Storage in Risk Limiting Dispatch: Control and Approximation

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***Abstract*— Integrating energy storage into the grid in scenarios with high penetration of renewable resources remains a significant challenge. Recently, Risk Limiting Dispatch (RLD) was proposed as a mechanism that utilizes information and market recourse to reduce reserve capacity requirements, emissions and other system operator objectives. Among other benefits, it included a set of simplified rules that can be readily utilized in existing system operator dispatch systems to provide reliable and computationally efficient decisions. RLD implements stochastic loss of load control to absorb the increased uncertainty in the grid. Storage is emerging as an alternative to mitigate the uncertainty in the grid. This paper extends the RLD framework to incorporate intrahour storage. It develops a closed form scheduling rule for optimal stochastic dispatch that incorporates a sequence of markets and real-time information. Simple approximations to the optimal rule that can be easily implemented in existing dispatch systems are proposed and evaluated. The approximation relies on continuous-time approximations of solutions for a discrete time optimal control problem. Numerical experiments are conducted to illustrate the proposed procedures.**

I. INTRODUCTION

The increased penetration of renewable generation in the grid increases the uncertainty that needs to be managed by the system operator [1]. Existing system dispatch procedures are designed assuming mild uncertainty, and utilize a worst-case schedule for generation by solving deterministic controls where random demands are replaced by their forecasted values added to '3σ' [2], where σ is the standard deviation of forecast error. Such rules require excess reserve capacity and result in increased emissions if σ is large [3].

An approach to mitigating the impact of increased uncertainty is to utilize energy storage [4]. Energy storage can be broadly classified into two groups depending on their scheduling characteristics [5], [6]: fast storage and slow storage. Fast storage can be utilized to mitigate intra-hourly variability of renewable supply. Slow storage can be utilized to transfer energy between different hours, so excess production can match peak demands. In this paper we address the integration of fast storage into power system dispatch.

Incorporating storage into a multiple stage market dispatch under significant uncertainty is a challenging problem. Existing approaches to stochastic dispatch rely on stochastic programming based on Monte Carlo scenario sampling that are hard to implement in large scale or do not incorporate market recourse [7], [8], [9], [10], [11], [12]. Moreover, the optimal decisions can be difficult to interpret in the context of system operations. Recent work has proposed utilizing robust optimization formulations with uncertainty sets [13], [14], but they do not capture multiple recourse opportunities and can result in conservative dispatch decisions. Instead, Risk Limiting Dispatch (RLD) was proposed in [15] to capture multiple operating goals and provide reliable and interpretable dispatch controls that can be readily incorporated in existing dispatch software. RLD incorporates real-time forecast statistics and recourse opportunities enabling the evaluation of improvements in forecasting and market design [16].

Fast storage integration with renewables has been studied in different scenarios. [17] examines the benefits of storage for renewable integration in a single bus model. Optimal contracting for co-located storage was studied in [18], [19] and the role of distributed storage was studied in [20], [21]. In this paper we develop RLD to incorporate fast storage. In particular, we utilize a careful analytic formulation to identify the structure of the optimal control and a numerical implementation that utilizes it. Approximate dispatch rules based on continuous time approximations are developed to facilitate implementation in existing dispatch software. To the best of our knowledge, this is the first paper to investigate incorporation of storage into a multiple recourse market setting for dispatch.

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The remainder of the paper is organized as follows. Section II states RLD with storage problem. Section III derives the optimal storage operation policy. Section IV establishes structural control results for dispatch. An approximation scheme is derived in Section V. Numerical results are presented in Section VI. Section VII concludes the paper.

II. Problem Statement

Supply must equal demand at the grid at each time instant. To meet this requirement, conventional energy generation is typically purchased at a sequence of markets at different time ahead of the delivery time to ensure that enough energy will be secured at the delivery time. In aggregated dispatch models, the goal is to find the optimal dispatch (*i.e.*, buy or sell) rule at different markets to satisfy the demand with conventional generation together with a random renewable generation at a delivery time interval. During this interval conventional generation is usually constant while renewable generation and loads varies randomly. Storage is used to smooth the net demand, that is, load minus the wind generation.

A. Model Formulation

There are R markets, modeled as stages, ahead of the delivery time interval. For example, stage 1 may occur 24 hours ahead of the delivery time; stage R occurs 15 minutes ahead of the delivery time; other stages occur in between. At each stage, an operator makes a decision to purchase s_r MWh of energy at stage r for the delivery time interval. Note s_r is a forward contract and may be interpreted as a contract for reserve capacity. The price c_r per unit of s_r is known in advance. These reserve capacities are different in two respects: the r-th capacity must be available in shorter time than the $(r-1)$-th capacity, and their prices are different. The operator may also sell reserve capacity at various stages. In such case, $s_r < 0$. We denote the index set for all dispatch stages as $\mathcal{R} = \{1, 2, \ldots, R\}$, and use r to denote one of the stages in $\mathcal{R}$. The restriction of whether buying or selling is allowed at each stage is given ahead of time, and may be summarized by a boolean vector, whose r-th entry is 0 if the market is a buying stage, and 1 if the stage is a selling stage.

Some constraints are imposed on the prices to avoid trivial solutions. For two buying dispatch stages $r_1,\ r_2 \in \mathcal{R}$ and $r_1 < r_2$, we require $0 < c_{r_1} < c_{r_2}$, i.e. price of purchasing power increases as the delivery deadline approaches. If $c_{r_1} > c_{r_2}$, it is worthwhile to defer the purchasing decision since more information is available at a lower price. Similarly, for two selling dispatch stages $r_1,\ r_2 \in \mathcal{R}$ and $r_1 < r_2$, we require $c_{r_1} > c_{r_2}$. Finally, to avoid arbitrage, for each buying stage r_1 and selling stage r_2, such that $r_1 < r_2$, we require $c_{r_1} > c_{r_2}$.

At each dispatch stage $r \in \mathcal{R}$, three events occur. First information Y_r is observed. In addition to the state at stage r, *i.e.*, $x_r \in Y_r$, the information set could also contain signals that help the prediction of the demand and wind generation. Examples include weather forecast and sensor measurement data that are available at the time of stage r. Notice $Y_r \subset Y_{r+1}$, for all $r \in \mathcal{R}$. Next a dispatch decision is made: The operator decides to purchase ($s_r > 0$) or sell ($s_r < 0$) from the r-th market. Lastly the total amount of power accumulated so far is computed

$$x_{r+1} = x_r + s_r,\ r \in \mathcal{R}. \tag{1}$$

The energy accumulated in R markets is supplied during a delivery time interval which is discretized into T stages. Let $\mathcal{T} := \{R+1, R+2, \ldots, R+T\}$ and use t to denote each element in $\mathcal{T}$. For each stage in the delivery time interval, the amount of energy supply from conventional generation is $x = x_{R+1}/T$.

A random wind generation W_t and a random load L_t are realized for each $t \in \mathcal{T}$. Let $D_t := L_t - W_t$ denote the net deficit at stage t. The deficit may be positive or negative. A monetary penalty is assessed to compensate the positive deficit or unmet demand. Different forms of the penalty will be discussed later in the section. The information set is extended to stage $R+T$, *i.e.*, for all $i \in \{2, \ldots, R, R+1, \ldots R+T\}$ we have $Y_{i-1} \subset Y_i$. We also define Y_{R+T+1} as the information available at the end of the entire period and $Y_{R+T} \subset Y_{R+T+1}$.

In each stage $t \in \mathcal{T}$, the storage operator can recharge $[u_t]_+$ or discharge $[u_t]_-$ units of energy subject to physical constraints of the storage device, where $[u_t]_+ = \max(u_t, 0)$, $[u_t]_- = \min(u_t, 0)$, and $u_t = [u_t]_+ + [u_t]_-$ is the variable representing the operation for storage at stage t. We denote the amount of energy stored in the storage device at stage t as b_t, the transition function for the storage device as $F(b_t, u_t)$, and the feasible set for the discharging/recharging operations as $\mathcal{U}(b_t)$. We denote the terminal cost as $g(\mathbf{D}, \mathbf{u}, x)$, which will be specified in Subsection II-C, where $\mathbf{D} = (D_{R+1}, D_{R+2}, \ldots, D_{R+T})$ and $\mathbf{u} = (u_{R+1}, u_{R+2}, \ldots, u_{R+T})$. A control policy $\phi = (\phi_1, \phi_2, \ldots, \phi_{R+T})$ is a sequence of functions each of which maps the information available at current stage to the action at the same stage, *i.e.*, ϕ_i is a Y_i-adapted function for every $i \in \mathcal{R} \cup \mathcal{T}$. We use $\phi^D := (\phi_1, \phi_2, \ldots, \phi_R)$ to represent the dispatch policy and $\phi^S := (\phi_{R+1}, \phi_{R+2}, \ldots, \phi_{R+T})$ to represent the storage operation policy.

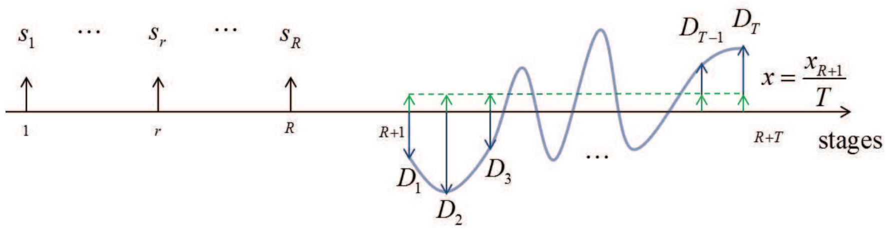


Fig. 1: Risk limiting dispatch with storage: The purchased conventional generation is supplied over the delivery time interval uniformly, while the wind generation and demand may vary over the time interval. A storage device is operated during the delivery time interval to minimize the terminal cost (penalty).

The RLD with storage problem can then be summarized as

$$\begin{aligned}
\text{minimize} \quad & \mathbb{E}\left[\sum_{r\in\mathcal{R}} c_r s_r + g(\mathbf{D},\mathbf{u},x)\right] && \text{(2a)}\\
\text{subject to} \quad & s_r = \phi_r(Y_r), && \text{(2b)}\\
& x_{r+1} = x_r + s_r, \quad r\in\mathcal{R}; && \text{(2c)}\\
& u_t = \phi_t(Y_t), && \text{(2d)}\\
& u_t \in \mathcal{U}(b_t), && \text{(2e)}\\
& b_{t+1} = F(b_t,u_t), \quad t\in\mathcal{T}. && \text{(2f)}
\end{aligned}$$

B. Storage

The storage device has finite *capacity* B (MWh). Energy loss between stages is modeled using the *storage efficiency* $\lambda \in [0,1]$, *i.e.*, the contribution of b_t units of stored energy at time $t+1$ is only λb_t. Energy conversion loss is modeled with parameters $\mu \in [0,1]$ and $\nu \in [0,1]$, denoting the *recharging efficiency* and *discharging efficiency*, respectively. If u units of electric energy are recharged into the storage device, only $u' = \mu u$ units of energy will be actually stored due to energy conversion loss. Similarly, if u' units of electric energy are discharged from the storage device, only $u = \nu u'$ units of the energy can be used to meet the net deficit realized.

The dynamics of the storage model is captured by the transition function

$$F(b_t,u_t) = \lambda\left(b_t + \mu[u_t]_+ + \frac{1}{\nu}[u_t]_-\right), \quad (3)$$

and the feasible action set is

$$\mathcal{U}(b_t) = \left\{u_t \middle| 0 \le [u_t]_+ \le \frac{1}{\mu}(B-b_t), -\nu b_t \le [u_t]_- \le 0\right\}. \quad (4)$$

To simplify the notation, we denote the feasible set for the vector $\mathbf{u}$ as $\mathcal{U}(\mathbf{b})$. Note we assume that the storage starts empty, *i.e.*, $b_{R+1} = 0$, and any energy remained in the storage after stage $R+T$ will be discarded at no cost/benefit.

C. Terminal Costs

Different terminal costs lead to different dispatch goals. In this paper, we focus on the value of loss load (VOLL). Other common penalties like loss of load probability (LOLP) and frequency drop charge can be extended to our framework easily and all our results can be applied to situations where these other penalties function are used, given they are convex in $(\mathbf{u}, x)$.

For VOLL, let

$$g_t(D_t,u_t,x) = c[D_t - x + u_t]_+$$

measure the shortfall to meet D_t when x units of energy are supplied and $-u_t$ units of energy are withdrawn from the storage at t, where c is the cost of unit shortfall. The total terminal cost over the delivery time interval then is

$$g(\mathbf{D},\mathbf{u},x) = \sum_{t\in\mathcal{T}} g_t(D_t,u_t,x) = c\sum_{t\in\mathcal{T}}[D_t - x + u_t]_+.$$

Notice $g_t(D_t,u_t,x) = [D_t - x + u_t]_+$ is convex in (x,u_t), for all values of D_t.

III. Optimal Storage Operation

Consider the storage operation problem given x_{R+1} units of energy are accumulated in R markets. Then the optimal storage operation problem is

$$\begin{aligned}
\text{minimize} \quad & \mathbb{E}\left[c\sum_{t\in\mathcal{T}}[D_t - x + u_t]_+\right],\\
\text{subject to} \quad & \text{(2d), (2e), (2f).}
\end{aligned}$$

The problem is solved with dynamic programming.

Lemma 3.1 (Optimal storage operation): The terminal cost-to-go function is $J_{R+T+1}(x, b_{R+T}) = 0$, and the cost-to-go for a storage operation stage $t \in \mathcal{T}$ is

$$\begin{aligned}
J_t(x,b_t) = \inf_{u_t\in\mathcal{U}(b_t)} & \mathbb{E}\{c[D_t - x + u_t]_+ \\
& + J_{t+1}(x,b_{t+1})|Y_t\}.
\end{aligned}$$

The cost-to-go function for each $t \in \mathcal{T} \cup \{R+T+1\}$ is convex in (x, b_t).

The optimal control policy for the storage operation is, for $t \in \mathcal{T}$,

$$[u_t^\star(b_t)]_+ = \min\left\{[x - D_t]_+, \frac{1}{\mu}(B - b_t)\right\},$$

$$[u_t^\star(b_t)]_- = -\min\left\{[D_t - x]_+, \nu b_t\right\}.$$

IV. Dispatch Threshold Rule

The previous section finds optimal storage control given any accumulated energy x_{R+1}. In this section, we turn to consider the optimal dispatch problem assuming the optimal storage control will be applied in the delivery interval.

We first define the cost-to-go function for the dispatch problem and state a standard dynamic programming result.

Lemma 4.1: The cost-to-go function for a dispatch stage $r \in \mathcal{R} \cup \{R+1\}$ is

$$J_{R+1}(x_{R+1}) = \inf_{\mathbf{u} \in \mathcal{U}(\mathbf{b})} \mathbb{E}\left\{g(\mathbf{D}, \mathbf{u}, x) | Y_{R+1}\right\},$$

$$J_r(x_r) = \inf_{s_r \in \mathcal{S}_r} \mathbb{E}\left\{c_r s_r + J_{r+1}(x_{r+1}) | Y_r\right\}, \ r \in \mathcal{R}, \tag{5}$$

where $\mathcal{S}_r = \{s | s \geq 0\}$ if r is a buying stage, $\mathcal{S}_r = \{s | s \leq 0\}$ if r is a selling stage. Further, if $s_r^\star = \phi_r^\star(Y_r)$ minimizes the right hand side of (5) for each x_r and r, then the dispatch policy $\phi^{D\star} = (\phi_1^\star, \phi_2^\star, \ldots, \phi_R^\star)$ is optimal.

This result allows us to solve the problem by finding the policy that minimizes the cost-to-go function. Now we state a convexity result which is necessary to establish the threshold rule.

Proposition 4.2: The cost-to-go function $J_r(x_r)$, for all $r \in \mathcal{R} \cup \{R+1\}$ is convex in x_r given $g(\mathbf{D}, \mathbf{u}, x)$ convex in $\mathbf{u}$ and x.

Relying on previous two results, we show that the dispatch rule for the RLD with storage problem can be characterized by simple thresholds.

Theorem 4.3: For each dispatch stage $r \in \mathcal{R}$, the optimal dispatch is

$$s_r^\star(x_r) = \begin{cases} [\psi_r - x_r]_+, & \text{if } r \text{ is a buying stage,} \\ [\psi_r - x_r]_-, & \text{if } r \text{ is a selling stage,} \end{cases} \tag{6}$$

where $\psi_r \in Y_r$ is a state independent variable that satisfies

$$c_r + \nabla \hat{J}_{r+1}(\psi_r) = 0,$$

with $\hat{J}_{r+1}(x) = \mathbb{E}[J_{r+1}(x) | Y_r]$. Hence ψ_r is uniquely defined as $\psi_r = \nabla \hat{J}_{r+1}^{-1}(-c_r)$.

Theorem 4.3 ensures that the optimal control for the dispatch could be characterized by the sequence of thresholds ψ_r for $r \in \mathcal{R}$. Therefore the important practical feature of RLD, *i.e.*, following simple sequence of thresholds is the optimal dispatch, carries over to the case with storage for convex terminal cost function.

V. Approximation Schemes

Computing the thresholds for dispatch requires a structured Monte Carlo estimation, that can be computationally costly and fails to reveal the structure of the relationship between various quantities of interest such as storage capacity, variability of the wind process and forecasting performance. In this section, we develop an approximate algorithm for dispatch by considering the thresholds for a continuous-time model for storage operation.

Before introducing the continuous-time model, we first reformulate the discrete-time counterpart. Without loss of generality, we assume $c = 1$. Let $V_t = \sum_{\tau=R+1}^{t} [D_t - x + u_t]_+$ and $Q_t = \sum_{\tau=R+1}^{t} [D_t - x + u_t]_-$ denote the cumulative VOLL cost and cumulative curtailment up to time t, respectively. Suppose that V_t and Q_t, $t \in \mathcal{T}$, are adapted to information Y_t, $t \in \mathcal{T}$. Then we can reformulate an optimal storage operation problem equivalent to the problem in Section III with V_t and Q_t as control variables, that is,

$$\begin{aligned}
\text{minimize} \quad & \mathbb{E}\left[V_{R+T+1}\right] \\
\text{subject to} \quad & b_{t+1} = b_{R+1} - \sum_{\tau=R+1}^{t} (D_\tau - x) + V_t + Q_t, \\
& 0 \leq b_{t+1} \leq B, \\
& V_t \geq V_{t-1} \geq \ldots \geq V_1 \geq 0, \\
& Q_t \leq Q_{t-1} \leq \ldots \leq Q_1 \leq 0, \\
& (V_t, Q_t) = \phi_t(Y_t).
\end{aligned}$$

Under the optimal policy in Lemma 3.1, the cumulative VOLL cost V_t increases only if storage is empty, that is, $b_{t+1} = 0$, and the cumulative curtailment Q_t increases only if storage is full, that is, $b_{t+1} = B$.

With the above reformulation, we are ready to introduce the continuous-time model. Assume that the delivery time is a continuous-time interval $\mathcal{T}^C := [R+1, R+T+1]$. Assume that given information set Y_{R+1}, the cumulative net deficit process D_t is a $(\hat{D}(Y_{R+1})/T, \sigma_{R+1}/\sqrt{T})$ Brownian motion, that is, D_{R+t+1} is a Gaussian random variable with mean $\hat{D}(Y_{R+1})t/T$ and variance $\sigma_{R+1}^2 t/T$. The cumulative VOLL cost V_t is adapted to information Y_t, continuous, and non-decreasing with $V_{R+1} = 0$. The cumulative curtailment Q_t is adapted to information Y_t, continuous, and non-increasing with $Q_{R+1} = 0$. Then the stored energy at time t is equal to

$$b_t = b_{R+1} - (D_t - xt) + V_t + Q_t$$

for $t \in \mathcal{T}^C$. Under the optimal policy, V_t increases only if $b_t = 0$, and Q_t decreases only if $b_t = B$. The stored energy process b_t is a reflected Brownian motion. We will approximate the total VOLL cost by the product of the long-term average VOLL cost and the delivery interval length. To find the long-term average cost, we use the properties of reflected Brownian motion in the following Lemma.

Lemma 5.1 ([22]): Let Z_t be a (μ, σ) Brownian motion with $Z_0 = 0$ and $b_t = Z_t + V_t + Q_t$ be a reflected Brownian motion in $[0, B]$ such that V_t and Q_t are adapted to the filtration induced by Z_t and satisfy

1) V_t is continuous and non-decreasing with $V_0 = 0$;
2) V_t increases only when $b_t = 0$;
3) Q_t is continuous and non-increasing with $Q_0 = 0$;
4) Q_t decreases only when $b_t = B$.

The long term average of V_t is equal to

$$\lim_{t\to\infty} \frac{1}{t}\mathbb{E}[V_t] = \frac{\sigma^2}{2B} h\left(\frac{2\mu B}{\sigma^2}\right),$$

where

$$h(x) = \begin{cases} \dfrac{x}{e^x - 1} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0. \end{cases}$$

Now we can approximate the VOLL cost for general c and its first-order derivative by

$$J_{R+1}(x_{R+1}) = c\mathbb{E}[V_T] \approx cT \lim_{t\to\infty} \frac{1}{t}\mathbb{E}[V_t] \tag{7a}$$

$$= \frac{c\sigma_{R+1}^2}{2B} h\left(\frac{2B}{\sigma_{R+1}^2}(x_{R+1} - \hat{D}(Y_{R+1}))\right),$$

$$\nabla J_{R+1}(x_{R+1}) \approx ch'\left(\frac{2B}{\sigma_{R+1}^2}(x_{R+1} - \hat{D}(Y_{R+1}))\right). \tag{7b}$$

Remark 5.2: Formulae (7) reveal the role played by storage explicitly. An important observation is that scaling B and σ_{R+1}^2 by the same constant does not affect $J_{R+1}(x_{R+1})$ and its derivative. That is, a system with more fluctuate wind (deeper penetration) and large storage can have the same terminal cost and thus dispatch thresholds with the another system with less fluctuate wind and small storage, given the ratio B/σ_{R+1}^2 is fixed. This quantifies the notion "storage firms the wind" in the context of dispatch.

Notice the approximate VOLL cost is convex. Thus, the approximate dispatch policy is still characterized by (6).

VI. Numerical Results

A. Setup

The comparison of the performance of different dispatch policies requires information about forecasting. Figure 2(a) shows a typical forecast error curve. Let $\sigma(t)$ be the standard deviation of the t-hours-ahead forecast error. The error ϵ_r explained from stage $r-1$ to stage r is assumed to be a Gaussian random variable with zero mean and variance $\sigma_r^2 = \sigma(t_{r-1})^2 - \sigma(t_r)^2$. We assume that at $t = 0.25$, the forecast error of the mean of the deficit contributes 20% of the error variance, and thus $\sigma_{R+1}^2 = 0.8\sigma(0.25)^2$. For the discrete-time model, the number of storage operation time intervals is $|\mathcal{T}| = 60$.

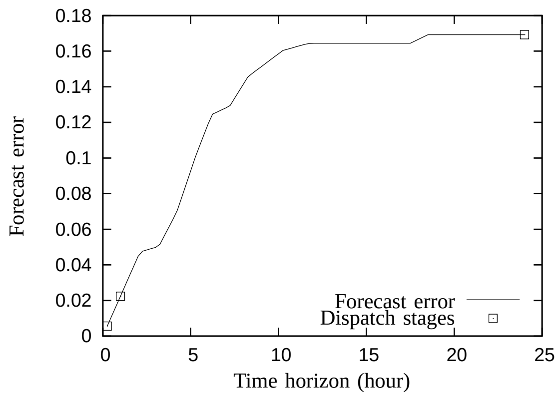
We consider 3-stage dispatch with day-ahead, hour-ahead, and 15-minutes-ahead stages. The prices of purchasing energy are suggested by average energy prices in California. We set the day-ahead price to $\$52$ per MWh, the hour-ahead price to $\$60$ per MWh, the 15-minutes-ahead price to $\$72$ per MWh, and the VOLL to $\$1000$ per MWh. The mean of the deficit D is normalized and is between -1 and $+1$. For a policy ϕ, the cost $J_\phi(D, B)$ is estimated by 2000 Monte Carlo runs of forecast errors.

B. Comparing dispatch approaches

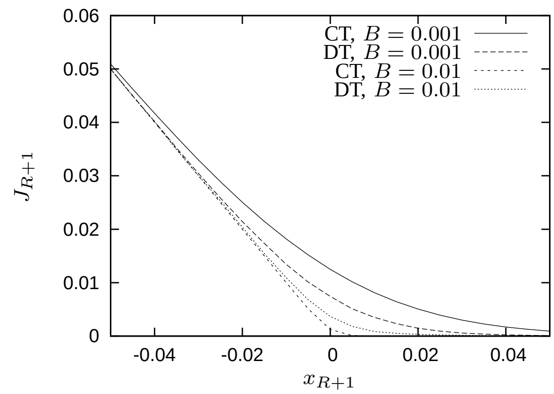
In addition to the optimal dispatch policy in Section IV and the approximate algorithm in Section V, we also consider the following two dispatch approaches as benchmarks. The 3σ-rule secures a constant margin over the forecast deficit which equals $3\sigma(t_r)$. The clairvoyant policy is the optimal dispatch given a perfect forecast and is obtained solving the deterministic version of Eqn. (2), with D_t equals to its realized value for $t \in \mathcal{T}$.

Denote the cost of the ideal policy by $J_0(D, B)$. For any policy ϕ, $J_\phi(D, B) \geq J_0(D, B)$, the difference to the clairvoyant policy (i.e. which knows the realization of the net demand) is the *integration cost*: $C_I = J_\phi(D, B) - J_0(D, B)$ [16]. Figure 2(b) shows the storage operation costs for the discrete-time model and the approximate continuous-time model. The approximate model overestimates the storage operation cost for small storage capacity since the discrete-time model does not consider the cost caused by the variation within each time interval. For large storage capacity, the continuous-time model underestimates the storage operation cost since it assumes that the probability distribution of the initial stored energy is steady-state distribution instead of zero assumed in the discrete-time model.

Figure 3(a) compares the cost $J_\phi(D, B)$ of the 3σ strategy, the optimal dispatch policy, the approximate policy, and the ideal policy for $B = 0.001$. The 3σ strategy has the highest cost. The cost of the approximate



(a) Forecast error curve



(b) Storage operation costs

Fig. 2: Illustration of the forecast error curve from Red Electrica Espana and the storage operation costs for the discrete-time (DT) and continuous-time (CT) models.

policy is slightly higher than the optimal cost. The integration costs with respect to the cost of the ideal policy are shown in Figure 3(b).

Figure 3(c) shows the cost $J_\phi(D, B)$ of the optimal dispatch policy and the approximate policy for $D = 0.4$. From Figure 2(b) it is observed the approximate model is not suitable for very small and very large storage capacities and thus has higher costs in those regimes.

VII. Conclusion

The paper developed risk limiting dispatch with fast storage. The optimal storage operation rule is given in closed form, and the optimal dispatch is characterized by a simple thresholds control. A gradient algorithm based on the structure of optimal control and Monte Carlo evaluation was developed. We also developed a simpler continuous time approximation to the storage operation problem, and find explicit formulae of the terminal cost-to-go as a function of the storage capacity B and the deficit process variance σ_{R+1}^2. The relation quantifies the notion that the storage smoothes the wind. The analytical development is illustrated and validated with numerical results. In future work, we consider extending the method to include slow storage, which requires modeling multiple simultaneous market decisions. We would also like to investigate incorporation of ramping constraints.

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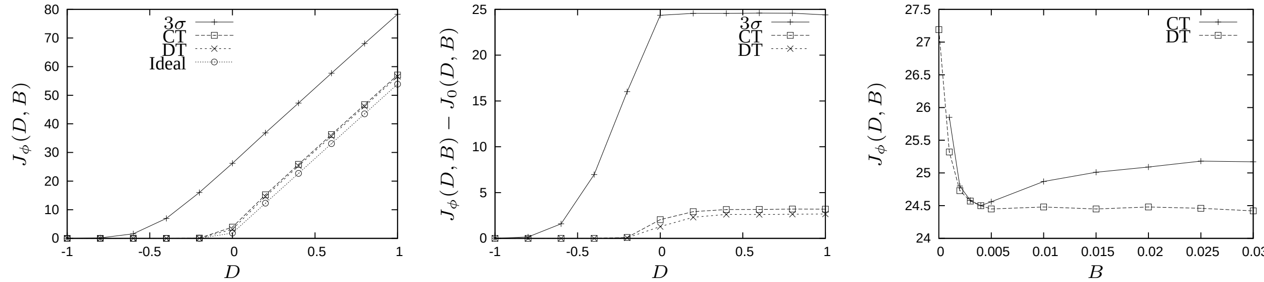

(a) Costs for $B = 0.001$ (b) Integration costs for $B = 0.001$ (c) Costs for $D = 0.4$

Fig. 3: The costs and integration costs for the 3σ strategy, the optimal dispatch policy, the approximate policy, and the ideal policy.

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