Problem 7.2 Equivalence of CSP and SAT

Answer:

1. According to the problem description,

SAT instance $P = (V, A)$, where V is a set of propositional variables and A is propositional formula

CSP instance $P' = (V', D', C')$

SAT to CSP:

- Variables in CSP are the propositional variables of SAT, thus $V' = V$
- Domains in CSP are the universe of SAT, thus $D' \in \{T, F\}$
- Constraints in CSP are the propositional formula of SAT, thus $C' = A$
- Assignments in CSP are the model of SAT $M \coloneqq \langle U, I \rangle$, where U is universe and I interpretation function
- And two bijections are as follows,
  - Solution of P' will be $f: I_\varphi(A) = T$ for some assignment φ
  - $f': I_\varphi(A) = F$ for some assignment φ

2. According to the problem description,

CSP instance $P = (V, D, C)$ and SAT instance $P' = (V', A')$

CSP to SAT:

- Propositional variables in SAT are the variables in CSP, thus $V' = V$
- Propositional formula in SAT is the constrains in CSP, thus $A' = C$
- And two bijections are as follows,
  - Solution of P' will be the consistent total assignment of CSP, thus

    $f: a, iff\ \forall C_{uv} \in C, \big(a(u), a(v)\big) \in C_{uv}$ where a is a consistent total variable assignment
  - $f': a, iff\ \forall C_{uv} \in C, \big(a(u), a(v)\big) \notin C_{uv}$ where a is an inconsistent variable assignment