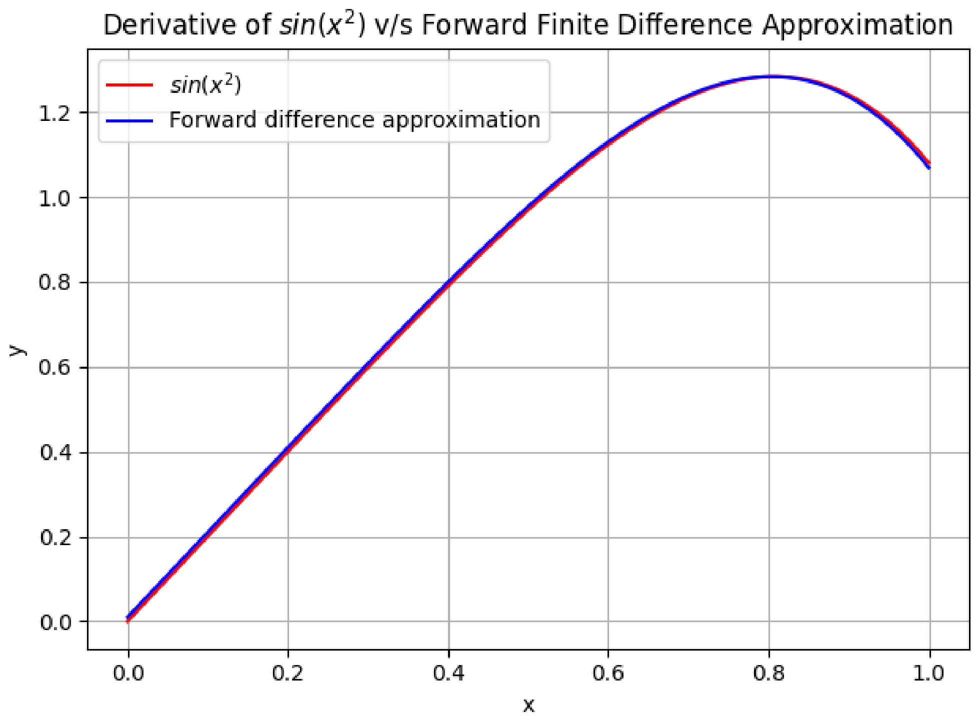
# CS5016: Computational Methods and Applications
# Assignment 4: Numerical Differentiation and Integration

**Sujit Mandava**
**112001043**

**Q1.** We implement the following functions to help us visualize the actual derivative and the forward difference approximation of $f(x) = sin(x^2)$:

1. $f$: Takes a single argument x and computes the value for the above function.
2. $f_{prime}$: Takes a single argument x and computes the integral of the above function, which is evaluated to $2x\ cos(x^2)$.
3. forward_difference: Takes two arguments x and h, and computes the value of $(f(x + h) - f(x))/h$.

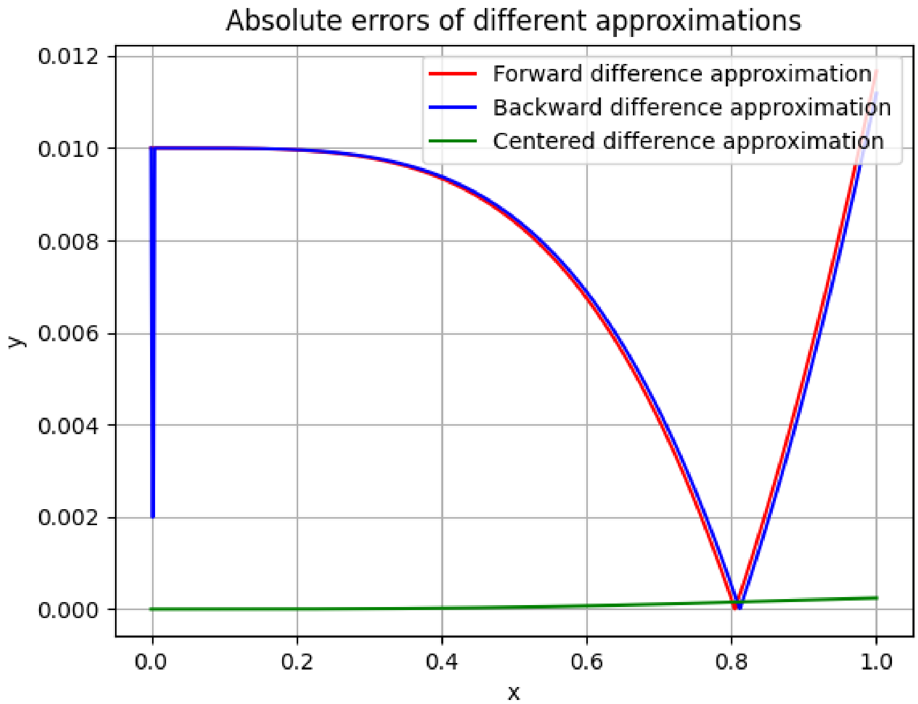
We generate 1000 points between 0 and 1 and compute the actual derivative and the forward difference of $f(x)$ at each point. We plot the obtained values to obtain the following graph:



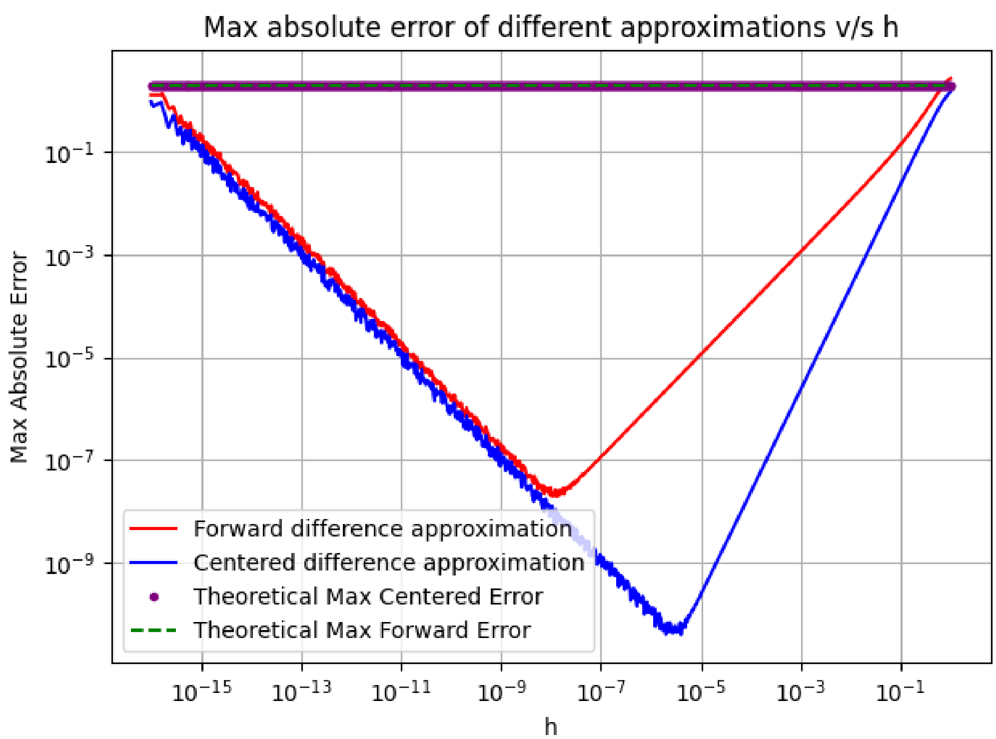

**Q2.** In addition to the functions mentioned above, we define two more functions:

4. backward-difference: Takes two arguments x and h, and computes the value of $(f(x) - f(x - h))/h$.
5. centered-difference: Takes two arguments x and h, and computes the value of $(f(x + h) - f(x - h))/2h$.

We generate 1000 points between 0 and 1 and compute the absolute error for each approximation at every point. Upon plotting the obtained values, we obtain the following graph:




**Q3.** We generate 1000 values of x between 0 and 1 and 1000 values of h to compute the different approximations. For every value of h, we find the maximum value of the absolute error of approximation. Upon obtaining this data, we plot it and obtain the following graph:

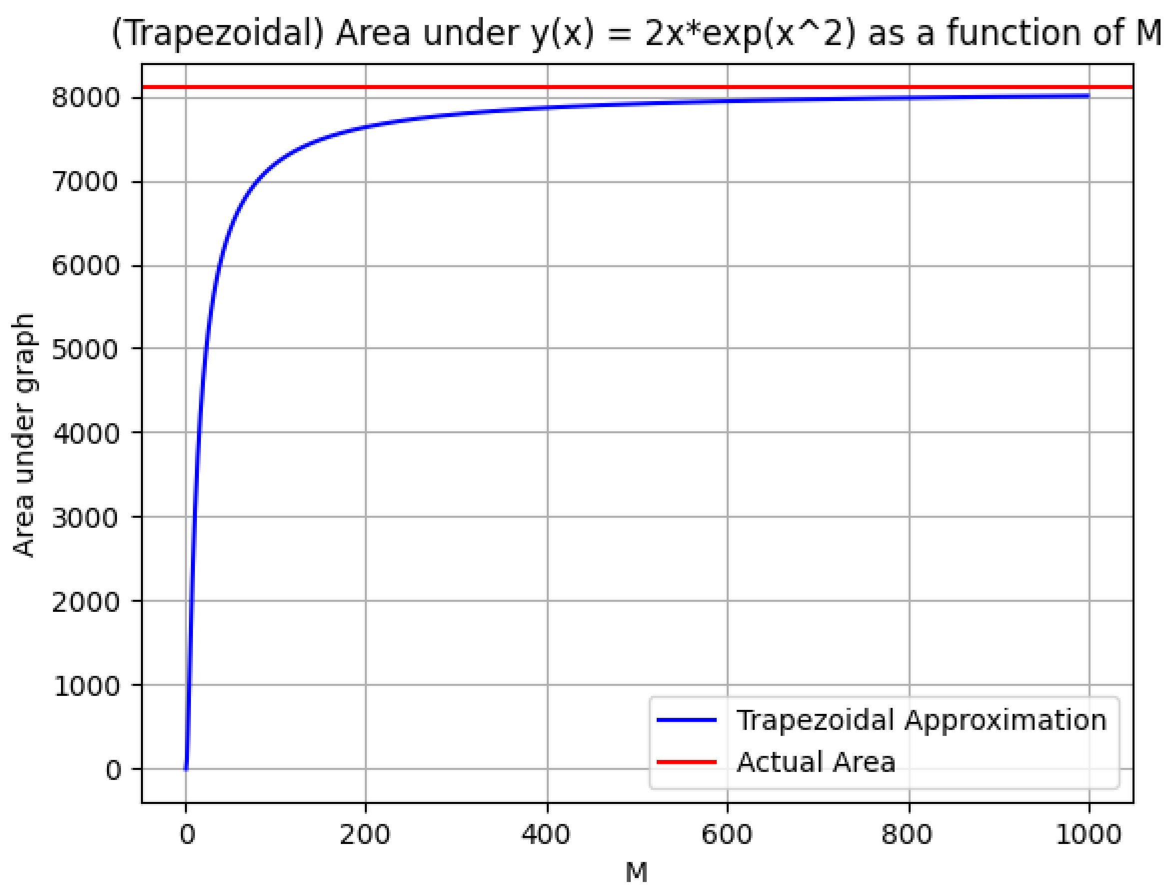


**Q4.** We use three functions to compute all the required values:

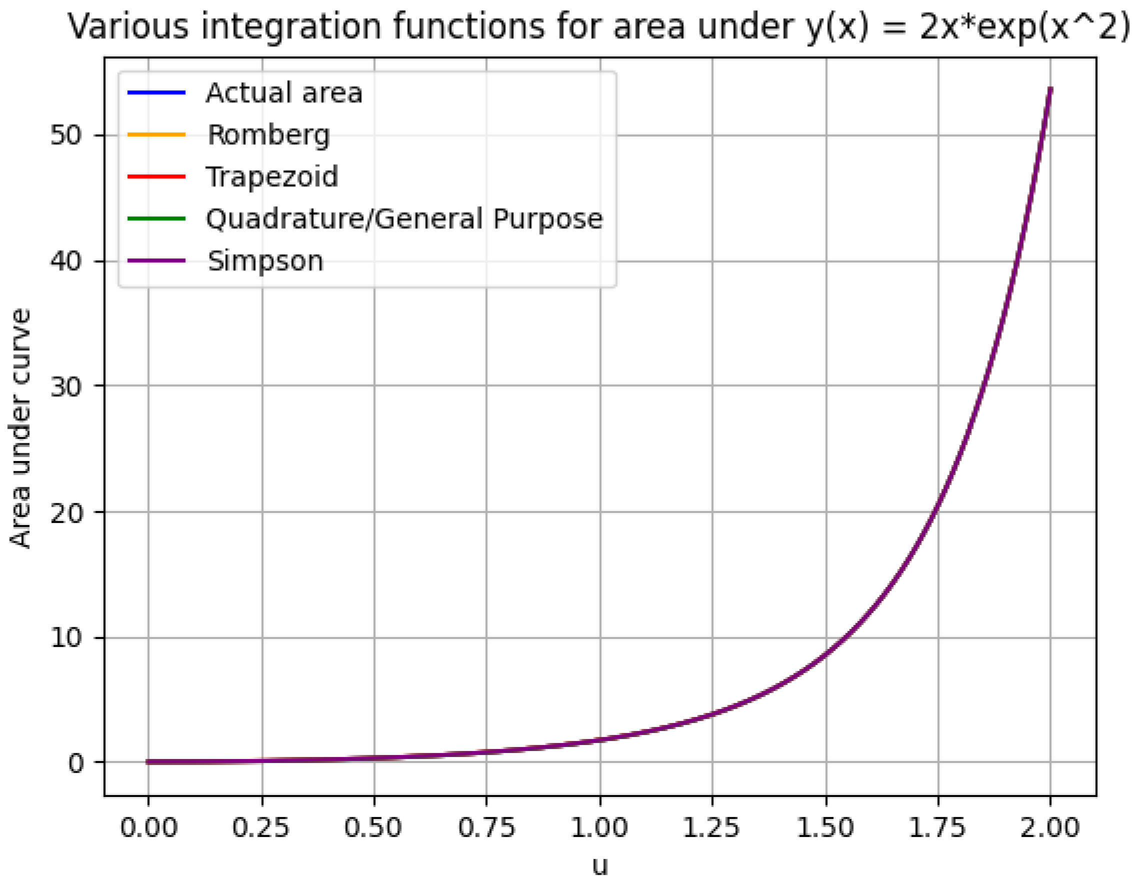
1. $f$: Takes a single argument x and evaluates the value of $2xe^{x^2}$.
2. $f_{prime}$: Takes a single argument x and evaluates the value of $e^{x^2}$.
3. integralM: Takes three arguments a, b, and M using which it computes the area under the curve $y(x) = f$ using the trapezoidal approximation method. The equation it uses to evaluate the area is

$$I_M(f) = \frac{b-a}{2M}\Sigma(f_{k+1}(x) + f_k(x))$$

Since $f_{prime}$ is the integral of $f$, we simply do $f_{prime}(3) - f_{prime}(1)$ to find the actual area under the graph $y(x) = f(x)$ and plot the obtained value. We then calculate the area under the graph using the trapezoidal approximation method for values of M ranging from 1 to 1000. Upon plotting the computed areas against M, we obtain the following graph:




**Q5.** We visualize 5 different integration methods by plotting the area under the graph $y(x) = 2xe^{x^2}$ in the range [0,u]. The different integration methods are the true integral, the Romberg method, Simpson method, General Purpose integral and Trapezoidal method. We calculate the area using each method for different values of u. We plot the obtained values and obtain the following graph:

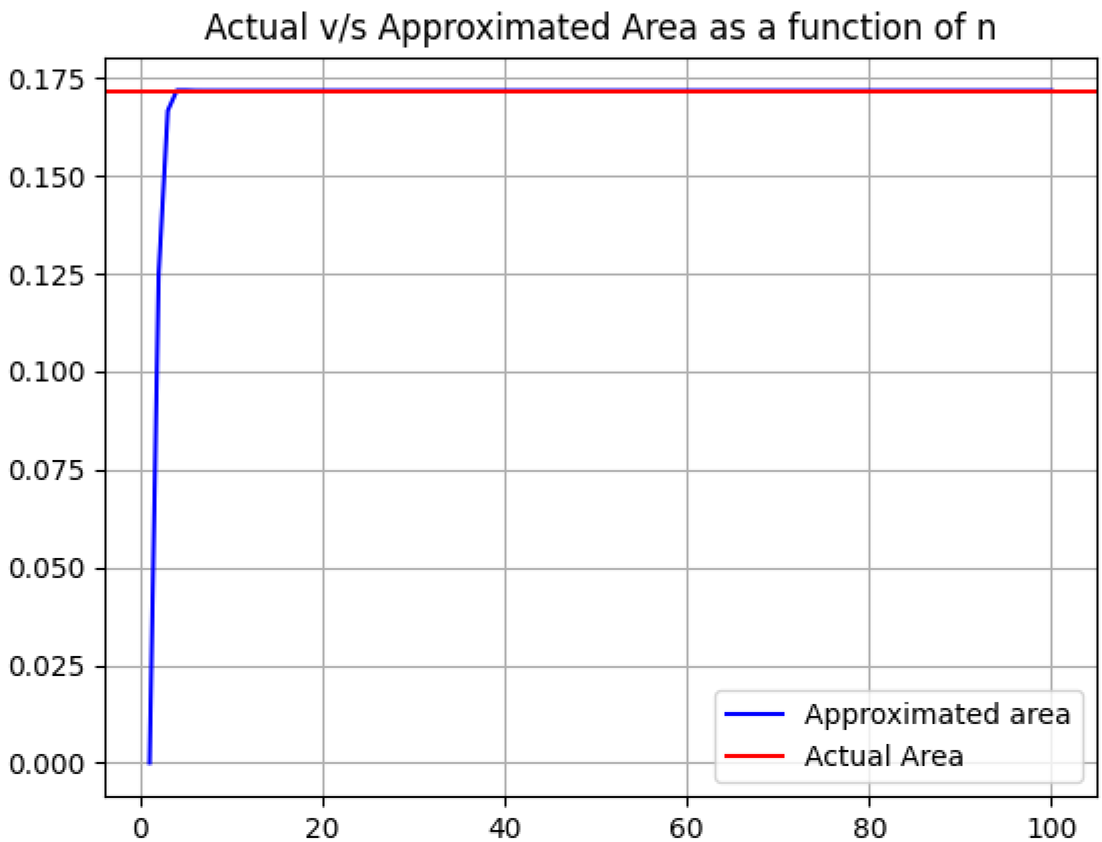



**Q6.** We add three new methods to the previously created Polynomial class, namely:

1. derivative: Takes no arguments; it takes the existing polynomial and returns an object of the Polynomial class that represents the derivative of the same.
2. integral: Takes no arguments; it takes the existing polynomial and returns an object of the Polynomial class that represents the integral of the same.
3. area: Takes two arguments a and b and uses them to calculate the area under the graph of the polynomial. It calls the integral function to find the integral of the polynomial. Then, it calls the evaluate method to find the value of the integral at $x = a$ and $x = b$ respectively, and finds the difference. This gives us the area under the graph between $x = a$ and $x = b$.

**Q7.** We define a new method in the enhanced Polynomial class, approxArea to find the area under the graph of $y(x) = e^{x} sin(x)$. We first calculate the true value of the area under the graph by using the integral of $y(x)$, which is $y^{1}(x) = \frac{e^{x}}{2} (sin(x) - cos(x))$ and computing $y^{1}(1/2) - y^{1}(0)$. We then compute the Taylor's Series expansion of the function up to one

hundred terms. We find the area under the obtained polynomial to find the (approximate) area under the graph of the function $y(x)$, without directly integrating the function itself.
We plot the values of the area under the graph for each value of n and obtain the following graph:




Observe that the approximated and the actual areas are almost identical after a certain value of n.