AI 1 2022/23
Assignment10: Knowledge Representation
– Given Jan. 21, Due Jan. 29 –

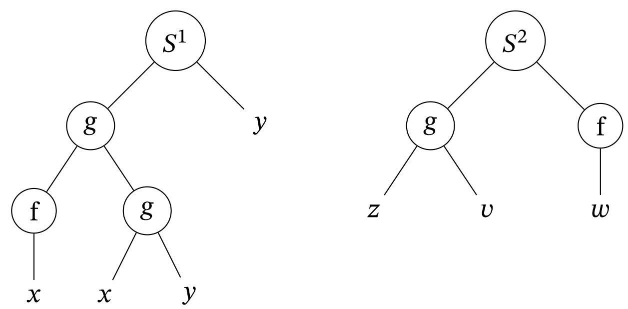
Problem 10.1 (Unification) 30 pt

Decide whether (and how or why not) the following pairs of terms are unifiable.
$S_1 \in \Sigma_2^p, S_2 \in \Sigma_3^p, f \in \Sigma_1^f, g \in \Sigma_2^f, c \in \Sigma_0^f$

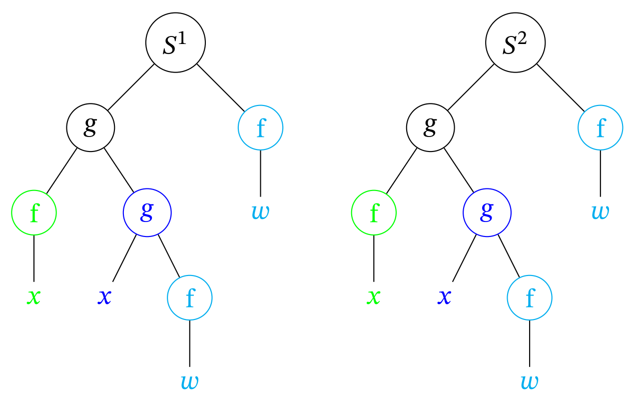
1. $S_1(g(f(x), g(x,y)), y)$ and $S_1(g(z,v), f(w))$
2. $S_2(g(f(x), g(x,u)), f(y), z)$ and $S_2(g(g(g(u,v), f(w)), f(c)), f(g(u,v)), f(c))$

Solution:

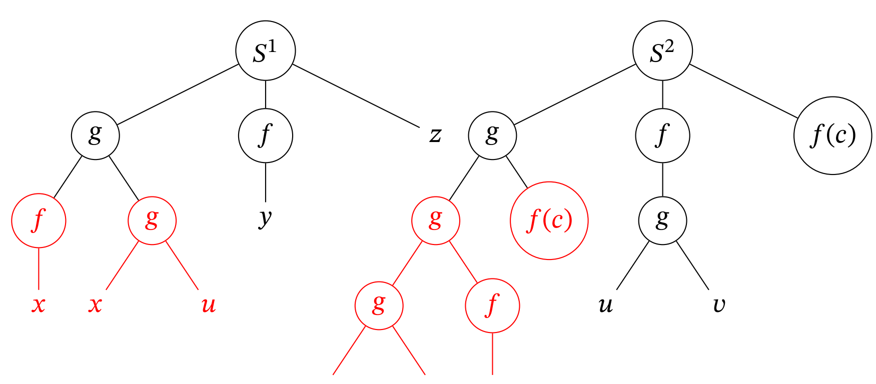
1. The term trees look like this:



   Obviously, we need to perform the following substitutions to make the two trees equal:



2. The term trees look like this:



   Obviously, the red subtrees can't be unified.

Problem 10.2 (First-Order Resolution) 35 pt

Prove the following formula using resolution.

$P \in \Sigma_1^p, R \in \Sigma_2^p, a, b \in \Sigma_0^f$

$$\exists X.\forall Y.\exists Z.\exists W.\, ((\neg P(Z) \wedge \neg R(b,a)) \vee \neg R(a,b) \vee R(W,a) \vee (P(Y) \wedge R(X,b)))$$

Solution: We negate:

$$\forall X.\exists Y.\forall Z.\forall W.(P(Z) \vee R(b,a)) \wedge R(a,b) \wedge \neg R(W,a) \wedge (\neg P(Y) \vee \neg R(X,b))$$

We skolemize:

$$(P(Z) \vee R(b,a)) \wedge R(a,b) \wedge \neg R(W,a) \wedge (\neg P(f_Y(X)) \vee \neg R(X,b))$$

This yields the clauses $\{P(Z)^T, R(b,a)^T\}, \{R(a,b)^T\}, \{R(W,a)^F\}, \{P(f_Y(X))^F, R(X,b)^F\}$.
We resolve:

$$\begin{gathered}\{P(Z)^T, R(b,a)^T\} + \{R(W,a)^F\}[b/W] \Longrightarrow \{P(Z)^T\} \\ \{R(a,b)^T\} + \{P(f_Y(X))^F, R(X,b)^F\}[a/X] \Longrightarrow \{P(f_Y(a))^F\} \\ \{P(Z)^T\}[f_Y(a)/Z] + \{P(f_Y(a))^F\} \Longrightarrow \{\}\end{gathered}$$

Problem 10.3 (First-Order Tableaux) 35 pt

Prove the following formula using the first-order free variable tableaux calculus.
We have $P \in \Sigma_1^p$.

$$\exists X.\,(P(X) \Rightarrow \forall Y.P(Y))$$

Solution:

(1)	$\exists X.(P(X) \Rightarrow \forall Y.P(Y))^F$	
(2)	$P(V_X) \Rightarrow \forall Y.P(Y)^F$	(from 1)
(3)	$P(V_X)^T$	(from 2)
(4)	$\forall Y.P(Y)^F$	(from 2)
(5)	$P(c_Y)^F$	(from 4)
(6)	$\perp[c_Y/V_X]$	
