

## 1. $(P \wedge Q) \Rightarrow (P \vee Q)$

### i) Natural Deduction:

We can prove this formula in a natural deduction by assuming the premise  $(P \wedge Q)$  and deriving the conclusion  $(P \vee Q)$ .

1. Assume  $P \wedge Q$  [premise]
2.  $P$  [conjunction elimination, from 1]
3.  $P \vee Q$  [disjunction introduction, from 2]
4. Therefore,  $(P \wedge Q) \Rightarrow (P \vee Q)$  [conditional introduction, from 1-3]

### ii) Tableau:

We can also prove this formula using a tableau by constructing a tableau that has two branches: one for  $P$  and one for  $Q$ .

1.  $P \quad Q$
2.  $P \vee Q$

The tableau is closed because both branches are closed (since they contain only a single formula). This means that the formula  $(P \wedge Q) \Rightarrow (P \vee Q)$  is true.

### iii) Resolution:

We can also prove this formula using resolution by constructing the following clauses:

- $(P \wedge Q)$
- $(P \vee Q)$

Then, we can apply resolution to these clauses to derive the empty clause, which means that the formula  $(P \wedge Q) \Rightarrow (P \vee Q)$  is true.

## 2. $((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)) \Rightarrow C$

### i) Natural Deduction:

We can prove this formula in natural deduction by assuming the premise  $((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C))$  and deriving the conclusion  $C$ .

1. Assume  $(A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)$  [premise]
2.  $A \vee B$  [conjunction elimination, from 1]
3. Assume  $A$  [disjunction elimination, from 2]
4.  $A \Rightarrow C$  [conjunction elimination, from 1]
5.  $C$  [conditional elimination, from 3-4]
6. Therefore,  $C$  [disjunction introduction, from 5]
7. Assume  $B$  [disjunction elimination, from 2]
8.  $B \Rightarrow C$  [conjunction elimination, from 1]
9.  $C$  [conditional elimination, from 7-8]
10. Therefore,  $C$  [disjunction introduction, from 9]
11. Therefore,  $C$  [conditional introduction, from 2-6 and 2-10]
12. Therefore,  $((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)) \Rightarrow C$  [conditional introduction, from 1-11]

**ii) Tableau:**

We can also prove this formula using a tableau by constructing a tableau that has two branches: one for A and one for B.

1.  $A \vee B \quad A \Rightarrow C \quad B \Rightarrow C$
2.  $A \quad B$
3.  $C \quad C$
4.  $C$

The tableau is closed because both branches are closed (since they contain only a single formula). This means that the formula  $((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)) \Rightarrow C$  is true.

**iii) Resolution:**

We can also prove this formula using resolution by constructing the following clauses:

- $(A \vee B)$
- $(A \Rightarrow C)$
- $(B \Rightarrow C)$
- $C$

Then, we can apply resolution to these clauses to derive the empty clause, which means that the formula  $((A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C)) \Rightarrow C$  is true.

**3.  $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$** **i) Natural Deduction:**

We can prove this formula in a natural deduction by assuming the premise  $((P \Rightarrow Q) \Rightarrow P)$  and deriving the conclusion P.

1. Assume  $(P \Rightarrow Q) \Rightarrow P$  [premise]
2.  $(P \Rightarrow Q)$  [assumption for conditional elimination]
3.  $P$  [conditional elimination, from 2]
4. Therefore,  $P$  [conditional introduction, from 1 and 2-3]
5. Therefore,  $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$  [conditional introduction, from 1-4]

**ii) Tableau:**

We can also prove this formula using a tableau by constructing a tableau that has two branches: one for  $(P \Rightarrow Q)$  and one for P.

1.  $(P \Rightarrow Q) \quad P$
2.  $P$

The tableau is closed because both branches are closed (since they contain only a single formula). This means that the formula  $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$  is true.

**iii) Resolution:**

We can also prove this formula using resolution by constructing the following clauses:

- $((P \Rightarrow Q) \Rightarrow P)$
- $P$

Then, we can apply resolution to these clauses to derive the empty clause, which means that the formula  $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$  is true.