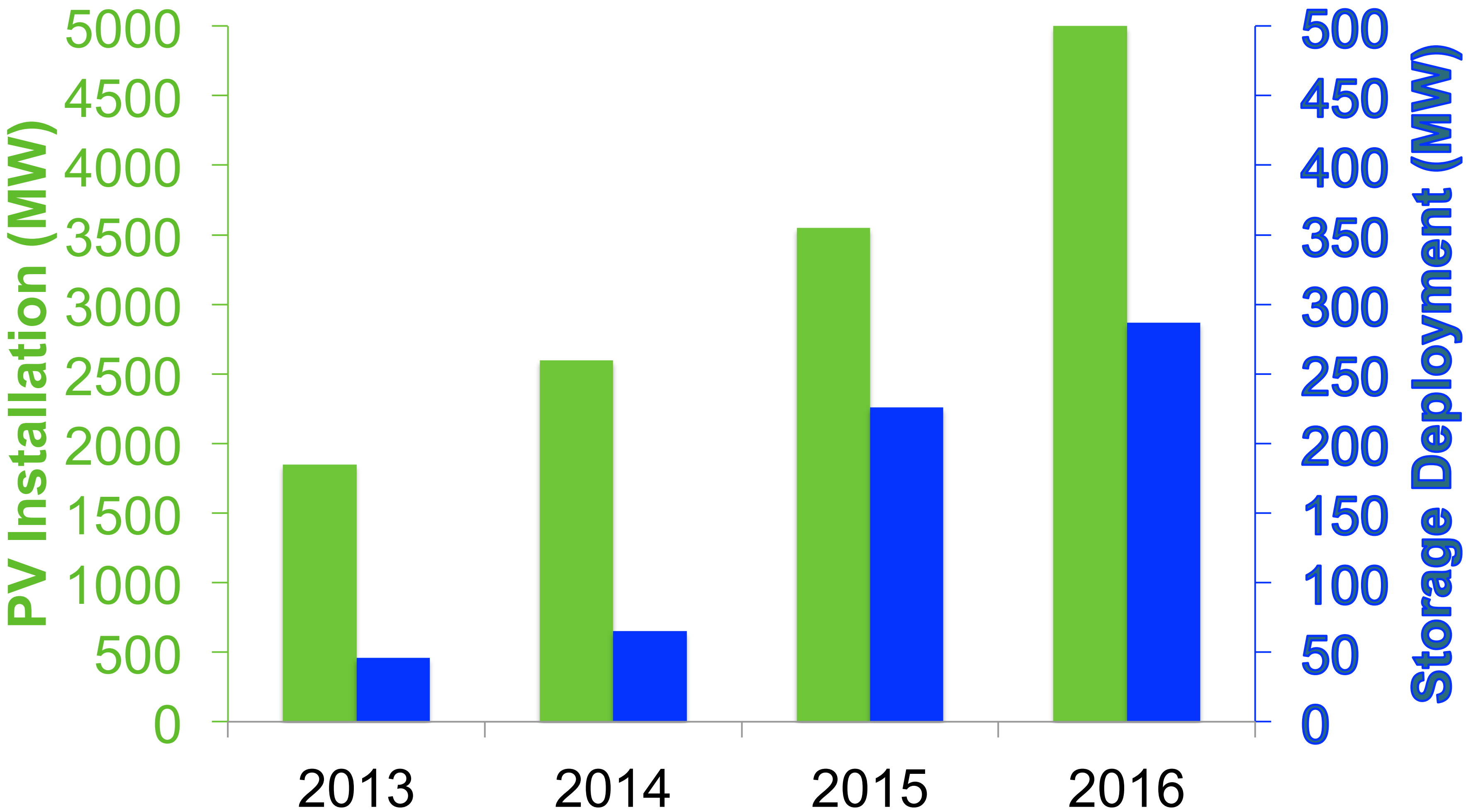
Submodularity of Energy Storage Placement in Power Network

Junjie Qin, ICME, Stanford
Joint work with Insoon Yang (USC) and Ram Rajagopal (Stanford)

Motivation

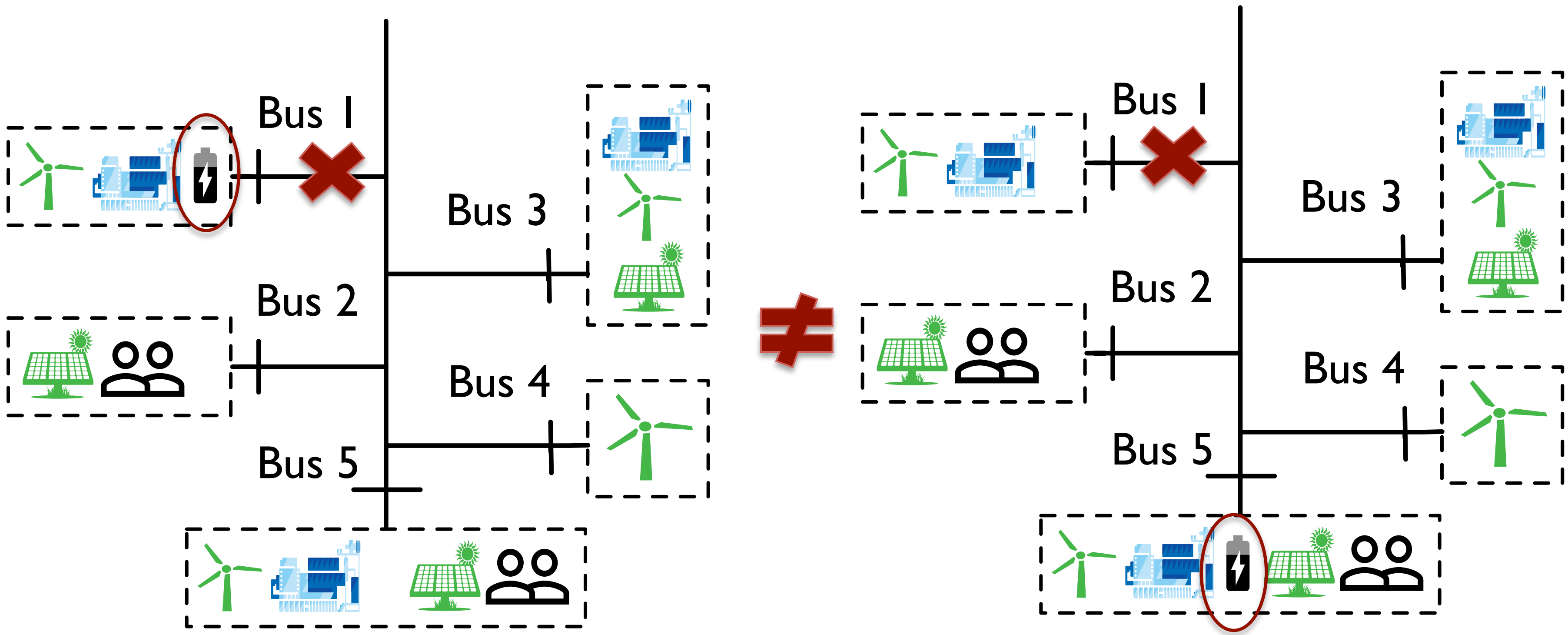
US distributed PV installation and storage deployment



Source: Rocky Mountain Institute and GTM Research

Motivation

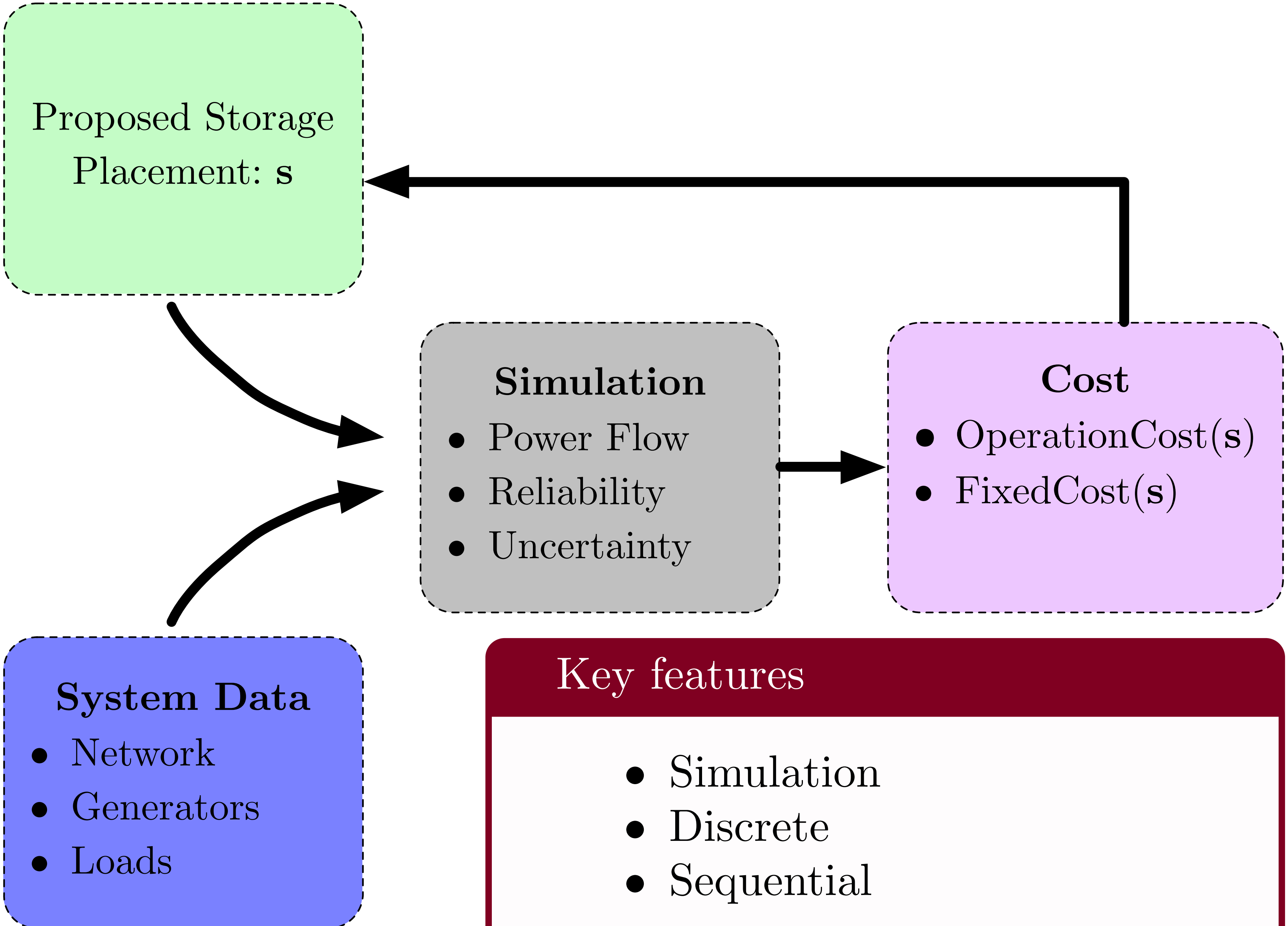
Location matters in storage deployment



Question

Where to deploy storage?

Practical Storage Placement



Literature: Continuous Optimization

$$\mathrm{OpCost}(\mathbf{0}) - \mathrm{OpCost}(\mathbf{s})$$

- Formulation

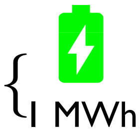
$$\max_{\mathbf{s} \in \mathbb{R}_{+}^{N}} \quad V(\mathbf{s})$$

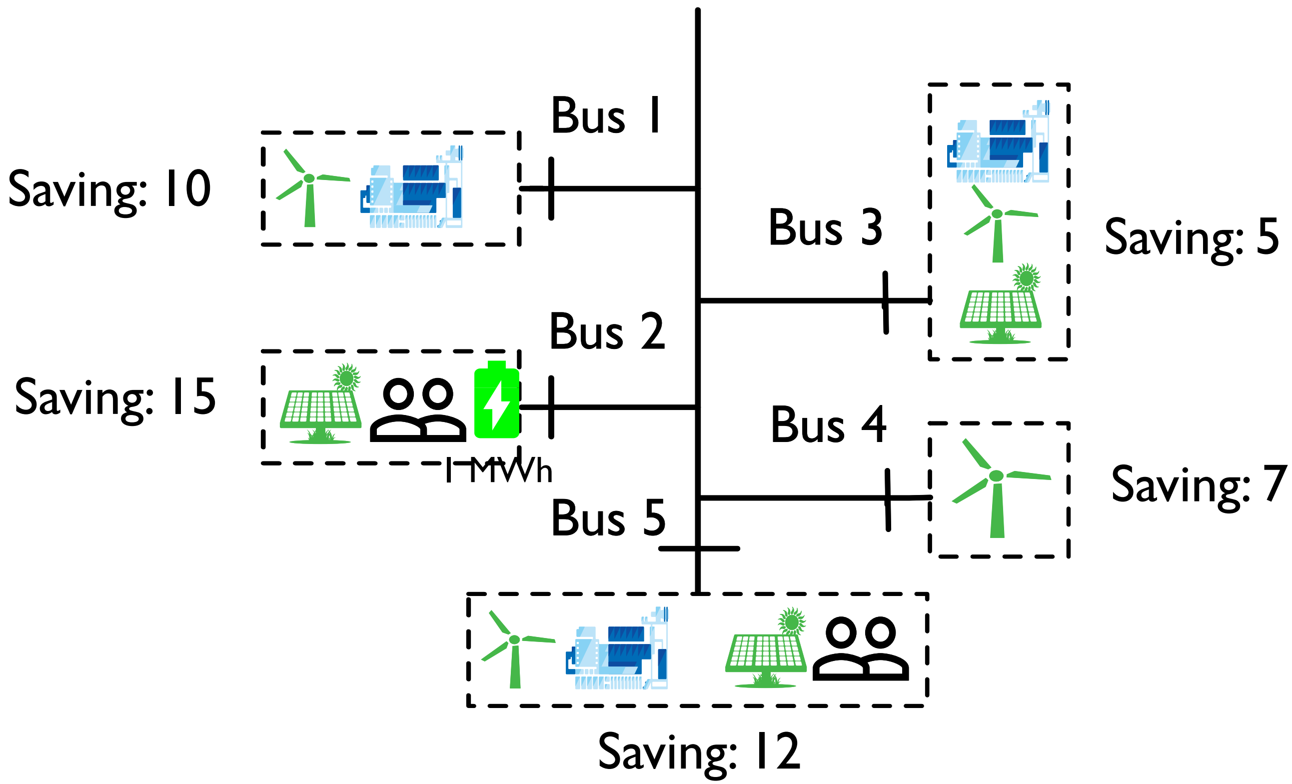
$$\text{s.t.} \quad \mathrm{FixedCost}(\mathbf{s}) \leq \mathrm{Budget}$$

- Structural results and simple computation
- Drawbacks
  - Simulation
  - Discrete
  - Sequential

Sjodin, Gayme & Topcu, ACC12; Bose et al., CDC12; Q & Rajagopal, PES13; Bose & Bitar, CDC14; Wogrin & Gayme, TPS15; Pandzic et al., TPS15; Thrampoulidis, Bose & Hassibi, TAC16; Tang & Low, CDC16...

Practice: Greedy Heuristic

- How to place {, } in network?



- Desired properties are satisfied
  - Simulation -- Discrete -- Sequential

Performance of Greedy Heuristic

- Greedy placement works well in simulation (Dvijotham, Cherkov & Backhaus, HICSS14)
- Performance benchmark: Discrete optimization

$$V^\star = \max_{\mathbf{s} \in \mathcal{S}} \quad V(\mathbf{s})$$

Achievable storage capacities

$$\text{s.t.} \quad \text{FixedCost}(\mathbf{s}) \leq \text{Budget}$$

Theorem (adapted from Nemhauser et al. 78)

If $V : \mathbb{R}^N \mapsto \mathbb{R}$ is nondecreasing and submodular, then

$$\frac{V^{\text{greedy}}}{V^\star} \geq 1 - \frac{1}{e}.$$

Value of Networked Storage

- $V(\mathbf{s}) = \mathrm{OpCost}(\mathbf{0}) - \mathrm{OpCost}(\mathbf{s})$
- Operation model

convex quadratic

power balance

$$\mathrm{OpCost}(\mathbf{s}) = \min_{u,g \in \mathbb{R}^{NT}} \quad \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \mathrm{GenCost}_{n,t}(g_n(t))$$

line flow limits

$$\begin{aligned} \text{s.t.} \quad & \mathbf{1}^\top(\mathbf{g}(t) - \mathbf{d}(t) + \mathbf{u}(t)) = 0, \quad t \in \mathcal{T}, \\ & -\bar{f} \leq H(\mathbf{g}(t) - \mathbf{d}(t) + \mathbf{u}(t)) \leq \bar{f}, \quad t \in \mathcal{T}, \\ & \mathbf{u}_n \in \mathcal{U}(s_n), \quad n \in \mathcal{N} \end{aligned}$$

admissible storage control

Proposition

$V : \mathbb{R}^N \mapsto \mathbb{R}$ is nondecreasing and concave, and $\nabla^2 V(\mathbf{s})$ exists almost everywhere.

Also see Bitar et al. 16 and Bose & Bitar 16

Substitutability of Networked Storage

Working Definition

$V : \mathbb{R}^N \mapsto \mathbb{R}$ is called submodular if wherever $\nabla^2 V(\mathbf{s})$ exists,

$$\frac{\partial}{\partial s_j} \underbrace{\left(\frac{\partial V(\mathbf{s})}{\partial s_i}\right)}_{\text{MV of storage at } i} \leq 0, \quad i, j \in \mathcal{N}.$$

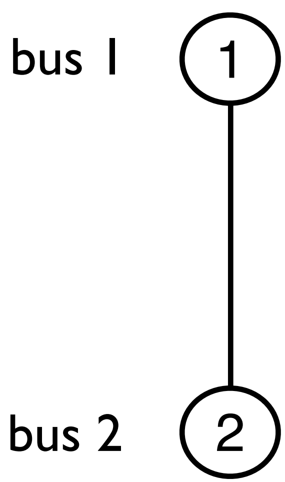
- Submodular if s_i substitutes s_j
- Supermodular if s_i complements s_j
- $i = j$: Substitutability holds by concavity
- $i \neq j$: Intuitive to have substitutability but hard to check

Complementarity of Networked Storage

Theorem

$V : \mathbb{R}^N \mapsto \mathbb{R}$ is not submodular in general.

Proof idea: Construct example where s_i complements s_j



Complementarity exists under certain congestion patterns!

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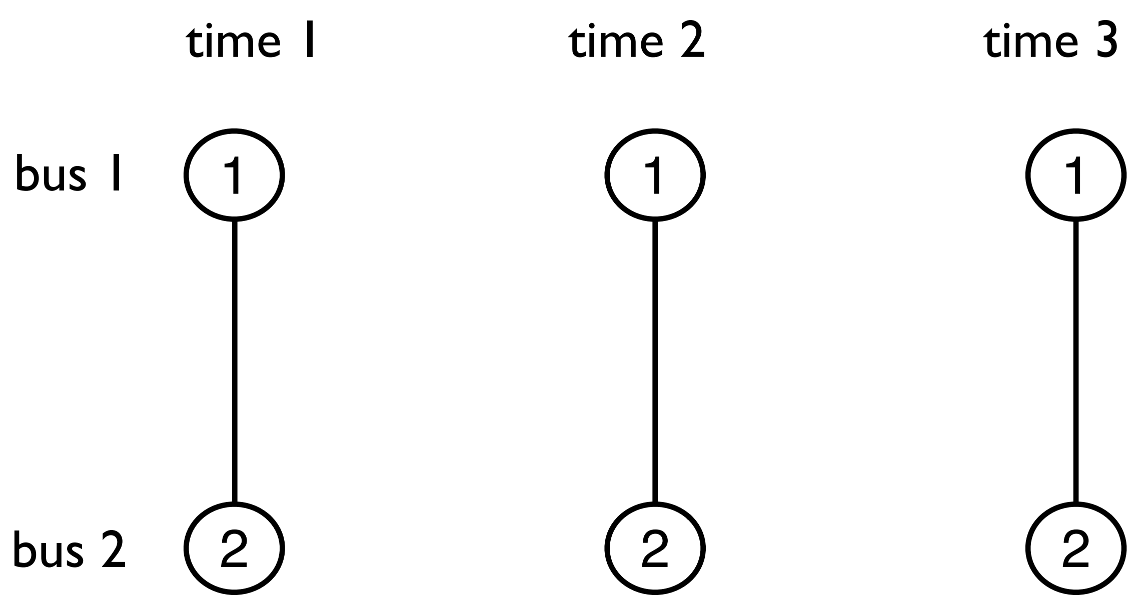

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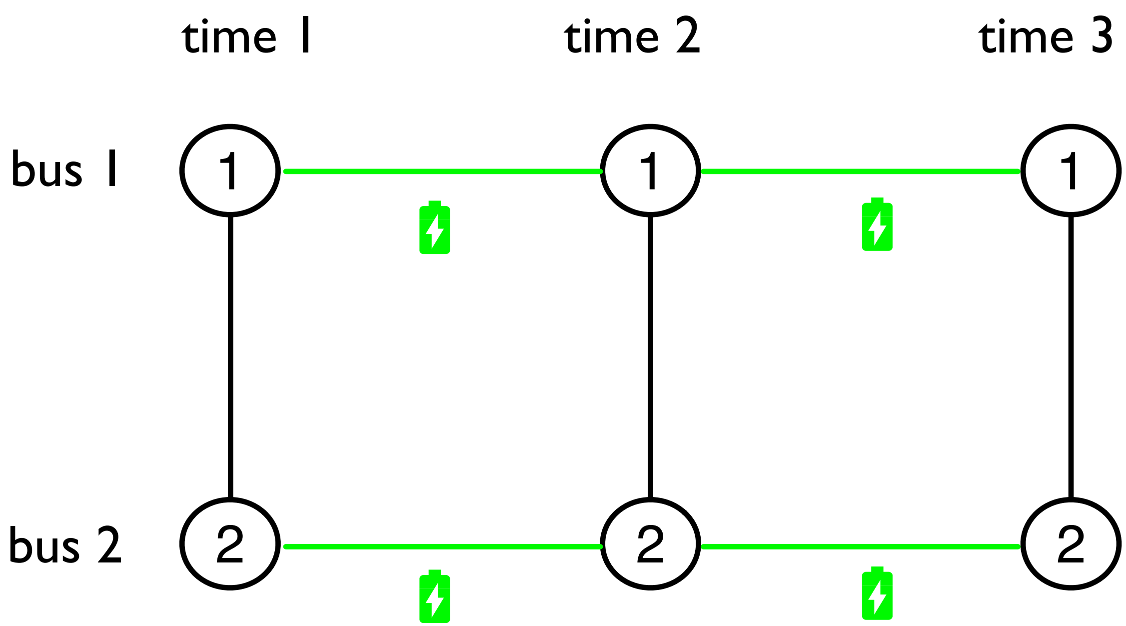

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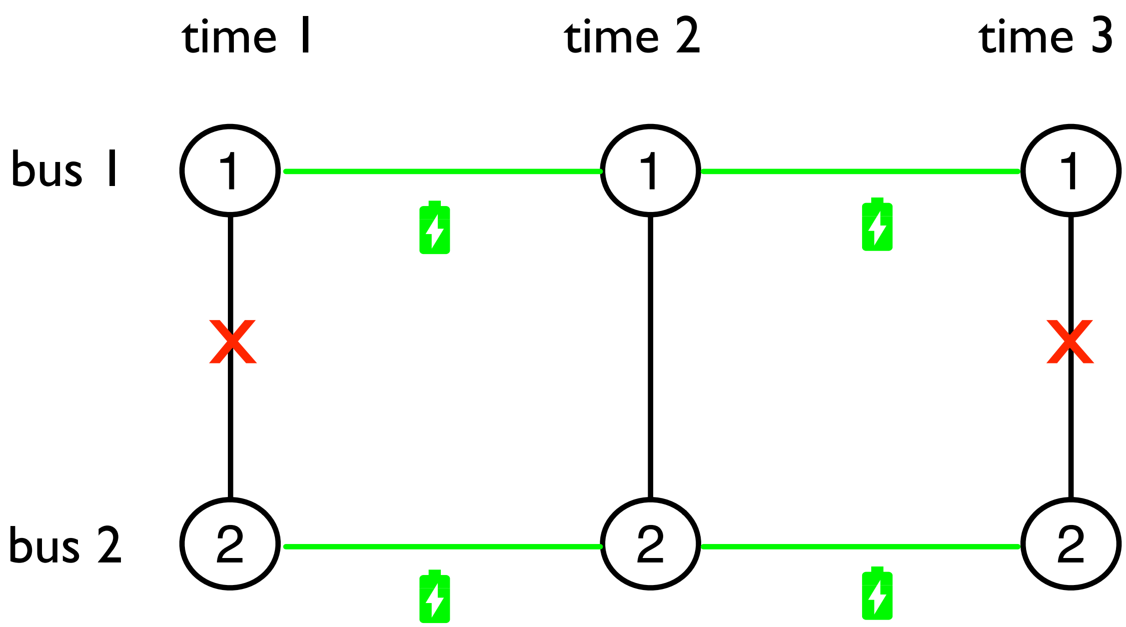

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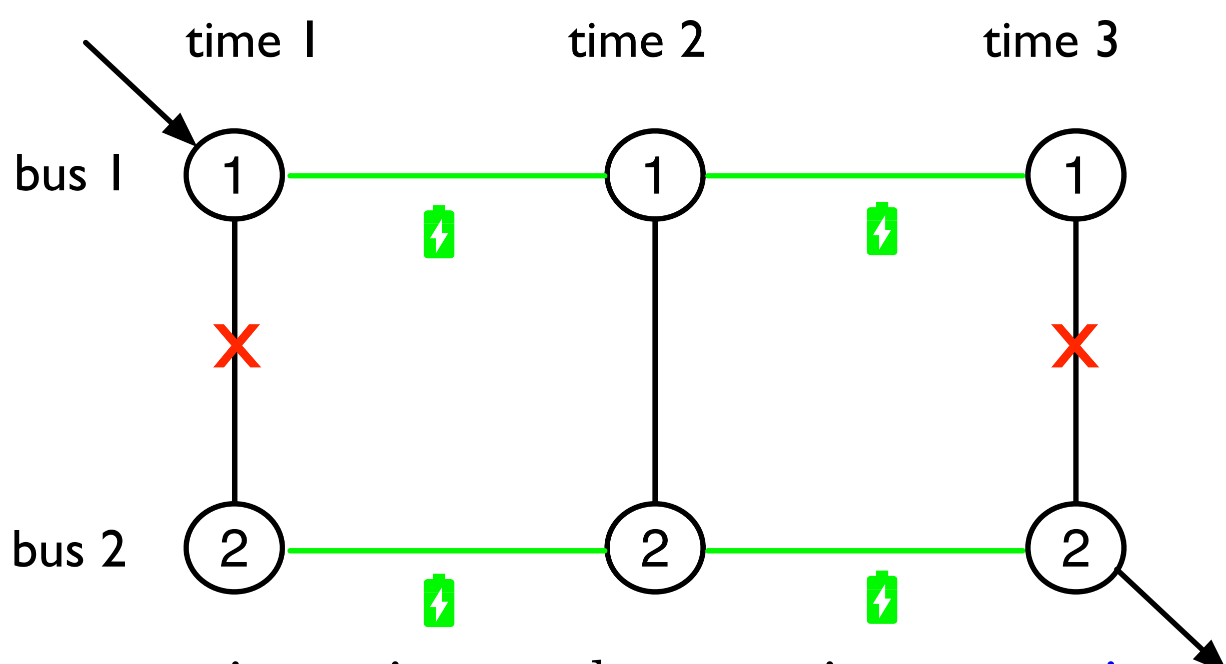

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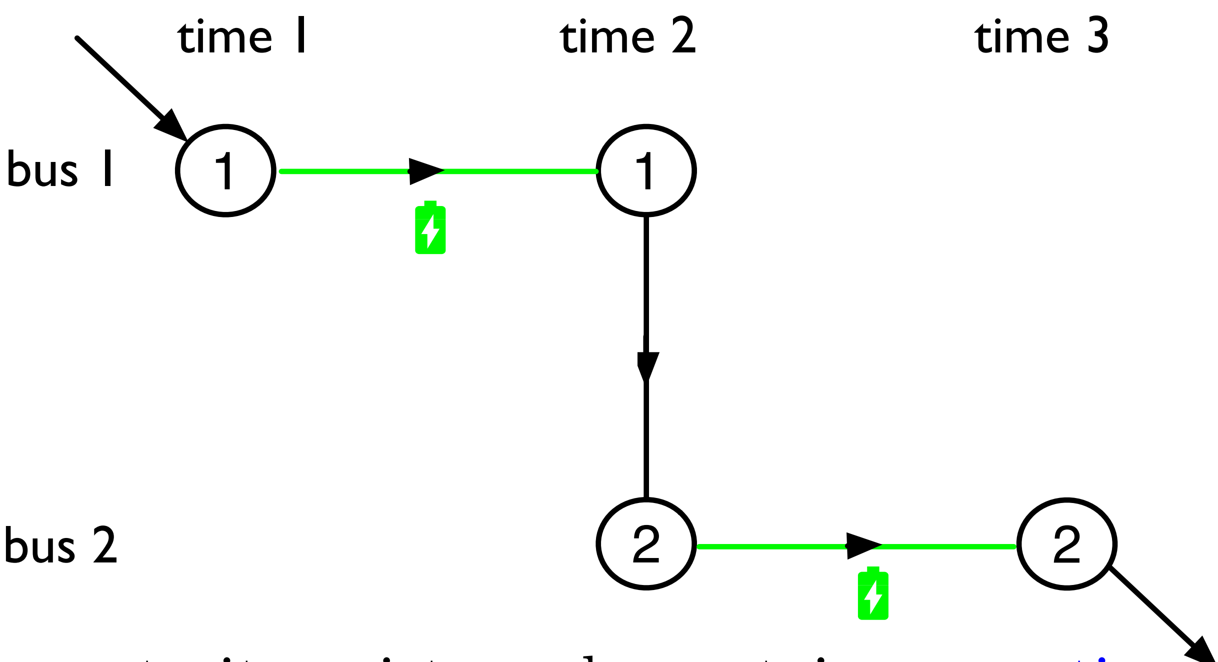

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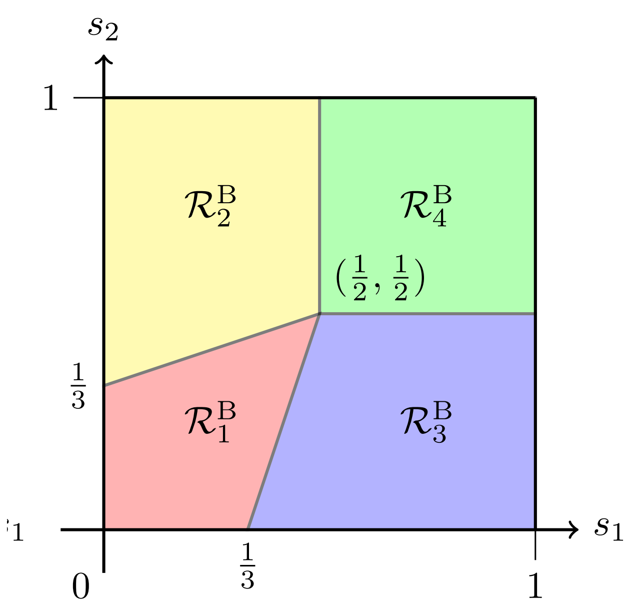
Certifying Submodularity

Certification Problem

Determine whether V is submodular given problem data and region of interest $[0, \bar{s}^{\max}]^N$.

Idea: Group $\mathbf{s}$ according to spatial-temporal congestion pattern



Theorem

(a) Congestion patterns define critical regions (CRs);

(b) Hessian is identical for all $\mathbf{s}$ in the same CR.

Hessian Computation

Hessian Decomposition

Whenever $\nabla^2 V(\mathbf{s})$ exists, it can be decomposed into

$$\nabla^2 V(\mathbf{s}) = \sum_{t \in \mathcal{T}} \mathrm{SC}_t(\mathbf{s}) \mathrm{NC}_t(\mathbf{s}) \mathrm{SC}_t^\top(\mathbf{s}),$$

where

- $\mathrm{SC}_t(\mathbf{s})$ depends only on storage congestion pattern,
- $\mathrm{NC}_t(\mathbf{s})$ depends only on problem data and network congestion pattern.

- $\mathrm{SC}(\mathbf{s})$ can be computed using only $\mathrm{LMP}(\mathbf{s})$
- $\mathrm{NC}(\mathbf{s})$ can be computed using only $\mathrm{LMP}(\mathbf{s})$ and input problem data if the network is a tree

Small Storage

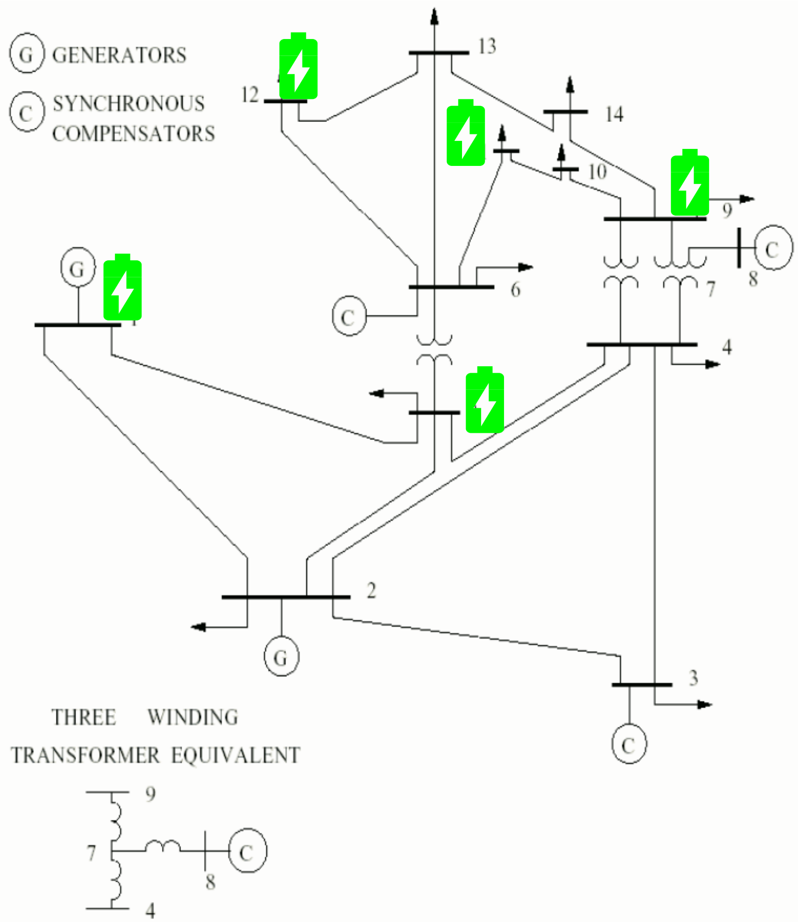
- Theory: The number of CRs can be large
- Practice: The amount of storage to place is small
  - 2015: 221 MW US storage deployment, 467 GW average US generation

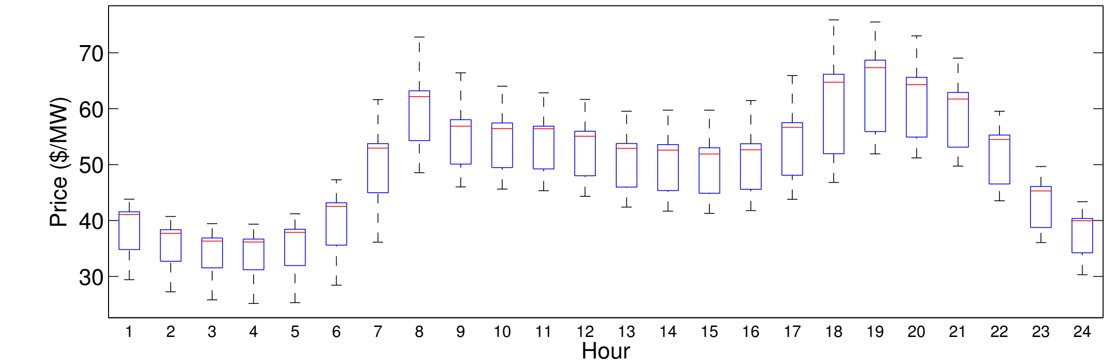
Small Storage Hypothesis

$[0, \bar{s}^{\max}]^N \subset \mathrm{CR}_1$, the CR containing $\mathbf{s} = \mathbf{0}$.

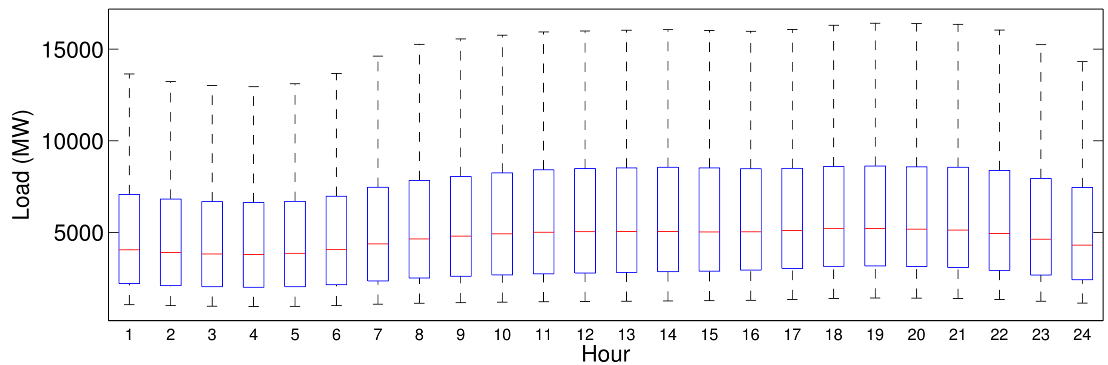
- Verification: Solve a simple LP
- Consequences:
  - Single CR
  - Determination of submodularity using current operation data

Numerical Example

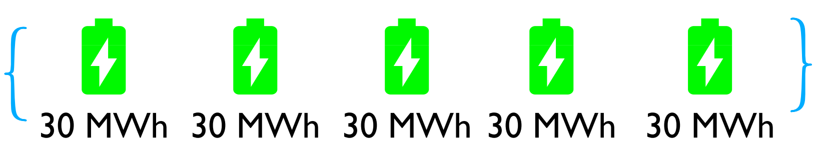




(a) Price



(b) Load



- Small Storage
- Submodularity
- Surprise: greedy finds exact OPT

Summary and Future Work

Summary

- Greedy placement
  - Practically desirable
  - Requires submodularity to have performance guarantee
- Complementarity exits under certain congestion pattern
- Methods & structural result for certifying submodularity

Future work

- Risk-averse placement
- Detection of complementarity and alg. for general case
- Certification with realistic operation models & simulation
- Implications on decentralized storage investments

Thank you for your attention!