BEM_114_PS4

May 19, 2024

[ ]:
```
import pandas as pd
import statsmodels.api as sm
from sklearn.linear_model import LinearRegression
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.dates import AutoDateLocator, AutoDateFormatter
```

[ ]:
```
def make_daily_dates(df):
  return [str(item)[:4] + '-' + str(item)[4:6] + '-' + str(item)[6:] for item␣
 ↪in df['Unnamed: 0']]

def make_monthly_dates(df):
  return [str(item)[:4] + '-' + str(item)[4:] for item in df['Unnamed: 0']]
```

[ ]:
```
ff5_daily = pd.read_csv('/content/F-F_Research_Data_5_Factors_2x3_daily.CSV')
mom_daily = pd.read_csv('/content/F-F_Momentum_Factor_daily.CSV')

ff5_monthly = pd.read_csv('/content/F-F_Research_Data_5_Factors_2x3.csv')
mom_monthly = pd.read_csv('/content/F-F_Momentum_Factor.CSV')

ff5_mom_monthly = pd.merge(ff5_monthly, mom_monthly, how = 'inner', on='Unnamed:
 ↪ 0')
ff5_mom_daily = pd.merge(ff5_daily, mom_daily, how = 'inner', on='Unnamed: 0')

ff5_mom_daily['date'] = make_daily_dates(ff5_mom_daily)
ff5_mom_monthly['date'] = make_monthly_dates(ff5_mom_monthly)

ff5_mom_daily.rename(columns = {'Mom   ': 'Mom'}, inplace = True)
ff5_mom_monthly.rename(columns = {'Mom   ': 'Mom'}, inplace = True)

aggregate_monthly_data = ff5_mom_monthly.copy()
```

1 1B

```
[ ]: factors = ['Mkt-RF', 'SMB', 'HML', 'RMW', 'CMA', 'Mom']

     for factor in factors:
       ff5_mom_daily[factor + '_rollvar'] = ff5_mom_daily[factor].rolling(window=22,␣
      ↪min_periods=22).var()
```

```
[ ]: ff5_mom_daily
```

```
[ ]:        Unnamed: 0  Mkt-RF   SMB   HML   RMW   CMA     RF   Mom        date  \
     0        19630701   -0.67  0.02 -0.35  0.03  0.13  0.012 -0.21  1963-07-01
     1        19630702    0.79 -0.28  0.28 -0.08 -0.21  0.012  0.42  1963-07-02
     2        19630703    0.63 -0.18 -0.10  0.13 -0.25  0.012  0.41  1963-07-03
     3        19630705    0.40  0.09 -0.28  0.07 -0.30  0.012  0.07  1963-07-05
     4        19630708   -0.63  0.07 -0.20 -0.27  0.06  0.012 -0.45  1963-07-08
     …             …      …     …     …     …     …      …     …          …
     15285    20240322   -0.23 -0.98 -0.53  0.29 -0.37  0.021  0.43  2024-03-22
     15286    20240325   -0.26 -0.10  0.88 -0.22 -0.17  0.021 -0.34  2024-03-25
     15287    20240326   -0.26  0.10 -0.13 -0.50  0.23  0.021  0.09  2024-03-26
     15288    20240327    0.88  1.29  0.91 -0.14  0.58  0.021 -1.34  2024-03-27
     15289    20240328    0.10  0.45  0.48 -0.07  0.09  0.021 -0.44  2024-03-28

            Mkt-RF_rollvar  SMB_rollvar  HML_rollvar  RMW_rollvar  CMA_rollvar  \
     0                 NaN          NaN          NaN          NaN          NaN
     1                 NaN          NaN          NaN          NaN          NaN
     2                 NaN          NaN          NaN          NaN          NaN
     3                 NaN          NaN          NaN          NaN          NaN
     4                 NaN          NaN          NaN          NaN          NaN
     …                   …            …            …            …            …
     15285        0.505910     0.708938     0.458988     0.309487     0.250368
     15286        0.349962     0.607054     0.395853     0.307150     0.185330
     15287        0.355926     0.601206     0.397961     0.319346     0.186243
     15288        0.376593     0.637681     0.421055     0.293502     0.199281
     15289        0.375942     0.568274     0.404085     0.226731     0.181229

            Mom_rollvar
     0              NaN
     1              NaN
     2              NaN
     3              NaN
     4              NaN
     …                …
     15285     0.683857
     15286     0.517176
     15287     0.516868
     15288     0.558861
```

```
15289     0.552656

[15290 rows x 15 columns]
```

2 1C

We can see from the results below that all factors have coefficients on the lagged variance to be significant at the 5% level according to the OLS summaries displayed below.

```
[ ]: # Collapse data to end of month level

ff5_mom_daily['date'] = pd.to_datetime(ff5_mom_daily['date'])
ff5_mom_daily.set_index('date', inplace = True)
end_of_month_data = ff5_mom_daily.resample('M').last()
# end_of_month_data = ff5_mom_daily.resample('M').mean()
```

```
[ ]: # Regress on future variance at lagged variance to determine if future variance
 ↪predicted by initial variance

for factor in factors:
  input = list(end_of_month_data[factor+'_rollvar'][:-1])
  output = list(end_of_month_data[factor+'_rollvar'][1:])
  model1 = sm.OLS(output, sm.add_constant(input)).fit()
  print(factor.upper() + " LAGGED VARIANCE REGRESSION RESULTS:")
  print(model1.summary())
```

```
MKT-RF LAGGED VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.207
Model:                            OLS   Adj. R-squared:                  0.206
Method:                 Least Squares   F-statistic:                     189.9
Date:                Tue, 14 May 2024   Prob (F-statistic):           1.51e-38
Time:                        21:21:35   Log-Likelihood:                -1519.8
No. Observations:                 728   AIC:                             3044.
Df Residuals:                     726   BIC:                             3053.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5672      0.080      7.073      0.000       0.410       0.725
x1             0.4553      0.033     13.781      0.000       0.390       0.520
==============================================================================
Omnibus:                     1236.538   Durbin-Watson:                   2.071
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           725257.224
Skew:                          10.642   Prob(JB):                         0.00
```

```
Kurtosis:                     156.155   Cond. No.                         2.78
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
SMB LAGGED VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.136
Model:                            OLS   Adj. R-squared:                  0.134
Method:                 Least Squares   F-statistic:                     114.0
Date:                Tue, 14 May 2024   Prob (F-statistic):           8.20e-25
Time:                        21:21:35   Log-Likelihood:                -463.72
No. Observations:                 728   AIC:                             931.4
Df Residuals:                     726   BIC:                             940.6
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1857      0.020      9.397      0.000       0.147       0.224
x1             0.3684      0.035     10.676      0.000       0.301       0.436
==============================================================================
Omnibus:                     1324.074   Durbin-Watson:                   2.111
Prob(Omnibus):                  0.000   Jarque-Bera (JB):          1503081.930
Skew:                          11.992   Prob(JB):                         0.00
Kurtosis:                     224.307   Cond. No.                         2.25
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
HML LAGGED VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.562
Model:                            OLS   Adj. R-squared:                  0.561
Method:                 Least Squares   F-statistic:                     930.5
Date:                Tue, 14 May 2024   Prob (F-statistic):          3.57e-132
Time:                        21:21:35   Log-Likelihood:                -431.95
No. Observations:                 728   AIC:                             867.9
Df Residuals:                     726   BIC:                             877.1
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
```

```
const          0.0841      0.018      4.621      0.000       0.048       0.120
x1             0.7494      0.025     30.504      0.000       0.701       0.798
==============================================================================
Omnibus:                      737.266   Durbin-Watson:                   2.470
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            65383.289
Skew:                           4.399   Prob(JB):                         0.00
Kurtosis:                      48.586   Cond. No.                         1.78
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
RMW LAGGED VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.522
Model:                            OLS   Adj. R-squared:                  0.522
Method:                 Least Squares   F-statistic:                     793.7
Date:                Tue, 14 May 2024   Prob (F-statistic):          1.43e-118
Time:                        21:21:35   Log-Likelihood:                 111.28
No. Observations:                 728   AIC:                            -218.6
Df Residuals:                     726   BIC:                            -209.4
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0438      0.009      5.035      0.000       0.027       0.061
x1             0.7226      0.026     28.173      0.000       0.672       0.773
==============================================================================
Omnibus:                      733.346   Durbin-Watson:                   2.192
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           122814.118
Skew:                           4.101   Prob(JB):                         0.00
Kurtosis:                      66.099   Cond. No.                         3.42
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
CMA LAGGED VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.473
Model:                            OLS   Adj. R-squared:                  0.472
Method:                 Least Squares   F-statistic:                     651.3
Date:                Tue, 14 May 2024   Prob (F-statistic):          4.91e-103
Time:                        21:21:35   Log-Likelihood:                 311.80
No. Observations:                 728   AIC:                            -619.6
```

```
Df Residuals:                     726   BIC:                            -610.4
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0441      0.007      6.324      0.000       0.030       0.058
x1             0.6876      0.027     25.520      0.000       0.635       0.740
==============================================================================
Omnibus:                      518.536   Durbin-Watson:                   2.343
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            27642.076
Skew:                           2.579   Prob(JB):                         0.00
Kurtosis:                      32.743   Cond. No.                         4.70
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
MOM LAGGED VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.396
Model:                            OLS   Adj. R-squared:                  0.395
Method:                 Least Squares   F-statistic:                     476.5
Date:                Tue, 14 May 2024   Prob (F-statistic):           1.33e-81
Time:                        21:21:35   Log-Likelihood:                -1033.1
No. Observations:                 728   AIC:                             2070.
Df Residuals:                     726   BIC:                             2079.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2165      0.041      5.315      0.000       0.137       0.297
x1             0.6294      0.029     21.829      0.000       0.573       0.686
==============================================================================
Omnibus:                      807.815   Durbin-Watson:                   2.335
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           103546.383
Skew:                           5.017   Prob(JB):                         0.00
Kurtosis:                      60.558   Cond. No.                         1.76
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[ ]:
```
ff5_monthly
```

[ ]:
```
     Unnamed: 0  Mkt-RF   SMB   HML   RMW   CMA    RF
0        196307   -0.39 -0.41 -0.97  0.68 -1.18  0.27
1        196308    5.07 -0.80  1.80  0.36 -0.35  0.25
2        196309   -1.57 -0.52  0.13 -0.71  0.29  0.27
3        196310    2.53 -1.39 -0.10  2.80 -2.01  0.29
4        196311   -0.85 -0.88  1.75 -0.51  2.24  0.27
..          …       …     …     …     …     …
724      202311    8.84 -0.12  1.64 -3.91 -1.00  0.44
725      202312    4.87  7.32  4.93 -3.07  1.32  0.43
726      202401    0.71 -5.74 -2.38  0.69 -0.96  0.47
727      202402    5.06 -0.78 -3.49 -1.99 -2.14  0.42
728      202403    2.83 -1.16  4.19  1.48  1.18  0.43

[729 rows x 7 columns]
```

For all factors, past variance significantly predicts future variance.

3 1D

[ ]:
```python
# Regress at returns for time t on variances at time t - 1 to determine if future returns predicgted by variance

for factor in factors:
  input = list(end_of_month_data[factor+'_rollvar'][:-1])
  output = list(ff5_mom_monthly[factor][1:])
  model1 = sm.OLS(output, sm.add_constant(input)).fit()
  print(factor.upper() + " FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:")
  print(model1.summary())
```

```
MKT-RF FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                  0.007564
Date:                Tue, 14 May 2024   Prob (F-statistic):              0.931
Time:                        21:24:17   Log-Likelihood:                -2126.6
No. Observations:                 728   AIC:                             4257.
Df Residuals:                     726   BIC:                             4266.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.5858      0.185      3.174      0.002       0.223       0.948
x1            -0.0066      0.076     -0.087      0.931      -0.156       0.143
==============================================================================
Omnibus:                       58.161   Durbin-Watson:                   1.912
```

```
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              118.267
Skew:                          -0.493   Prob(JB):                     2.08e-26
Kurtosis:                       4.711   Cond. No.                         2.78
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
SMB FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.001
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.3840
Date:                Tue, 14 May 2024   Prob (F-statistic):              0.536
Time:                        21:24:17   Log-Likelihood:                -1841.1
No. Observations:                 728   AIC:                             3686.
Df Residuals:                     726   BIC:                             3695.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1626      0.131      1.241      0.215      -0.095       0.420
x1             0.1418      0.229      0.620      0.536      -0.307       0.591
==============================================================================
Omnibus:                       67.382   Durbin-Watson:                   1.860
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              275.941
Skew:                           0.317   Prob(JB):                     1.20e-60
Kurtosis:                       5.949   Cond. No.                         2.25
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
HML FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.001
Method:                 Least Squares   F-statistic:                    0.3245
Date:                Tue, 14 May 2024   Prob (F-statistic):              0.569
Time:                        21:24:17   Log-Likelihood:                -1831.8
No. Observations:                 728   AIC:                             3668.
Df Residuals:                     726   BIC:                             3677.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
```

```
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.3219      0.125      2.585      0.010       0.077       0.566
x1            -0.0957      0.168     -0.570      0.569      -0.426       0.234
==============================================================================
Omnibus:                       44.049   Durbin-Watson:                   1.661
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              158.124
Skew:                           0.105   Prob(JB):                     4.61e-35
Kurtosis:                       5.274   Cond. No.                         1.78
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
RMW FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.018
Model:                            OLS   Adj. R-squared:                  0.016
Method:                 Least Squares   F-statistic:                     13.19
Date:                Tue, 14 May 2024   Prob (F-statistic):           0.000301
Time:                        21:24:17   Log-Likelihood:                -1608.0
No. Observations:                 728   AIC:                             3220.
Df Residuals:                     726   BIC:                             3229.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1268      0.092      1.376      0.169      -0.054       0.308
x1             0.9881      0.272      3.632      0.000       0.454       1.522
==============================================================================
Omnibus:                      209.166   Durbin-Watson:                   1.734
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4104.097
Skew:                          -0.771   Prob(JB):                         0.00
Kurtosis:                      14.529   Cond. No.                         3.42
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
CMA FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.022
Model:                            OLS   Adj. R-squared:                  0.021
Method:                 Least Squares   F-statistic:                     16.48
Date:                Tue, 14 May 2024   Prob (F-statistic):           5.45e-05
```

```
Time:                        21:24:17   Log-Likelihood:                -1556.1
No. Observations:                 728   AIC:                             3116.
Df Residuals:                     726   BIC:                             3125.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0712      0.091      0.785      0.432      -0.107       0.249
x1             1.4231      0.351      4.060      0.000       0.735       2.111
==============================================================================
Omnibus:                       23.162   Durbin-Watson:                   1.774
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               51.834
Skew:                          -0.091   Prob(JB):                     5.55e-12
Kurtosis:                       4.295   Cond. No.                         4.70
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
MOM FUTURE RETURNS ON VARIANCE REGRESSION RESULTS:
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.033
Model:                            OLS   Adj. R-squared:                  0.031
Method:                 Least Squares   F-statistic:                     24.58
Date:                Tue, 14 May 2024   Prob (F-statistic):           8.88e-07
Time:                        21:24:17   Log-Likelihood:                -2067.5
No. Observations:                 728   AIC:                             4139.
Df Residuals:                     726   BIC:                             4148.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9555      0.169      5.664      0.000       0.624       1.287
x1            -0.5920      0.119     -4.958      0.000      -0.826      -0.358
==============================================================================
Omnibus:                      162.415   Durbin-Watson:                   2.022
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1347.295
Skew:                          -0.751   Prob(JB):                    2.75e-293
Kurtosis:                       9.493   Cond. No.                         1.76
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

There is not a consistant pattern between variance in factors and market returns.

4 1E

[ ]:
```
# # Creating weights

# for factor in factors:
#   mean_var = np.mean(end_of_month_data[factor + '_rollvar'])
#   ff5_mom_monthly[factor + '_weight'] = list(mean_var /␣
↪end_of_month_data[factor + '_rollvar'])
#   ff5_mom_monthly[factor + '_weight'] = ff5_mom_monthly[factor + '_weight'].
↪shift(1)
#   ff5_mom_monthly[factor + '_rf_weight'] = 1 - ff5_mom_monthly[factor +␣
↪'_weight']

# # ff5_mom_monthly.head(30)
```

[ ]:
```
end_of_month_data.head()
# ff5_mom_monthly.head()
```

[ ]:
```
            Unnamed: 0  Mkt-RF   SMB   HML   RMW   CMA    RF   Mom  \
date
1963-07-31    19630731   -0.13  0.11 -0.03 -0.13  0.30 0.012  0.01
1963-08-31    19630830    0.44  0.15 -0.13  0.15 -0.21 0.011 -0.11
1963-09-30    19630930   -0.60  0.21  0.08  0.24  0.13 0.014 -0.56
1963-10-31    19631031    0.21 -0.03  0.08  0.10 -0.26 0.013  0.11
1963-11-30    19631129    1.34  0.30  0.29 -0.31 -0.15 0.015 -0.09

            Mkt-RF_rollvar  SMB_rollvar  HML_rollvar  RMW_rollvar  \
date
1963-07-31        0.223726     0.020938     0.037883     0.022160
1963-08-31        0.086949     0.033653     0.039540     0.013295
1963-09-30        0.154223     0.052577     0.035055     0.028740
1963-10-31        0.171368     0.077462     0.116457     0.061698
1963-11-30        1.493974     0.207698     0.079971     0.092893

            CMA_rollvar  Mom_rollvar
date
1963-07-31     0.031652     0.077050
1963-08-31     0.023129     0.047702
1963-09-30     0.028083     0.072686
1963-10-31     0.085728     0.094405
1963-11-30     0.081876     0.639916
```

[ ]:
```
# Calculate the long-run average variance for each factor over the entire␣
↪sample period
```

```
long_run_avg_var = {factor: end_of_month_data[factor + '_rollvar'].mean() for␣
↪factor in factors}
LEVERAGE = 1.5
LEVERAGE = 1.2
# LEVERAGE = 1
# print(long_run_avg_var)
# Calculate the weights for each factor based on the most recent rolling␣
↪variance at the end of each month
for factor in factors:
    # The weight for each month is based on the long-run average variance␣
↪divided by the rolling variance from the end of the previous month

    ff5_mom_monthly[factor + '_weight'] = list(long_run_avg_var[factor] /␣
↪(end_of_month_data[factor + '_rollvar']))
    # ff5_mom_monthly[factor + '_weight'] = np.minimum(ff5_mom_monthly[factor +␣
↪'_weight'], LEVERAGE)
    ff5_mom_monthly[factor + '_weight'] = ff5_mom_monthly[factor + '_weight'].
↪shift(1)
    ff5_mom_monthly[factor + '_rf_weight'] = 1 - ff5_mom_monthly[factor +␣
↪'_weight']


    # print(factor + " weights:")
    # print(ff5_mom_monthly[[factor + '_weight', factor + '_rf_weight']].
↪head(-25))

    # print(factor + " weights:")
    # print()
    # print(ff5_mom_monthly[[factor + '_weight', factor + '_rf_weight']].
↪head(30))

# Due to the shift in calculation, the first row will have NaN weights which␣
↪should be handled if necessary
# print(end_of_month_data.head())
# ff5_mom_monthly.head(10)
```

[ ]:
```
print(ff5_mom_monthly['Mom_weight'])
```

```
0            NaN
1       7.560256
2      12.211626
3       8.014243
4       6.170476
          …
724     1.037258
725     0.547110
726     0.723806
```

```
727     1.001564
728     0.652168
Name: Mom_weight, Length: 729, dtype: float64
```

[ ]:
```python
ff5_mom_monthly[ff5_mom_monthly['Unnamed: 0'] < 202304]
```

[ ]:
```
     Unnamed: 0  Mkt-RF   SMB   HML   RMW   CMA    RF    Mom     date  \
0        196307   -0.39 -0.41 -0.97  0.68 -1.18  0.27   0.90  1963-07
1        196308    5.07 -0.80  1.80  0.36 -0.35  0.25   1.01  1963-08
2        196309   -1.57 -0.52  0.13 -0.71  0.29  0.27   0.19  1963-09
3        196310    2.53 -1.39 -0.10  2.80 -2.01  0.29   3.12  1963-10
4        196311   -0.85 -0.88  1.75 -0.51  2.24  0.27  -0.74  1963-11
..          …       …     …     …     …     …     …      …        …
712      202211    4.60 -2.74  1.38  6.38  3.18  0.29  -1.98  2022-11
713      202212   -6.41 -0.15  1.37  0.25  4.20  0.33   4.55  2022-12
714      202301    6.65  4.41 -4.01 -2.44 -4.47  0.35 -16.01  2023-01
715      202302   -2.58  0.66 -0.81  1.01 -1.33  0.34   0.15  2023-02
716      202303    2.51 -6.94 -8.85  2.24 -2.37  0.36  -2.47  2023-03

     Mkt-RF_weight  …  CMA_weight  CMA_rf_weight  Mom_weight  Mom_rf_weight  \
0              NaN  …         NaN            NaN         NaN            NaN
1         4.648926  …    4.436600      -3.436600    7.560256      -6.560256
2        11.961976  …    6.071510      -5.071510   12.211626     -11.211626
3         6.744035  …    5.000403      -4.000403    8.014243      -7.014243
4         6.069292  …    1.638058      -0.638058    6.170476      -5.170476
..             …   …         …              …          …              …
712       0.329631  …    0.290298       0.709702    0.374611       0.625389
713       0.326837  …    0.159744       0.840256    0.114565       0.885435
714       0.567171  …    0.318249       0.681751    0.337354       0.662646
715       0.890784  …    0.373564       0.626436    0.393579       0.606421
716       0.931522  …    0.240823       0.759177    0.214625       0.785375

     Mkt-RF_return  SMB_return  HML_return  RMW_return  CMA_return  Mom_return
0              NaN         NaN         NaN         NaN         NaN         NaN
1        22.657823  -14.449207   13.905104    1.028684   -2.411960    5.995795
2       -21.740036   -6.610864   -0.911674  -11.293256    0.391430   -0.706930
3        15.396638   -9.075994   -3.423037   13.989730  -11.210927   22.970309
4        -6.527607   -4.081592    4.511355   -1.713134    3.496974   -5.962181
..             …          …           …           …           …           …
712       1.710710   -3.021643    0.824265    1.877965    1.128962   -0.560366
713      -1.872884   -0.248597    0.612366    0.309848    0.948211    0.813462
714       3.923179    7.035222   -1.406490   -0.784840   -1.183962   -5.169108
715      -2.261088    0.781287   -0.503209    0.688619   -0.283852    0.265220
716       2.362772   -5.681699   -3.064905    1.427898   -0.297446   -0.247389

[717 rows x 27 columns]
```

```
[ ]: # Creating returns
print(ff5_mom_monthly['Mkt-RF'].mean())
for factor in factors:
  ff5_mom_monthly[factor + '_return'] = (ff5_mom_monthly[factor] *␣
↪ff5_mom_monthly[factor + '_weight'])
  # print(factor + " mean returns: " + str(np.mean(ff5_mom_monthly[factor +␣
↪'_return'])))
  # print(factor + " reg returns: " + str(np.mean(ff5_mom_monthly[factor])))

  # print(ff5_mom_monthly[["date",factor, factor + '_weight', factor +␣
↪'_return', "RF"]].head(5))
  # print()
```

```
0.5775582990397805
```

5 1F

```
[ ]: def estim_CAPM(portfolio_ret, data_unique_dates):
  model1 = sm.OLS(portfolio_ret, sm.add_constant(data_unique_dates['Mkt-RF'])).
↪fit()
  alpha, beta = model1.params
  print("CAPM ESTIMATES")
  # print(model1.summary())
  # print(f'alpha: {model1.params["const"]}')
  conf_int = model1.conf_int(alpha=0.05)  # Default is 95% confidence interval
  alpha_conf_int = conf_int.loc["const"]  # Confidence interval for the alpha

  # Print the results including the 95th percentile bounds for alpha
  # print("FF5 + MOMENTUM ESTIMATES")
  print(f'alpha: {round(alpha, 3)}')
  print(f'Alpha 95% Confidence Interval: {round(alpha_conf_int[0], 3)} to␣
↪{round(alpha_conf_int[1], 3)}')

def estim_FF3(portfolio_ret, data_unique_dates):
  model1=sm.OLS(portfolio_ret, sm.add_constant(data_unique_dates[['Mkt-RF',␣
↪'SMB', 'HML']])).fit()
  alpha, beta_1, beta_2, beta_3 = model1.params
  print("FF3 ESTIMATES")
  # print(model1.summary())
  # print(f'alpha: {model1.params["const"]}')

  conf_int = model1.conf_int(alpha=0.05)  # Default is 95% confidence interval
  alpha_conf_int = conf_int.loc["const"]  # Confidence interval for the alpha

  # Print the results including the 95th percentile bounds for alpha
  # print("FF5 + MOMENTUM ESTIMATES")
  print(f'alpha: {round(alpha, 3)}')
```

```
  print(f'Alpha 95% Confidence Interval: {round(alpha_conf_int[0], 3)} to␣
↪{round(alpha_conf_int[1], 3)}')

# def estim_FF5mom(portfolio_ret, data_unique_dates):
#   model1=sm.OLS(portfolio_ret, sm.add_constant(data_unique_dates[['Mkt-RF',␣
↪'SMB', 'HML', 'RMW', 'CMA', 'Mom']])).fit()
#   alpha, beta_1, beta_2, beta_3, beta_4, beta_5, beta_6 = model1.params
#   print("FF5 + MOMENTUM ESTIMATES")
#   print(f'alpha: {model1.params["const"]}')
#   print(model1.summary())

def estim_FF5mom(portfolio_ret, data_unique_dates):
    # Running the regression with Fama-French 5 Factors + Momentum
    model1 = sm.OLS(portfolio_ret, sm.add_constant(data_unique_dates[['Mkt-RF',␣
↪'SMB', 'HML', 'RMW', 'CMA', 'Mom']])).fit()
    alpha = model1.params["const"]

    # Extract the 95% confidence interval for the alpha
    conf_int = model1.conf_int(alpha=0.05)  # Default is 95% confidence interval
    alpha_conf_int = conf_int.loc["const"]  # Confidence interval for the alpha

    # Print the results including the 95th percentile bounds for alpha
    print("FF5 + MOMENTUM ESTIMATES")
    print(f'alpha: {round(alpha, 3)}')
    print(f'Alpha 95% Confidence Interval: {round(alpha_conf_int[0], 3)} to␣
↪{round(alpha_conf_int[1], 3)}')
    # print(model1.summary())
```

[ ]:

```
# Run models on each factor

for factor in factors:
  print(factor.upper() + " MODEL ESTIMATION RESULTS:")
  estim_CAPM(ff5_mom_monthly[factor + '_return'].dropna(), ff5_mom_monthly.
↪dropna())
  estim_FF3(ff5_mom_monthly[factor + '_return'].dropna(), ff5_mom_monthly.
↪dropna())
  estim_FF5mom(ff5_mom_monthly[factor + '_return'].dropna(), ff5_mom_monthly.
↪dropna())
  print("________________________________________________")
```

```
MKT-RF MODEL ESTIMATION RESULTS:
CAPM ESTIMATES
alpha: 0.494
Alpha 95% Confidence Interval: -0.113 to 1.1
FF3 ESTIMATES
alpha: 0.468
Alpha 95% Confidence Interval: -0.144 to 1.08
```

```
FF5 + MOMENTUM ESTIMATES
alpha: 0.077
Alpha 95% Confidence Interval: -0.551 to 0.704
------------------------------------------------
SMB MODEL ESTIMATION RESULTS:
CAPM ESTIMATES
alpha: 0.136
Alpha 95% Confidence Interval: -0.418 to 0.689
FF3 ESTIMATES
alpha: -0.04
Alpha 95% Confidence Interval: -0.451 to 0.371
FF5 + MOMENTUM ESTIMATES
alpha: -0.21
Alpha 95% Confidence Interval: -0.634 to 0.214
------------------------------------------------
HML MODEL ESTIMATION RESULTS:
CAPM ESTIMATES
alpha: 1.13
Alpha 95% Confidence Interval: 0.436 to 1.823
FF3 ESTIMATES
alpha: 0.401
Alpha 95% Confidence Interval: -0.153 to 0.956
FF5 + MOMENTUM ESTIMATES
alpha: 0.764
Alpha 95% Confidence Interval: 0.212 to 1.316
------------------------------------------------
RMW MODEL ESTIMATION RESULTS:
CAPM ESTIMATES
alpha: 0.874
Alpha 95% Confidence Interval: 0.492 to 1.256
FF3 ESTIMATES
alpha: 1.034
Alpha 95% Confidence Interval: 0.665 to 1.403
FF5 + MOMENTUM ESTIMATES
alpha: 0.604
Alpha 95% Confidence Interval: 0.291 to 0.918
------------------------------------------------
CMA MODEL ESTIMATION RESULTS:
CAPM ESTIMATES
alpha: 0.533
Alpha 95% Confidence Interval: 0.238 to 0.828
FF3 ESTIMATES
alpha: 0.296
Alpha 95% Confidence Interval: 0.034 to 0.559
FF5 + MOMENTUM ESTIMATES
alpha: 0.199
Alpha 95% Confidence Interval: -0.021 to 0.419
------------------------------------------------
```

```
MOM MODEL ESTIMATION RESULTS:
CAPM ESTIMATES
alpha: 4.01
Alpha 95% Confidence Interval: 2.991 to 5.029
FF3 ESTIMATES
alpha: 4.202
Alpha 95% Confidence Interval: 3.18 to 5.225
FF5 + MOMENTUM ESTIMATES
alpha: 2.786
Alpha 95% Confidence Interval: 1.893 to 3.679
------------------------------------------------
```

Based on our results, the only strategy that produces consistant and positive alpha across tests is the MOM estimation model. All the other strategies produce either insignificant or negative alpha in some tests. We acknowlege that this is not the result that we anticipated. We believe that the issue here may be related to the uncapped leverage that this implementation of the strategy employs. The way we are calculating the weights as described in the spec allows for this and we believe this is hurting our alpha generation

6 Question 2

[ ]:
```python
# Invert variance

for factor in factors:
  aggregate_monthly_data[factor+'_invar'] = list(1 / end_of_month_data[factor +␣
↪'_rollvar'])
```

[ ]:
```python
# Create weights for each date and shift by 1 since weights at t - 1 determine␣
↪returns at t

def create_weights(group):
  columns = [col for col in group.columns if 'invar' in col]

  # Sum the values across the selected columns for each row
  variance_sums = (group[columns].sum(axis=1))

  for factor in factors:
    group[factor + '_weight'] = group[factor + "_invar"] / variance_sums

  return group


aggregate_monthly_data = aggregate_monthly_data.groupby('date').
↪apply(create_weights)
aggregate_monthly_data = aggregate_monthly_data.reset_index(drop = True)

for factor in factors:
```

```
  aggregate_monthly_data[factor + "_weight"] = aggregate_monthly_data[factor +␣
 ↪'_weight'].shift(1)

aggregate_monthly_data = aggregate_monthly_data.dropna()
```

[ ]:
```
# Calculate returns

def create_returns(group):
  ret = 0
  for factor in factors:
    ret += group[factor] * group[factor + '_weight']

  group['returns'] = ret

  return group


aggregate_monthly_data = aggregate_monthly_data.groupby('date').
 ↪apply(create_returns)
aggregate_monthly_data = aggregate_monthly_data.reset_index(drop = True)
```

[ ]:
```
# Estimate models on aggregate strategy

estim_CAPM(aggregate_monthly_data['returns'], aggregate_monthly_data)
estim_FF3(aggregate_monthly_data['returns'], aggregate_monthly_data)
estim_FF5mom(aggregate_monthly_data['returns'], aggregate_monthly_data)
```

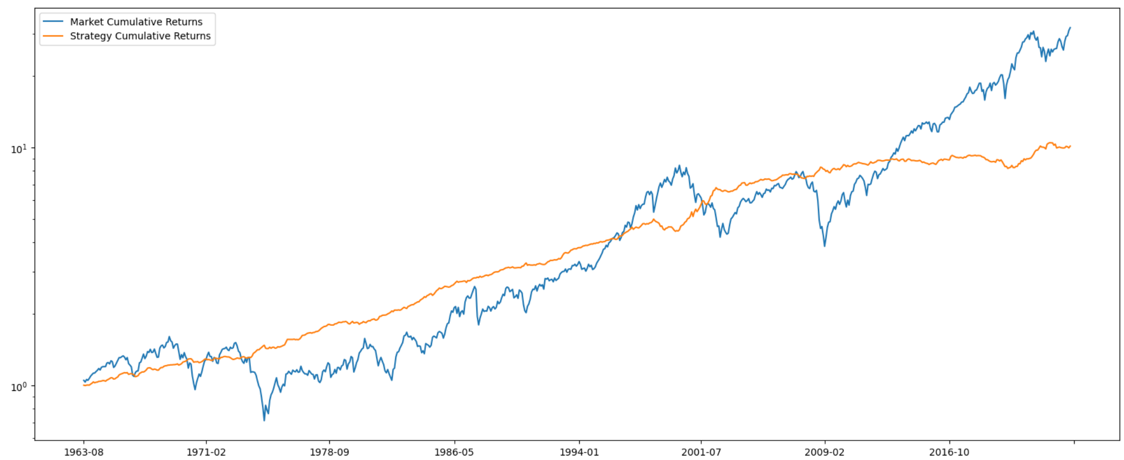
```
CAPM ESTIMATES
alpha: 0.333
Alpha 95% Confidence Interval: 0.257 to 0.41
FF3 ESTIMATES
alpha: 0.251
Alpha 95% Confidence Interval: 0.191 to 0.311
FF5 + MOMENTUM ESTIMATES
alpha: 0.076
Alpha 95% Confidence Interval: 0.033 to 0.119
```

The aggregate portfolio produced significant alpha, which is better than all the individual portfolios except for the momentum.

[ ]:
```
# Calculate cumulative market and strategy portfolio values

market_rets = list(1 + aggregate_monthly_data['Mkt-RF'] / 100)
strat_rets = list(1 + aggregate_monthly_data['returns'] / 100)

market_cumulative = [1 * np.prod(market_rets[0:i+1]) for i in␣
 ↪range(len(market_rets))]
```

```
strat_cumulative = [1 * np.prod(strat_rets[0:i+1]) for i in␣
↪range(len(strat_rets))]
```

[ ]:
```
import matplotlib.pyplot as plt
from matplotlib.dates import AutoDateLocator

plt.figure(figsize=(20, 8))
x_axis = list(aggregate_monthly_data['date'])
plt.plot(x_axis, market_cumulative, label='Market Cumulative Returns')
plt.plot(x_axis, strat_cumulative, label='Strategy Cumulative Returns')
plt.gca().xaxis.set_major_locator(AutoDateLocator())
plt.gca().set_yscale('log')  # Set the y-axis to a logarithmic scale
plt.legend()  # Optionally add a legend
plt.show()
```



The volatility managed portfolio produces alpha becuase it has a lower variance than the market, but a low market beta. Since the turn of the century, the returns have significantly decreased. This is likely due to capital flowing into the strategy, leading to theory behind this strategy being priced into the market.