg P(f_Y(X)) \vee \neg R(X,b))$$

Resolution:

$$\{P(Z)^T, R(b,a)^T\} + \{R(W,a)^F\}[b/W] \Longrightarrow \{P(Z)^T\}$$

$$\{R(a,b)^T\} + \{P(f_Y(X))^F, R(X,b)^F\}[a/X] \Longrightarrow \{P(f_Y(a))^F\}$$

$$\{P(Z)^T\} + \{P(f_Y(a))^F\}[f_Y(a)/Z] \Longrightarrow \{\}$$