Artificial Intelligence 1
Assignment5: Constraint Satisfaction
– Given Nov. 28, Due Dec. 4 –

Problem 5.1 (3 Rooks on a Small Board) 40 pt

Consider the following problem: We want to place 3 rooks (german: Turm) on a 4×7 chess-board such that no two rooks threaten each other. (Rooks can move horizontally and vertically as far as they like.)

Model the problem above as a *constraint satisfaction problem* $\langle V, D, C\rangle$. Explain your model briefly by saying how rook placements correspond to the *variable assignments* for the problem.

Make sure you give a formally exact definition, i.e., explicitly define the sets V and all sets D_v. You can describe each *constraint* as a set of tuples or as a formula.

Solution: $V = \{a_1, a_2, b_1, b_2, c_1, c_2\}$
$D_{a_1} = D_{b_1} = D_{c_1} = \{1, 2, 3, 4\}$
$D_{a_2} = D_{b_2} = D_{c_2} = \{1, 2, 3, 4, 5, 6, 7\}$
Constraints in C:

- $v_1 \neq w_1$ for all $(v, w) \in \{(a, b), (a, c), (b, c)\}$
- $v_2 \neq w_2$ for all $(v, w) \in \{(a, b), (a, c), (b, c)\}$

The assignments to (a_1, a_2), (b_1, b_2), and (c_1, c_2) correspond to the coordinates of the squares where the rooks are placed.

Problem 5.2 (CSP as a Search Problem) 30 pt

We consider a binary CSP P with

- a set V of variables
- a family D of domains D_v for $v \in V$
- a family C of constraints $C_{uv} \subseteq D_u \times D_v$ for $u, v \in V$, $u \neq v$ where C_{uv} is the dual of C_{vu}

Note: We assume here that a constraint C_{uv} is given for all pairs of unequal variables — if we want to omit a constraint, we can simply assume $C_{uv} = D_u \times D_v$ or $C_{uv} = \texttt{true}$, i.e., all pairs are allowed and thus there is no constraint. That could be problematic in implementations, but is practical on paper.

Define the search problem (S, A, T, I, G) corresponding to P.

Note: This problem formalizes the informal statement that CSPs are a special case of search problems.

Solution: The search problem is defined as follows:

- states are the assignments, i.e., S is the set of the partial mappings with domain V that map each $v \in V$ to an element of D_v
  More formally, we can write this as

  $$S = \{a : V \rightharpoonup \bigcup_{v \in V} D_v \mid \forall v \in \mathrm{dom}(a).a(v) \in D_v\}$$

  Typically, V is finite. In that case, we can use the simpler definition

  $$S = \prod_{v \in V} (D_v \cup \{\perp\})$$

  where $a(v) = \perp$ represents that a is partial at v.

- An action assigns to a variable a concrete value of its domain. So

  $$A = \{(v, x) \mid v \in V, x \in D_v\}$$

- The transition model simply updates the assignment: $T((v, x), a) = \{a'\}$ where $a'(v) = x$ and $a'(w) = a(w)$ if $w \neq v$.
  Note: alternatively, we could put $T((v, x), a) = \emptyset$ if a already assigns a value to v.

- In the initial state, no variable is assigned: $I = \{i\}$ where i is undefined everywhere.

- The terminal states are the solutions, i.e., T is the set of all $a \in S$ such that
  - a is total
  - for all $u, v \in V$ with $u \neq v$, we have $(a(u), a(v)) \in C_{uv}$

Problem 5.3 (Basic Definitions) 30 pt

Consider the following binary CSP:

- $V = \{a, b, c, d\}$
- $D_a = \texttt{bool}, D_b = D_c = \{0, 1, 2, 3\}, D_d = \{0, 1, 2, 3, 4, 5, 6\}$
- Constraints:
  - if a, then $b \leq 2$
  - if $c < 2$, then a
  - $b + c < 4$
  - $b > d$
  - $d = 2c$

1. Give all solutions.
2. Give an inconsistent total assignment.
3. Give all consistent partial assignments α such that $\mathrm{dom}(\alpha) \subseteq \{a, b\}$.

Solution:

- There are 2 solutions: a true, $b \in \{1, 2\}$, $c = 0$, $d = 0$
- Any total assignment that is not a solution (see previous question).
- We classify the possibly assignments a by their domain:
  - $\mathrm{dom}(\alpha) = \emptyset$: 1 assignment, namely α undefined everywhere
  - $\mathrm{dom}(\alpha) = \{a\}$: 2 assignments, namely $\alpha(a) \in D_a$, undefined elsewhere
  - $\mathrm{dom}(\alpha) = \{b\}$: 4 assignments, namely $\alpha(b) \in D_b$, undefined elsewhere
  - $\mathrm{dom}(\alpha) = \{a, b\}$: 7 assignments, namely
    * 4 assignments with $\alpha(a) = false$, $\alpha(b) \in D_b$, undefined elsewhere
    * 3 assignments with $\alpha(a) = true$, $\alpha(b) \in \{0, 1, 2\}$, undefined elsewhere

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