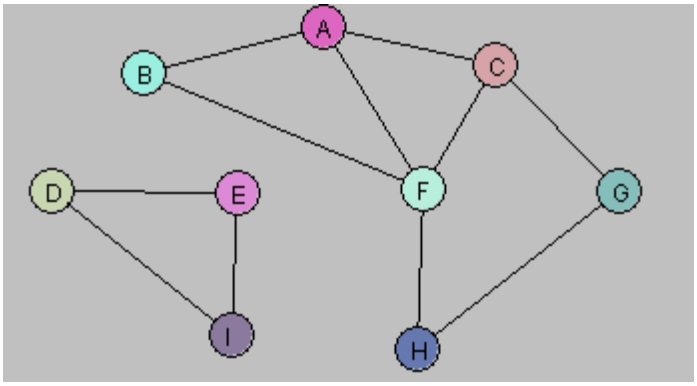
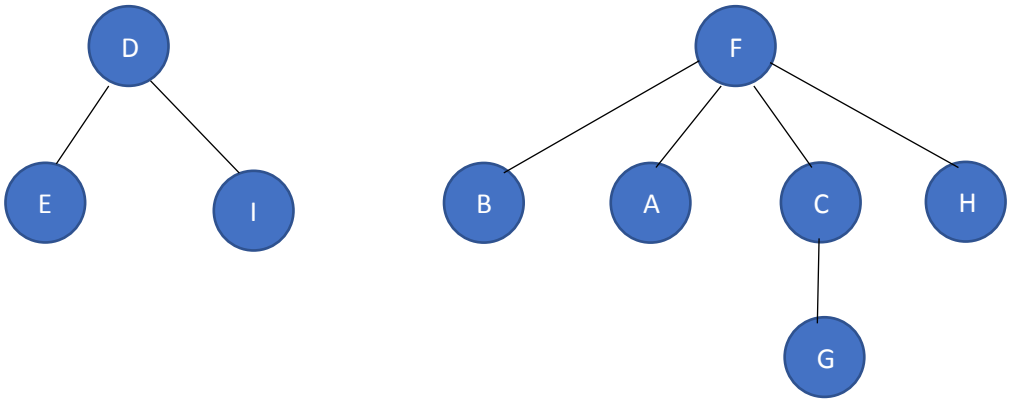
Algorithm: Lab11 (By Sujiv Shrestha ID:610145)
Problem 1.

1. Answer questions about the graph G = (V,E) displayed below.



A. Is the graph G connected? If not, what are the connected components for G?
Answer: The graph G is disconnected. The connected components for G are G1 = ({D,E,I}, {D-E,D-I,I-E}) and G2 = ({B,A,C,F,G,H},{B-A, A-C, A-F, B-F, F-C, F-H, C-G, G-H})

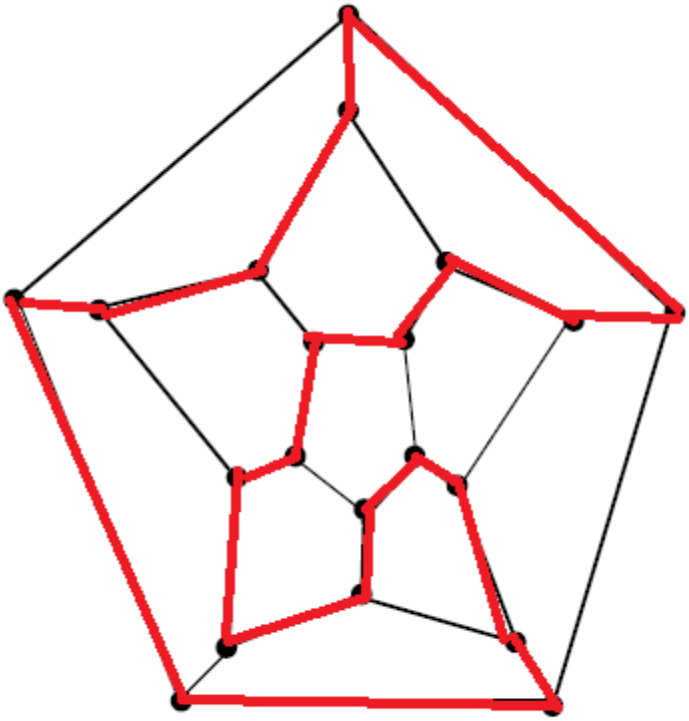
B. Draw a spanning tree/forest for G.



C. Is G a Hamiltonian graph?
Answer: G is not a Hamiltonian graph because in Hamiltonian graph there should be atleast one cyclic path (Hamiltonian cycle) connecting all the vertices in that graph but here the graph is disconnected.

D. Is there a Vertex Cover of size less than or equal to 5 for G? If so, what is the Vertex Cover?
Answer: Yes, there is a Vertex Cover of size less than or equal to 5 for G. That Vertex Cover is V={D,E,F,G,A}

Problem 2.
2. *Hamiltonian Graphs.* The following graph has a Hamiltonian cycle. Find it.



Problem 3.
3. *Vertex Covers.* Create an algorithm for computing the smallest size of a vertex cover for a graph. The input of your algorithm is a set V of vertices along with a set E of edges. Assume you have the following functions available (no need to implement these):

- computeEndpoints(edge) – returns the vertices that are at the endpoints of the input edge
- belongsTo(vertex, set) – returns true if the input vertex is a member of the given set

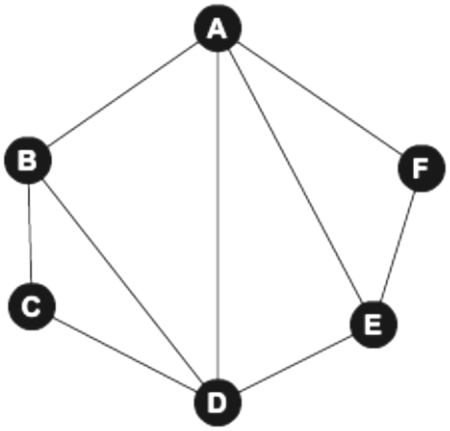
Hint: Loop through all subsets of V. For each subset W, check to see if W is a vertex cover. Do this by looping through all edges; for each edge e, check to see if at least one of its endpoints lies in W.

Solution:

```
Algorithm vertexCover(V, E)
        Input set of vertex V and set of Edges E
        Output vertex Cover VC
        VC←emptyList of vertex
        while (¬E.isEmpty()) do
                E1 ← E.removeFirst()
                (V1,V2) ←computeEndpoints(E1)
                if(¬belongsTo(V1, VC) and ¬belongsTo(V2, VC)) then
                        VC.insertLast(V2)
        return VC
```

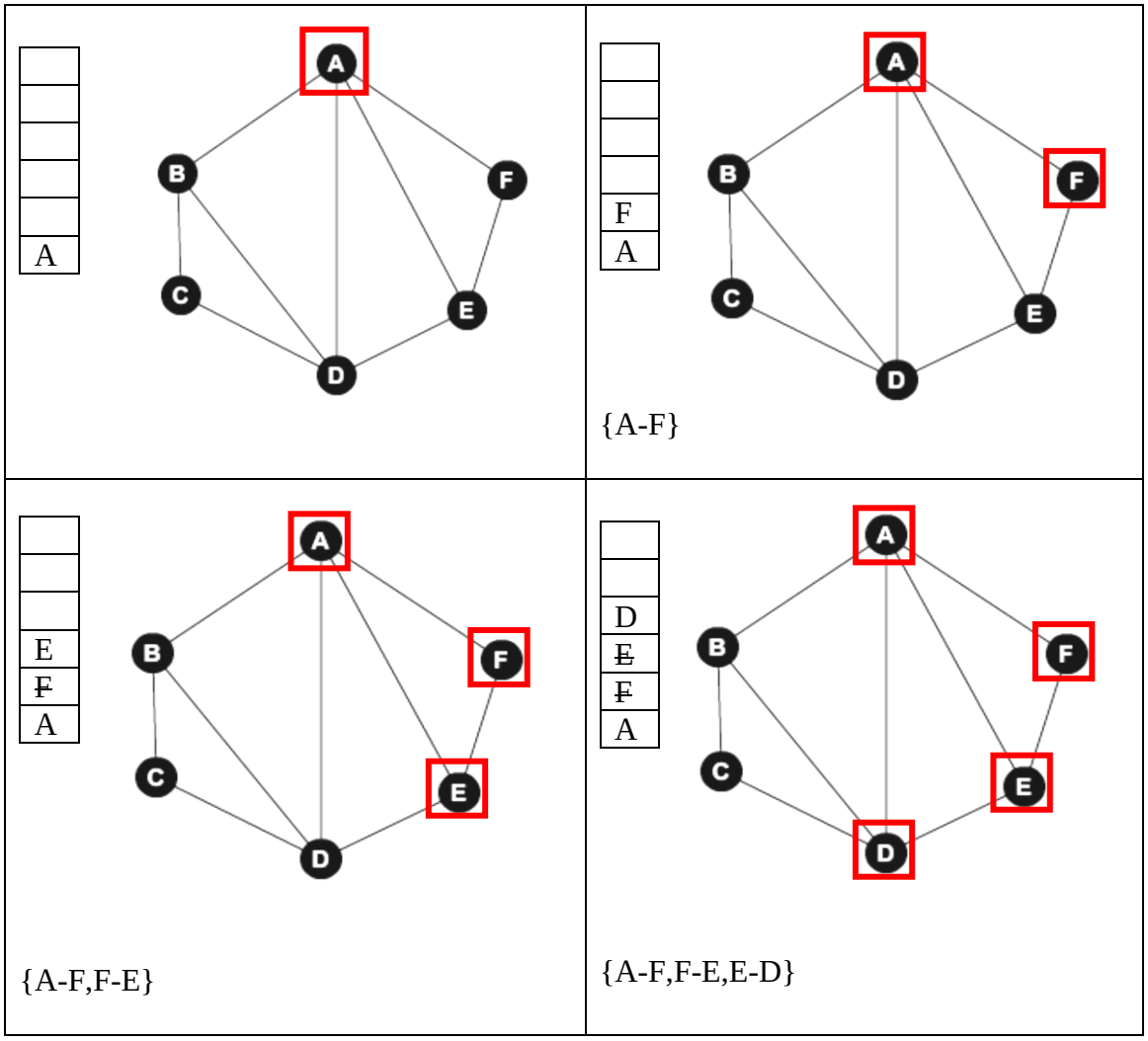
Problem 4.

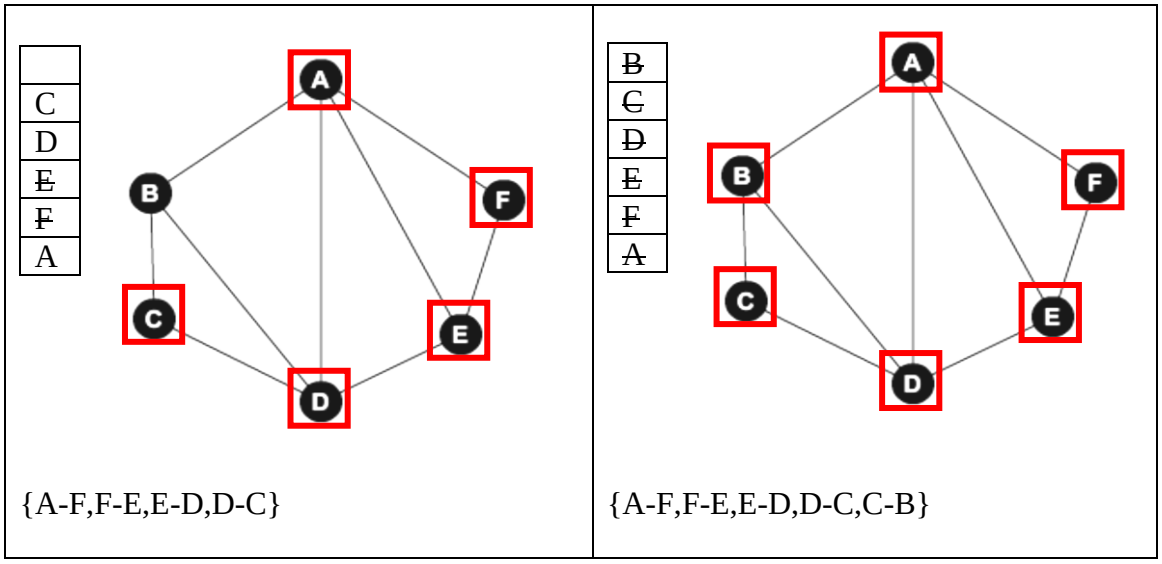
4. Compute two spanning trees for the graphs below using algorithms we discuss in class. (You can start with vertex A) Are the two spanning trees same?



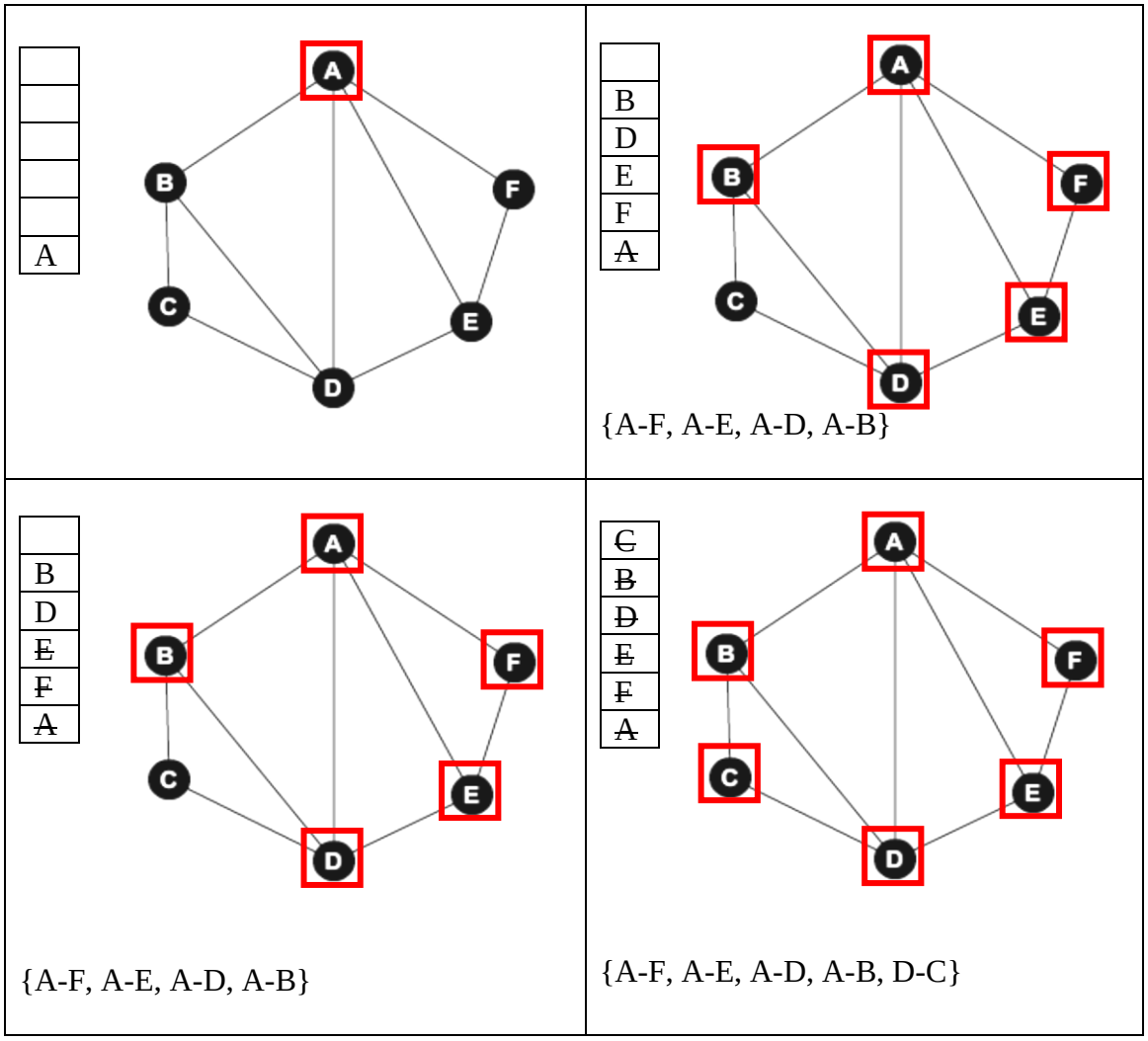
Solution:

A. Spanning tree (Depth-First Search algorithm)





B. Spanning tree (Breadth-First Search Algorithm)



Problem 5.

5. Write the pesudo-code for compute connected components algorithm discussed in class. Your algorithm can be built on top of DFS discussed in the slides.

Solution:

Algorithm: AbstractGraphSearch
    while (some vertex not yet visited) **do**
        handleInitialVertext()
        singleComponentLoop()
        additionalProcessing()

Algorithm: additionalProcessing
    Increase currentComponentNumber by 1

Algorithm: processVertex(Vertex v)
    Input: Vertex v of a graph
    componentMap[currentComponentNumber].insertLast(v)
    vertexComponentMap.add(v,currentComponentNumber)

Algorithm: computeConnectedComponents
    Input: //Nothing
    Output: total number of connected components in the graph
    AbstractGraphSearch.start()
    return currentComponentNumber

Java Implementation

```
public class ConnectedComponentSearch extends DepthFirstSearch {
	ArrayList<List<Vertex>> componentMap = new ArrayList<>();
	HashMap<Vertex,Integer> vertexComponentMap = new HashMap<>();
	int currentComponentNumber = 0;
	public ConnectedComponentSearch(Graph graph) {
		super(graph);
	}

	@Override
	protected void handleInitialVertex() {
		if(currentComponentNumber<=componentMap.size())
			componentMap.add(new ArrayList<>());
		super.handleInitialVertex();
	}



	@Override
	public void processVertex(Vertex v) {
		//super.processVertex(v);
		componentMap.get(currentComponentNumber).add(v);
		vertexComponentMap.put(v, currentComponentNumber);
	}

	@Override
	public void additionalProcessing() {
		super.additionalProcessing();
		//if(someVertexUnvisited()) {
			currentComponentNumber++;
```

```
                        //componentMap.add(new ArrayList<>());
                        //System.out.println(someVertexUnvisited());
                //}
        }

        public int computeConnectedComponent() {
                super.start();
                return currentComponentNumber;
        }
}
```

//inside main() of Main.java

```
                List<Pair> l = new ArrayList<Pair>();
                l.add(new Pair("A","B"));
                l.add(new Pair("B","C"));
                l.add(new Pair("A","D"));
                l.add(new Pair("B","D"));
                l.add(new Pair("E", "F"));

                Graph g = new Graph(l);
                ConnectedComponentSearch c = new ConnectedComponentSearch(g);
                System.out.println("Total Components:"+c.computeConnectedComponent());
                System.out.println(c.componentMap);
                System.out.println(c.vertexComponentMap);
```

```
Output:
Total Components:2
[[A, B, C, D], [E, F]]
{A=0, B=0, C=0, D=0, E=1, F=1}
```

Problem 6.

6. Write the pesudo-code for the algorithm, discussed in class, that computes the shortest path length between two vertices in a graph. You can assume that:

a. The graph is connected.

b. A version of BFS is provided that accepts a specified starting vertex.

Solution:

Algorithm: AbstractGraphSearch

while (some vertex not yet visited) **do**

handleInitialVertext()

singleComponentLoop()

additionalProcessing()

Algorithm: handleInitialVertex

parentMap.add(start,start)

levelsMap.add(start,0)

queue.add(start)

mark start as visited

Algorithm: processVertex(Vertex v)

Input: Vertex v of a graph

parent← parentMap.get(v)

levelsMap.add(v, levelsMap.get(parent)+1)

Algorithm: processEdge(Edge edge)
Input: Edge edge of a graph
parentMap.add(edge.v, edge.u)

Algorithm: computeShortestPath
Input: start point Vertex 'start' and end point Vertex 'end' of path
Output: length of the shortest possible path from start to end
shortPath←empty list of Vertex
AbstractGraphSearch.start()
traversal←end
While(¬parentMap.get(traversal)=traversal) **do**
shortPath.insertFirst(traversal)
traversal←parentMap.get(traversal)
shortPath.insertFirst(start)
return shortPath.size() -1

Java Implementation

```
public class ShortPathLength extends BreadthFirstSearch {
	HashMap<Vertex,Integer> levelsMap = new HashMap<>();
	HashMap<Vertex,Vertex> parentMap = new HashMap<>();
	Vertex start, end;

	public ShortPathLength(Graph g) {
		super(g);
	}

	public List<Vertex> computeShortestPath(Vertex s, Vertex e){
		start = s;
		end = e;
		Vertex trav = e;
		super.start();
		List<Vertex> shortPath = new ArrayList<Vertex>();
		System.out.println(parentMap);
		System.out.println(levelsMap);
		while(parentMap.get(trav)!=trav) {
			shortPath.add(0,trav);
			trav = parentMap.get(trav);
		}
		shortPath.add(0,start);
		System.out.println("Shortest path length is:"+levelsMap.get(end));
		return shortPath;
	}

	@Override
	protected void handleInitialVertex() {
		setHasBeenVisited(start);
		levelsMap.put(start, 0);
		parentMap.put(start,start);
		queue.add(start);
	}

	@Override
	protected void singleComponentLoop() {
		while(!queue.isEmpty()){
			Vertex v = nextUnvisitedAdjacent(queue.peek());
			while(v!=null) {
				System.out.println("Vertex: "+v);
```

```
                        setHasBeenVisited(v);
                        processEdge(new Edge(queue.peek(),v));
                        processVertex(v);
                        queue.add(v);
                        v=nextUnvisitedAdjacent(queue.peek());
                        System.out.println("Queue: "+queue);
                    }
                    queue.remove();
                }
        }

        @Override
        protected void processVertex(Vertex w){
                levelsMap.put(w, levelsMap.get(parentMap.get(w))+1);
        }

        @Override
        protected void processEdge(Edge edge) {
                parentMap.put(edge.v, edge.u);
        }
}

//In main() of Main.java
                List<Pair> l = new ArrayList<Pair>();
                l.add(new Pair("A","B"));
                l.add(new Pair("B","C"));
                l.add(new Pair("A","D"));
                l.add(new Pair("B","D"));
                l.add(new Pair("D","E"));
                l.add(new Pair("E", "F"));
                Graph g = new Graph(l);

                ShortPathLength spl = new ShortPathLength(g);
                System.out.println("Short Path is:"+spl.computeShortestPath(new
                Vertex("C"), new Vertex("F")));
Output:
Shortest path length is:4
Short Path is:[C, B, D, E, F]
```