Problem 2.4 Mathematical Notation

Answer:

Consider 0 as a natural number, hence $\mathbb{N} = \{0, 1, 2, 3, \dots \dots\}$

1. The set containing all natural numbers, $S = \{x: x\epsilon\mathbb{N}\}$
2. The set containing the set of natural numbers, $S = \{\{x\}: x\epsilon\mathbb{N}\}$
3. The set containing all square numbers, $S = \{x^2: x\epsilon\mathbb{N}\}$
4. The set containing all even natural numbers,

   $S = \{2x: x\epsilon\mathbb{N}\}$, or $S = \{x: x\epsilon\mathbb{N}, 2|x\}$
5. The set containing all even square numbers, $S = \{x^2: x\epsilon\mathbb{N}, 2|x\}$
6. The 3-tuple of 0, 1, and 2, which is triplet,

   $T = \langle x_1, x_2, x_3\rangle, where\ x_i\epsilon\mathbb{N}, 0 \leq x_i \leq 2, i = 1,2,3$
7. The n-tuple of all numbers from 0 to n – 1,

   $T = \langle x_1, x_2, x_3, \dots \dots, x_n\rangle, where\ x_i\epsilon\mathbb{N}, 0 \leq x_i \leq n - 1, i = 1,2,3, \dots, n$
8. The set of pairs of natural numbers and their squares, $S = \{(x, y): x\epsilon\mathbb{N}, y = x^2\}$
9. The pair of sets of natural numbers and square numbers,

   $P = (x, y), where\ x = \{n: n\epsilon\mathbb{N}\}, y = \{n^2: n\epsilon\mathbb{N}\}$
10. Algebraic structure $\langle\mathbb{N}, +\rangle$ is monoid, as the identity element $0\epsilon\mathbb{N}$ $[\forall a\epsilon\mathbb{N}, a + 0 = a]$

    So, the monoid of natural numbers under addition, $\langle\mathbb{N}, +, 0\rangle$
11. Algebraic structure $\langle\mathbb{N}, +\rangle$ and $\langle\mathbb{N}, *\rangle$ are monoid, as the identity element $0\epsilon\mathbb{N}$ $[\forall a\epsilon\mathbb{N}, a + 0 = a]$ and $1\epsilon\mathbb{N}$ $[\forall a\epsilon\mathbb{N}, a * 1 = a]$ respectively. So, the pair of monoids of the natural numbers under addition and under multiplication, $P = (\langle\mathbb{N}, +, 0\rangle, \langle\mathbb{N}, *, 1\rangle)$.
12. The set of the monoids of the natural numbers under addition and under multiplication,

    $S = \{\langle\mathbb{N}, +, 0\rangle, \langle\mathbb{N}, *, 1\rangle\}$
13. Given a monoid $\langle U, o, e\rangle$, the set of elements that are not the neutral element,

    $S = \{x: x\epsilon U, x\ o\ e \neq x\}$
14. Given a monoid $\langle U, o, e\rangle$, the monoid in which the operation is flipped,

    $\langle U, o, e\rangle$ are new monoid, where the identity element $e\epsilon U$ and $\forall a\epsilon U, a\ o\ a^{-1} = e$