Last Name: First Name:

Matriculation Number:

Seat:

Exam
Artificial Intelligence 1

August 1, 2022

	To be used for grading, do not write here													
prob.	1.1	2.1	2.2	3.1	4.1	4.2	5.1	5.2	5.3	6.1	7.1	7.2	Sum	grade
total	10	8	7	6	7	8	5	8	8	9	12	8	96	
reached														

Organizational Information

Please read the following directions carefully and acknowledge them with your signature.

1. Please place your student ID card and a photo ID on the table for checking
2. You can reach 96 points if you fully solve all problems. You will only need 90 points for a perfect score, i.e. 6 points are bonus points.
3. No resources or tools are allowed except for a pen.
4. You have 90 min (sharp) for the test
5. Write the solutions directly on the sheets, no other paper will be graded.
6. If you have to abort the exam for health reasons, your inability to sit the exam must be certified by an examination at the University Hospital. Please notify the exam proctors and have them give you the respective form.
7. Please make sure that your copy of the exam is complete (20 pages excluding cover sheet and organizational information pages) and has a clear print. **Do not forget to add your personal information on the cover sheet and to sign this declaration.**

Declaration: With my signature I certify having received the full exam document and having read the organizational information above.

Erlangen, August 1, 2022 ....

(signature)

Organisatorisches

Bitte lesen die folgenden Anweisungen genau und bestätigen Sie diese mit Ihrer Unterschrift.

1. Bitte legen Sie Ihren Studierendenausweis und einen Lichtbildausweis zur Personenkontrolle bereit!
2. Sie können 96 Punkte erreichen, wenn Sie alle Aufgaben vollständig lösen. Allerdings zählen 90 Punkte bereits als volle Punktzahl, d.h. 6 Punkte sind Bonuspunkte.
3. Es sind keine Hilfsmittel erlaubt außer einem Stift.
4. Die Bearbeitungszeit beträgt genau 90 min.
5. Schreiben Sie die Lösungen direkt auf die ausgeteilten Aufgabenblätter. Andere Blätter werden nicht bewertet.
6. Wenn Sie die Prüfung aus gesundheitlichen Gründen abbrechen müssen, so muss Ihre Prüfungsunfähigkeit durch eine Untersuchung in der Universitätsklinik nachgewiesen werden. Melden Sie sich in jedem Fall bei der Aufsicht und lassen Sie sich das entsprechende Formular aushändigen.
7. Überprüfen Sie Ihr Exemplar der Klausur auf Vollständigkeit (20 Seiten exklusive Deckblatt und Hinweise) und einwandfreies Druckbild! **Vergessen Sie nicht, auf dem Deckblatt die Angaben zur Person einzutragen und diese Erklärung zu unterschreiben!**

Erklärung: Durch meine Unterschrift bestätige ich den Empfang der vollständigen Klausurunterlagen und die Kenntnisnahme der obigen Informationen.

Erlangen, August 1, 2022 ....

(Unterschrift)

Please consider the following guidelines to avoid losing points:

- If you continue an answer on another page, clearly give the problem number on the new page and a page reference on the old page.
- If you do not want something to be graded, clearly cross it out.
- The instructions "Give X", "List X" or similar mean that only X is needed. If you additionally justify your answer, we will try to give you partial credit for a wrong answer (but there is no guarantee that we will).
- The instruction "Assume X" means that X is information that you may use in your answer.
- The instruction "Model X as a Y" means that you have to describe X formally and exactly as an instance of Y using the definition of Y from the lecture.
- If you are uncertain how long or complex an answer should be, use the number of points as an indication: 1 point roughly corresponds to 1 minute.

1 Prolog

Problem 1.1 (Prolog) 10 pt

Consider the following Prolog code for propositional logic, where we represent the propositional variables as pv(N) for natural numbers N.

```
isNat(zero).
isNat(succ(N)) :- isNat(N).

isForm(conj(F,G)) :- isForm(F), isForm(G).
isForm(neg(F)) :- isForm(F).
isForm(pv(N)) :- isNat(N).
```

1. What do the following queries return: 3 pt

   (a) `isForm(conj(pv(zero),pv(zero)))` true

   (b) `isForm(X)` Nothing, it will backtracks foreaver

   If a query returns multiple results, only give the first two.

2. Write a Prolog program `uses` such that `uses(F,N)` holds if the propositional variable with number N occurs in the formula F. 5 pt

3. Assume we change the `isForm` predicate to 2 pt

   ```
   isForm(conj(F,G)) :- isForm(G), isForm(F).
   ```

   Explain if and how this affects which results are returned by the query `isForm(conj(pv(M),pv(N)))`.

½

it will find diff formulae $P_n \wedge P_0$ instead of $P_0 \wedge P_n$ for increasing n

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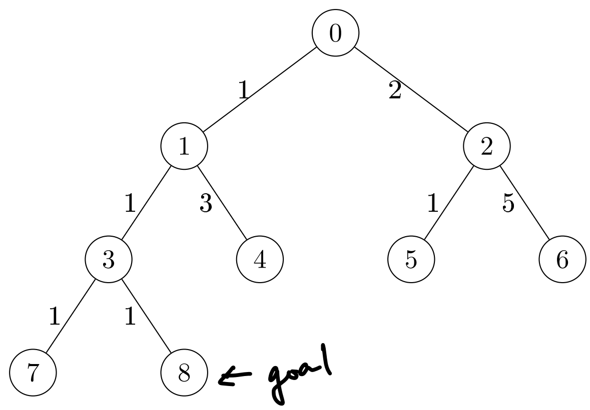
Q2.

```
uses( conj(F,_), N) :- uses(F,N).
uses( conj(_,G), N) :- uses(G,N).
uses( neg(F), N) :- uses(F,N).
uses( pv(N), N).
```

2 Search

Problem 2.1 (Search Algorithms) 8 pt

Consider the following tree with root 0:



Every edge is labeled with its cost.
As a heuristic for estimating the distance from node n to a goal node, we use $h(n) = 8 - n$.

Assume you have already expanded the root node. List the **next** 4 nodes that will be expanded using

1. depth-first search 1 pt

   0, 1, 3, 7, 8

2. breadth-first search 1 pt

   0, 1, 2, 3, 4

3. uniform-cost search 2 pt

   f ~~1~~ ~~2~~ ~~3~~ 4 5 6 7 8
   c 1 2 2 4 3 7 3 3
   v: 0 1 2 3 5

4. greedy search 2 pt

   f : ~~1~~ ~~2~~ ~~5~~ ~~6~~ 3 4
   h(n) : 7 6 3 2 5 4
   v: 0 2 6 5 1

5. A^*-search 2 pt

   f : ~~1~~ 2 ~~3~~ 4 ~~7~~ ~~8~~
   g(n) + h(n) : 1+7 2+6 2+5 4+4 3+1 3+0
   v : 0 1 3 8 7

If there is a tie, first expand the node with the smaller number.

Problem 2.2 (Search Problems) 7 pt

Consider the search problem (S, A, T, I, G) where

- $S = \mathbb{Z}$
- $A = \{-2, -1, 0, 1, 2\}$
- $T(a, s) = \{a \cdot s\}$
- $I = \{1\}$
- $G = \{8\}$

1. Give the state resulting from applying the action sequence $-1, 2, -2$ to the initial state. 1 pt

$1 \xrightarrow{-1} -1 \xrightarrow{2} -2 \xrightarrow{-2} 4$

2. Give a solution to the problem. 3 pt

$\underbrace{2,2,2}_{8}$

3. Assume we try actions in the order $\underbrace{-2, -1, 0, 1, 2}$. Will DFS find a solution? Why (not)? 2 pt

$1 \xrightarrow{-2} -2 \xrightarrow{-2} 4 \xrightarrow{-2} -8 \xrightarrow{-2} 16$

No, it will only try -2 and find the states 1, -2, 4, -8, 16, ...

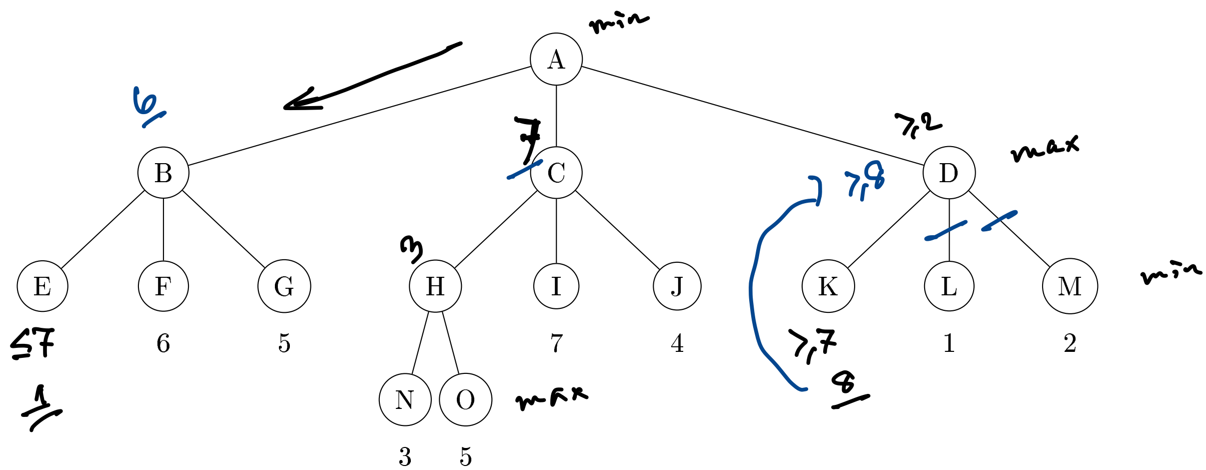
4. Give all states that are reachable from the initial state. 1 pt

- any power of 2 → $2^0, 2^1, \ldots$
- and there negation → $-2^0, -2^1, \ldots$
- and 0

3 Adversarial Search

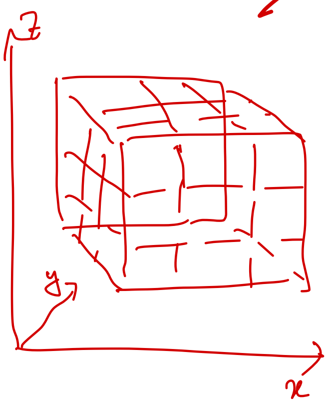
Problem 3.1 (Minimax) 6 pt

Consider the following minimax game tree (**without** alpha-beta pruning) for the **minimizing player**'s turn. The values at the leafs are the static evaluation function values of those states; some of those values are currently missing.



1. Label the nodes H and C with their minimax values. H=3, C=7 2 pt
2. Label the nodes E and K with evaluation function values that result in the player choosing move B. E ≤7 , K ≥7 2 pt
3. Now assume E is labeled 1 and K is labeled 8.
   Which out of K, L, M are α-β-search pruned (i.e., not visited)?
   Assume children of a node are visited in alphabetical order. L & M 2 pt

4 Constraint Satisfaction/Propagation



Problem 4.1 (Pieces on a 3D Board) 7 pt

Consider a cubic board of size $3 \times 3 \times 3$ (i.e., there are 27 spots to place a piece). We want to place 2 pieces in such a way that no pieces touch via a face in any direction (up/down, left/right, forward/backward). Touches via an edge or a corner are allowed.

1. Model the problem as a constraint satisfaction problem (V, D, C). 6 pt
2. Explain your model briefly by saying how piece placements correspond to the assignments for the problem. 1 pt

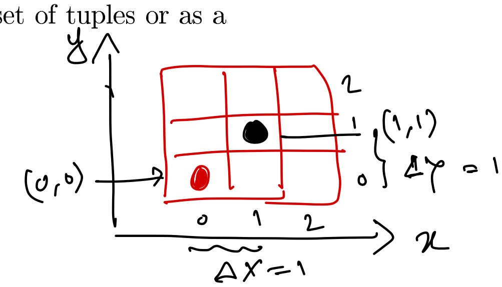
Make sure you give a formally exact definition, i.e., explicitly define the sets V and all sets D_v. You can describe each constraint as a set of tuples or as a formula.

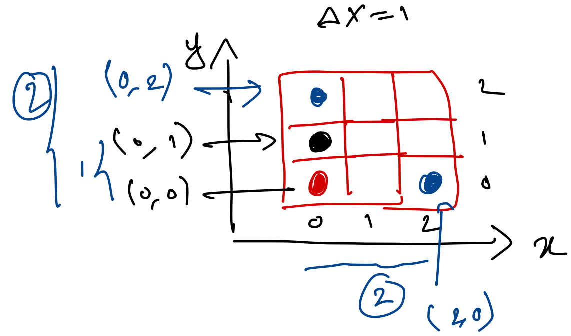
01. $\mathcal{V} = \overbrace{x_1, y_1, z_1}^{1p}, \overbrace{x_2, y_2, z_2}^{2p}$

$D_v = \{0, 1, 2\}$ for all $v \in \mathcal{V}$

C :

$|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2| > 1$





02. (x_1, y_1, z_1) & (x_2, y_2, z_2)

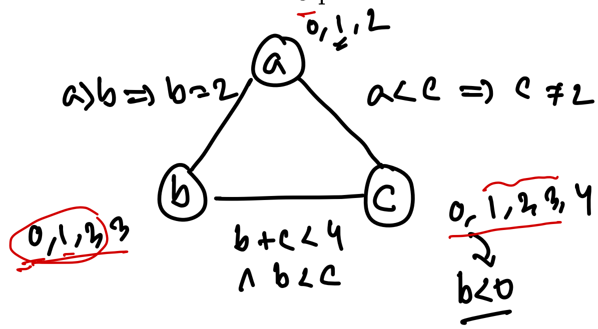
assign ⟷ corresponding position where two pieces are placed.

⇕ coordinates

?

Problem 4.2 (Solving) 8 pt

Consider the following binary CSP:



- $V = \{a, b, c\}$
- $D_a = \{0, 1, 2\}$, $D_b = \{0, 1, 2, 3\}$, $D_c = \{0, 1, 2, 3, 4\}$
- Constraints:
  - if $a > b$, then $b = 2$
  - if $a < c$, then $c \neq 2$
  - $b + c < 4$ and $b < c$

1. Give all pairs (v, w) of variables such that v is **not** arc-consistent relative to w. 3 pt

   (a,b) ✓ (b,c) ✗ (a,c) ✓

   (b,a) ✓ (c,b) ✗ (c,a) ✓

2. Give all solutions. 3 pt

   ?

   $a = 0, b = 0, c \in \{1, 3\}$

3. Assume we assign a to be 1, and apply forward-checking. Give the resulting domains D_b and D_c. 2 pt

   $D_b = \{1, 2, 3\}$

   $D_c = \{0, 1, 3, 4\}$

5 Logic

Problem 5.1 (Propositional Logic) 5 pt

We use the propositional variables P, Q, R. Consider the formula A given by

$$(P \vee Q) \Rightarrow ((R \vee P) \wedge (R \Rightarrow Q))$$

1. Using the assignment $\varphi(P) = F$, $\varphi(Q) = T$, and $\varphi(R) = T$, give the value $I_\varphi(A)$. 2 pt

$$(F \vee T) \to ((\underbrace{T \vee F}_{T}) \wedge (\underbrace{T \to T}_{T}))$$

T

$I_\varphi(A) = T$

2. Argue whether A is valid (i.e., give a proof or a counter-example). 3 pt

$(P \vee Q) \to ((R \vee P) \wedge (R \to Q))^F$

$(P \vee Q)^T$

$((R \vee P) \wedge (R \to Q))^F$

$P^T \mid Q^T \;\|\; (R \vee P)^F \mid R \to Q^F$

$\bot \quad \bot \quad R^F \quad R^T$

$P^F \quad Q^F$

$\bot$

??

$(R \vee P) \wedge (R \to Q)^F$

? $\mid R \to Q^F$

$R^T \to$

$Q^F \to$

$(P \vee Q)^T$

$P^T \mid Q^T$ ×

$[P^T, R^T, Q^F]$

counter example

Problem 5.2 (Definitions) 8 pt

Consider the following signature of first-order logic:

- binary function symbol f
- unary predicate symbol P

Give counter-examples for the following statements:

1. For every satisfiable formula A, the formula $\neg A$ is unsatisfiable. 2 pt

??

non-theorem non-contradiction

e.g. $A = \forall x\, P(x)$

2. Every model that satisfies $\forall x.P(f(x,x))$ also satisfies $\forall x.\forall y.P(f(x,y))$. 3 pt

$U = \mathbb{N}$, $\mathcal{I}(P) = \text{even}$, $\mathcal{I}(f) = +$

o+o = e, e+e = e, o+e = o

$x \in U$ → $f(x,x) = x+x = 2x$ → always even

$y \in U$ → $f(x,y) = x+y$ → even/odd

3. Consider the model with universe $\mathbb{Z}$, $I(P) = \{n \in \mathbb{Z} | n \geq 5\}$, and $I(f)(m,n) = m - n$. Then for every assignment φ, we have 3 pt

$$I_\varphi(P(f(x,y)) \Rightarrow P(f(y,x))) = T.$$

??

$f(m,n) = m-n$

$P(f(x,y)) \to P(f(y,x))$ F

$P(f(x,y))$ T

$f(x,y) > 5$

$x - y > 5$

$\boxed{\varphi(x) - \varphi(y) > 5}$

$P(f(y,x))$ F

$f(y,x) \leq 5$

$y - x \leq 5$

??

$\boxed{\varphi(y) - \varphi(x) \leq 5}$

$y = 7$, $x = 5$

$7 - 5 = 2 \leq 5$

$P(5-7) \to P(7-5)$

$P(-2) \to P(2)$

$F \to F$

T ✗

$x = 12$, $y = 5$

$12 - 5 = 7 > 5$

$P(12-5) \to P(5-12)$

$P(7) \to P(-7)$

$T \to F$

F ✓

$\neg(y-x) > 5$

$y - x < -5$

$\neg\, x - y > 5$

$y - x \leq 5$

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Problem 5.3 (Proving) 8 pt

Prove the following formula using tableaux calculus:

$$(\forall x.P(x) \Rightarrow \neg Q(x)) \Rightarrow \big((\exists y.P(y)) \Rightarrow \neg\forall z.Q(z)\big)$$

1. $\left(\forall x.P(x) \rightarrow \neg Q(x)\right) \rightarrow \left((\exists y.P(y)) \rightarrow \neg\forall z.Q(z)\right)^F$
2. $\forall x.P(x) \rightarrow \neg Q(z)^T$ from (1)
3. $\exists y.P(y) \rightarrow \neg\forall z.Q(z)^F$ "
4. $\exists y.P(y)^T$ from (3)
5. $\neg\forall z.Q(z)^F$ "
6. $P(c)^T$ from (4)
7. $\neg Q(c)^F \;\|\; Q(c)^T$ from (5)
8. $P(c) \rightarrow \neg Q(c)^T$ from (2)
9. $P(c)^F \;\big|\; \neg Q(c)^T$ split on (8)

$P(c)^F$	$\neg Q(c)^T$
	$Q(c)^F$ from (9.2)
$\perp$	$\perp$
close via $P(c)$	close via $Q(c)$

6 Knowledge Representation

Problem 6.1 (Specifying Properties in ALC) 9 pt

Consider the following ALC setting:

- concepts: `human`, `child`, `grownup`, `animal`
- relations: `isChildOf`, `owns`

We abbreviate every concept/relation by its first letter.

1. Give an ALC ABox that is not consistent with the axiom $h \sqcap \exists o.a = \bot$. 2 pt

   $x : h$, $y : a$, $x\ o\ y$
   $\hookrightarrow o(x,y)$

2. Give an ALC TBox that formalizes the following properties 3 pt

   - Children and grownups are humans, and humans are children or grownups. No one is both a child and a grownup.

     $h = c \sqcup g$

     $c \sqcap g = \bot$

   - Humans cannot be owned.

     $\exists o.h = \bot$

3. Give an ALC formalization for the concept of children who have a parent who owns an animal. 2 pt

   $c \sqcap \exists i.\exists o.a$

4. Give the translation to first-order logic of the ALC statement $(\forall i.h) \sqsubseteq (\exists i.g)$. 2 pt

   PL¹

   $\forall x.\left(\forall y.\, i(x,y) \Rightarrow h(y)\right) \Rightarrow \left(\exists y.\, i(x,y) \wedge g(y)\right)$

7 Planning

Problem 7.1 (STRIPS) 12 pt

Consider a STRIPS task $\mathcal{T} = \langle P, A, I, G\rangle$. Recall that the actions $a \in A$ are triples (pre_a, add_a, del_a).

1. Explain the meaning of $\underbrace{pre_a,\ add_a, \text{ and } del_a}_{\text{set of facts}}$. 4 pt

pre$_a$ = must hold for a to applicable

add$_a$ = after applying a, those in add$_a$ hold

del$_a$ = ________ , those in del$_a$ do not hold

2. What happens if $G \subseteq I$? 2 pt

goal condition is already satisfied in I.

So, the empty seq. of actions is a plan for $\mathcal{T}$

3. What is the definition and motivation of the delete relaxation? 3 pt

def^n: it makes del$_i$:= ϕ, empty

Motivation: As, del$_i$:= ϕ, facts never become false, thus precondition are easier to satisfy and optimal plans become shorter & easier to find. So, it can be used as heuristic.

4. We can define a deterministic search problem $\langle S', A', T', I', G' \rangle$ where 3 pt

- S' is the power set of P
- $I' = I$
- the solutions are exactly the plans for $\mathcal{T}$.

Give

(a) the set $A'(s)$ of actions applicable in state s:

$A'(s)$ is set of actions, applicable if $pre_a \subseteq s$ for $a \in A$

(b) the state $T'(a, s)$ resulting from applying action a in state s:

$T'(a,s) = \{ s \xrightarrow{a} apply(s,a) \mid pre_a \subseteq s \}$

or, $T'(a,s) = s \setminus del_a \cup add_a$

(c) the set G' of goal states:

?? G' contains all supersets of G

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Problem 7.2 (Partial Order Planning) 8 pt

Consider the planning task (P, A, I, G) where

- facts $P = \{a, b, c, d\}$
- actions $A = \{U, V, W\}$ where the preconditions (above the box) and effects (below the box) of the actions are given by

$$\begin{array}{ccc} a & b & a \\ \boxed{U} & \boxed{V} & \boxed{W} \\ b & \neg a, c & \neg a, d \end{array}$$

- initial state $I = \{a\}$
- goal $G = \{c, d\}$

Our goal is to build a partially ordered plan. Recall that each step is an action or the start step or the finish step.

1. Give the start step and finish step. 2 pt

$$\begin{array}{c} \\ \boxed{\text{start}} \\ a \end{array} \qquad \begin{array}{c} c, d \\ \boxed{\text{finish}} \\ \\ \end{array}$$

2. Give all causal links between the steps. 2 pt

start $\xrightarrow{a}$ U, start $\xrightarrow{a}$ W, U $\xrightarrow{b}$ V,
V $\xrightarrow{c}$ finish, W $\xrightarrow{d}$ finish

3. Give an example of a step that clobbers a link. 2 pt

both V, W clobbers a link $\xrightarrow{a}$

4. Give the temporal ordering that yields a partially ordered plan that solves the task. 2 pt

start $\xrightarrow{a}$ U	start $\xrightarrow{a}$ W
W	V
U $\prec$ W	W $\prec$ V

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