On Certainty Equivalence of Demand Charge Reduction Using Storage

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***Abstract*—We investigate the problem of optimal storage control for the purpose of jointly reducing the volumetric energy charge and a whole-period relevant demand charge. We systematically investigate the structure of the optimal solution set of this storage control problem, and we for the first time establish the equivalence among the optimal solutions of the original stochastic control problem, a corresponding stochastic programming program and its deterministic counterpart, which greatly simplifies the process of finding optimal solutions for the original problem. Based on the equivalence among different problems, we propose an algorithm to obtain an optimal solution of the stochastic control program which merely needs offline information. We validate our approach by testing on over PG&E residential customers' data and show substantial cost reduction under the tariff with both energy charge and demand charge.**

I. Introduction

Renewables are expected to supply more than 50% of electricity demand by 2050 in various parts of the world [1], [2], [3]. The intermittent nature of wind and solar energy challenges conventional *reserve-based* operational schemes for the electric power grid as they are designed for small uncertainty scenarios [4]. Given that forecast errors for wind and solar generation can be $5-10$ times larger than those for load [5], 50% renewable penetration will result in substantially higher reserve requirements, which can be cost prohibitive and offset the environmental benefits of renewables as these reserve requirements are usually fulfilled by fast-ramping fossil-fueled generators.

Energy storage provides an alternative solution to maintain supply demand balance by transporting energy across time [6]. Recent years have witnessed a strong positive trend for energy storage. On one hand, partly as a consequence of the increasing public and commercial interest in electric vehicles, the cost of energy storage is decreasing rapidly [7]. On the other hand, many states have set policies to incentivize or mandate the adoption of energy storage. Examples include California's 1.3 GW energy storage procurement mandate by 2020 [8] and New Jersey's Renewable Electric Storage Incentive Program [9]. This trend has led to significant industrial impacts, as evidenced by the record-breaking growth of US storage installations in 2015 [10].

The question of optimal control of energy storage has been investigated with a variety of settings. The problem of optimally arbitraging a stochastic price sequence using storage is analyzed using dynamic programming in [11], where it is shown that the optimal control policy can be characterized by a sequence of price thresholds whose recursive expressions are also obtained. Faghih *et al*. [12] study the same problem with a more realistic storage model in which the power (ramping) limits of the storage devices are modeled. Problems of using energy storage to minimize energy imbalance and thus reduce the system risk are studied in various contexts: In [13], [14] for reducing reserve energy requirements in power system dispatch, in [15], [16] for operating storage co-located with a wind farm, in [17], [18] for operating storage co-located with end-user demands, and in [19], [20] for storage with demand response. Most of these studies obtain closed-form expressions or structural characterization of the optimal storage control policy by exploiting the specific form of the cost function considered. In a rather general setting, an online algorithm is constructed in [21] for the problem of single storage control with any convex cost function and random demand, where a bound is provided for a priori estimate of the algorithm's performance.

Common to all these prior work is the assumption that the cost function is additively separable across time periods. Under this assumption, the problem of storage control is greatly simplified, as one can focus on the storage dynamics, which are often considered to be the only source of coupling of decision variables in different time periods. Although this assumption is accurate for many use cases of storage when the cost or benefit is accounted for period by period, this is not always the case, especially for behind-the-meter applications of storage. In particular, a nonseparable cost function is relevant when the tariff of the electricity utility company includes payment components depending on (monthly) aggregate statistics of electricity usage in addition to the total energy consumption.

A preeminent example is that of *demand charge*, which charges the customer a demand price $q > 0$ multiplied by the *maximum* or peak demand of the customer consumed in any hour of the month. As the demand prices charged by utility companies can be as high as 100 times of energy prices, demand charge could account for more than 50% of the monthly electricity bill of a commercial customer. As a consequence, reducing the peak demand has been deemed the "most economic use" of energy storage when demand charge is in place[22]. Furthermore, while the majority of smaller residential customers have never been subject to demand charge, recent trend of adoption of *residential demand charge* by a number of utility companies indicates that the use of storage for reducing demand charge could have impact beyond the commercial electricity customers.

We present here a study of the problem of stochastic control of energy storage for the purpose of jointly reducing the (volumetric) energy charge and demand charge. The stochastic demand in our model could be used to model the net demand, which is the actual demand minus the renewable generation from, for instance, rooftop solar generation. The paper contributes to the existing literature in the following ways: (i) We provide a stylized formulation of the problem, which reduces the challenge of time-coupling cost functions into a standard stochastic control problem with an enlarged state space. (ii) Using novel probabilistic arguments, we establish that this problem is *certainty equivalent* (CE) whenever the deterministic counterpart of the problem has a unique solution. Our proof approach is significantly different from dynamic programming based arguments, which are widely used to establish common certainty equivalence results such as that for LQG. (iii) When the deterministic counterpart of the problem has multiple solutions, some solutions can lead to better performance for the stochastic control program than others. In this case, we provide an algorithm approaching the optimal solution, and could be proved that is strictly better than some CE solution.

Our paper is closely related to recent work [23], which formulates the demand charge reduction problem and proposes stochastic dynamic programming based algorithm for solving the problem. Although their formulation is more general than ours in that they allow random price and use a more detailed storage model, we have obtained sharper analytical results and much simpler algorithm. The solution approaches taken in these two papers are also completely different.

A. Organization

The paper is organized as follows: Section II proposes the problem formulation. Section III investigates the structure of optimal control and establish the equivalence among the optimal solutions of the original stochastic control problem, a corresponding stochastic programming program and its deterministic counterpart. Section IV provides an algorithm constructing an optimal solution for the stochastic problem and proves its improvement over the solution of deterministic counterpart. Experimental results are in section V. Section VI concludes the paper.

B. Notation

We use lower case English and Greek letters, such as a, μ to denote scalars, use lower case bold English and Greek letters, such as $\mathbf{a}, \boldsymbol{\mu}$ to denote vectors, and use upper case letters such as L to denote matrices. We also use upper case letters such as J to denote objective functions in optimization problems. We use $t \in \mathcal{T} := \{1, \ldots, T\}$ to index time periods. For any Euclidean vector space $\mathbb{R}^d$, we use $\mathbf{1} \in \mathbb{R}^d$ to denote the all-one vector. For any real number $x \in \mathbb{R}$, $(x)^+ := \max(x, 0)$ denotes the positive part of x and $(x)^- := -\min(x, 0)$ to denote the negative part of x so that $x = (x)^+ - (x)^-$. For any vector $\mathbf{x} \in \mathbb{R}^d$ and $1 \leq d_1 \leq d_2 \leq d$, we denote $\mathbf{x}_{d_1:d_2} := (x_{d_1}, \ldots, x_{d_2})$. For two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$, we use $\mathbf{x} \circ \mathbf{y}$ to denote the Hadamard (element-wise) product of these two vectors. That is, if $\mathbf{z} = \mathbf{x} \circ \mathbf{y}$, then $z_i = x_i y_i$.

II. Problem Formulation

We consider the problem of operating energy storage over a finite horizon $t \in \mathcal{T}$ to minimize a user's electricity bill, which includes an energy component and a peak demand component.

For every time period t, the user has a net demand d_t which is the actual demand minus distributed generation such as rooftop solar. We assume that at the beginning of the horizon, by utilizing historical data, we have a certain demand forecast $\widehat{d}_t$ so that the actual demand is

$$d_t = \widehat{d}_t + \epsilon_t, \quad t \in \mathcal{T}. \tag{1}$$

Here the demand forecast should encapsulate the seasonality and many other deterministic features of the demand process, whereas the forecast error ϵ_t is assumed to be zero-mean and independently and identically distributed (i.i.d.), following some prescribed distribution with a probability density function $f(x)$ and the corresponding cumulative distribution function $F(x)$. Note that the distribution of forecast error F is either supplied by a probabilistic forecasting procedure or fitted using historical forecast error data. Albeit the error distribution may itself be inaccurate, it has been proven a useful way to model the stochasticity of the system.

We take a stylized energy storage model, for which the state of charge is denoted by b_t. The storage dynamics are

$$b_{t+1} = b_t + u_t, \quad t \in \mathcal{T}, \tag{2}$$

where u_t represents the charging and discharging operation with $u_t > 0$ modeling charging and $u_t < 0$ modeling discharging. We consider a slow time scale setup, where each discrete time slot represents an hour or longer such that the power limit of the storage is not constrained. The energy limit of the storage can then be modeled as

$$\underline{b} \leq b_t \leq \overline{b}, \quad t \in \mathcal{T} \cup \{T+1\}, \tag{3}$$

where $\underline{b}$ is the minimal allowed state of charge, and $\overline{b}$ is the energy capacity of storage. Given the finite horizon formulation of the problem, it is sometimes desired to impose additional requirements on the terminal storage level (*e.g.*, by requiring the end-of-month storage level to equal the initial storage level so that the problem can be periodically solved for each month). To this end, we require that

$$\underline{b}_{T+1} \leq b_{T+1} \leq \overline{b}_{T+1}, \tag{4}$$

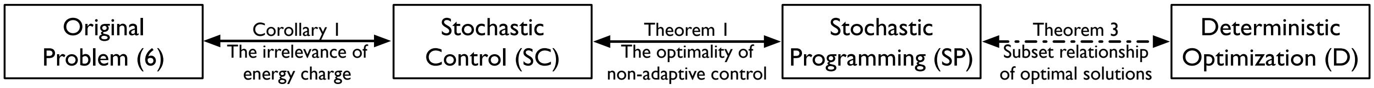

Fig. 1: Illustration of the relationships between different control problems

where $\underline{b} \le \underline{b}_{T+1} \le \overline{b}_{T+1} \le \overline{b}$ are given parameters.

The total cost over the horizon is calculated as follows: For every time period t, the energy charge is calculated as $p(d_t + u_t)$, where $p > 0$ is the energy price specified by the tariff. For the entire horizon, the peak demand charge is calculated as $q \cdot \max_{t \in \mathcal{T}}(d_t + u_t)$, where $q > 0$ is the rate for (peak) demand charge. Thus the ex-post cost function is

$$C(\mathbf{u}, \mathbf{d}) := \sum_{t \in \mathcal{T}} p(d_t + u_t) + q \cdot \max_{t \in \mathcal{T}}(d_t + u_t). \quad (5)$$

The decision process evolves as follows:

1) Before the first time period $t = 1$, the demand forecast, the distribution of forecast error, and the storage parameters are known. The storage starts at a preset level $b_1 \in [\underline{b}, \overline{b}]$.
2) At the beginning of period t, the demand of the previous time period d_{t-1} is revealed. The storage operator needs to issue an instruction for storage control in period t, i.e., u_t.
3) During the period t, the state of charge is updated according to (2).
4) After time period T (denoted as period $T+1$ and referred to as the terminal period), the total electricity bill is calculated according to (5).

Let $z_t := \max_{1 \le \tau < t}(d_\tau + u_\tau)$ be the running maximum demand prior to time period t, with $z_1 := -\infty$. Given the battery dynamics, it is easy to observe that the system is Markov, with (continuous) state variable (b_t, z_t), so that it suffices to optimize over the space of policies of the form $u_t = \pi_t(b_t, z_t)$[1], $t \in \mathcal{T}$ [24]. Thus the stochastic control (SC) problem for storage operation takes the form of

$$\begin{aligned}
J^\star := \min_{\boldsymbol{\pi} \in \Pi} \quad & J(\boldsymbol{\pi}) = \mathbb{E}\left[\sum_{t \in \mathcal{T}} p(d_t + u_t) + q z_{T+1}\right] && (6a)\\
\text{s.t.} \quad & b_{t+1} = b_t + u_t, \qquad t \in \mathcal{T}, && (6b)\\
& z_{t+1} = \max(z_t, d_t + u_t), \qquad t \in \mathcal{T}, && (6c)\\
& u_t = \pi_t(b_t, z_t), \qquad t \in \mathcal{T}, && (6d)\\
& \underline{b} \le b_t \le \overline{b}, \qquad t \in \mathcal{T}, && (6e)\\
& \underline{b}_{T+1} \le b_{T+1} \le \overline{b}_{T+1}, && (6f)
\end{aligned}$$

where $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_T)$, and $\Pi = \Pi_1 \times \cdots \times \Pi_T$ is the set of admissible control policies with Π_t being the set of all functions that map the state (b_t, z_t) to a feasible storage control action u_t.

[1]The policy is allowed to implicitly depend on static information that is available prior to the first time period such as demand forecast $\widehat{d}_t$, $t \in \mathcal{T}$.

III. The Structure of Optimal Control

Directly solving the original SC problem (6) is very challenging due to the difficulty of evaluating Bellman's recursion. In this section, we investigate the structure of the stochastic control problem (6), in particular, we establish the equivalence between the stochastic control problem (SC) and a suitably defined stochastic programming problem (SP$_1$), for any fixed forecast error distribution F. Furthermore, we prove that for any distribution, the optimal solution of (SP$_1$) is an optimal solution of (D). The series of relationship between different optimization problems are illustrated in Fig. 1.

A. Irrelevance of Energy Charge

We start by establishing that we can modify the constraint on the terminal battery state (6f) to get a simpler equivalent stochastic control program, for which the energy charges of all feasible control policies are the same. In particular, we show that for any optimal control policy sequence $\boldsymbol{\pi}$, the policy at the terminal period π_T is always discharging the battery to its lower bound regardless of the state (b_T, z_T).

Lemma 1. *For any optimal control policy sequence $\boldsymbol{\pi}^\star \in \Pi$ of* (6)*, the terminal control takes the form of $u_T^\star = \pi_T^\star(b_T, z_T) = \underline{b}_{T+1} - b_T$, independent of the running maximum demand z_T.*

Proof. It suffices to prove that, for all control policy sequence $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_T)$ such that the induced control action at the terminal period $u_T \ne u_T^\star$ with positive probability, the modified control policy sequence $\boldsymbol{\pi}' = (\pi_1', \ldots, \pi_T') := (\pi_1, \ldots, \pi_{T-1}, \pi_T^\star)$ has a cost $J(\boldsymbol{\pi}') < J(\boldsymbol{\pi})$. In particular, we show that for any realization of demand sequence $\mathbf{d}$, that $u_T' \ne u_T$, the ex-post cost induced by $\boldsymbol{\pi}$ is always larger than that induced by $\boldsymbol{\pi}'$. Keeping the demand realization $\mathbf{d}$ fixed, let $\mathbf{u}$ and $\mathbf{u}'$ be the control action sequences induced by $\boldsymbol{\pi}$ and $\boldsymbol{\pi}'$, respectively. From the construction of $\boldsymbol{\pi}$ and $\boldsymbol{\pi}'$, $u_t' = u_t$ for $t = 1, \ldots, T-1$, and $u_T' = \underline{b}_{T+1} - b_T$. From the storage dynamics, we have $\mathbf{1}^\top \mathbf{u}' = \underline{b}_{T+1} - b_1$. Therefore

$$u_T' = \underline{b}_{T+1} - b_T = \underline{b}_{T+1} - (b_1 + \mathbf{1}^\top \mathbf{u} - u_T) < u_T,$$

as $u_T' \ne u_T$ by the assumption on $\mathbf{d}$. It follows that

$$\begin{aligned}
& C(\mathbf{u}, \mathbf{d}) - C(\mathbf{u}', \mathbf{d})\\
=& \sum_{t \in \mathcal{T}} p(d_t + u_t) + q \cdot \max_{t \in \mathcal{T}}(d_t + u_t)\\
& - \sum_{t \in \mathcal{T}} p(d_t + u_t') - q \cdot \max_{t \in \mathcal{T}}(d_t + u_t')\\
=& p(u_T - u_T') + q \cdot \max_{t \in \mathcal{T}}(d_t + u_t) - q \cdot \max_{t \in \mathcal{T}}(d_t + u_t')\\
>& p(u_T - u_T') + 0 = p\left(\mathbf{1}^\top \mathbf{u} - \underline{b}_{T+1} + b_1\right) \ge 0.
\end{aligned}$$

Since for any other demand realization $\tilde{\mathbf{d}}$, we have $u'_T = u_T$, $C(\mathbf{u}, \tilde{\mathbf{d}}) = C(\mathbf{u}', \tilde{\mathbf{d}})$. As the probability that $u'_T \neq u_T$ is positive, we conclude that $J_F(\boldsymbol{\pi}') < J_F(\boldsymbol{\pi})$. □

Let the set of all feasible control action sequences be

$$\mathcal{U} := \left\{ \mathbf{u} \in \mathbb{R}^T : \underline{b}_{t+1} \leq b_1 + \sum_{\tau=1}^{t} u_\tau \leq \bar{b}_{t+1},\ t \in \mathcal{T} \right\},$$

with $(\underline{b}_t, \bar{b}_t) := (\underline{b}, \bar{b})$ for all $t \in \mathcal{T}$ and $(\underline{b}_{T+1}, \bar{b}_{T+1})$ as defined before. An immediate consequence of Lemma 1 is the following corollary:

Corollary 1. *The control action sequence induced by any optimal control policy sequence is in $\mathcal{U}^{\mathrm{R}}$, the subset of $\mathcal{U}$ defined as $\mathcal{U}^{\mathrm{R}} := \mathcal{U} \cap \{\mathbf{u} \in \mathbb{R}^T : \mathbf{1}^\top \mathbf{u} = \underline{b}_{T+1} - b_1\}$.*

Corollary 1 reveals that we can modify the constraint (6f) to $b_{T+1} = \underline{b}_{T+1}$ without loss of optimality. Thus we will use the equality constraint, $b_{T+1} = \underline{b}_{T+1}$, for the stochastic control problem (6) in what follows.

Another consequence of Corollary 1 is that it allows us to focus on the expected maximum net demand $\mathbb{E}[z_{T+1}]$ in (6), as any element of $\mathcal{U}^{\mathrm{R}}$, including all the control sequence $\mathbf{u}$ induced by the optimal policy sequence of stochastic control program (6), will have the same initial and terminal battery state, b_1 and b_{T+1}, respectively. Therefore the total energy charges $\sum_{t\in\mathcal{T}} p(d_t + u_t) = p(\mathbf{1}^\top \mathbf{d} + b_{T+1} - b_1)$ are the same. To summarize, for identifying the optimal policy sequence, the stochastic control problem (6) is equivalent to the following reduced problem on which we focus in the sequel

$$
\begin{aligned}
Z^\star_{\mathrm{SC}} := \min_{\boldsymbol{\pi}\in\Pi} \quad & Z_{\mathrm{SC}}(\boldsymbol{\pi}) = \mathbb{E}_F\left[z_{T+1}\right] && \text{(SC-a)}\\
\text{s.t.} \quad & b_{t+1} = b_t + u_t, \quad t \in \mathcal{T}, && \text{(SC-b)}\\
& z_{t+1} = \max(z_t, d_t + u_t), \quad t \in \mathcal{T}, && \text{(SC-c)}\\
& u_t = \pi_t(b_t, z_t), \quad t \in \mathcal{T}, && \text{(SC-d)}\\
& \mathbf{u} \in \mathcal{U}^{\mathrm{R}}. && \text{(SC-e)}
\end{aligned}
$$

As in the standard literature of discrete time stochastic control [24], of central importance to problem (SC) is the *cost-to-go functions*, defined via the Bellman recursion backward from the terminal stage:

$$Z^\star_{\mathrm{SC}}(b, z; T) = \mathbb{E}\left[\max(z, d_T + \underline{b}_{T+1} - b)\right], \tag{7}$$

$$
\begin{aligned}
& Z^\star_{\mathrm{SC}}(b, z; t)\\
:= & \min_{u\in[\underline{b}-b, \bar{b}-b]} \mathbb{E}\left[Z^\star_{\mathrm{SC}}\left(b+u, \max(z, d_t+u); t+1\right)\right],
\end{aligned} \tag{8}
$$

for $t \in \mathcal{T}\backslash\{T\}$, where we have overloaded the symbol $Z^\star_{\mathrm{SC}}$ for notational convenience. Given the cost-to-go functions, an optimal policy for each time period $t \in \mathcal{T}\backslash\{T\}$ is defined as

$$\pi^\star_t(b, z) \in \operatorname*{argmin}_{u\in[\underline{b}-b, \bar{b}-b]} \mathbb{E}\left[Z^\star_{\mathrm{SC}}(b+u, \max(z, d_t+u); t+1)\right], \tag{9}$$

and an optimal policy for the terminal period is

$$\pi^\star_T(b, z) = \underline{b}_{T+1} - b \tag{10}$$

in view of Corollary 1.

B. Optimality of Non-Adaptive Control

The stochastic control problem (SC) is still challenging to solve as the optimal policy $\pi^\star_t$ could depend on the running maximum state z_t, which evolves stochastically driven by the realization of d_t. We show in this subsection that in fact we can restrict the problem to policies that are non-adaptive to z_t. As a consequence, we can establish the optimality of certain *static policies* that compute a deterministic *control action sequence* $\mathbf{u}$ simply using information that is available offline (before period 1) without knowing the realization of $\mathbf{d}$.

To this end, we consider a family of *stochastic programming* problems (SP) that will bridge the stochastic control problem (SC) and its deterministic counterpart as defined later in the paper. Note that the distinctive feature of SP in contrast to SC is that it optimizes for the optimal control action sequence $\mathbf{u}$ instead of the optimal control policy sequence $\boldsymbol{\pi}$. In general SP is intrinsically less adaptive than SC and leads to sub-optimal policies for SC. Nevertheless, we will show that this is not the case for (SC) defined here by first defining a sequence of SP problems referred to as *rolling horizon stochastic programming* problems (RH-SP)[25] and then showing that the adaptation to z_t brings no benefit for these problems.

Given a state at the t-th period, (b_t, z_t), we define the t-th RH-SP problem for storage operation as

$$
\begin{aligned}
Z^\star_{\mathrm{SP}}(b_t, z_t; t) := \min_{\mathbf{u}_{t:T}} \quad & Z_{\mathrm{SP}}(\mathbf{u}_{t:T};\, b_t, z_t, t) = \mathbb{E}_F\left[z_{T+1}\right] && (\mathrm{SP}_t\text{-a})\\
\text{s.t.} \quad & b_{\tau+1} = b_\tau + u_\tau, && (\mathrm{SP}_t\text{-b})\\
& z_{\tau+1} = \max(z_\tau, d_\tau + u_\tau), && (\mathrm{SP}_t\text{-c})\\
& \underline{b} \leq b_\tau \leq \bar{b}, && (\mathrm{SP}_t\text{-d})\\
& b_{T+1} = \underline{b}_{T+1}, && (\mathrm{SP}_t\text{-e})
\end{aligned}
$$

where the constraints (SP_t-b), (SP_t-c) and (SP_t-d) hold for all $\tau \in \{t, \ldots, T\}$, and the expectation in (SP_t-a) is taken over the random variables $d_t, \ldots, d_T$. We first show the uniqueness of the optimal solution of (SP_t) under mild conditions for the forecast error distribution.

Lemma 2. *For each fixed state (b_t, z_t), the objective function of (SP_t) is a strictly convex function of $\mathbf{u}_{t:T}$ in the feasible set of (SP_t). Thus there exists a unique solution of (SP_t), denoted by $\mathbf{u}^\star_{t:T}(b_t, z_t)$.*

Proof. The objective function of (SP_t) is

$$
\begin{aligned}
& Z_{\mathrm{SP}}(\mathbf{u}_{t:T}; b_t, z_t, t)\\
=& \mathbb{E}\left(\max\left[z_t, u_t + \widehat{d}_t + \epsilon_t, \ldots, u_T + \widehat{d}_T + \epsilon_T\right]\right),
\end{aligned} \tag{11}
$$

which is convex in $\mathbf{u}_{t:T}$ as it is the expected value of a convex piecewise linear function. We show that the function in fact is strictly convex in $\mathbf{u}_{t:T}$ for $\mathbf{u}_{t:T}$ belonging to the feasible set of (SP_t), so that (SP_t) has a unique solution [26]. Consider a realization of the demand process $\mathbf{d}_{t:T}$ and two feasible control action sequences $\mathbf{u}^1_{t:T}$ and $\mathbf{u}^2_{t:T}$ such that $\mathbf{u}^1_{t:T} \neq \mathbf{u}^2_{t:T}$. For all

$\eta \in (0,1)$, let $\mathbf{u}_T^3 := \eta \mathbf{u}_T^1 + (1-\eta)\mathbf{u}_T^2$. Then by convexity of the piecewise linear function below, we have

$$\begin{aligned}&\max\left[z_t, u_t^3 + d_t, \ldots, u_T^3 + d_T\right] \\ \leq &\eta\left(\max\left[z_t, u_t^1 + d_t, \ldots, u_T^1 + d_T\right]\right) \\ &+ (1-\eta)\left(\max\left[z_t, u_t^2 + d_t, \ldots, u_T^2 + d_T\right]\right),\end{aligned} \tag{12}$$

where the equality holds if and only if there exists a $\tau^\star \in \{t, \ldots, T\}$ such that

$$u_{\tau^\star}^i + d_{\tau^\star} = \max\left[z_t, u_t^i + d_t, \ldots, u_T^i + d_T\right], \tag{i}$$

for both $i = 1$ and $i = 2$, or

$$z_t = \max\left[z_t, u_t^i + d_t, \ldots, u_T^i + d_T\right], \tag{ii}$$

for both $i = 1$ and $i = 2$. Put $\mathcal{E}_1$ and $\mathcal{E}_2$ to be the set of $\mathbf{d}_{t:T}$ realizations satisfying condition (i) and (ii), respectively. We show that $\mathbb{P}(\mathbf{d}_{t:T} \in \mathcal{E}) < 1$, where $\mathcal{E} = \mathcal{E}_1 \cup \mathcal{E}_2$. Consider two cases:

- *Case 1*: $\mathbb{P}(\mathcal{E}_2) > 0$. Here we know that for both $\mathbf{u}_{t:T}^1$ and $\mathbf{u}_{t:T}^2$, there exist corresponding sets of $\mathbf{d}_{t:T}$ realizations on which z_t achieve the maximum in (ii), denoted $\mathcal{D}_0^1$ and $\mathcal{D}_0^2$, respectively. As $\mathcal{E}_2 = \mathcal{D}_0^1 \cap \mathcal{D}_0^2$, we know that $\mathbb{P}(\mathcal{D}_0^i) \geq \mathbb{P}(\mathcal{E}_2) > 0$, for $i = 1, 2$. However, as

$$\begin{aligned}\mathcal{D}_0^i &= \left\{\mathbf{d}_{t:T} : z_t = \max\left[z_t, u_t^i + d_t, \ldots, u_T^i + d_T\right]\right\} \\ &= \left\{\mathbf{d}_{t:T} : u_\tau^i + d_\tau \leq z_t, \tau = t, \ldots, T\right\} \\ &= \left\{d_t : d_t \leq z_t - u_t^i\right\} \times \cdots \times \left\{d_T : d_T \leq z_t - u_T^i\right\},\end{aligned}$$

 it follows that the difference between $\mathcal{D}_0^1$ and $\mathcal{D}_0^2$ must have non-zero probability, or in other words, $\mathbb{P}((\mathcal{D}_0^1 - \mathcal{D}_0^2) \cup (\mathcal{D}_0^2 - \mathcal{D}_0^1)) > 0$, given that $\mathbf{d}_{t:T}$ has continuous distribution and $\mathbf{u}_{t:T}^1 \neq \mathbf{u}_{t:T}^2$. Thus in this case, there exists a set of $\mathbf{d}_{t:T}$ realizations with positive probability on which z_t achieves the maximum of (ii) for one of $\mathbf{u}_{t:T}^i$, $i = 1, 2$ but not for the other one. As (12) takes strict inequality on this set, we conclude that (11) is strictly convex in this case. Note that $((\mathcal{D}_0^1 - \mathcal{D}_0^2) \cup (\mathcal{D}_0^2 - \mathcal{D}_0^1)) \cap \mathcal{E} = \emptyset$, we have $\mathbb{P}(\mathbf{d}_{t:T} \in \mathcal{E}) < 1$.
- *Case 2*: $\mathbb{P}(\mathcal{E}_2) = 0$. If the probability of only one of the $\mathcal{D}_0^i$ is zero and that of the other is positive, arguments similar to those in Case 1 hold. Thus we focus on the case in such $\mathbb{P}(\mathcal{D}_0^i) = 0$ for both $i = 1, 2$. In this case, there exists a $\tau^\star$ such that with positive probability $u_{\tau^\star}^i + d_{\tau^\star}$ achieves the maximum of (i) for both $i = 1, 2$. Let $\mathcal{D}_{\tau^\star}^i$ be the set of $\mathbf{d}_{t:T}$ realizations on which $u_{\tau^\star}^i + d_{\tau^\star}$ achieves the maximum for the corresponding $\mathbf{u}_{t:T}^i$. We have

$$\begin{aligned}\mathcal{D}_{\tau^\star}^i &= \left\{\mathbf{d}_{t:T} : u_{\tau^\star}^i + d_{\tau^\star} = \max\left[u_t^i + d_t, \ldots, u_T^i + d_T\right]\right\} \\ &= \left\{\mathbf{d}_{t:T} : u_{\tau^\star}^i + d_{\tau^\star} \geq u_\tau^i + d_\tau, \tau \neq \tau^\star\right\}.\end{aligned}$$

 For any fixed realization of $d_{\tau^\star}$, let $\mathcal{D}_{-\tau^\star}^i(d_{\tau^\star})$ be the set of realizations of other entries of $\mathbf{d}_{t:T}$ such that $d_{\tau^\star}$ achieves the maximum for the corresponding $\mathbf{u}_{t:T}^i$, which can be written as

$$\mathcal{D}_{-\tau^\star}^i(d_{\tau^\star}) = \prod_{\tau \neq \tau^\star} \left\{d_\tau \leq d_{\tau^\star} + (u_{\tau^\star}^i - u_\tau^i)\right\}.$$

 By the constraint (SP$_t$-e), the sum of all entries of any feasible $\mathbf{u}_{t:T}$ is the same constant. Thus for any two different feasible control sequences $\mathbf{u}_{t:T}^1$ and $\mathbf{u}_{t:T}^2$, it must be the case that $u_{\tau^\star}^1 - u_\tau^1 \neq u_{\tau^\star}^2 - u_\tau^2$ for some $\tau \in \{t, \ldots, T\}$, $\tau \neq \tau^\star$. It follows that the difference between the sets $\mathcal{D}_{\tau^\star}^1$ and $\mathcal{D}_{\tau^\star}^2$ has a positive probability as $\mathbf{d}_{t:T}$ has a continuous distribution. Note that $((\mathcal{D}_{\tau^\star}^1 - \mathcal{D}_{\tau^\star}^2) \cup (\mathcal{D}_{\tau^\star}^2 - \mathcal{D}_{\tau^\star}^1)) \cap \mathcal{E} = \emptyset$, we have $\mathbb{P}(\mathbf{d}_{t:T} \in \mathcal{E}) < 1$.

To summarize, we thus conclude that (11) is strictly convex on the feasible set of (SP$_t$). □

Equipped with the uniqueness property of the solution of (SP$_t$) , we proceed to establish the key result in this subsection, namely the optimal action sequence of (SP$_t$) does not depend on the running maximum demand z_t:

Lemma 3. *For any $t \in \mathcal{T}$, any feasible battery state b_t and any two running maximum states $z_t, \tilde{z}_t \in \mathbb{R}$, we have*

$$\mathbf{u}_{t:T}^\star(b_t, z_t) = \mathbf{u}_{t:T}^\star(b_t, \tilde{z}_t). \tag{13}$$

Proof. We prove the result by showing that there exist primal-dual pairs satisfying the KKT conditions of (SP$_t$) with z_t and $\tilde{z}_t$ that share the same primal solution.

We rewrite (SP$_t$) by using vector form to conveniently construct KKT condition:

$$\min_{\mathbf{u}_{t:T} \in \mathbb{R}^k} \quad Z_{\mathrm{SP}}(\mathbf{u}_{t:T}; b_t, z_t, t) \tag{14a}$$
$$\text{s.t.} \quad \gamma : \mathbf{1}^\top \mathbf{u}_{t:T} = \underline{b}_{T+1} - b_t, \tag{14b}$$
$$\boldsymbol{\nu} : L\mathbf{u}_{t:T} \geq \underline{\mathbf{b}} - b_t \mathbf{1}, \tag{14c}$$
$$\boldsymbol{\mu} : L\mathbf{u}_{t:T} \leq \overline{\mathbf{b}} - b_t \mathbf{1}, \tag{14d}$$

where $k = T - t + 1$, $\underline{\mathbf{b}} = [\underline{b}, \ldots, \underline{b}, \underline{b}_{T+1}] \in \mathbb{R}^k$, $\overline{\mathbf{b}} = [\bar{b}, \ldots, \bar{b}, \underline{b}_{T+1}] \in \mathbb{R}^k$, $L \in \mathbb{R}^{k \times k}$ is a lower triangular matrix such that $l_{ij} = 1$ if $i \geq j$ and $l_{ij} = 0$ otherwise, and the corresponding dual variables are labeled in front of each constraint. It is obvious that $L\mathbf{u}_{t:T}$ is the cumulative sum of $\mathbf{u}_{t:T}$. Within this proof we use subscripts from t to T for all vectors, instead of using the natural subscripts from 1 to k. From Lemma 2, we can let $\mathbf{u}_{t:T}^\star(b_t, z_t)$ be the unique minimizer of the above optimization for given b_t and z_t.

Consider the KKT condition of the (SP$_t$) with z_t:

$$\mathbf{1}^\top \mathbf{u}_{t:T} = \underline{b}_{T+1} - b_t, \tag{15a}$$
$$L\mathbf{u}_{t:T} \geq \underline{\mathbf{b}} - b_t \mathbf{1}, \tag{15b}$$
$$L\mathbf{u}_{t:T} \leq \overline{\mathbf{b}} - b_t \mathbf{1}, \tag{15c}$$
$$\mathbf{p} - \gamma \mathbf{1} + L^\top(\boldsymbol{\mu} - \boldsymbol{\nu}) = 0, \tag{15d}$$
$$\boldsymbol{\nu}, \boldsymbol{\mu} \geq 0, \tag{15e}$$
$$\boldsymbol{\nu} \circ (L\mathbf{u}_{t:T} - \underline{\mathbf{b}} + b_t \mathbf{1}) = 0, \tag{15f}$$
$$\boldsymbol{\mu} \circ (L\mathbf{u}_{t:T} - \overline{\mathbf{b}} + b_t \mathbf{1}) = 0. \tag{15g}$$

Notice that, by [27], the objective function (11) of (SP$_t$) is differentiable provided that the distribution of forecast error has a continuous distribution (which follows from the fact that F has a density f). Therefore we can calculate the partial derivative of the objective function (11) with respect to u_τ, which is expressed as $\mathbf{p}$ in (15d), where the elements in $\mathbf{p}$ are:

$$p_\tau = \mathbb{P}\left(d_\tau + u_\tau = \max\left[z_t, d_t + u_t, \ldots, d_T + u_T\right]\right),$$

$\tau \in \{t, \ldots, T\}$. In KKT condition (15), (15a) – (15c) correspond to primal feasibility, (15d) – (15e) correspond to dual feasibility, and (15f) – (15g) correspond to complimentary slackness.

Suppose that $(\mathbf{u}^\star_{t:T}, \gamma, \boldsymbol{\mu}, \boldsymbol{\nu})$ is a primal-dual pair satisfying the optimality conditions (15) for given z_t. The dual feasibility condition (15d) can be written as

$$\boldsymbol{\mu} - \boldsymbol{\nu} = L^{-\top}(\gamma \mathbf{1} - \mathbf{p}).$$

At most one of the μ_τ and ν_τ can be positive for each $\tau \in \{t, \ldots, T\}$, follows from the fact that for each term in (15b) and (15c), one of the constraints must be slack, and the complimentary slackness condition. Therefore (15d) is also equivalent to

$$\mu_\tau = (p_{\tau+1} - p_\tau)_+ \quad \text{and} \quad \nu_\tau = (p_{\tau+1} - p_\tau)_-,$$

for all $\tau \in \{t, \ldots, T\}$, where $p_{T+1} := \gamma$. Here we use the property that the inverse of a cumulative sum matrix L is just a bidiagonal matrix, with main diagonal entries 1 and the upper diagonal entries -1.

Now we consider an arbitrary $\tilde{z}_t > z_t$, and we construct a primal-dual pair $(\mathbf{u}^\star_{t:T}, \tilde{\gamma}, \tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\nu}})$ satisfying the optimality condition (15) for $\tilde{z}_t$. The primal feasibility condition, (15a) – (15c), does not change for different z_t's, therefore $\mathbf{u}^\star_{t:T}$ is also primal feasible for the optimality condition (15a) – (15c) for $\tilde{z}_t$.

Furthermore, we denote the partial derivative of the objective function (11) for $\tilde{z}_t$ by $\tilde{\mathbf{p}} \in \mathbb{R}^k$, and the entries of $\tilde{\mathbf{p}}$ are defined as

$$\tilde{p}_\tau = \mathbb{P}\left(d_\tau + u_\tau = \max\left[\tilde{z}_t, d_t + u_t, \ldots, d_T + u_T\right]\right),$$

$\tau \in \{t, \ldots, T\}$. We construct $\tilde{\boldsymbol{\mu}}$ and $\tilde{\boldsymbol{\nu}}$ by assigning their entries as:

$$\tilde{\mu}_\tau = (\tilde{p}_{\tau+1} - \tilde{p}_\tau)_+ \quad \text{and} \quad \tilde{\nu}_\tau = (\tilde{p}_{\tau+1} - \tilde{p}_\tau)_-,$$

where $\tilde{p}_{T+1} = \tilde{\gamma}$ with $\tilde{\gamma}$ to be determined. The construction guarantees that $(\tilde{\gamma}, \tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\nu}})$ satisfies the dual feasibility conditions (15d) – (15e).

Last but not least, we prove that $\tilde{\boldsymbol{\mu}}$ and $\tilde{\boldsymbol{\nu}}$ also satisfy the complimentary slackness condition (15f) – (15g) by validating that the relative order of the entries of $\mathbf{p}$ and $\tilde{\mathbf{p}}$ is the same. In other words, we prove that for any $\tau_1, \tau_2 \in \{t, \ldots, T\}$, if $p_{\tau_1} \geq p_{\tau_2}$, then $\tilde{p}_{\tau_1} \geq \tilde{p}_{\tau_2}$, then the complimentary slackness condition of $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ guarantees which of $\tilde{\boldsymbol{\mu}}$ and $\tilde{\boldsymbol{\nu}}$. It suffices to prove that if

$$\begin{aligned}&\mathbb{P}(d_{\tau_1} + u_{\tau_1} = \max\left(d_t + u_t, \ldots, d_T + u_T\right)) \\ \geq &\mathbb{P}(d_{\tau_2} + u_{\tau_2} = \max\left(d_t + u_t, \ldots, d_T + u_T\right)).\end{aligned}$$

then for any $z \in \mathbb{R}$,

$$\begin{aligned}&\mathbb{P}(d_{\tau_1} + u_{\tau_1} = \max\left(d_t + u_t, \ldots, d_T + u_T, z\right)) \\ \geq &\mathbb{P}(d_{\tau_2} + u_{\tau_2} = \max\left(d_t + u_t, \ldots, d_T + u_T, z\right)).\end{aligned}$$

Note that

$$\begin{aligned}&\max(d_t + u_t, ..., d_T + u_T, z) \\ = &\max(d_{\tau_1} + u_{\tau_1}, d_{\tau_2} + u_{\tau_2}, z, \max_{\tau \in \{t,\ldots,T\} \setminus \{\tau_1, \tau_2\}} (d_\tau + u_\tau)).\end{aligned}$$

As

$$d_i + u_i = \hat{d}_i + u_i + \epsilon_i, i = \tau_1, \tau_2,$$

and ϵ_i i.i.d., by symmetry $p_{\tau_1} \geq p_{\tau_2}$ is equivalent to

$$\hat{d}_{\tau_1} + u_{\tau_1} \geq \hat{d}_{\tau_2} + u_{\tau_2},$$

for any $z \in \mathbb{R}$, then by symmetry again we can get $\tilde{p}_{\tau_1} \geq \tilde{p}_{\tau_2}$. By tedious algebraic manipulation, $\tilde{\gamma} = 0$, $\tilde{\mu}_T = 0$, $\tilde{\nu}_T = \tilde{p}_T$ satisfy the complimentary slackness condition for the terminal period.

To summarize, we construct the primal-dual pair $(\mathbf{u}^\star_{t:T}, \tilde{\gamma}, \tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\nu}})$ satisfying all KKT conditions (15) for $\tilde{z}_t$. As the stochastic programming problem (SP$_t$) is convex, $(\mathbf{u}^\star_{t:T}, \tilde{\gamma}, \tilde{\boldsymbol{\mu}}, \tilde{\boldsymbol{\nu}})$ is an optimal primal-dual point and therefore $\mathbf{u}^\star_{t:T}(\tilde{z}_t) = \mathbf{u}^\star_{t:T}(z_t)$ by the uniqueness assumption.

□

Given the uniqueness and z_t-independence of the optimal solution of the stochastic programming problem (SP$_t$), we show the optimality of the nonadaptive control policy of the stochastic control problem (SC):

Theorem 1 (The Optimality of Non-Adaptive Control)**.** *There exists a deterministic Markov policy that is optimal for the stochastic control program* (SC) *and does not depend on the realization of the stochastic states z_t's, which means such an optimal control policy does not need to be adaptive to z_t. In other words, there exists*

$$\pi_t(b_t, z_t) = \pi_t(b_t)$$

independent of z_t that is optimal for (SC) *for all $t \in \mathcal{T}$.*

Proof. We prove Theorem 1 by backward induction. The optimal control policy at the terminal period is non-adaptive, which is shown in (10). Then we assume Theorem 1 is true for periods $t+1, \ldots, T$, which allows us to denote the optimal policies for all $(z_{t+1}, b_{t+1}), \ldots, (z_T, b_T)$ as $\pi^\star_{t+1}(b_{t+1}), \ldots, \pi^\star_T(b_T)$. Notice that, if the optimal policy for periods $t+1, \ldots, T$ are all non-adaptive, the optimal policy can determine the optimal solution of the (t+1)-th rolling horizon stochastic programming problem (SP$_{t+1}$), denoted as $\mathbf{u}^\star_{t+1:T}(b_{t+1})$, because the randomness of actual demand does not affect the evolution of battery states. In particular, we have the optimal actions for $t+1,\dots,T$:

$$\begin{aligned} &u^\star_{t+1} = \pi^\star_{t+1}(b_{t+1}),\\ &u^\star_{t+2} = \pi^\star_{t+2}(b_{t+2}) = \pi^\star_{t+2}(b_{t+1}+u^\star_{t+1}),\\ &\quad\dots,\\ &u^\star_T = \pi^\star_T(b_T) = \underline{b}_{T+1} - b_{t+1} - \sum_{\tau=t+1}^{T-1} u^\star_\tau. \end{aligned} \tag{16}$$

The optimal policy at time period t, $\pi^\star_t(b_t,z_t)$, is determined by minimizing the Bellman value function (8) and (9). Furthermore, for given b_t and z_t, the optimal policy $\pi^\star_t$ determines the optimal action $u^\star_t$ and the battery charging state $b_{t+1} := b_t + u^\star_t$. From (16), all following optimal actions at time periods $t+1,\dots,T$ are also determined. However, from Lemma 2 and Lemma 3, given b_t and z_t, there exists a unique optimal action sequence $\mathbf{u}^\star_{t:T}(b_t)$ that minimizes the t-th period RH-SP problem (SP$_t$), independent from z_t. We denote the action of $\mathbf{u}^\star_{t:T}(b_t)$ at period t by $\tilde{u}^\star_t(b_t)$. We assert that $\pi^\star_t(b_t,z_t) := \tilde{u}^\star_t(b_t)$, since from the uniqueness, for all $\pi_t(b_t,z_t) \neq \tilde{u}^\star_t(b_t)$,

$$\begin{aligned} &\mathbb{E}\left[Z^\star_{\mathrm{SC}}\left(b_t+\pi_t(b_t,z_t), \max(z_t, d_t+\pi_t(b_t,z_t)); t+1\right)\right]\\ \geq&\mathbb{E}\left[Z^\star_{\mathrm{SC}}\left(b_t+\tilde{u}^\star_t(b_t), \max(z_t, d_t+\tilde{u}^\star_t(b_t)); t+1\right)\right]. \end{aligned}$$

□

Theorem 1 states that, instead of considering the difficult stochastic control problem (SC), we can solve the original problem by solving only the first rolling horizon stochastic programming problem (SP$_1$), and determine all optimal action sequences at the starting period, merely from the information of the starting battery state b_1, the terminal charging state $\underline{b}_{T+1}$, and the forecast error distribution F.

C. On the Optimality of Certainty Equivalence Control

As the stochastic programming formulation (SP$_1$), is less daunting than the stochastic control formulation (SC), standard algorithms based on stochastic approximation [28] or sample average approximation [29] exist. However, the complexities of such methods usually scale with the number of time periods under consideration which can be large in our case as demand charge is usually calculated in a monthly basis. Furthermore, the consistency of the aforementioned methods relies critically on the ability to sample from the *true error distribution* F which may not be available in practice. Thus in the rest of this section, we proceed to establish sharp structural results of (SP$_1$), connecting its solution to solutions of its deterministic counterpart defined as follows

$$\begin{aligned} Z^\star_{\mathrm{D}} := \min_{\mathbf{u}} \quad & Z_{\mathrm{D}}(\mathbf{u}) = z_{T+1} && \text{(D-a)}\\ \text{s.t.} \quad & \mathbf{u} \in \mathcal{U}^{\mathrm{R}}, && \text{(D-b)}\\ & z_{t+1} = \max(z_t, d_t+u_t), \quad t\in\mathcal{T}. && \text{(D-c)} \end{aligned}$$

To gain intuition, we start by considering a simple case where the storage is *adequate* so that using storage control we can get a modified demand sequence with a constant mean. The constant mean scenario is of concern because of the following impossibility result:

Lemma 4. *Using storage, it is impossible to reduce the electricity bill if the d_t's are i.i.d., and the initial and terminal battery storage state are the same. In other words, given that $\widehat{d}_t$'s are constant for $t\in\mathcal{T}$ and $b_1 = \underline{b}_{T+1}$, we have $Z^\star_{\mathrm{SC}} = Z^\star_{\mathrm{SP}_1} = Z_{\mathrm{SP}_1}(\mathbf{0})$, where $Z_{\mathrm{SP}_1}(\mathbf{0})$ is the expected maximum net demand when $u_t \equiv 0$ for all $t\in\mathcal{T}$,*

$$Z_{\mathrm{SP}_1}(\mathbf{u}) := \lim_{z\to-\infty} Z_{\mathrm{SP}}(\mathbf{u}; b_1, z, 1),$$

and

$$Z^\star_{\mathrm{SP}_1} := \min_{\mathbf{u}\in\mathcal{U}^{\mathrm{R}}} Z_{\mathrm{SP}_1}(\mathbf{u}).$$

The initial battery state b_1, terminal battery state $\underline{b}_{T+1}$ and the forecast error distribution F are implicitly defined for a given stochastic programming problem (SP$_1$).

Proof. We need to show that applying any feasible charging and discharging control action sequence will result in a cost no smaller than $Z_{\mathrm{SP}_1}(\mathbf{0})$, when d_t's are i.i.d. and $b_1 = \underline{b}_{T+1}$. It suffices to prove that

$$\mathbb{E}\left[\max_{t\in\mathcal{T}}(d_t+u_t)\right] \geq \mathbb{E}\left[\max_{t\in\mathcal{T}} d_t\right]$$

holds for any u_t sequence that $\sum_{t\in\mathcal{T}} u_t = b_{T+1} - b_1 = 0$. We prove this inequality using a symmetry argument. Let Ω be the event space and denote

$$\Gamma := \{\gamma : \gamma \text{ is a permutation of } \{1,\dots,T\}\}.$$

Let γ_ω be the permutation defined such that for any $\omega\in\Omega$

$$d_{\gamma_\omega(1)}(\omega) \geq \cdots \geq d_{\gamma_\omega(T)}(\omega),$$

i.e., random variable $d_{\gamma_\omega(t)}(\omega)$ is the t-th order statistic of $(d_1,\dots,d_T)$. By the fact that d_t's are i.i.d. random variables, we know that γ_ω is uniform on Γ. It follows that

$$\mathbb{E}\left[\max_{t\in\mathcal{T}}(d_t+u_t)\right] = \sum_{\gamma\in\Gamma}\frac{1}{|\Gamma|}\mathbb{E}\left[\max_{t\in\mathcal{T}}(d_{\gamma(t)}+u_{\gamma(t)})\middle|\gamma\right].$$

Meanwhile, notice that

$$\begin{aligned} &\mathbb{E}\left[\max_{t\in\mathcal{T}}(d_{\gamma(t)}+u_{\gamma(t)})\middle|\gamma\right]\\ &= \mathbb{E}\left[\max\left\{d_{\gamma(1)}, \max_{t=2}^{T}(d_{\gamma(t)}+u_{\gamma(t)}-u_{\gamma(1)})\right\}+u_{\gamma(1)}\middle|\gamma\right]\\ &\geq \mathbb{E}\left[d_{\gamma(1)}+u_{\gamma(1)}\middle|\gamma\right], \end{aligned}$$

and therefore

$$\begin{aligned} \mathbb{E}\left[\max_{t\in\mathcal{T}}(d_t+u_t)\right] &\geq \sum_{\gamma\in\Gamma}\frac{1}{|\Gamma|}\mathbb{E}\left[d_{\gamma(1)}+u_{\gamma(1)}\middle|\gamma\right]\\ &= \mathbb{E}\left[\max_{t\in\mathcal{T}} d_t\right] + \sum_{t\in\mathcal{T}}\frac{1}{T}u_t = \mathbb{E}\left[\max_{t\in\mathcal{T}} d_t\right]. \end{aligned}$$

□

Lemma 4 states that if the original forecast demand process $\widehat{\mathbf{d}}$ is flat, then using storage cannot further reduce the cost of

(SP$_1$). This also leads to a *certificate of optimality*, namely if there exists a feasible storage control that produces a flat modified demand, then such storage control is optimal for (SP$_1$). We formalize this statement as follows:

Definition 1 (Perfectly Adequate Storage)**.** *A storage model, specified by parameters* $(\underline{b}, \overline{b}, b_1, \underline{b}_{T+1})$ *is deemed (perfectly) adequate for forecast demand* $\widehat{\mathbf{d}}$*, if for the set* $\mathcal{U}^{\mathrm{R}}$ *defined using these storage parameters, there exists* $\mathbf{u} \in \mathcal{U}^{\mathrm{R}}$*, such that for all* $s, t \in \mathcal{T}$*,*

$$\widehat{d}_s + u_s = \widehat{d}_t + u_t. \tag{17}$$

This storage control sequence $\mathbf{u}$ *is deemed a* (perfectly) flattening control.

Lemma 5. *If the storage is perfectly adequate for the forecast demand* $\widehat{\mathbf{d}}$*, then the perfectly flattening control is optimal for* (SP$_1$).

Thus for the special case of adequate storage, Lemma 5 concludes that the optimal control action sequence for (SP$_1$) (and so is (SC)) is the offline control action sequence that perfectly flattens the forecast demand process. Notice that this also establishes the *certainty equivalence* of (SP$_1$) (and (SC)) for the special case of adequate storage, as it is easy to see that the flattening control is also an optimal solution to the deterministic counterpart (D).

Theorem 2 (Certainty Equivalence, Adequate Storage Case)**.** *The stochastic control problem* (SC) *is certainty equivalent when the storage is adequate. That is, the certainty equivalence control obtained from solving* (D) *is optimal for* (SC) *for adequate storage.*

An intuitive extrapolation of results from the adequate storage case is that the flatter the modified forecast demand, the better the control sequence. This intuition holds rigorously. In particular, for two battery control action sequences which are different only on two periods, we have the following result.

Lemma 6. *For any forecast error distribution* F *and any control action sequence* $\mathbf{v}, \mathbf{w} \in \mathcal{U}^{\mathrm{R}}$ *such that* $v_t = w_t$ *for all* $t \in \mathcal{T} \setminus \{i, j\}$*, we have*

$$Z_{\mathrm{SP}_1}(\mathbf{v}) > Z_{\mathrm{SP}_1}(\mathbf{w})$$

if and only if

$$\left|\widehat{d}_i + v_i - \widehat{d}_j - v_j\right| > \left|\widehat{d}_i + w_i - \widehat{d}_j - w_j\right|. \tag{18}$$

Proof. By denoting the maximum net demand during the periods that the two control actions are the same by a random variable

$$Y := \max_{t \in \mathcal{T}\setminus\{i,j\}} (d_t + v_t) = \max_{t \in \mathcal{T}\setminus\{i,j\}} (d_t + w_t),$$

it suffices to prove that

$$\mathbb{E}\max(d_i + v_i, d_j + v_j, Y) > \mathbb{E}\max(d_i + w_i, d_j + w_j, Y)$$

if and only if

$$\left|\widehat{d}_i + v_i - \widehat{d}_j - v_j\right| > \left|\widehat{d}_i + w_i - \widehat{d}_j - w_j\right|.$$

Notice that $v_i + v_j = w_i + w_j$, we denote

$$\widehat{d}_0 = \frac{1}{2}\left(\widehat{d}_i + v_i + \widehat{d}_j + v_j\right) = \frac{1}{2}\left(\widehat{d}_i + w_i + \widehat{d}_j + w_j\right),$$

and then $d_i' := \widehat{d}_0 + \epsilon_i$ and $d_j' := \widehat{d}_0 + \epsilon_j$ are i.i.d. random variables. We also denote $\delta_v = \widehat{d}_i + v_i - \widehat{d}_0$ and $\delta_w = \widehat{d}_i + w_i - \widehat{d}_0$. The claim above is then equivalent to

$$\mathbb{E}\max\left(d_i' + \delta_v, d_j' - \delta_v, Y\right) > \mathbb{E}\max\left(d_i' + \delta_w, d_j' - \delta_w, Y\right)$$

if and only if $|\delta_v| > |\delta_v|$.

The claim is a consequence of the properties of the function

$$h(x) := \mathbb{E}\left[\max\left(d_1 + x, d_2 - x, Y\right)\right],$$

where d_1 and d_2 are any i.i.d. continuous random variables, and Y is an arbitrary continuous random variable independent of d_1 and d_2. In particular, we have the claim holds, if $h(x)$ is strictly convex, monotonically increasing for $x \in [0, +\infty)$, and monotonically decreasing for $x \in (-\infty, 0]$. Indeed, since Y is independent of d_1 and d_2, $h(\cdot)$ is an even function by symmetry. Furthermore, $h(x)$ is convex since it is an expectation of a convex piecewise affine function [26]. The fact that $h(x)$ is in fact strictly convex can be established using similar arguments as in the proof of Lemma 2. Finally, for any $x \in \mathbb{R}$, we have

$$h(x) = (h(x) + h(-x))/2 \geq h(0),$$

and so it achieves its minimum at $x = 0$. By strict convexity, we conclude that $h(x)$ is increasing on $x \geq 0$ and decreasing on $x \leq 0$. □

Given a control sequence $\mathbf{u} \in \mathcal{U}^{\mathrm{R}}$, Lemma 6 suggests that if there exists a *two-period modification* $\mathbf{\Delta u}$ such that $\widetilde{\mathbf{u}} := \mathbf{u} + \mathbf{\Delta u} \in \mathcal{U}^{\mathrm{R}}$, and the new control $\widetilde{\mathbf{u}}$ has a lower net demand difference in the modification periods in view of (18), then $\widetilde{\mathbf{u}}$ has a lower cost in terms of $Z_{\mathrm{SP}_1}(\cdot)$ than $\mathbf{u}$. Two major consequences are drawn from this observation. The first is an iterative procedure that reduces the cost of any given control sequence $\mathbf{u}$ without the need of evaluating $Z_{\mathrm{SP}_1}(\cdot)$; see Section IV. The other is a systematic connection between the cost of the deterministic counterpart (D) and that of (SP$_1$), which we state next.

Lemma 7. *For any* $\mathbf{u} \in \mathcal{U}^{\mathrm{R}}$ *and for any forecast error distribution* F*,* $Z_{\mathrm{SP}_1}(\mathbf{u}) > Z^\star_{\mathrm{SP}_1}$ *if* $Z_{\mathrm{D}}(\mathbf{u}) > Z^\star_{\mathrm{D}}$.

Proof. Consider a non-optimal control action sequence of (D), $\mathbf{u}$, then

$$Z_{\mathrm{D}}(\mathbf{u}) = \max_{t \in \mathcal{T}} \widehat{d}_t + u_t > Z^\star_{\mathrm{D}}.$$

For convenience, we assume that there is only one period τ such that $\widehat{d}_\tau + u_\tau = \max_{t \in \mathcal{T}} \widehat{d}_t + u_t > Z^\star_{\mathrm{SP}_1}$, and more general cases could be deducted in a similar way. For such control $\mathbf{u}$, either

$$b_\tau = b_1 + \sum_{i=1}^{\tau} u_i < \overline{b} \quad \text{or} \quad b_{\tau+1} = b_\tau + u_\tau > \underline{b}$$

must be true. Otherwise if $b_\tau = \bar{b}$ and $b_{\tau+1} = \underline{b}$, then we have

$$\widehat{d}_\tau + u_\tau = \widehat{d}_\tau + b_{\tau+1} - b_\tau = \widehat{d}_\tau + \underline{b} - \bar{b} \le \widehat{d}_\tau + u^\star_\tau \le Z^\star_{\mathrm{D}},$$

where $\mathbf{u}^\star$ is an optimal control sequence for (D), the first inequality comes from the limitation of feasible actions, and the second inequality is directly from the definition of $Z^\star_{\mathrm{D}}$. This contradicts with the fact that

$$\widehat{d}_\tau + u_\tau = Z_{\mathrm{D}}(\mathbf{u}) > Z^\star_{\mathrm{D}}.$$

If $b_\tau < \bar{b}$ and by assumption we have $\widehat{d}_{\tau-1} + u_{\tau-1} < \widehat{d}_\tau + u_\tau$, then let

$$\delta = \frac{1}{2}\left(\widehat{d}_\tau + u_\tau - \widehat{d}_{\tau-1} - u_{\tau-1}\right) > 0,$$

and consider $\mathbf{u}'$ that $u_i = u'_i$ for all $i \in \mathcal{T}\backslash\{\tau-1, \tau\}$, $u'_{\tau-1} = u_{\tau-1} + \delta$ and $u'_\tau = u_\tau - \delta$. Then directly from Lemma 6, we know $Z_{\mathrm{SP}_1}(\mathbf{u}) > Z_{\mathrm{SP}_1}(\mathbf{u}') \ge Z^\star_{\mathrm{SP}_1}$.Similar arguments show that $Z_{\mathrm{SP}_1}(\mathbf{u}) > Z^\star_{\mathrm{SP}_1}$ for the case $b_{\tau+1} > \underline{b}$. □

Lemma 7 establishes that the preferences over control actions $\mathbf{u} \in \mathcal{U}^{\mathrm{R}}$ defined based on cost function $Z_{\mathrm{SP}_1}(\cdot)$ is induced by that defined based on $Z_{\mathrm{D}}(\cdot)$. A direct consequence is the following *weak certainty equivalence* property:

Theorem 3 (Weak Certainty Equivalence)**.** *There exists a solution of the deterministic counterpart* (D) *that is optimal for* (SC). *In other words, let $\mathbf{u}^\star_{\mathrm{SP}_1}$ be the unique optimal solution of* (SP$_1$) *and $\mathcal{U}^\star_{\mathrm{D}}$ be the set of optimal solutions of* (D). *Then we have $\mathbf{u}^\star_{\mathrm{SP}_1} \in \mathcal{U}^\star_{\mathrm{D}}$.*

Of course, when (D) has a unique solution, this implies the usual notion of certainty equivalence.

Corollary 2. *If $|\mathcal{U}^\star_{\mathrm{D}}| = 1$, then $\mathbf{u}^\star_{\mathrm{D}} = \mathbf{u}^\star_{\mathrm{SP}_1}$, where $\mathbf{u}^\star_{\mathrm{D}}$ is the solution of* (D).

However, as (D) can be re-written as a linear program and in general it is possible for (D) to has non-vertex solutions (in which case there exists an uncountable number of solutions), we expect that Theorem 3 has a much wider applicability than that of Corollary 2.

IV. Algorithm

Theorem 3 guarantees that the optimal solution of (SC) can be identified from the set of optimal solutions of the deterministic counterpart (D). However, modern optimization solvers usually only provide *an* optimal solution instead of the set of *all* optimal solutions. Furthermore, to locate the best optimal solution of (D) as measured by cost function Z_{SP_1} requires sampling from the true error distribution F, which may not be available in practice.

As a by-product of Lemma 6, in this section we develop an *improvement procedure* for any given solution of (D) which may be produced by any existed optimization solver. The idea is to randomly select two time periods, and attempt to apply a cost-reducing two-period modification while remaining optimal for (D). The resulting control, if any such modification is found, is guaranteed to have a lower cost as measured by Z_{SP_1}. Notice that although this might not be the most satisfactory solution method for (SC), it is remarkable that the we can achieve cost reduction without ever evaluating the actual cost function. Numerical evaluation in the next section shows that the performance of the resulting control is usually near optimal within 5 seconds.

Algorithm 1 STSolver

1: **procedure** STSOLVER($\widehat{\mathbf{d}}, \beta, \delta$)
2:     $T \leftarrow$ Length of $\widehat{\mathbf{d}}$
3:     $\widehat{\mathbf{u}}^\star \leftarrow$ CESOLVER($\widehat{\mathbf{d}}, \beta$)
4:     **while** stopping criteria is not fulfilled **do**
5:         $\widehat{\mathbf{u}}' \leftarrow \widehat{\mathbf{u}}^\star$
6:         Randomly pick $i, j \in \mathcal{T}$
7:         **if** $\widehat{d}_i + \widehat{u}^\star_i > \widehat{d}_j + \widehat{u}^\star_j + \delta$ **then**
8:             $\widehat{u}'_i \leftarrow \widehat{u}^\star_i - \delta$, $\widehat{u}'_j \leftarrow \widehat{u}^\star_j + \delta$
9:         **else if** $\widehat{d}_j + \widehat{u}^\star_j > \widehat{d}_i + \widehat{u}^\star_i + \delta$ **then**
10:             $\widehat{u}'_i \leftarrow \widehat{u}^\star_i + \delta$, $\widehat{u}'_j \leftarrow \widehat{u}^\star_j - \delta$
11:         **end if**
12:         **if** $\widehat{\mathbf{u}}' \in \mathcal{U}^{\mathrm{R}}$ **then**
13:             $\widehat{\mathbf{u}}^\star \leftarrow \widehat{\mathbf{u}}'$
14:         **end if**
15:     **end while**
16:     **return** $\widehat{\mathbf{u}}^\star$
17: **end procedure**

The inputs are $\widehat{\mathbf{d}}$, the forecasted load at time stamp $1, \ldots, T$, δ, the update step size δ, and a vector $\beta = [\beta_1, \beta_2, \beta_3, \beta_4]$ containing the storage parameters: $\beta_1 = \underline{b}$ is the storage state lower bound, $\beta_2 = \bar{b}$ is the storage state upper bound, $\beta_3 = b_1$ is the initial storage state and $\beta_4 = \underline{b}_{T+1}$ is the terminal storage state.

The deterministic counterpart (D), a simple linear programming problem, could be efficiently solved by many standardized methods, which are denoted as CESOLVER in line 3. After obtaining one optimal solution from CESOLVER, we randomly pick a pair of control actions and use the result of Lemma 6, to go to a strictly better solution in terms of $Z_{\mathrm{SP}_1}(\cdot)$, no matter what the forecast error distribution is, until the number of unsuccessful trails of picking action pairs is reached.

V. Experimental Results

We study an example of demand charge reduction in residential setting considering the recent interest among utility companies to implement residential demand charge and the emergence of home battery systems such as Tesla Powerwall. The proposed algorithm is tested against a representative demand profile, derived from A Pacific Gas and Electric (PG&E) dataset which contains anonymized and secure smart meter readings on electricity consumption for 1923 residential customers for a period of one year spanning from 08/01/2010 to 07/31/2011 at 1 hour intervals. The average daily energy consumption is 28.0 kWh, and we assume the storage device we plan to use is a Tesla Powerwall battery pack with 6.4 kWh energy storage capacity. The current flat electricity rate for PG&E customer is \$0.243/kWh. The current demand charge rate for medium- and large-scale users is \$17.84/kW for PG&E [30], and \$18.62/kW for South California Edison(SCE) [31]. We use \$17/kW as the projected peak demand charge in our experiments.

In the experiments, the period of study is one week with the total time periods 168. The Past-3-week averaging method [32] is used for forecasting a single user's load a week ahead. Fig. 2 shows the hourly forecast load and true load for one customer from 7/25/2011 to 7/31/2011. We use the forecast load and assume the forecast errors are sampled from i.i.d. zero mean Gaussian distribution in all the following algorithm implementations.

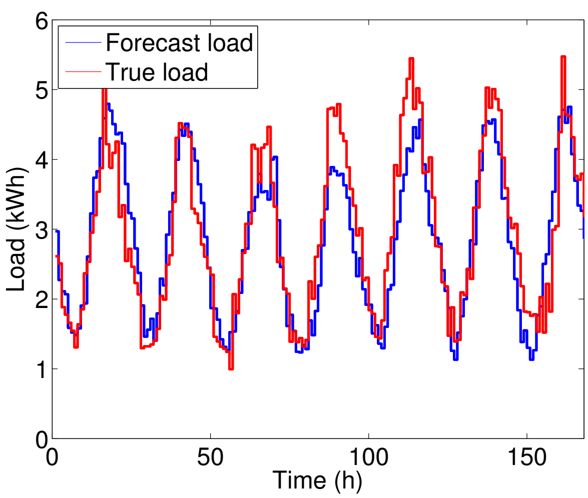

Fig. 2: Forecast and true load for single customer

For comparison, besides the proposed STSolver we also implement an standard Monte Carlo approach for the stochastic programming problem (SP_1), which solves a deterministic convex optimization replacing the expected value in the objective by its sample average approximation [33].

During each iteration in Algorithm 1, STSolver only needs to compare two numbers and judge whether a control action sequence is feasible. In contrast, the complexity of the Monte Carlo method scales with the number of samples. Therefore it is expected that the proposed STSolver spends much less time than the Monte Carlo approach. The running times of these two methods are compared in Fig. 3. One can observe that the time used for each increased sample of Monte Carlo method is roughly an order of magnitude larger than that used for each update of the proposed method.

We then benchmark the cost of the proposed STSolver and the standard Monte Carlo algorithm. We evaluate the expected modified peak demand $Z_{\mathrm{SP}_1}(\mathbf{u})$ by repeated sampling from the forecast error distribution, and compute $Z_{\mathrm{SP}_1}(\mathbf{u})$ for different number of updates/samples for these two methods. The empirical $Z_{\mathrm{SP}_1}(\mathbf{u})$ versus the running time is shown in Fig. 4. The expected maximum demand of 4 iterations converges to around 3.85kWh, while the original maximum

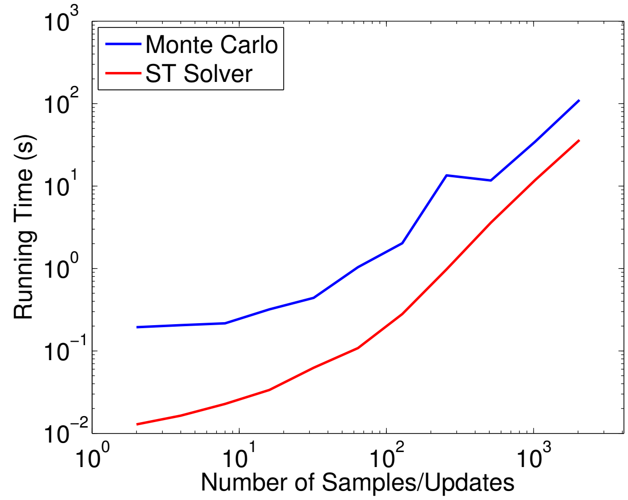

Fig. 3: Running time versus the number of updates (for ST Solver) or the number of samples (for Monte Carlo method)

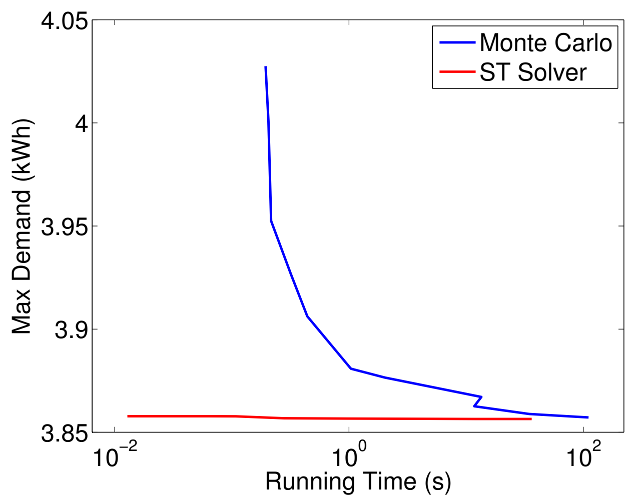

Fig. 4: Performance versus running time

demand of forecast load is 4.80kWh. We observe that cost reduction of using the stochastic improvement procedure is not significant comparing to the initial solution obtained from solving the deterministic counterpart, which indicates that the Weak Certainty Equivalence in Theorem 3 is in fact "strong" in the sense that the variation of the actual expected cost among the solutions of (D) is relatively small for the given load profiles and forecast error standard deviation.

We then plot the original and modified hourly forecast load and true load profile using the control action sequence from the solution of STSolver in Fig. 5 and Fig. 6, respectively. The forecast peak demand is decreased from 4.80 kW to 3.70 kW. Moreover, it is clear that the proposed STSolver not only reduces the highest forecast peak but also reduces the sub-peaks and forms a number of "plateaus." During the

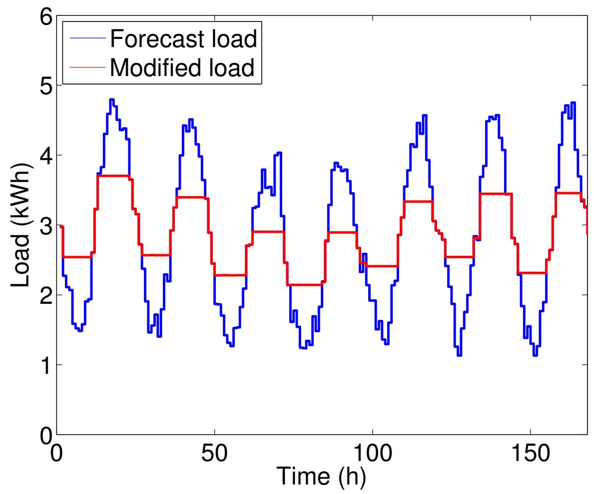

Fig. 5: Original and modified forecast load for ST Solver

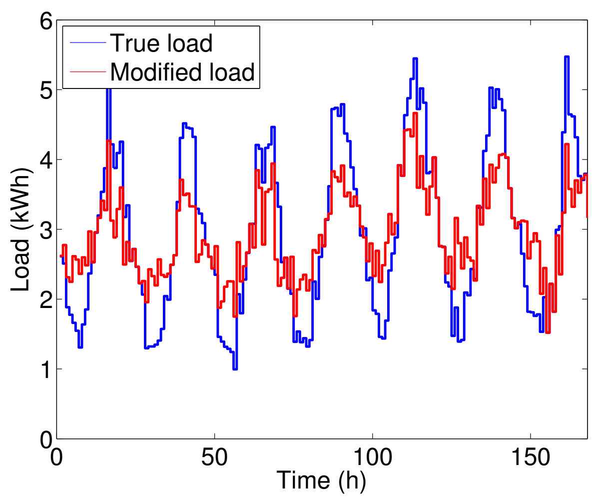

Fig. 6: Original and modified true load for ST Solver

last week of July 2011, the customer's total energy charge is \$120.40, and the maximum demand is 5.47 kW at the 161st hour. If the user had installed a Tesla Powerwall and deployed the proposed optimal control sequence for the stochastic program (SP_1), the net demand maximum would have been reduced to 4.63kW, at the 113rd hour of the week. As a comparison, if the user just take one optimal control action for the deterministic optimization given forecast load, the net maximum demand would have been 4.83kW at the 113rd hour The difference in demand charge for the two control sequences is \$ 2.11 for a single peak for a single customer. The simulation reveals that the proposed STSolver can effectively improve the deterministic optimization's optimal control actions.

The potential amount of performance improvement of running our algorithm is determined by the amount of randomness in the forecast error. We conduct a simulation of the relationship between the forecast error's standard deviation and the ST solver's improvement over an initial solution of (D). The results are shown in Fig. 7.

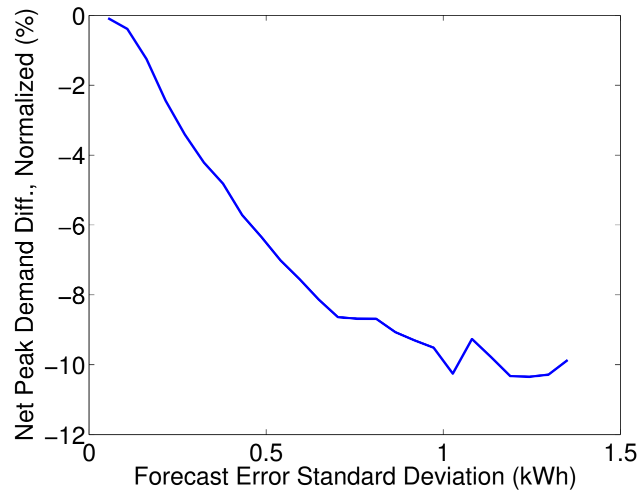

Fig. 7: Performance difference between an arbitrary solution of the deterministic counterpart and the improved solution computed using ST solver versus the forecast error's standard deviation

When there is no forecast error, the two methods' performance is the same. The proposed STSolver over-performs the initial deterministic solution when the error's standard deviation increases. This provides a validation of the advantage of using our proposed STSolver, especially when the forecast error is large, which is just the case for residential level users.

VI. Conclusion

In this paper, we formulate the problem of demand charge reduction using storage as a stochastic control program and systematically investigate its structure. In particular, we for the first time establish the certainty equivalence among the optimal solutions of the original stochastic control problem, a corresponding stochastic programming program and its deterministic counterpart. We them propose an algorithm that starts from the offline optimal solution and iteratively improves the solution approaching toward the optimal solution of the original stochastic control problem. The method is robust, in the sense that during each update, the objective function is monotonically decreasing, and the solution of our algorithm is strictly better than offline optimal solution. Our approach is significantly different from widely used dynamic programming based arguments and extremely simple as it even does need to know the actual distribution of the forecast error. Simulation shows that its speed is fast enough for real-time operations. Our experimental study using PG&E data validates the superior performance of our proposed algorithm.

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