Multi-type Resource Allocation with Partial Preferences

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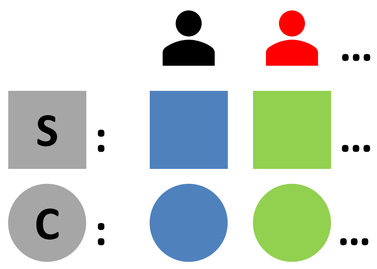
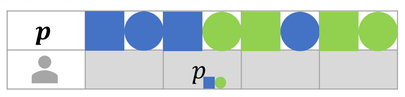
Multi-type Resource Allocation (MTRA)

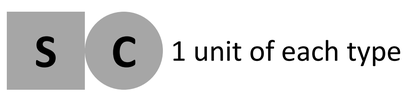
n agents

p types of items
(n of each type)



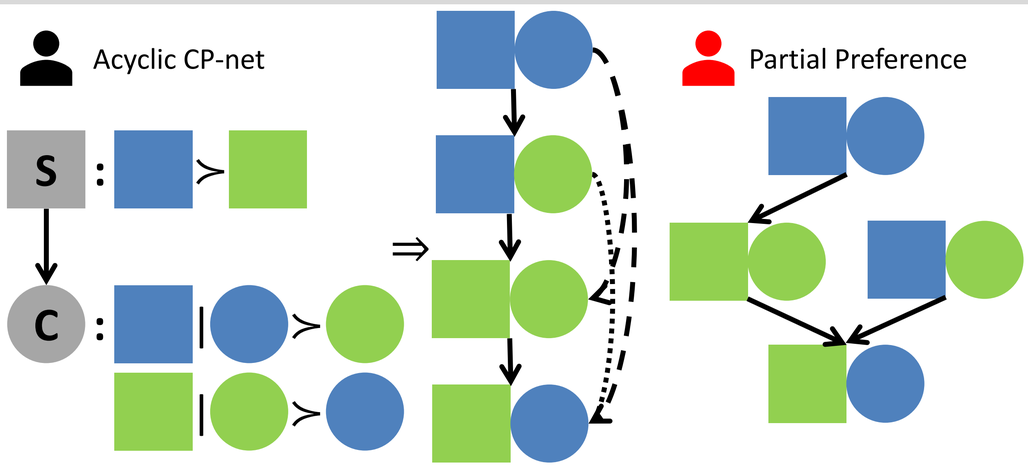
Assignments: Each agents' *allocation* is a collection of fractional *bundles*





1 unit of each type

Partial Preferences



Compute an assignment that is fair and economically efficient

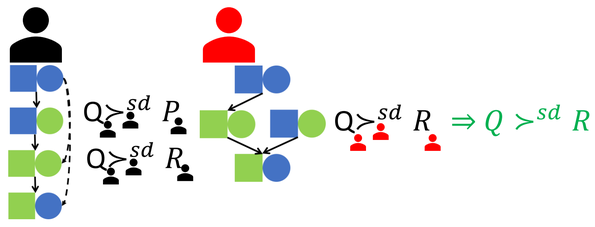
Upper Contour Set @ bundle x w.r.t. $\succ$ is $\mathrm{U}(\succ, x) = \{\mathrm{y}: \mathrm{y} \succ x \text{ or } \mathrm{y} = \mathrm{x}\}$

Allocation p ***stochastically dominates*** q w.r.t. $\succ$, if @ *every* bundle x,

$\sum_{y \in U(\succ, x)} p_y \geq \sum_{y \in U(\succ, x)} q_y$

Assignment P ***stochastically dominates*** Q if $p_j \succ_j^{sd} q_j$, for every agent j

P				
	0.5	0	0.25	0.25
	0	0.5	0.25	0.25
Q				
	0.5	0	0	0.5
	0.5	0	0	0.5
R				
	0	0.5	0.5	0
	0.5	0	0	0.5



Fairness, Efficiency, Strategyproofness, and Indivisibility

A fractional assignment P satisfies:

- decomposability: if it is a probability distribution over *discrete* assignments
- equal treatment of equals: agents with identical preferences receive identical allocations
- sd-envy-freeness: if for every pair of agents j, j', $P_j \succ_j^{sd} P_{j'}$
- sd-efficiency: *no* assignment Q s.t. for every agent j, $\mathrm{Q}_j \succ_j^{sd} P_j$
- ordinal fairness: if for every pair of agents j, j', and every bundle x, s.t. $P_{j,x} > 0$, we have $\sum_{\hat{x} \in U(\succ_j, x)} P_{j,\hat{x}} \leq \sum_{\hat{x} \in U(\succ_{j'}, x)} P_{j',\hat{x}}$
- weak-sd-envy-freeness: if for every pair of agents j, j', $P_j \succ_j^{sd} P_j \Rightarrow P_j = P_{j'}$
- ex-post-efficiency: a probability distribution over sd-efficient *discrete* assignments

A mechanism f satisfies:

- sd-strategyproofness: if $f(\succ) \succ_j^{sd} f(\succ')$, for every agent j, every misreport
- sd-weak-strategyproofness: if $f(\succ') \succ_j^{sd} f(\succ) \Rightarrow f(\succ')(j) = f(\succ)(j)$

In general, mechanism f satisfies property X if $f(\succ)$ satisfies X for every profile $\succ$

Assignments are NOT guaranteed to be decomposable when there are multiple types of items

NO mechanism is sd-efficient AND sd-envy-free under general partial preferences

Multi-type Random Priority (MRP)

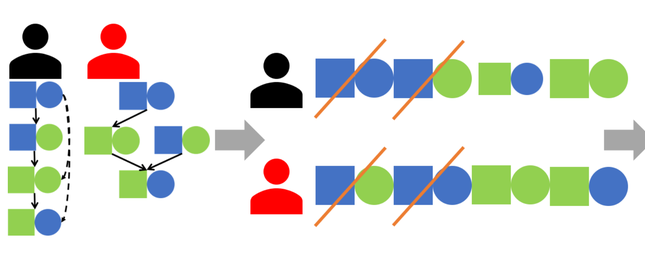
Extends the Random Priority (RP) mechanism [Abdulkadiroglu and Sonmez, 1998]

- Topologically sort partial order $\succ_j$ to linear order $\succ_j'$
- Pick priority ordering σ over agents uniformly at random
- Agents arrive according to σ, and are allocated their favorite remaining *bundle*
  - Remove agent and all items in bundle

Multi-type Probabilistic Serial (MPS)

Extends Probabilistic Serial (PS) mechanism [Bogomolnaia and Moulin, 2001]

- Topologically sort partial order $\succ_j$ to linear order $\succ_j'$
- While there is a remaining item:
  - ALL agents simultaneously consume their favorite remaining bundle (per $\succ_j'$) at an equal, uniform rate until supply of any item being consumed is exhausted



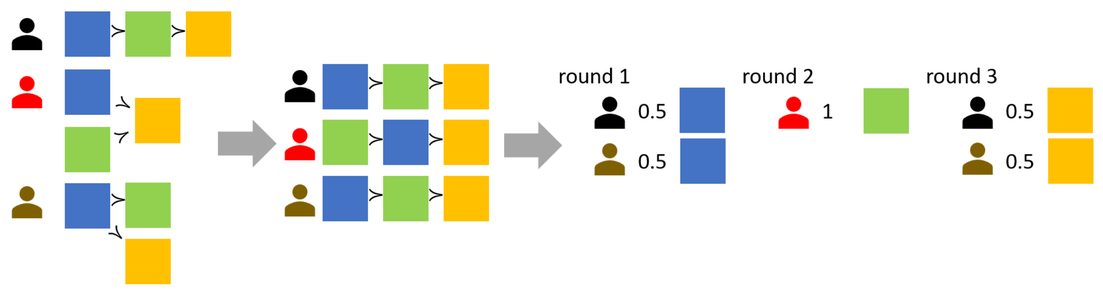
round 1				
	0.5	0	0	0
	0	0.5	0	0

round 2				
	0.5	0	0.5	0
	0	0.5	0	0.5

Multi-type General Dictatorship (MGD)

Hybrid of MRP and MPS

- Topologically sort partial order $\succ_j$ to linear order $\succ_j'$
- For $j = 1, \ldots, n$ do:
- Agent j *invites* all other agents j' s.t. $\succ_{j'} = \succ_j'$ to simultaneously consume their favorite remaining bundle until some item being consumed is exhausted



Fairness, Efficiency, and Non-Manipulability

Mechanism and Preference Domain		SE	EPE	OF	SEF	WSEF	ETE	UI	SS	WSS	DC
MRP	General partial preferences	N†	Y	N‡	N†	Y	Y	N	N	Y	Y
	CP-nets	N†	Y	N‡	N†	Y	Y	Y	Y	Y	Y
	CP-nets with shared dependency graph	N†	Y	N‡	N†	Y	Y	Y	Y	Y	Y
MPS	General partial preferences	Y	N	N	N	Y	Y	N	N†	N	N
	CP-nets	Y	N	Y	Y	Y	Y	Y	N†	N	N
	CP-nets with shared dependency graph	Y	N	Y	Y	Y	Y	Y	N†	Y	N
MGD	General partial preferences	Y	Y	N‡	N	N	Y	N	N	N	Y
	CP-nets	Y	Y	N‡	N	N	Y	N	N	N	Y
	CP-nets with shared dependency graph	Y	Y	N‡	N	N	Y	N	N	N	Y

SE: *sd-efficiency* EPE: *ex-post-efficiency* (W)SEF: (*weak-*)*sd-envy-freeness*
OF: *ordinal fairness* UI: *upper-invariance* (W)SS: (*weak-*)*sd-strategyproofness*
DC: *decomposability* ETE: *equal treatment of equals*

Results annotated with † are due to [Bogomolnaia and Moulin, 2001], and those annotated ‡ are due to [Hashimoto et al., 2014]

Future Work

- Characterizing MRP and MPS
- Stronger properties under natural restrictions on the domain of preferences