AI 1 2022/23
Assignment11: Knowledge Representation
– Given Jan. 26, Due Feb. 5 –

Problem 11.1 (CompLog Students in ALC) 30 pt

Using ALC, first give a list of suitable primitive concepts and roles and then use them to represent the following:

1. the concept of students that are registered for at least one course
2. the concept of students that are only registered for courses taught by a woman
3. the fact that only students are registered for courses

Give the result of translating the ALC formulas into first-order logic. (Recall that a fact/concept is translated into a first-order formula with 0/1 free variables.)

Solution: primitive concepts: `student`, `woman`, `course`
roles: `registeredfor`, `taughtby`

ALC representations and first-order translations:

1. $\mathtt{student} \sqcap \exists \mathtt{registeredfor}.\mathtt{course}$
   $\mathtt{student}(x) \wedge \exists y.\mathtt{registeredfor}(x, y) \wedge \mathtt{course}(y)$
2. $\mathtt{student} \sqcap \forall \mathtt{registeredfor}.(\mathtt{course} \sqcap \exists \mathtt{taughtby}.\mathtt{woman})$
   $\mathtt{student}(x) \wedge \forall y.\mathtt{registeredfor}(x, y) \Rightarrow (\mathtt{course}(y) \wedge \exists z.\mathtt{taughtby}(y, z) \wedge \mathtt{woman}(z))$
3. $\exists \mathtt{registeredfor}.\mathtt{course} \sqsubseteq \mathtt{student}$
   $\forall x.(\exists y.\mathtt{registeredfor}(x, y) \wedge \mathtt{course}(y)) \Rightarrow \mathtt{student}(x)$

Problem 11.2 (ALC Semantics) 30 pt

Consider the ALC concepts $\forall R.(C \sqcap D)$ and $(\forall R.C) \sqcap (\forall R.D)$.

1. By applying the semantics of ALC, show that the two are equivalent.
2. Translate both formulas to first-order logic and state which FOL formula we would need to prove (e.g., with the ND calculus) to show that the two are equivalent.

Solution:

1. We have:

$$\llbracket \forall R.(C \sqcap D) \rrbracket$$
$$= \{x \in \mathcal{D} | \text{for all } y \in \mathcal{D},\ \text{if } (x,y) \in \llbracket R \rrbracket, \text{then } y \in \llbracket C \sqcap D \rrbracket\}$$
$$= \{x \in \mathcal{D} | \text{for all } y \in \mathcal{D},\ \text{if } (x,y) \in \llbracket R \rrbracket, \text{then } y \in \llbracket C \rrbracket \cap \llbracket D \rrbracket\}$$

$$\llbracket (\forall R.C) \sqcap (\forall R.D) \rrbracket$$
$$= \llbracket \forall R.C \rrbracket \cap \llbracket \forall R.D \rrbracket$$
$$= \{x \in \mathcal{D} | \text{for all } y \in \mathcal{D},\ \text{if } (x,y) \in \llbracket R \rrbracket, \text{then } y \in \llbracket C \rrbracket \cap \{x \in \mathcal{D} | \text{for all } y \in \mathcal{D},\ \text{if } (x,y) \in \llbracket R \rrbracket, \text{then } y \in \llbracket D \rrbracket\}\}$$

Now to prove that sets are equal, consider an $x \in \mathcal{D}$ and see that both conditions are equivalent to

$$\text{for all } y \in \mathcal{D},\ \text{if } (x,y) \in \llbracket R \rrbracket, \text{then } y \in \llbracket C \rrbracket \text{and } y \in \llbracket D \rrbracket$$

2. The translation yields

$$C_1(x) = \forall y.R(x,y) \Rightarrow (C(y) \wedge D(y))$$
$$C_2(x) = (\forall y.R(x,y) \Rightarrow C(y)) \wedge (\forall y.R(x,y) \Rightarrow D(y))$$

We need to show

$$\forall x.C_1(x) \Leftrightarrow C_2(x)$$

Problem 11.3 (ALC TBox) 40 pt

Consider ALC with the following

- primitive concepts: `woman`, `man`
- roles: `has_child`, `has_parent`, `has_sibling`, `has_spouse`

Give an ALC TBox that defines the concepts `person`, `parent`, `mother`, `father`, `grandmother`, `aunt`, `uncle`, `sister`, `brother`, `onlychild`, `cousin`, `nephew`, `niece`, `fatherinlaw`, `motherinlaw`.

Solution: $\mathtt{person} = \mathtt{man} \sqcup \mathtt{woman}$
$\mathtt{parent} = \mathtt{person} \sqcap \exists \mathtt{has\_child.person}$
$\mathtt{mother} = \mathtt{woman} \sqcap \mathtt{parent}$
$\mathtt{father} = \mathtt{man} \sqcap \mathtt{parent}$
$\mathtt{grandmother} = \mathtt{woman} \sqcap \exists \mathtt{has\_child.parent}$
$\mathtt{aunt} = \mathtt{woman} \sqcap \exists \mathtt{has\_sibling.parent}$
$\mathtt{uncle} = \mathtt{man} \sqcap \exists \mathtt{has\_sibling.parent}$
$\mathtt{sister} = \mathtt{woman} \sqcap \exists \mathtt{has\_sibling.person}$
$\mathtt{brother} = \mathtt{man} \sqcap \exists \mathtt{has\_sibling.person}$
$\mathtt{onlychild} = \mathtt{person} \sqcap \overline{\mathtt{brother} \sqcup \mathtt{sister}}$
$\mathtt{cousin} = \mathtt{person} \sqcap \exists \mathtt{has\_parent.} \exists \mathtt{has\_sibling.parent}$
$\mathtt{nephew} = \mathtt{man} \sqcap \exists \mathtt{has\_parent.} \exists \mathtt{has\_sibling.person}$
$\mathtt{niece} = \mathtt{woman} \sqcap \exists \mathtt{has\_parent.} \exists \mathtt{has\_sibling.person}$
$\mathtt{fatherinlaw} = \mathtt{man} \sqcap \exists \mathtt{has\_child.} \exists \mathtt{has\_spouse.person}$
$\mathtt{motherinlaw} = \mathtt{woman} \sqcap \exists \mathtt{has\_child.} \exists \mathtt{has\_spouse.person}$