

# Astrostatistics

Monday, 04 February 2019

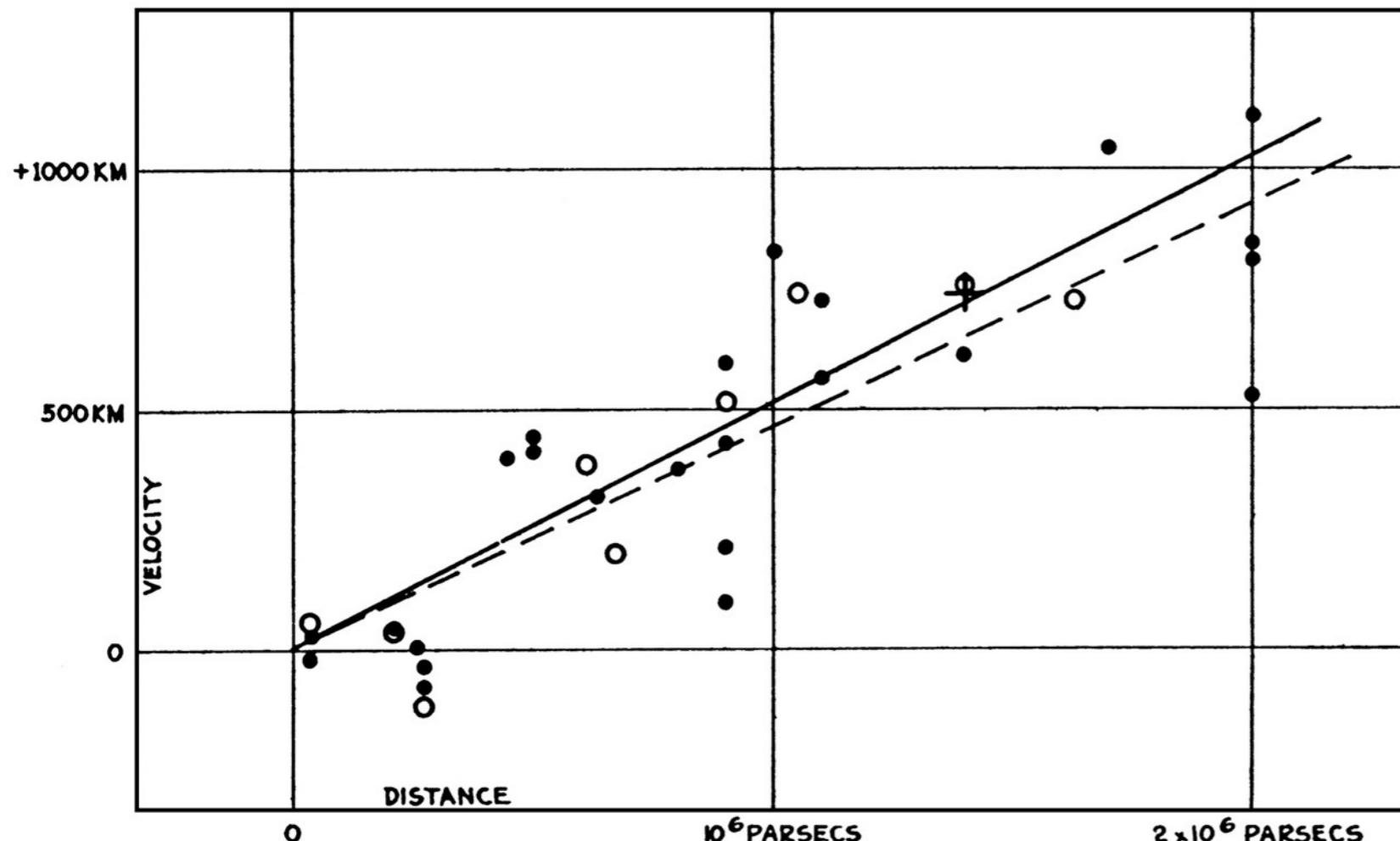
- Statistics Foundations
  - Ivezic Ch 4 “Classical Statistical Inference” & Ch 5 “Bayesian Statistical Inference”
  - F&B Ch 3 “Statistical Inference”
- Review (on your own) properties of multivariate Gaussian random variables and densities  
(see `multivariate_gaussian_notes.pdf` on website)
- and also other standard distributions (F&B Ch 4 and Ivezic Ch 3)

# Determining Astronomical Distances using Standard Candles

1. Estimate or model Luminosity  $L$  of a Class of Astronomical Objects
2. Measure the apparent brightness or flux  $F$
3. Derive the distance  $D$  to Object using Inverse Square Law:  $F = L / (4\pi D^2)$
4. Optical Astronomer's units:  $\mu = m - M$

$m$  = apparent magnitude [log apparent brightness flux],  
 $M$  = absolute magnitude [log Luminosity],  
 $\mu$  = distance modulus [log distance].

# The Expanding Universe: Galaxies are moving apart! Hubble's Law (1929)



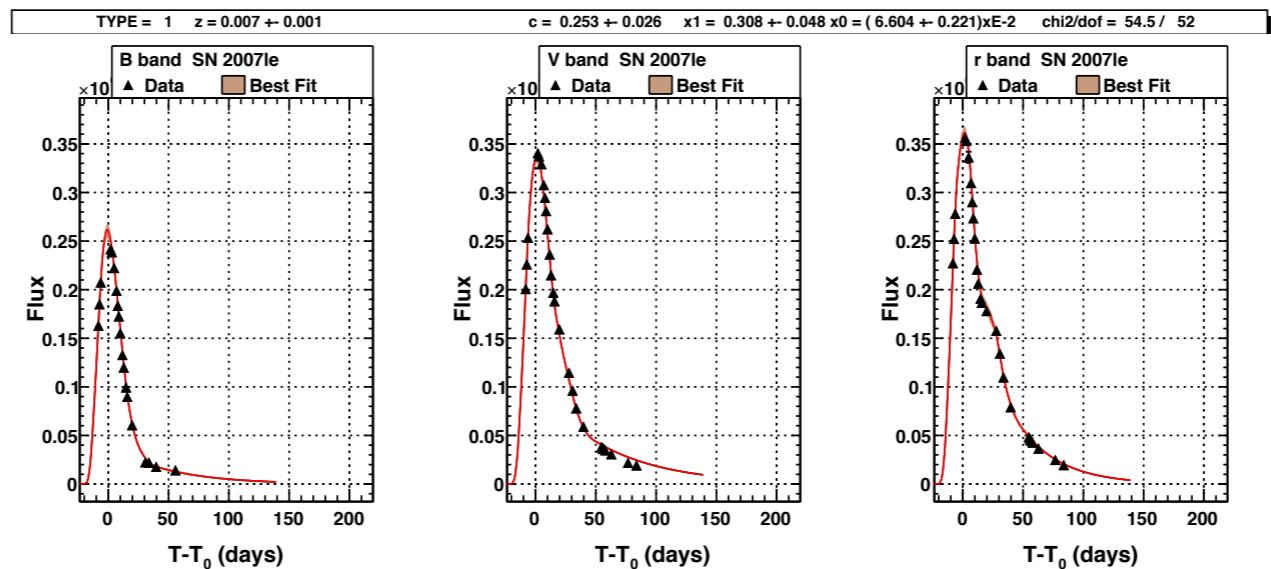
Einstein & Hubble

Distance  $\propto$  Velocity (Redshift)

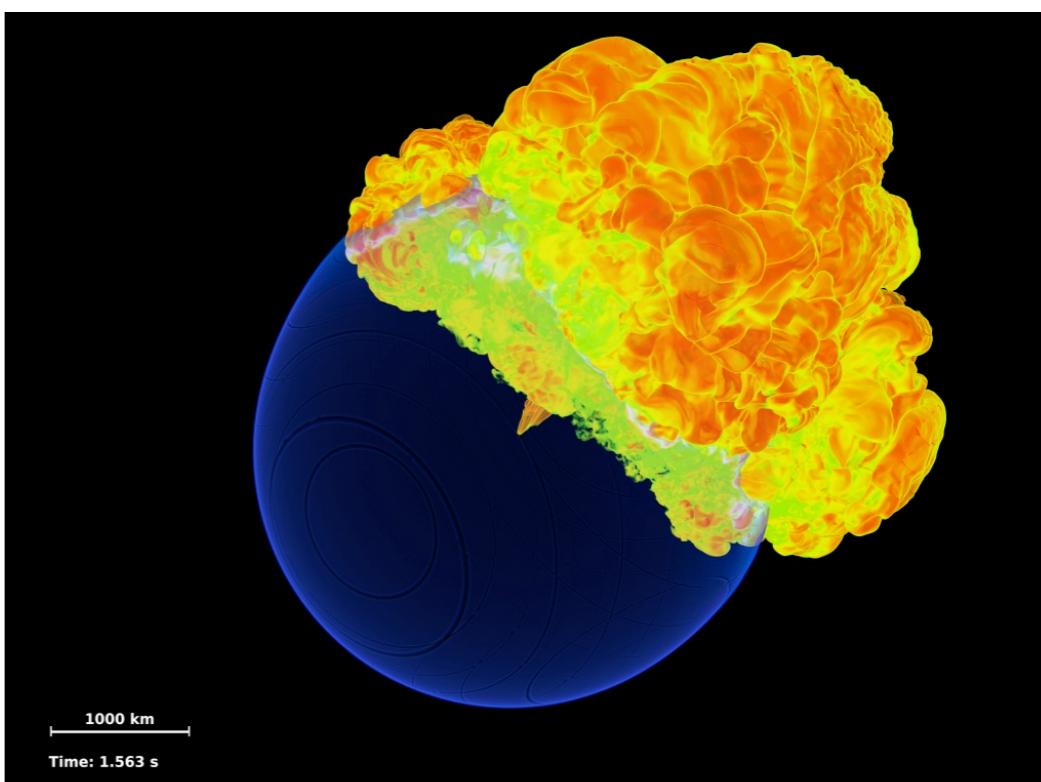
# Type Ia Supernovae (SN Ia) are Almost Standard Candles



Credit: High-Z Supernova Search Team, HST, NASA

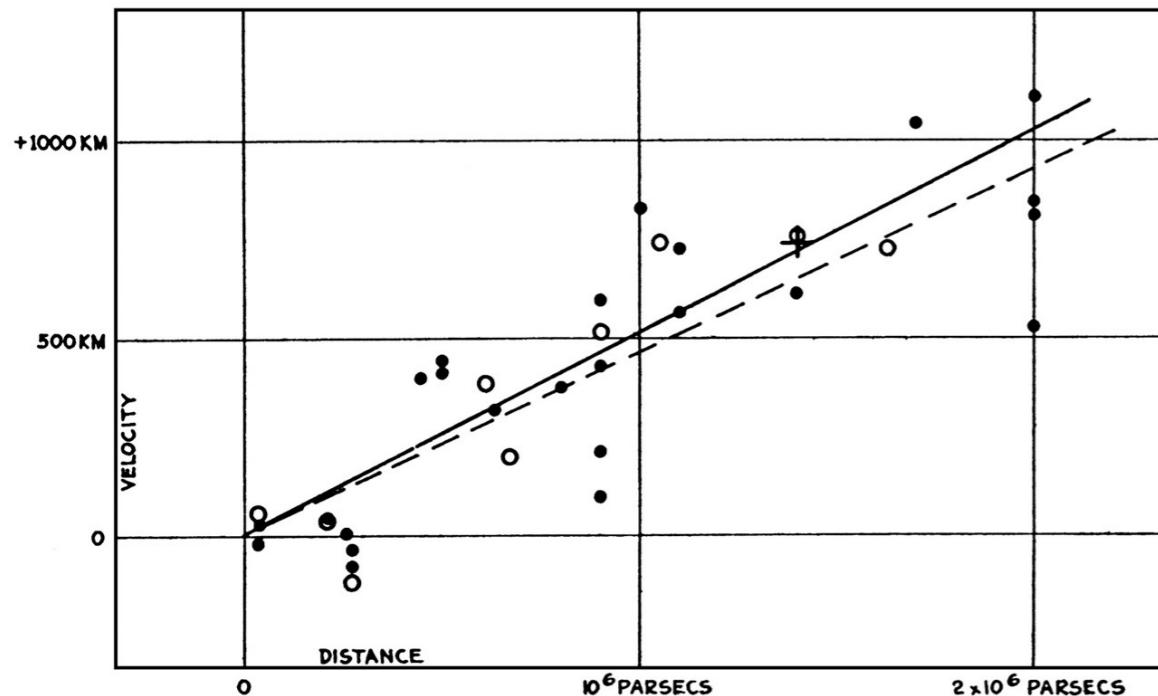


- Progenitor: C/O White Dwarf Star accreting mass leads to instability
- Thermonuclear Explosion: Deflagration/ Detonation
- Nickel to Cobalt to Iron Decay + radiative transfer powers the light curve

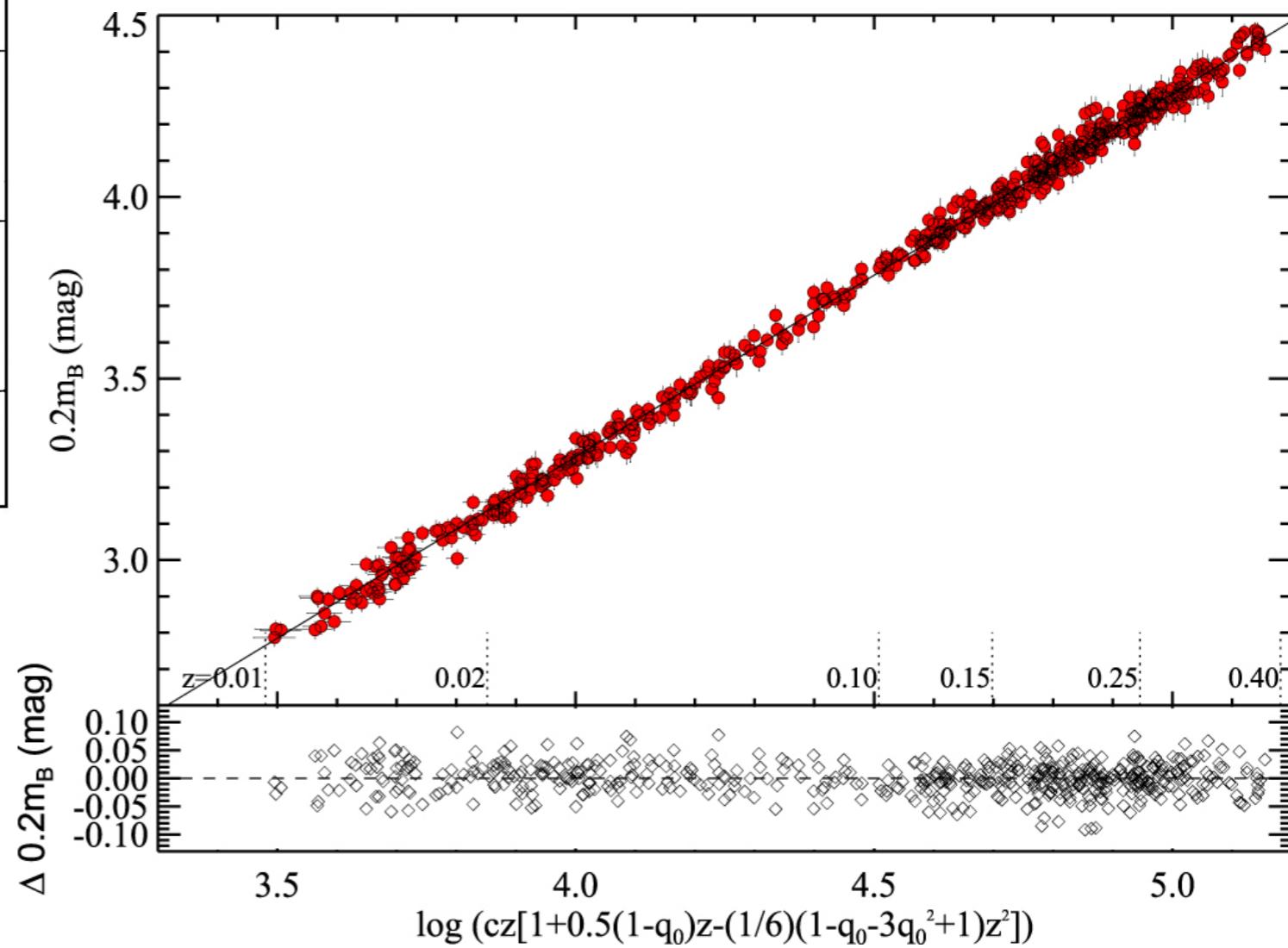


# Hubble Constant

$$\text{Distance} = H_0 \times \text{velocity}$$



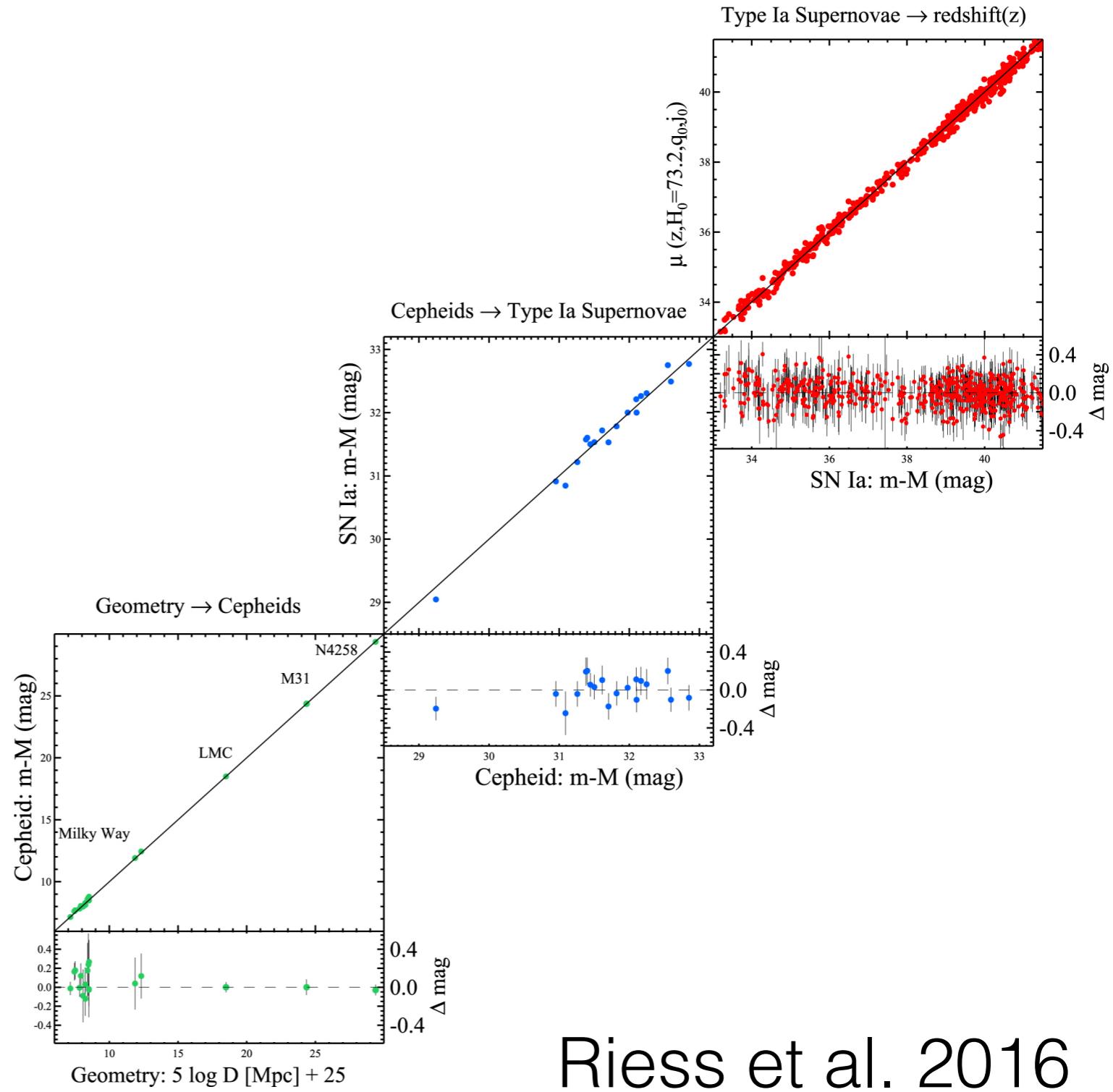
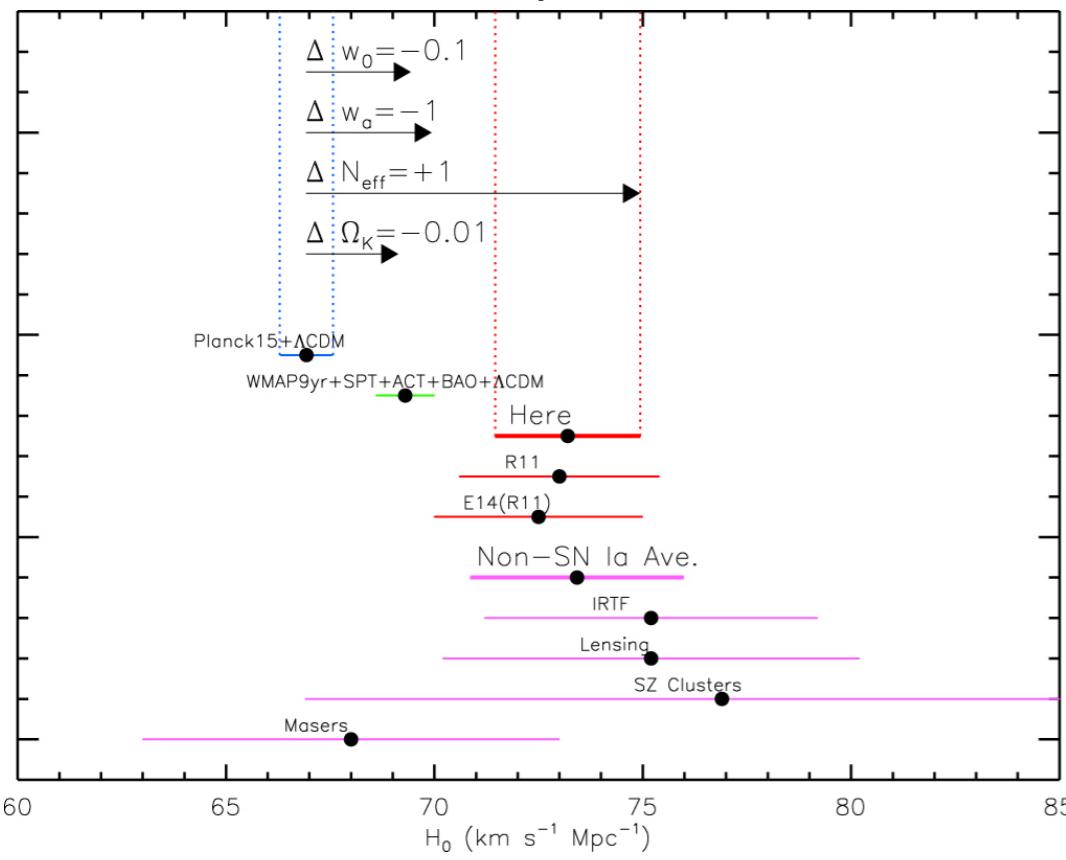
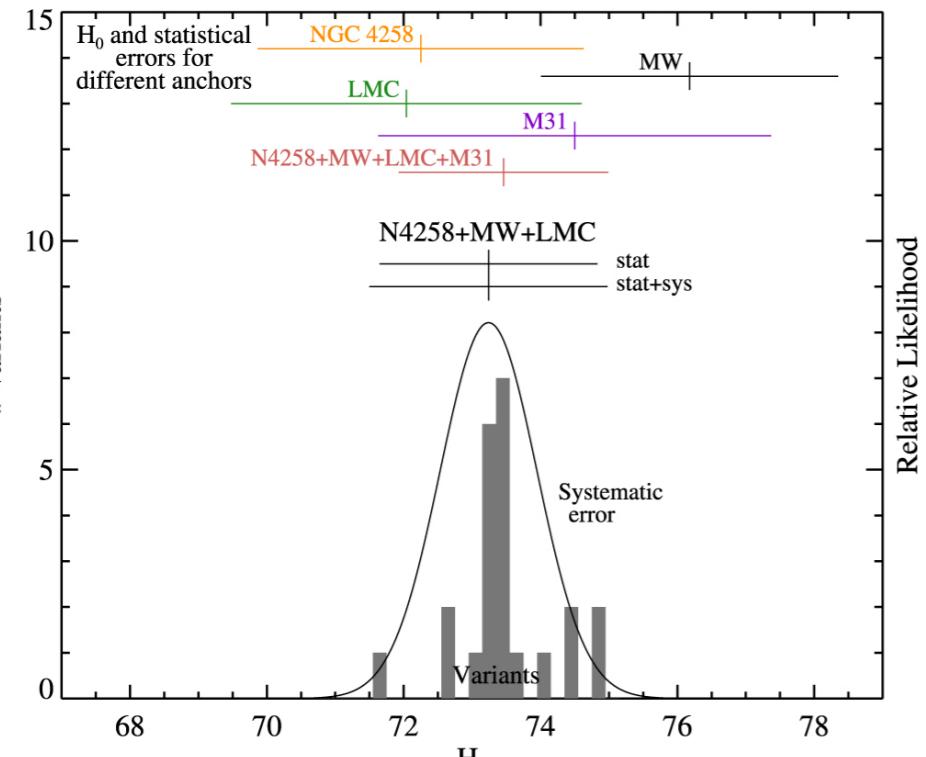
Hubble (1929)



Riess et al. 2016

# Hubble Constant

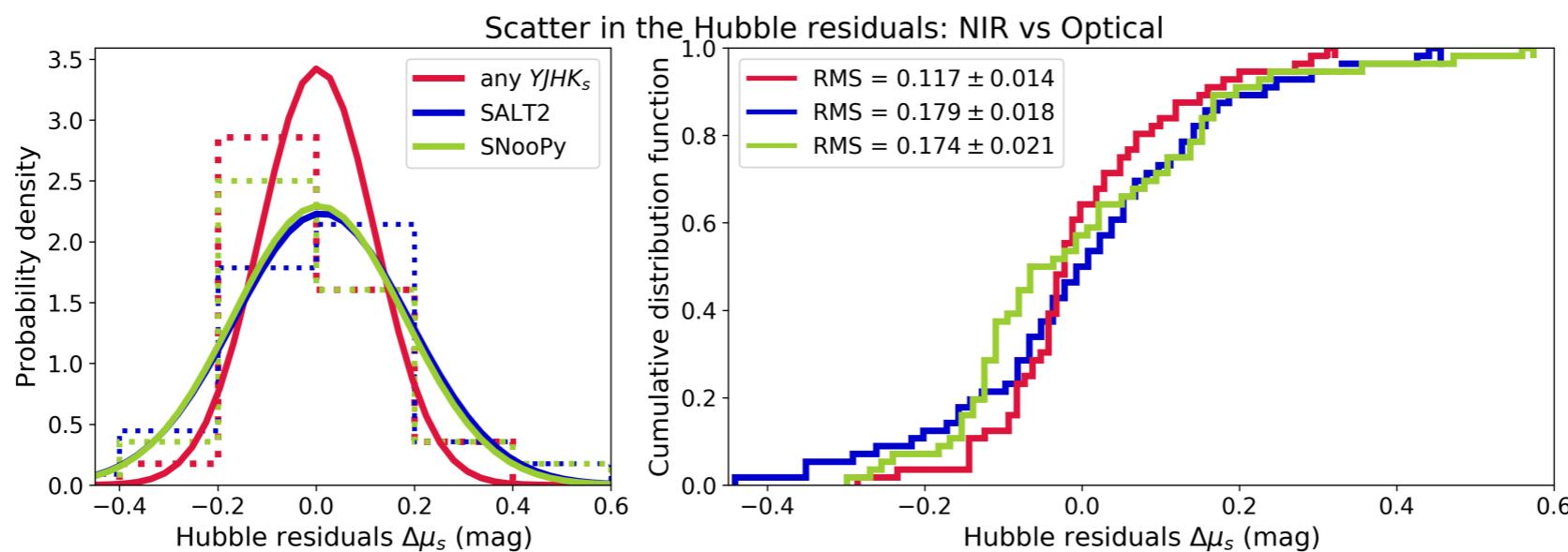
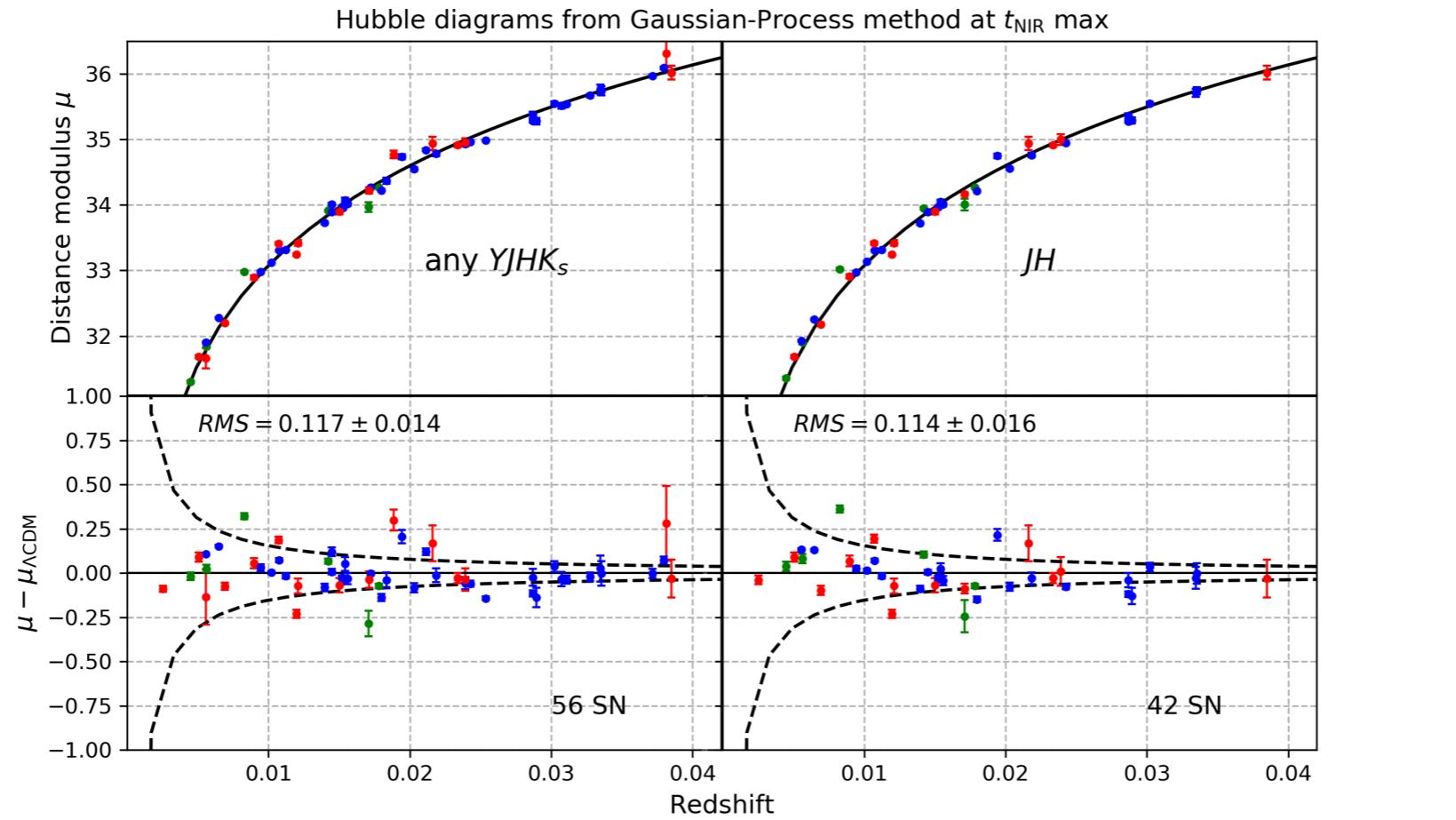
Distance =  $H_0 \times \text{velocity}$



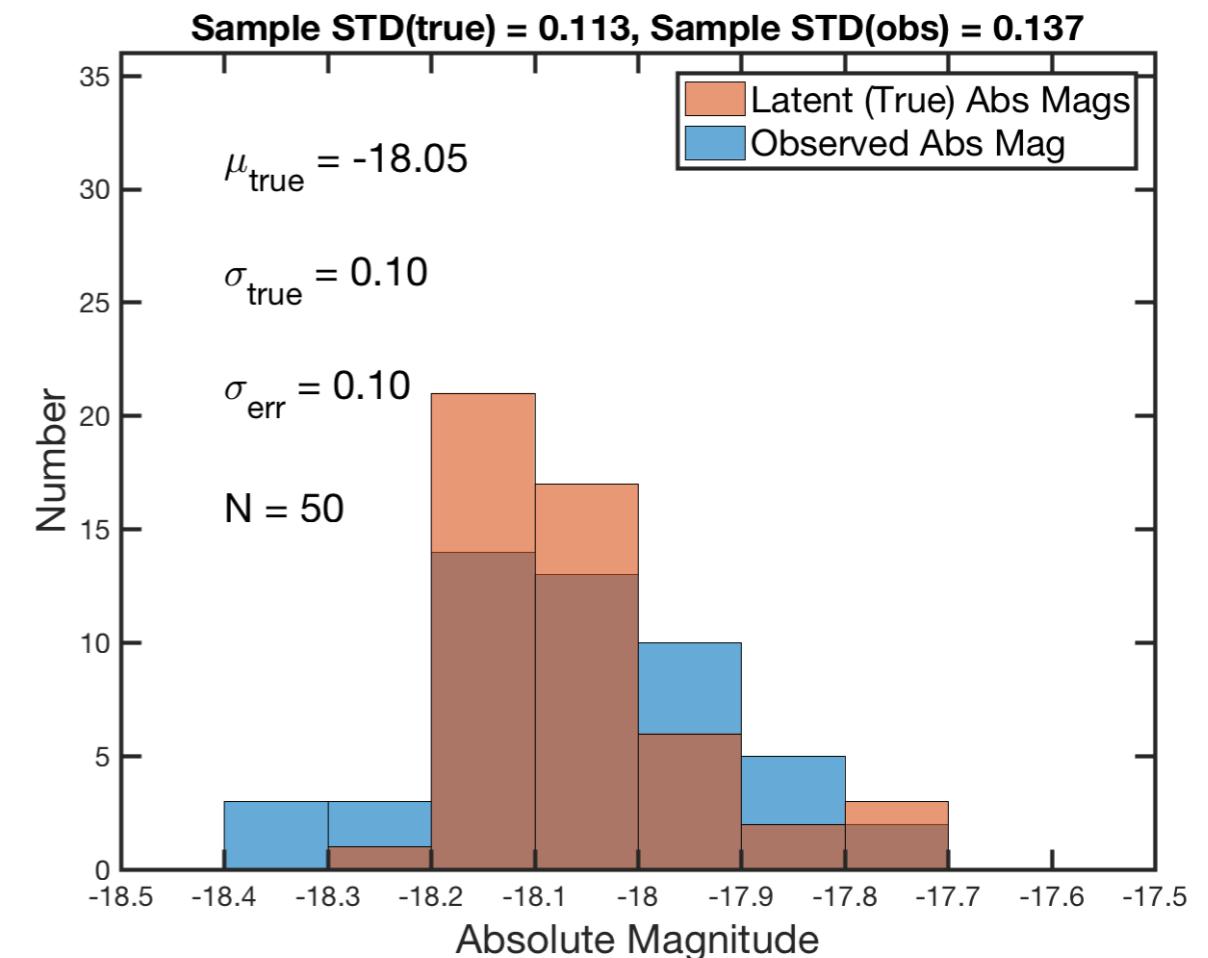
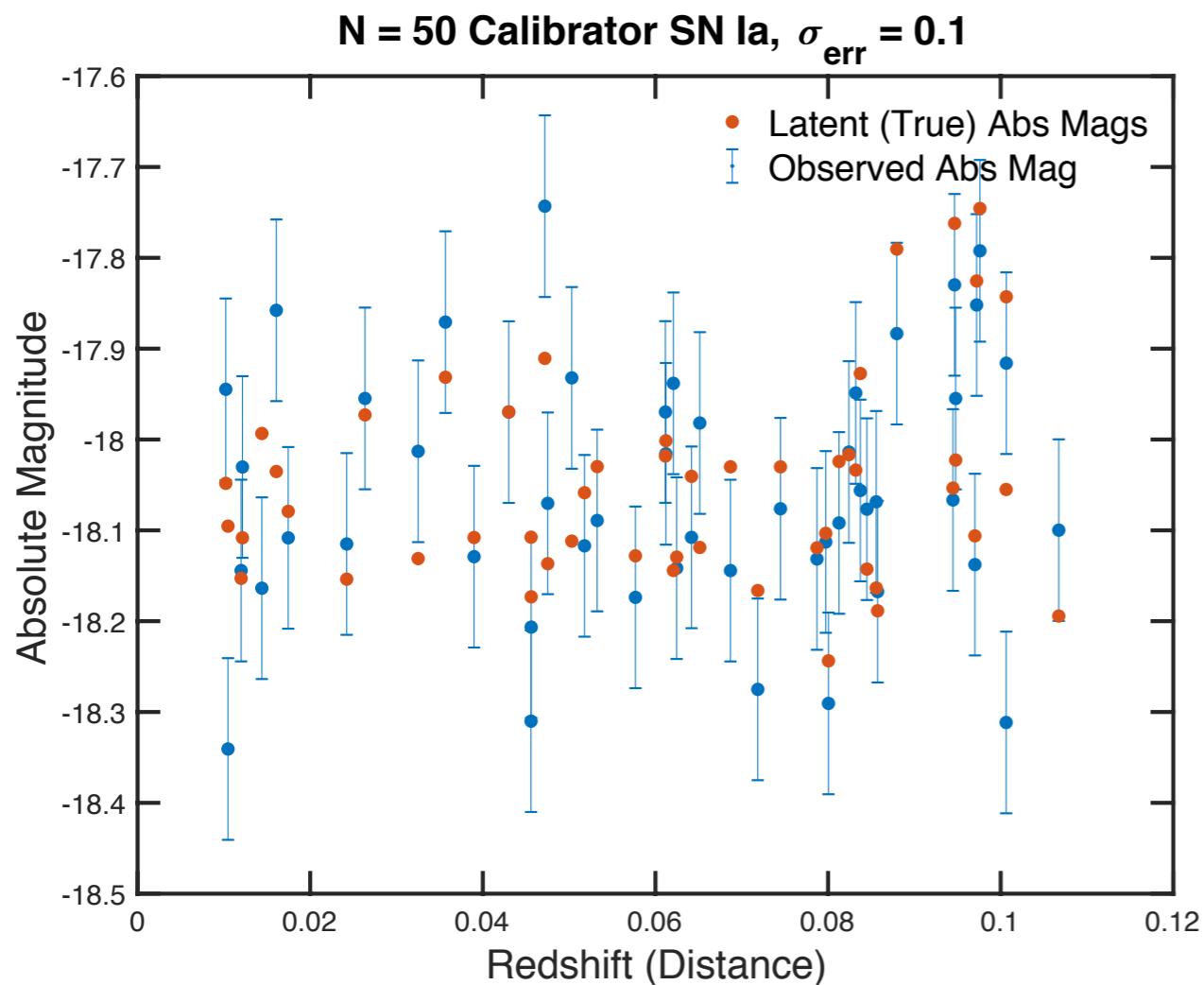
Riess et al. 2016

# Calibrating SN Ia Standard Candles

(Avelino, Friedman, Mandel et al. 2019)



Want to Calibrate SN Ia (N=50)  
 determine  $M_0$ ,  $\sigma_{\text{int}}$   
 from data with measurement error std dev  $\sigma_{\text{err}} = 0.1$

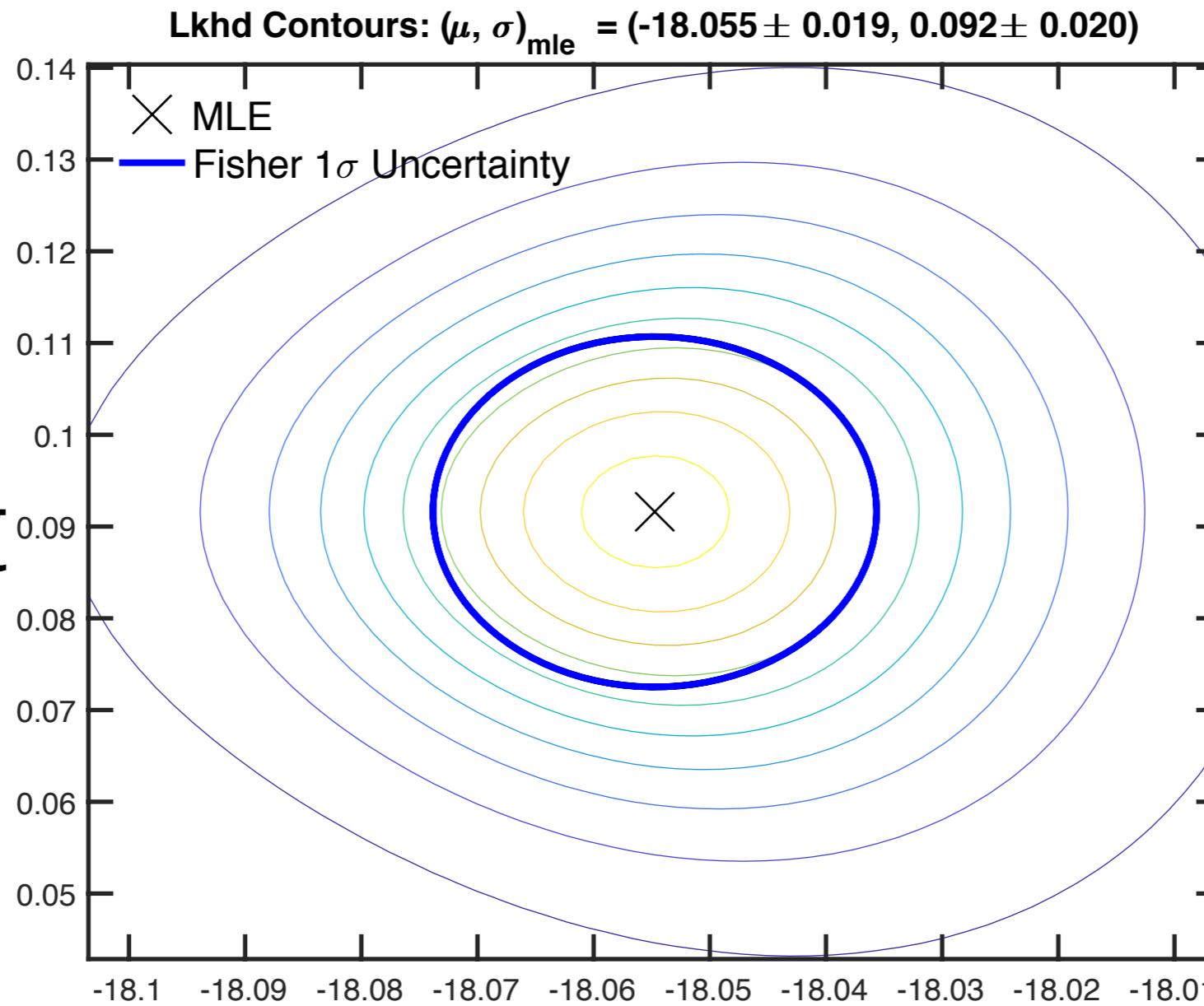


# Maximum Likelihood with measurement error

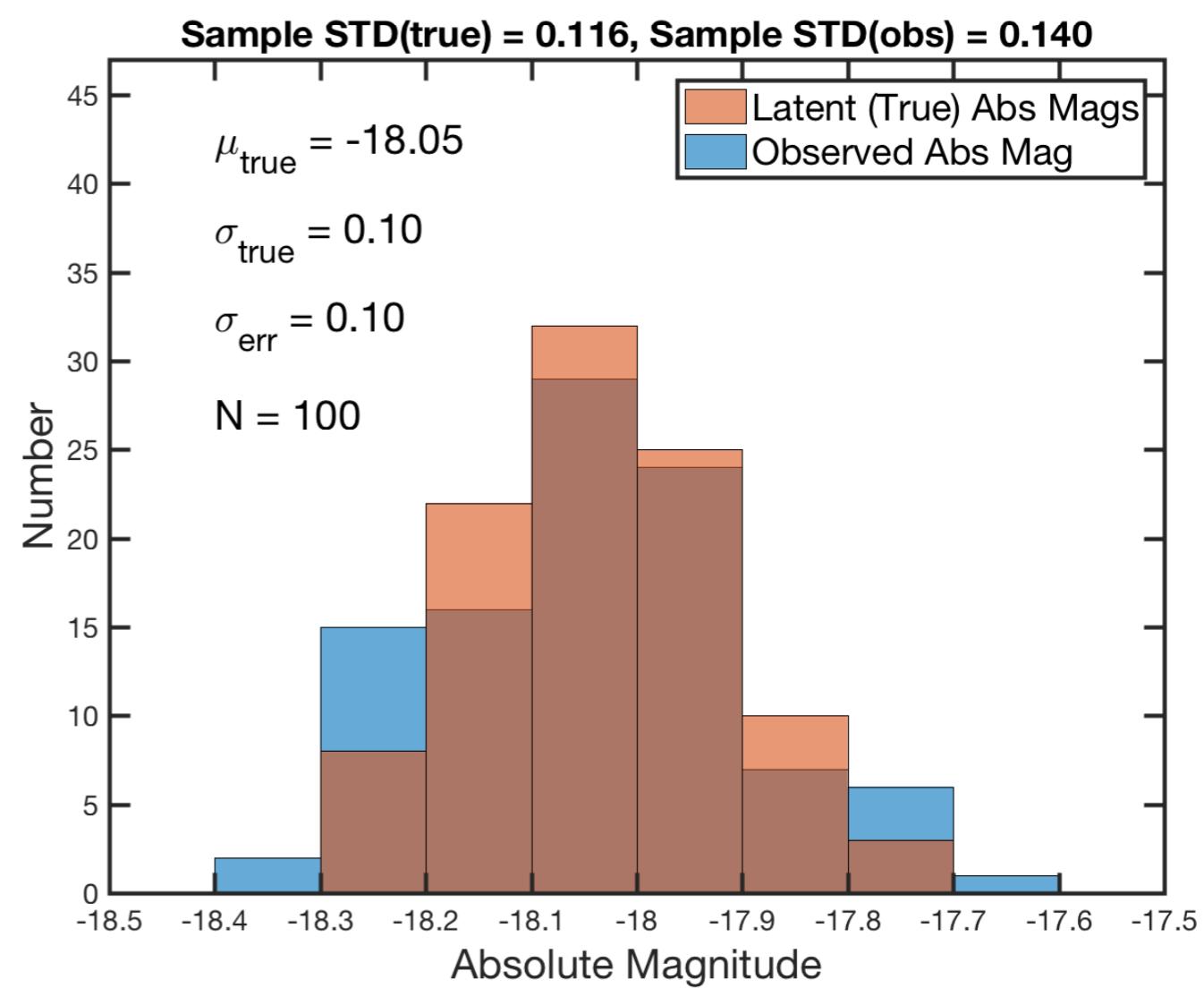
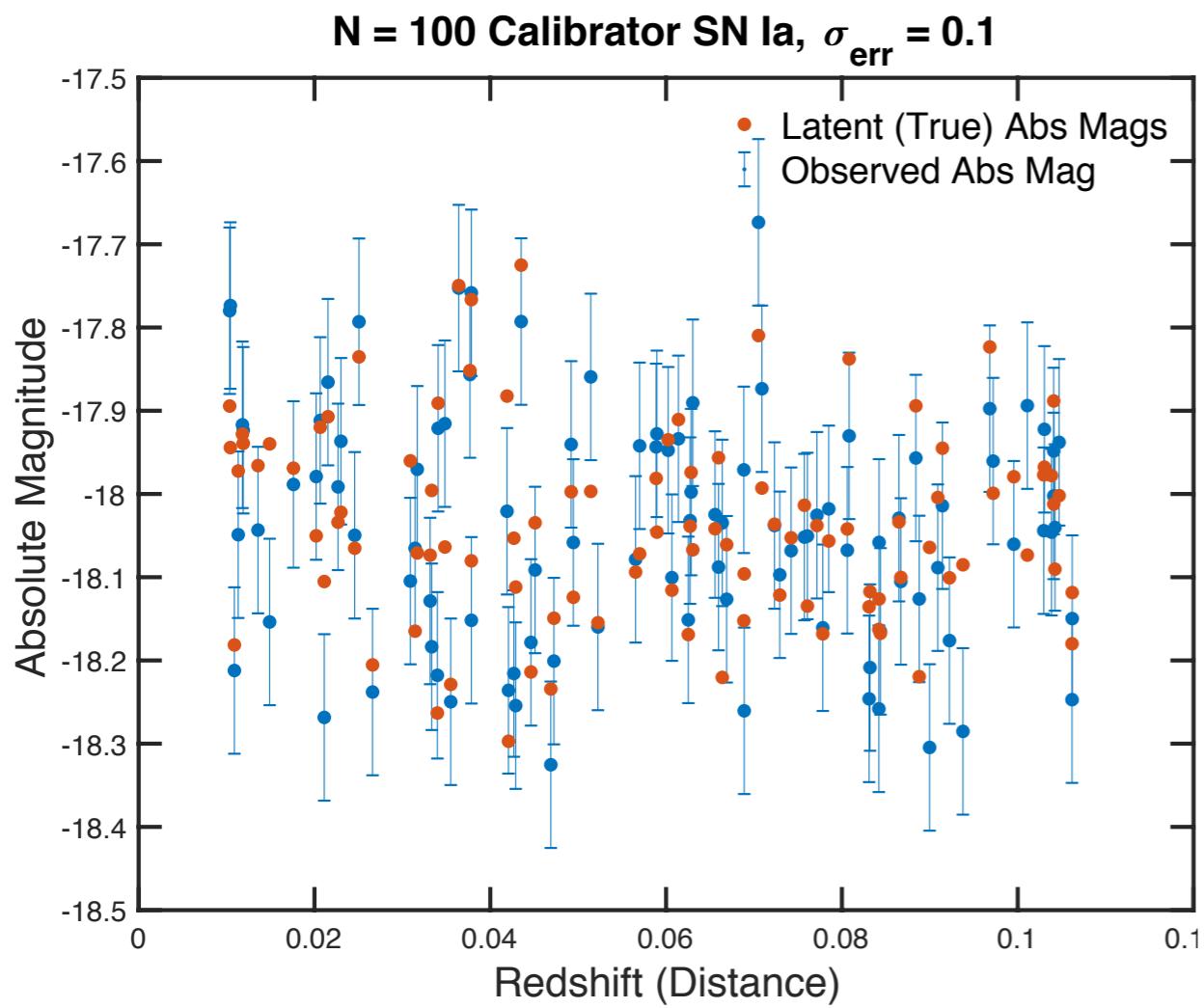
$N = 50$

$\sigma_{\text{int}}$

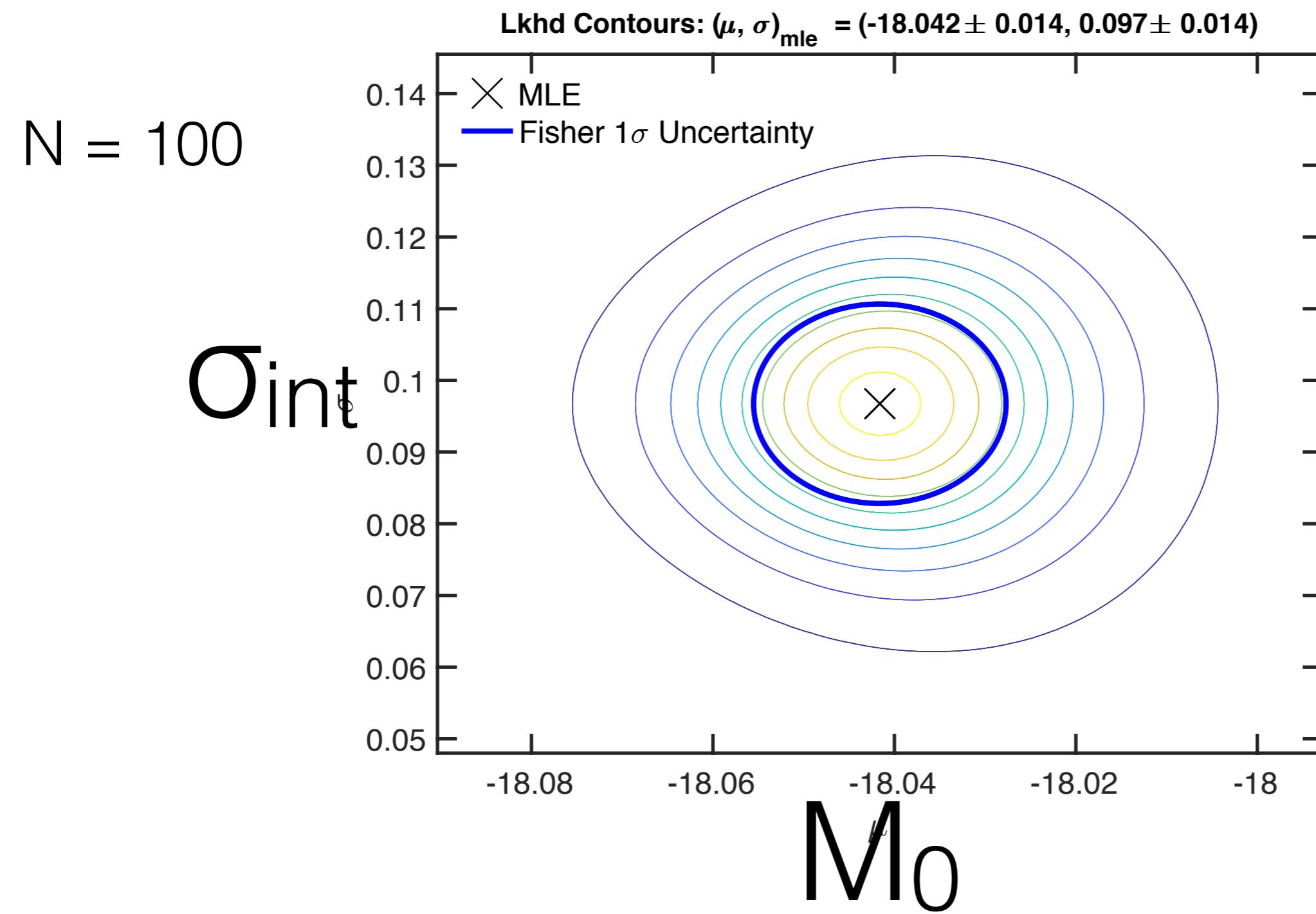
$M_0^\mu$



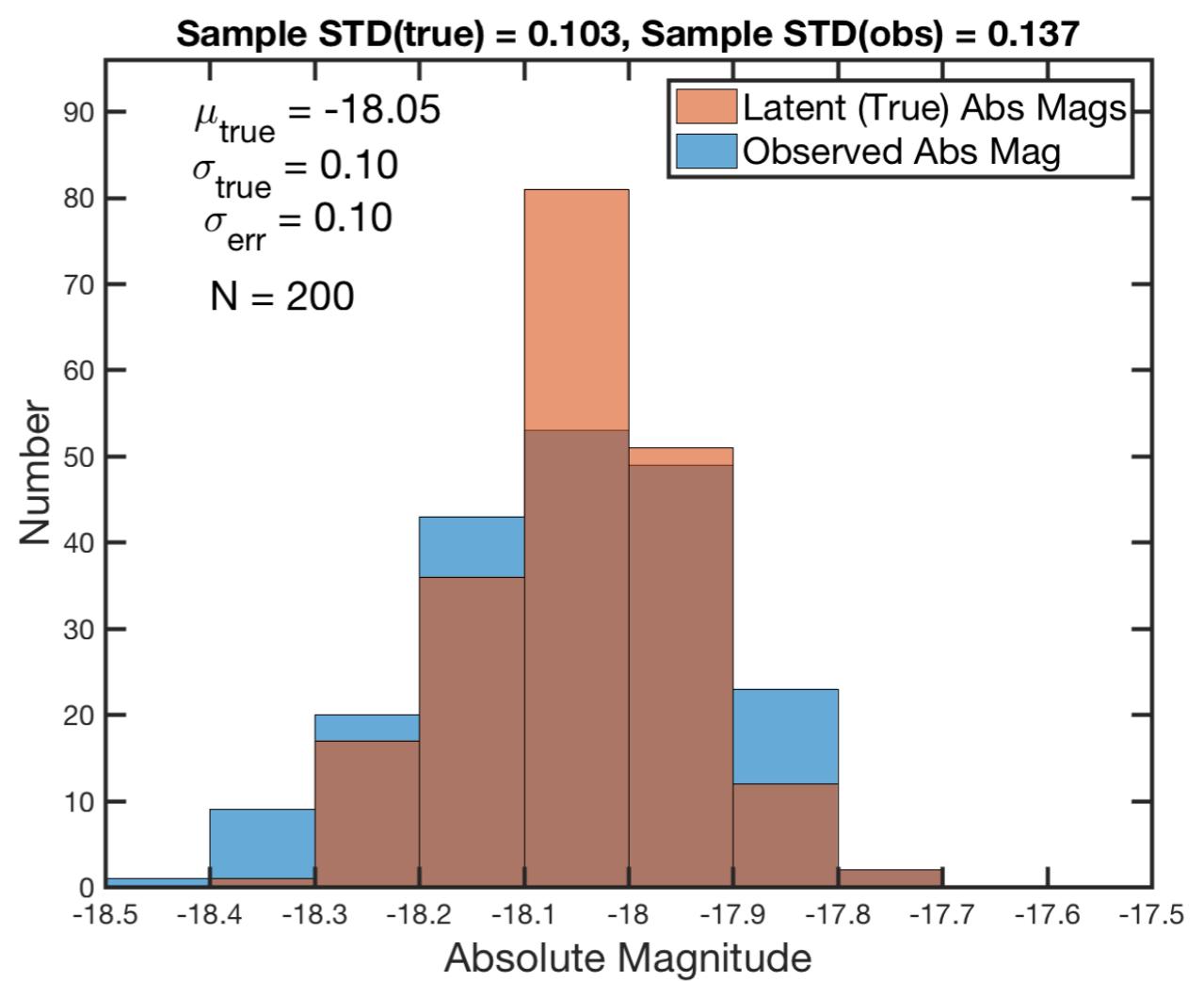
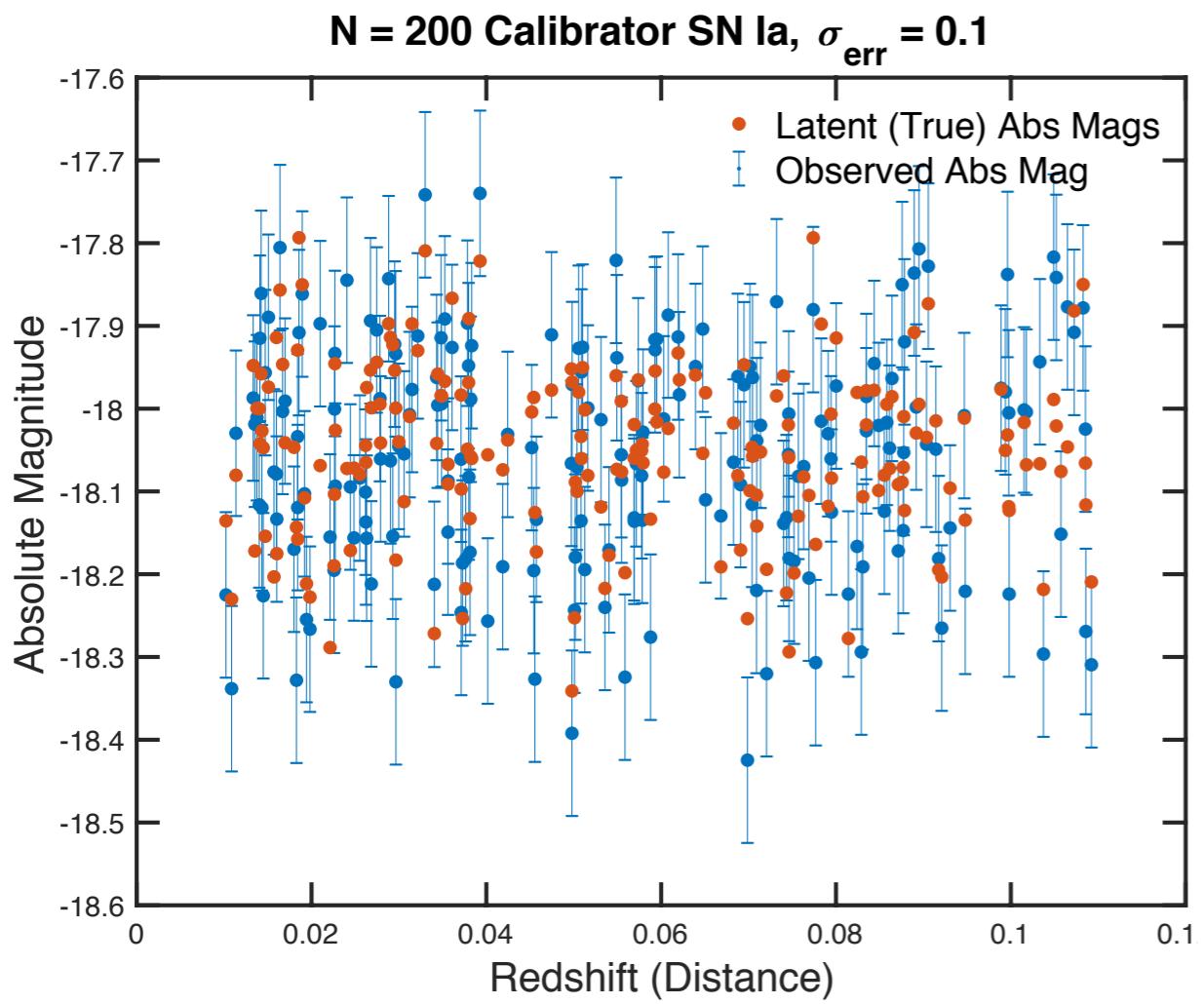
# Want to Calibrate SN Ia (N=100) determine $M_0$ , $\sigma_{\text{int}}$



# Maximum Likelihood with measurement error



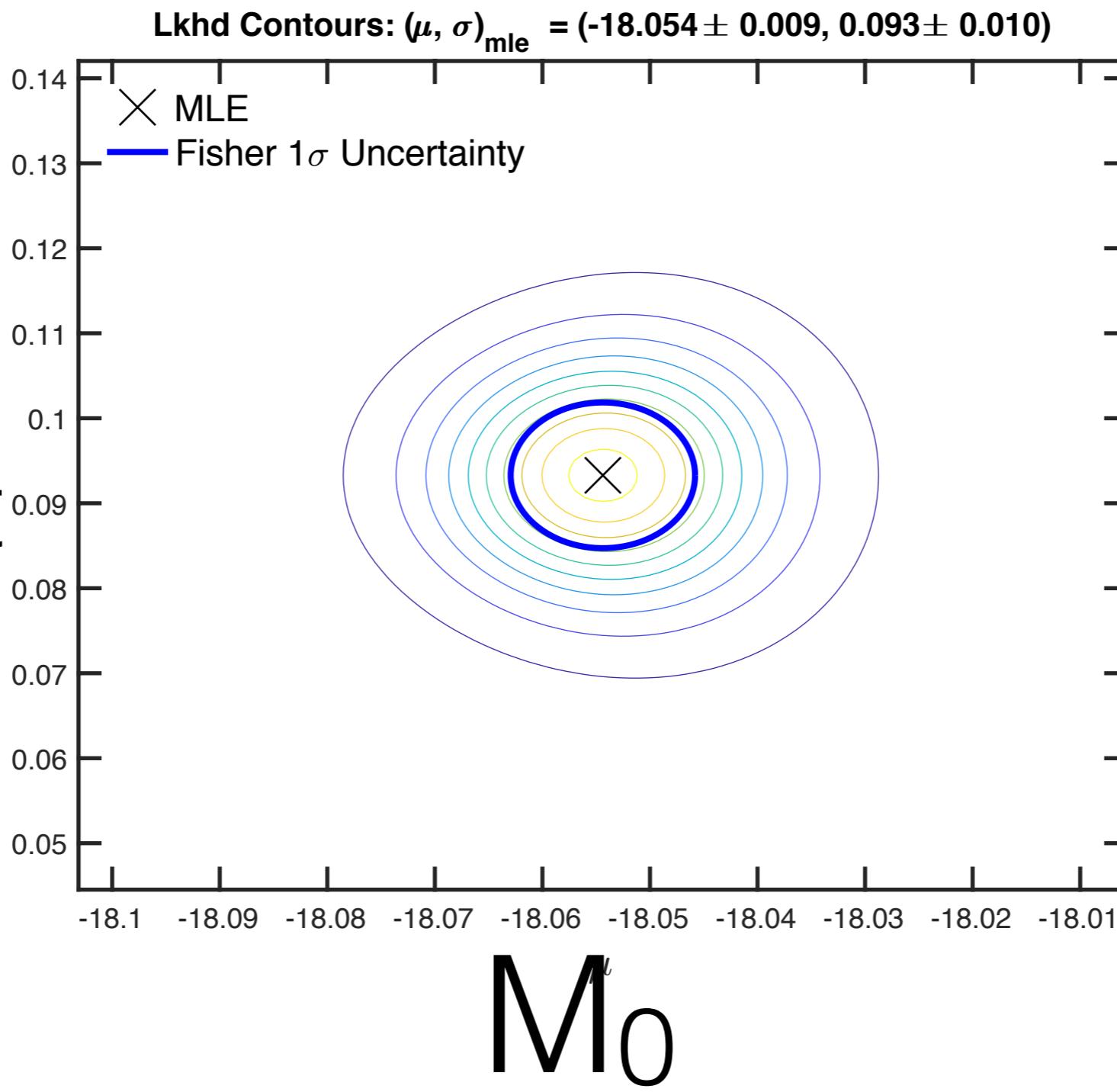
# Want to Calibrate SN Ia (N=200) determine $M_0$ , $\sigma_{\text{int}}$



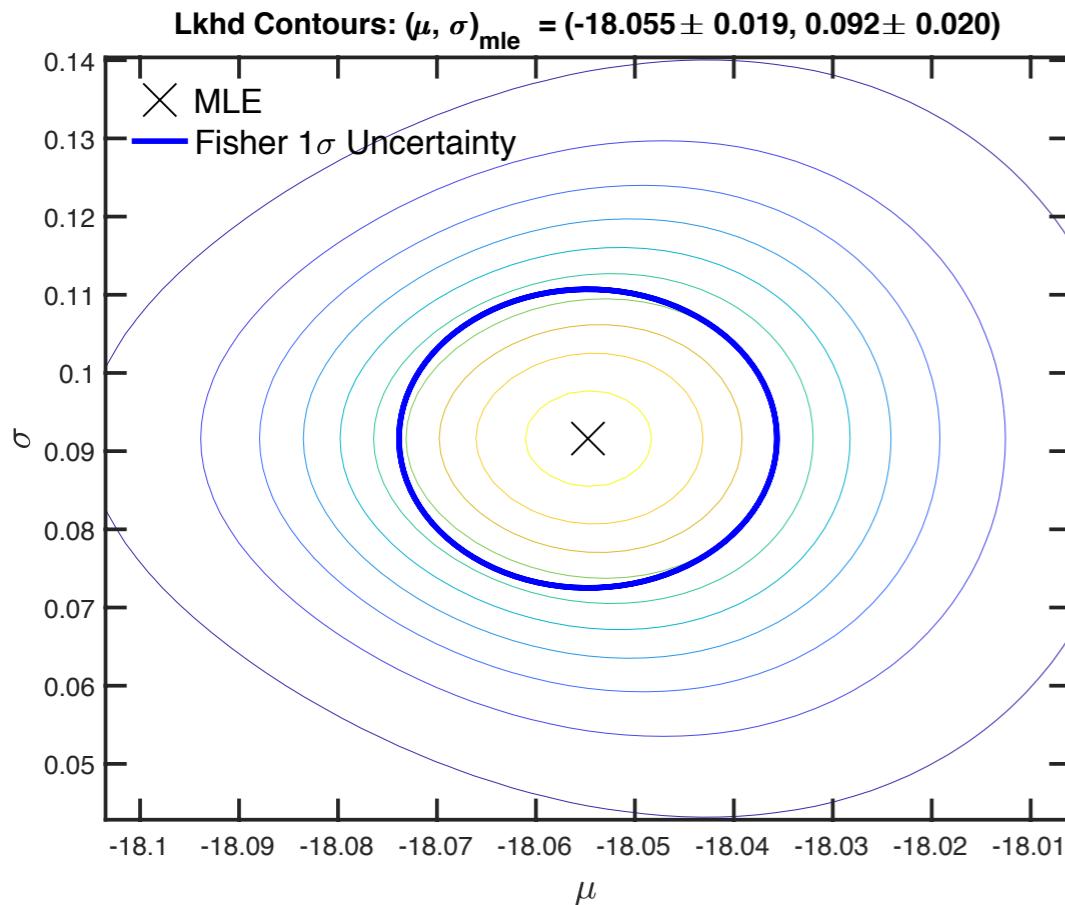
# Maximum Likelihood with measurement error

$N = 200$

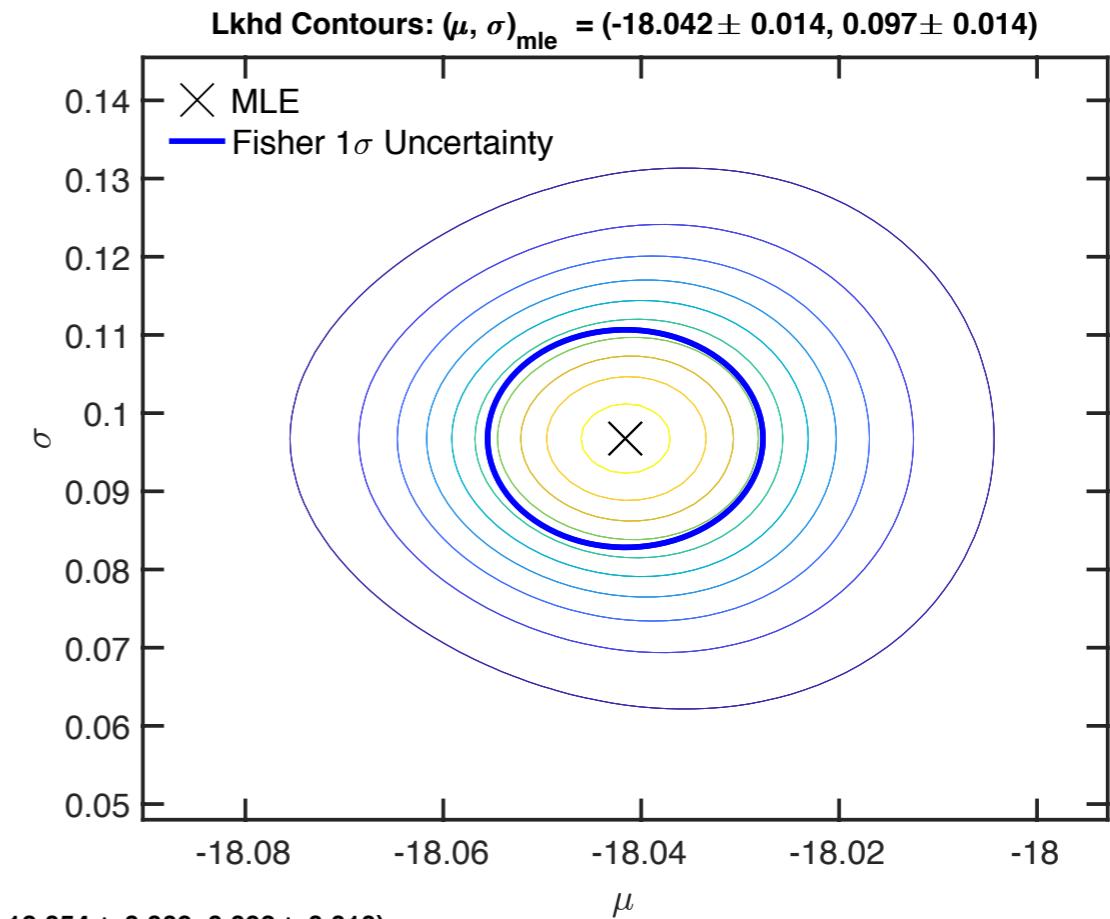
$\sigma_{\text{int}}$



# Constraints vs. sample size

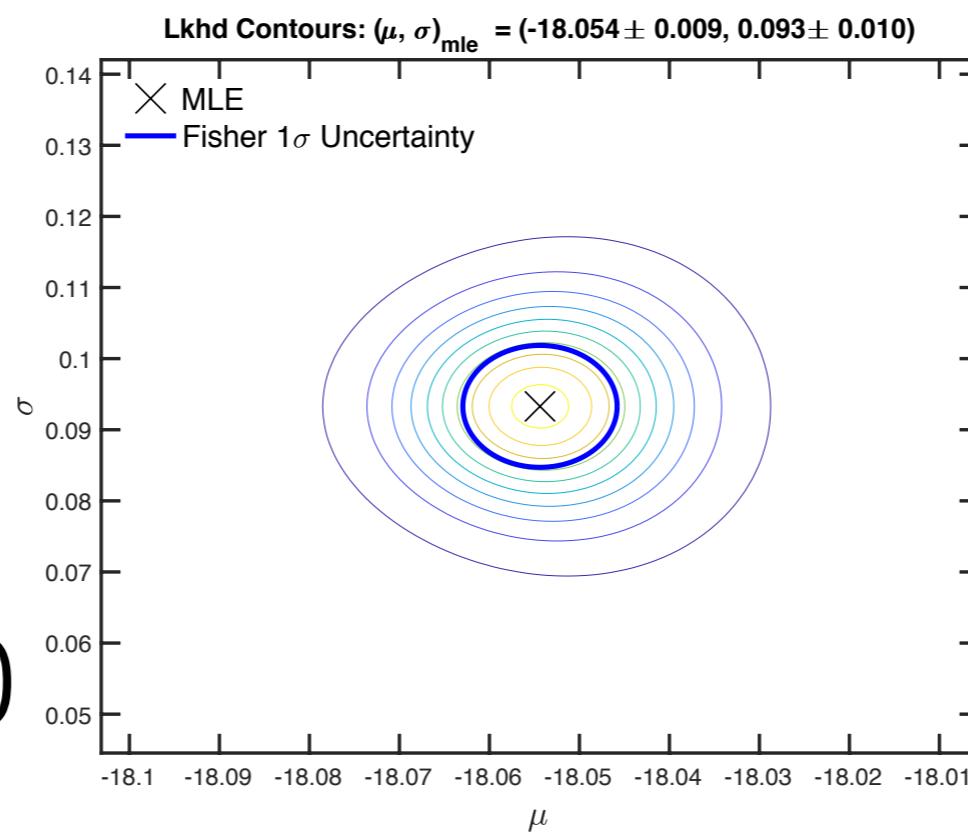


N=50

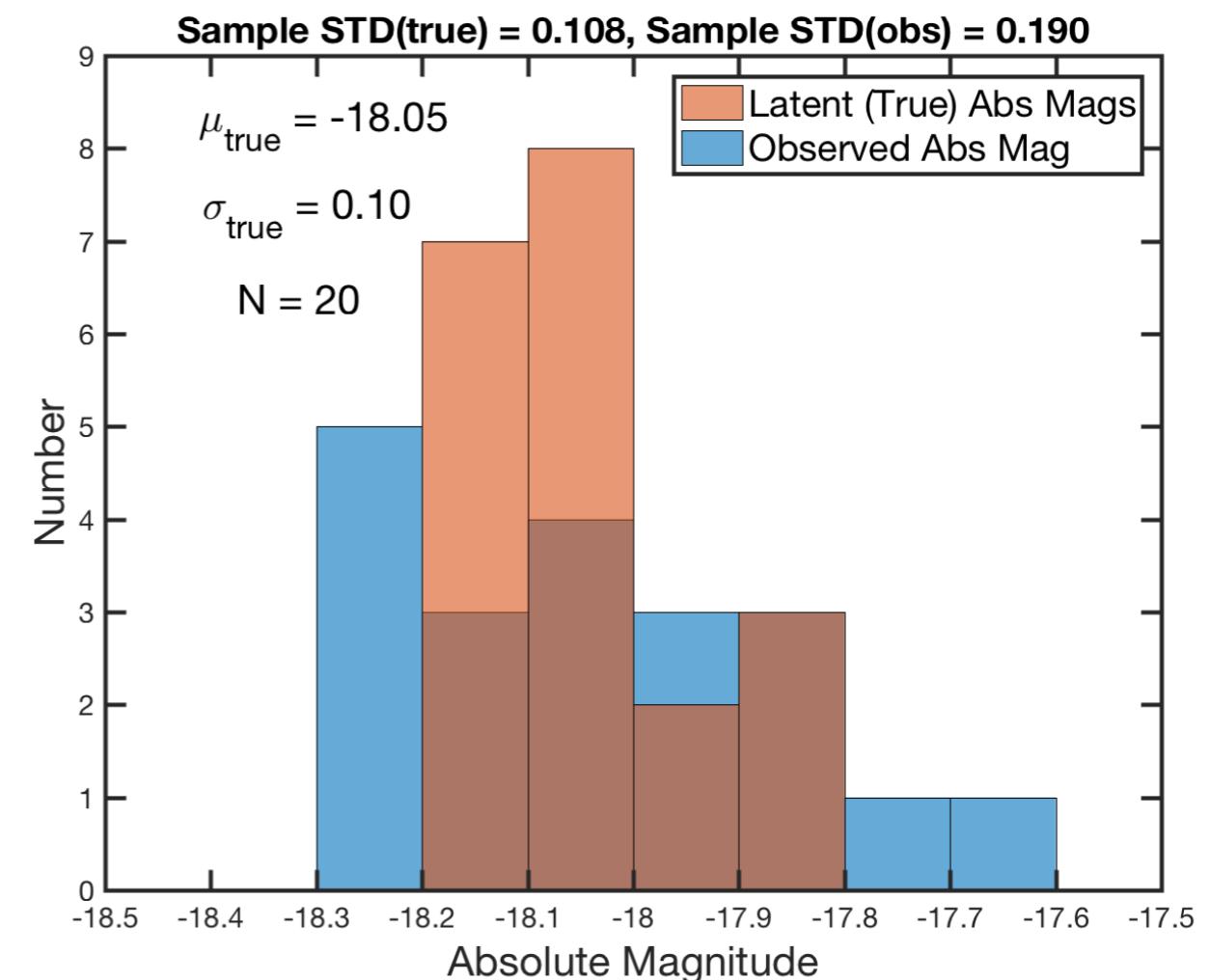
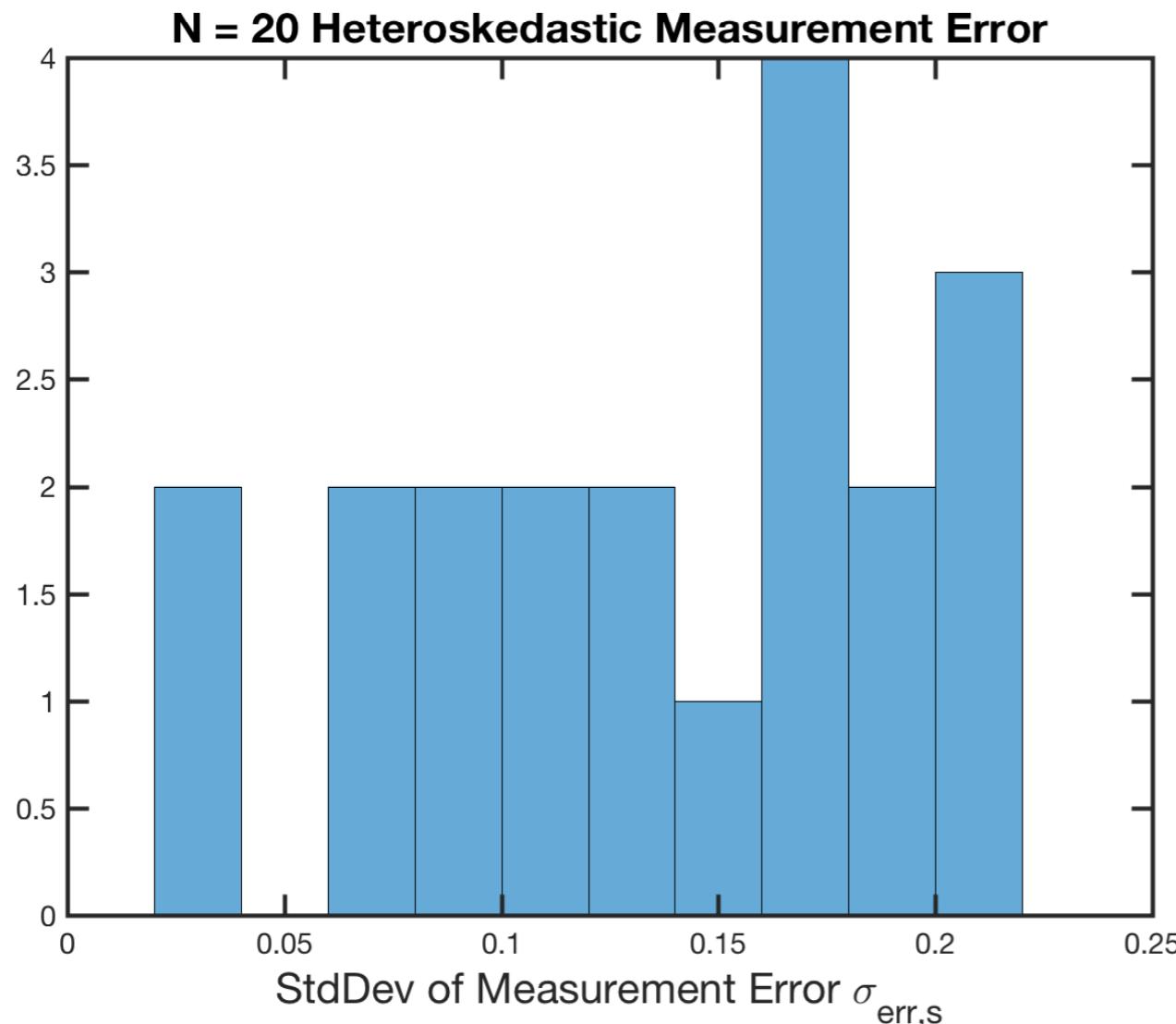


N=100

N=200



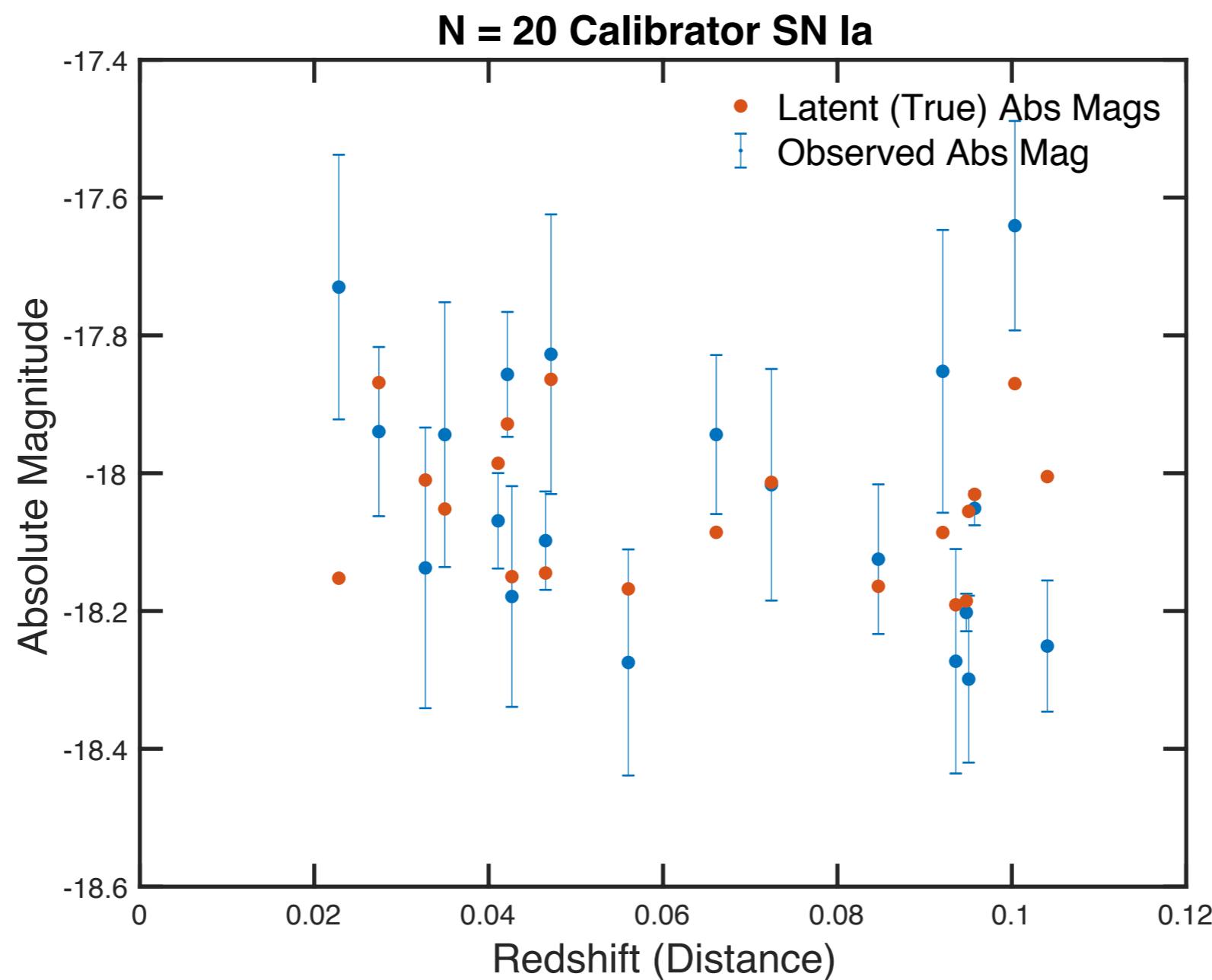
# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to $0.21$



$N = 20$

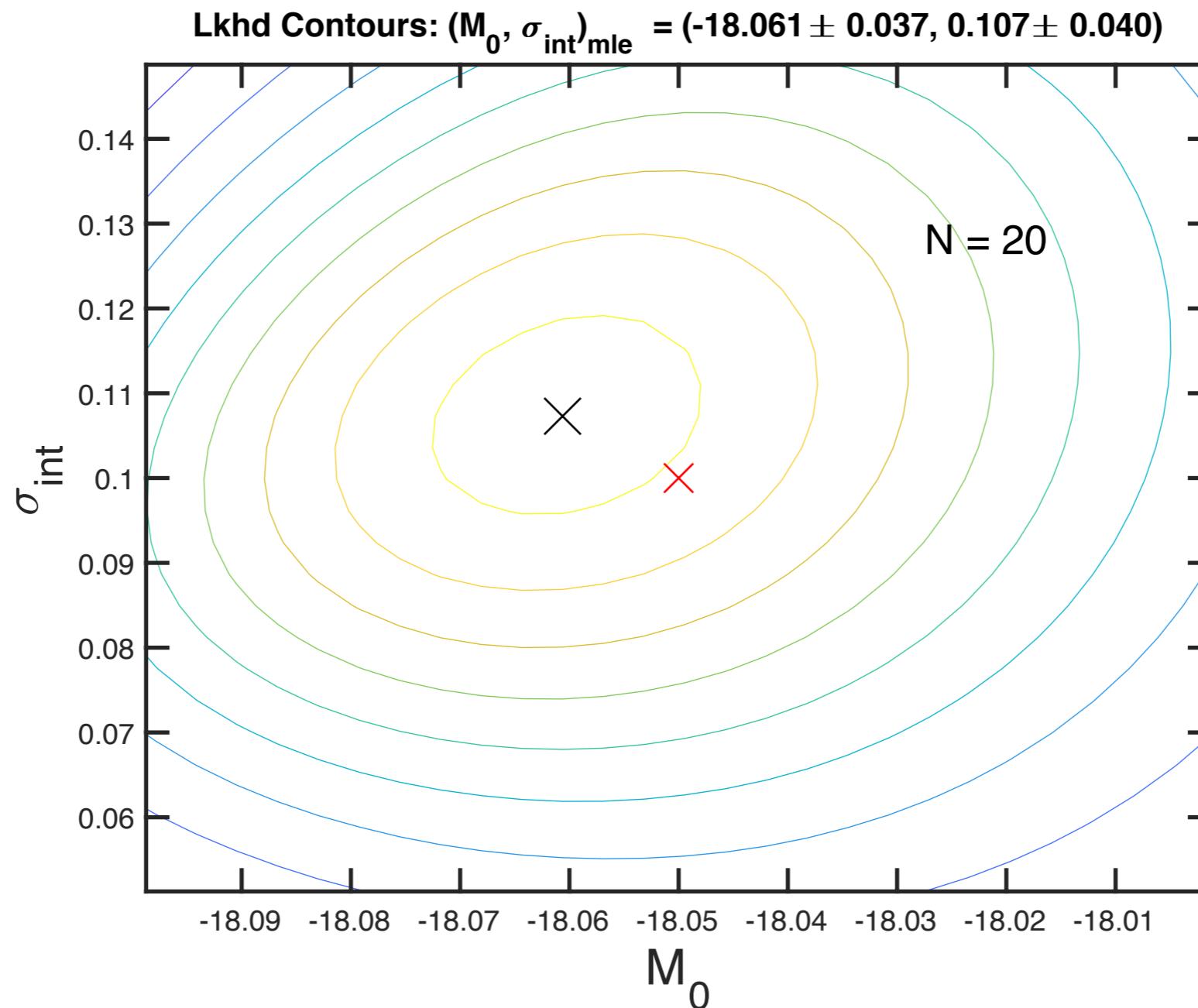
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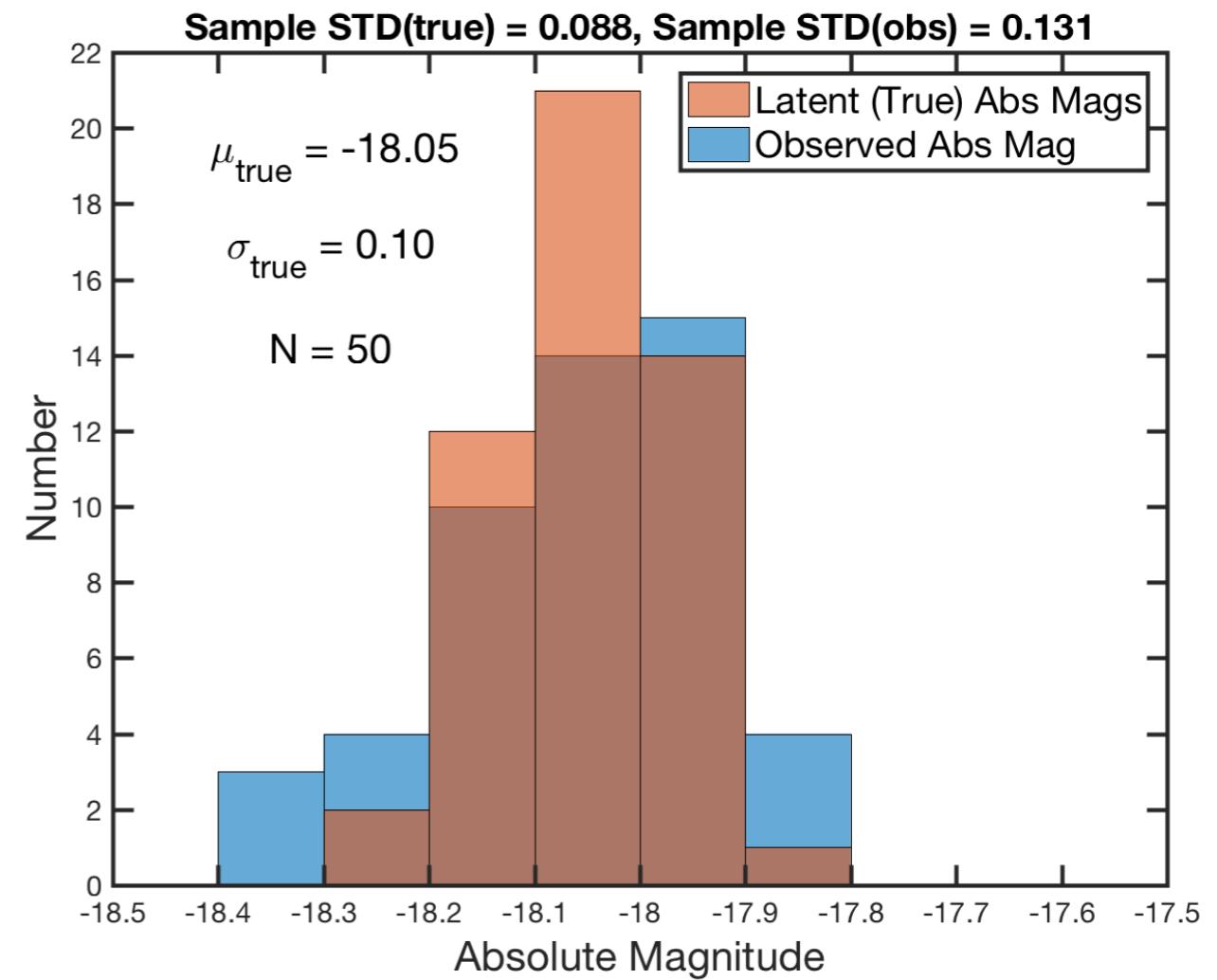
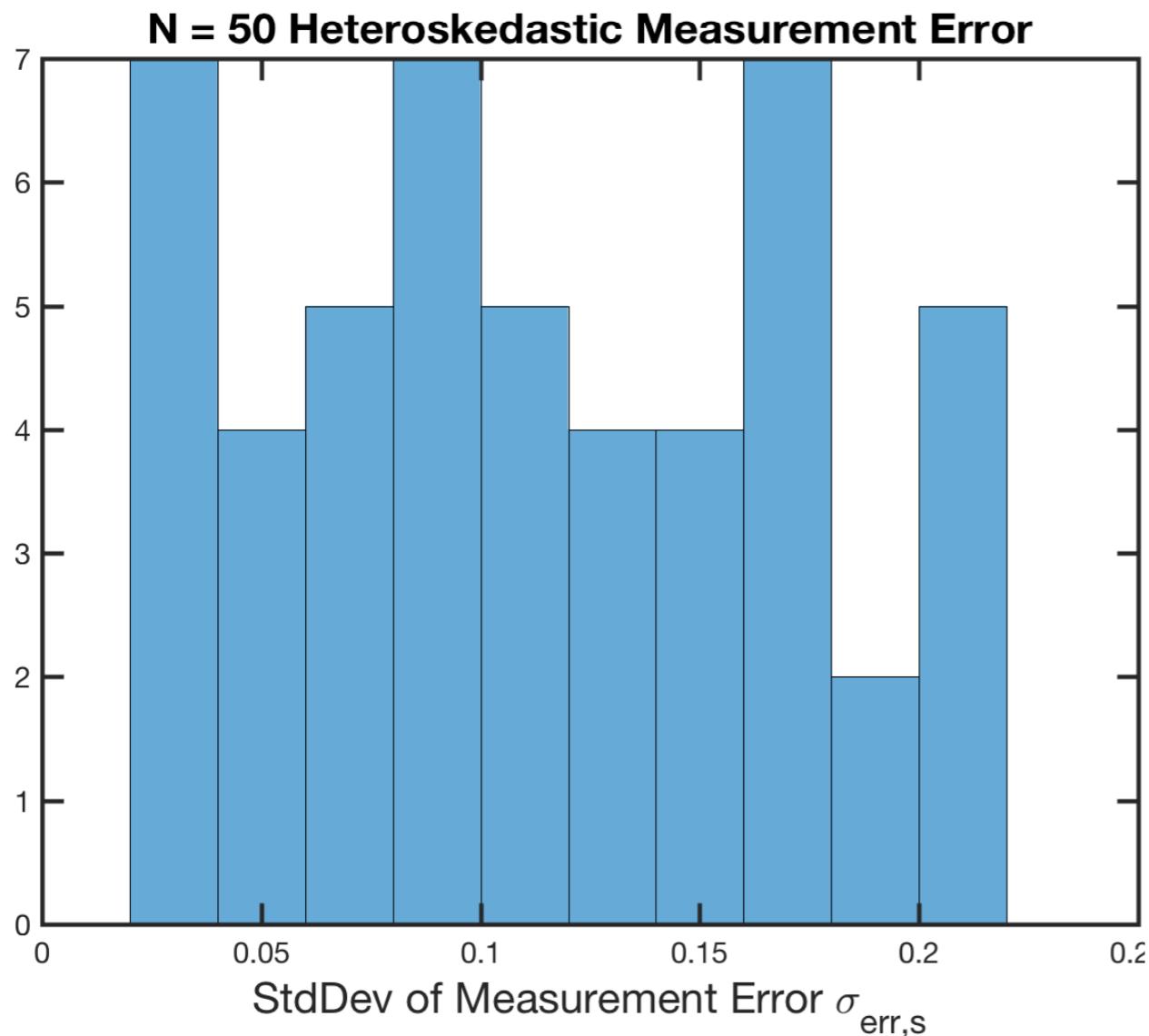


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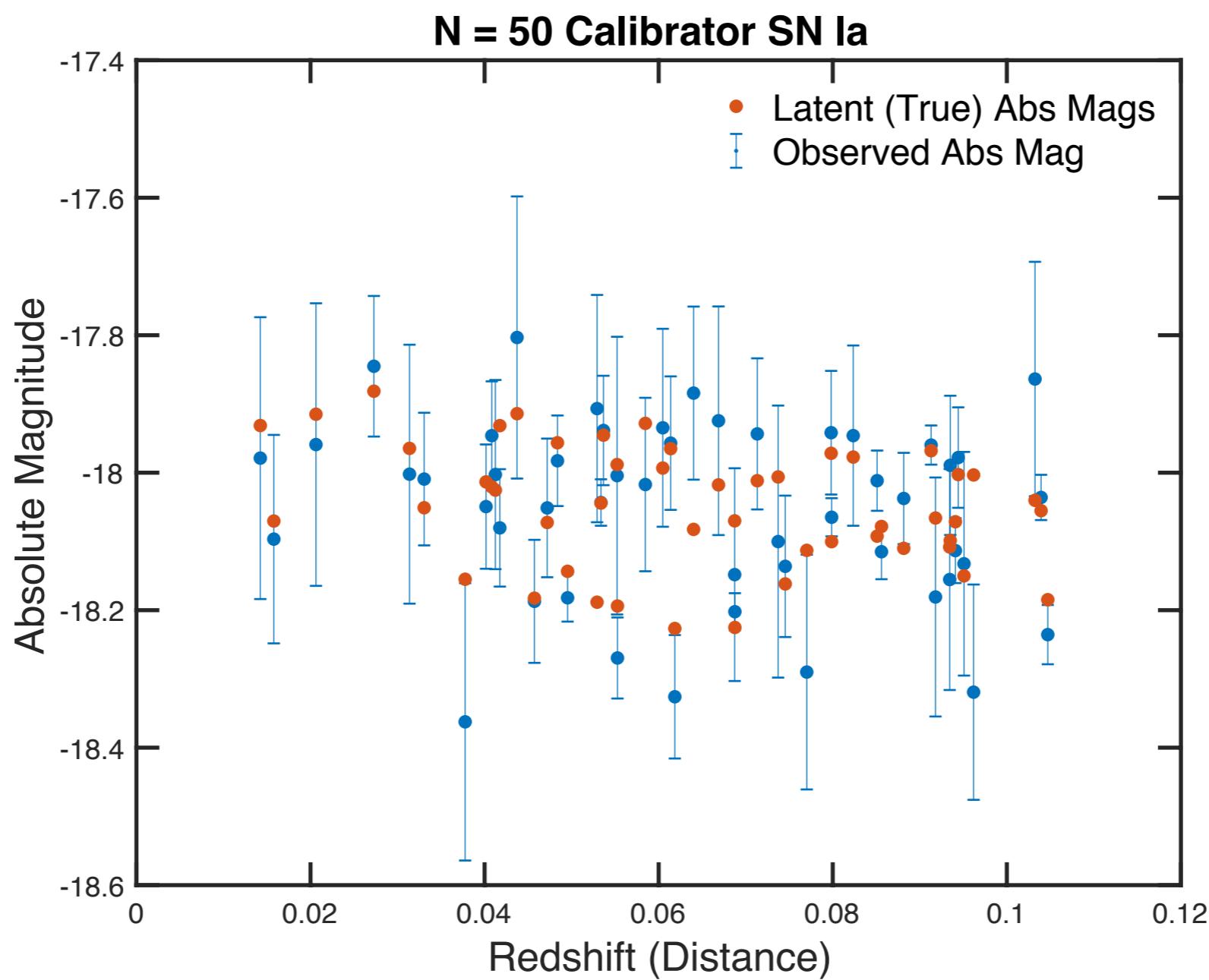
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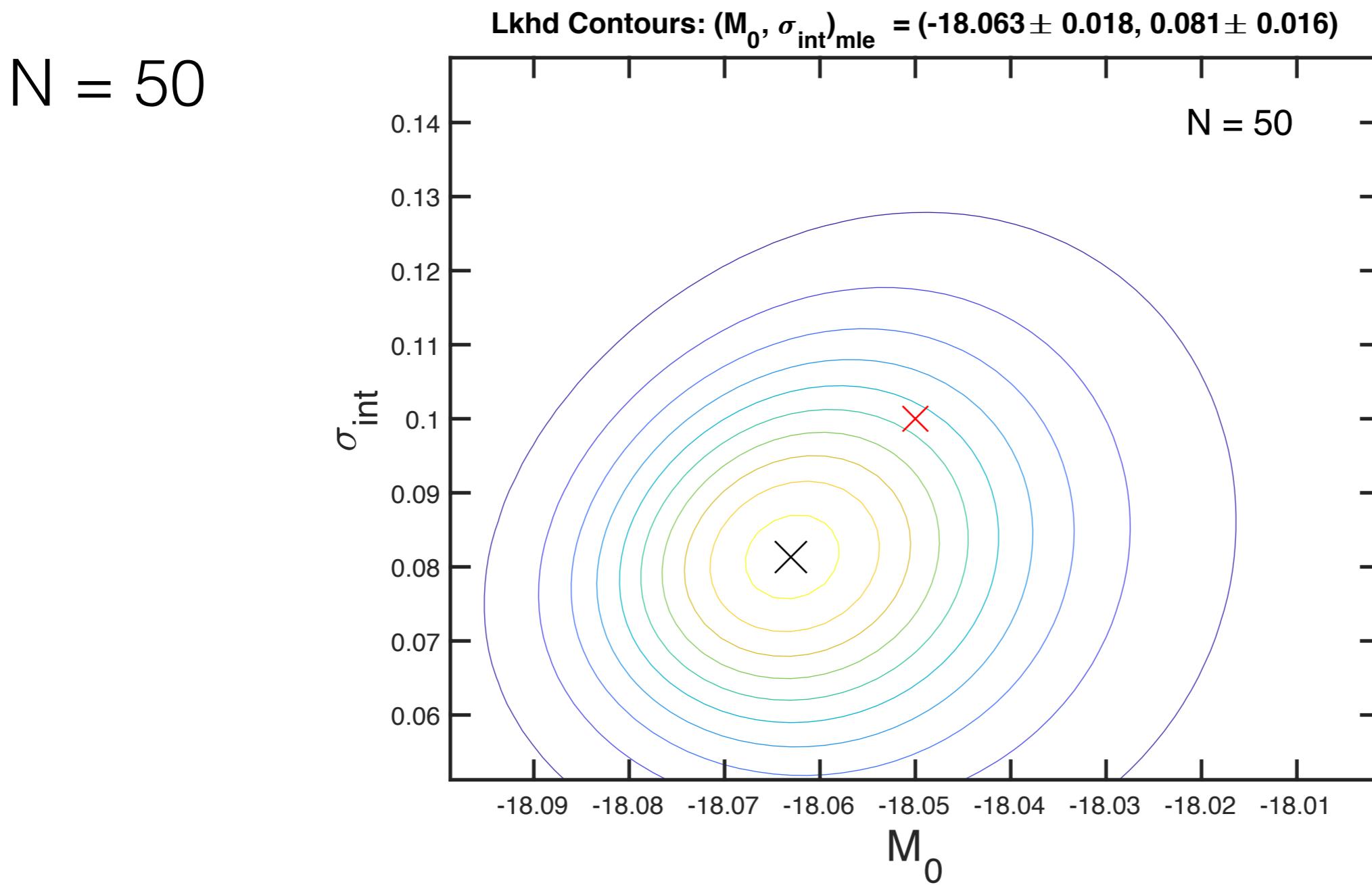
$N = 50$

# Maximum Likelihood with heteroskedastic measurement error with std devs $\sigma_{\text{err}} = 0.01$ to 0.21

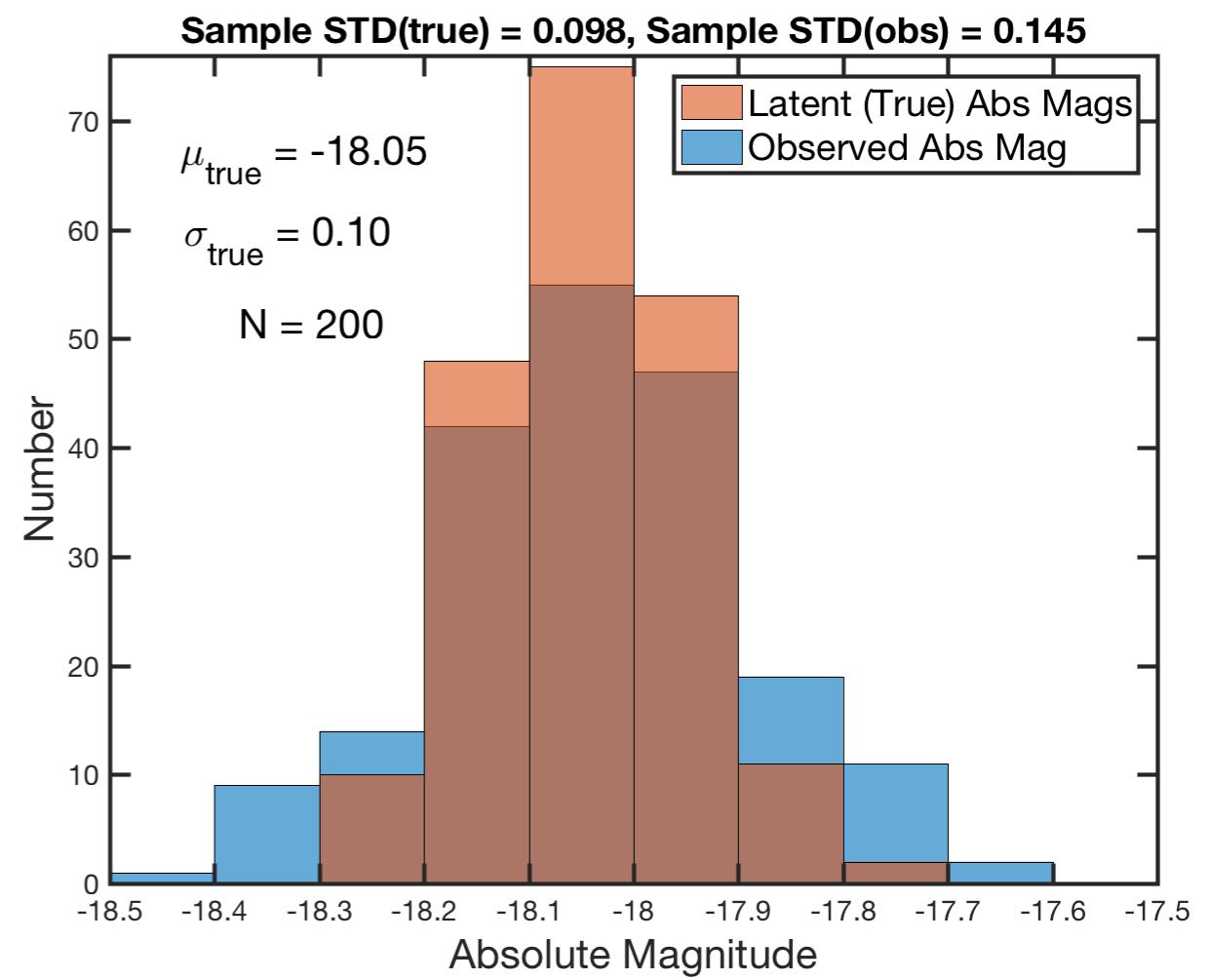
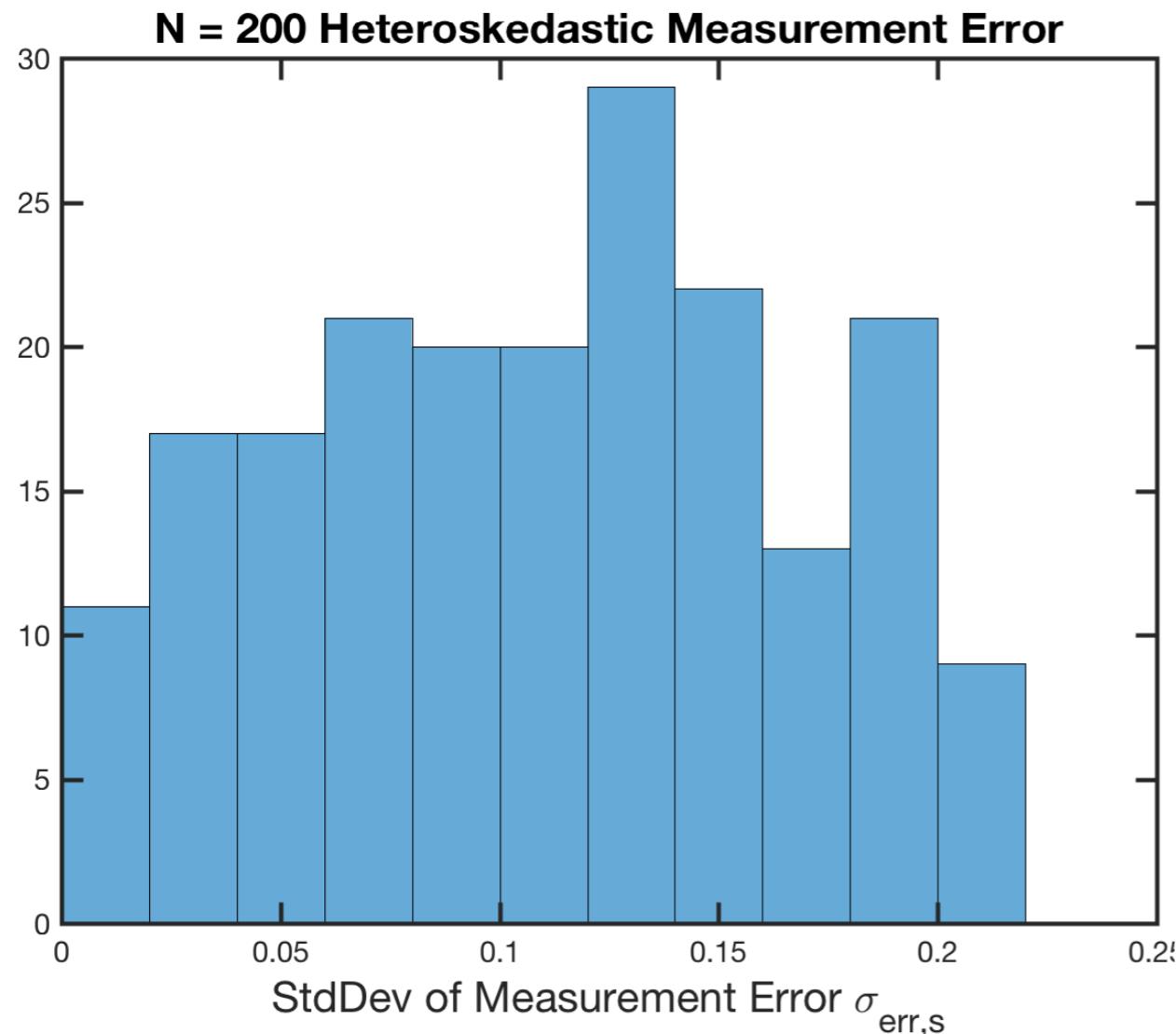
$N = 50$



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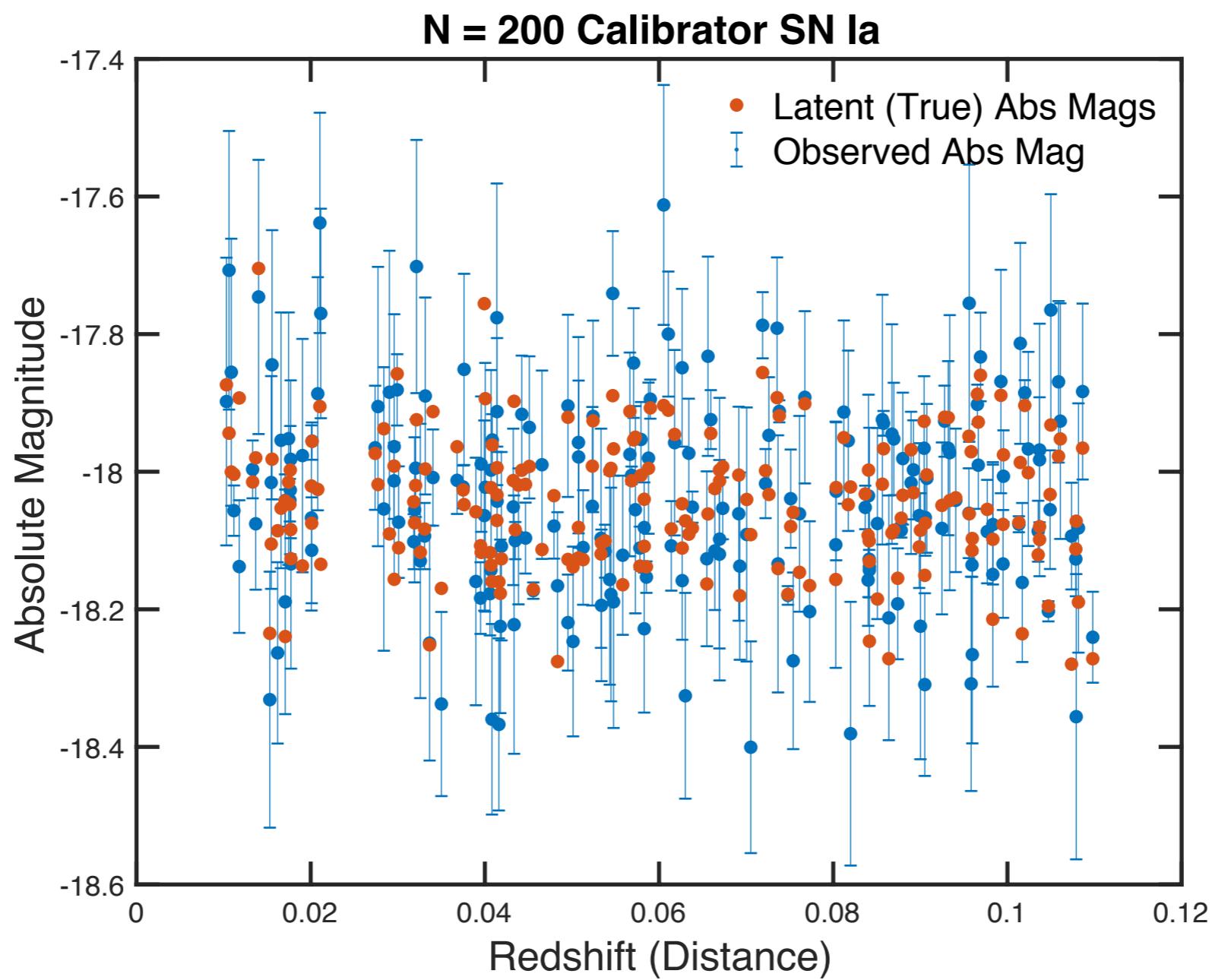
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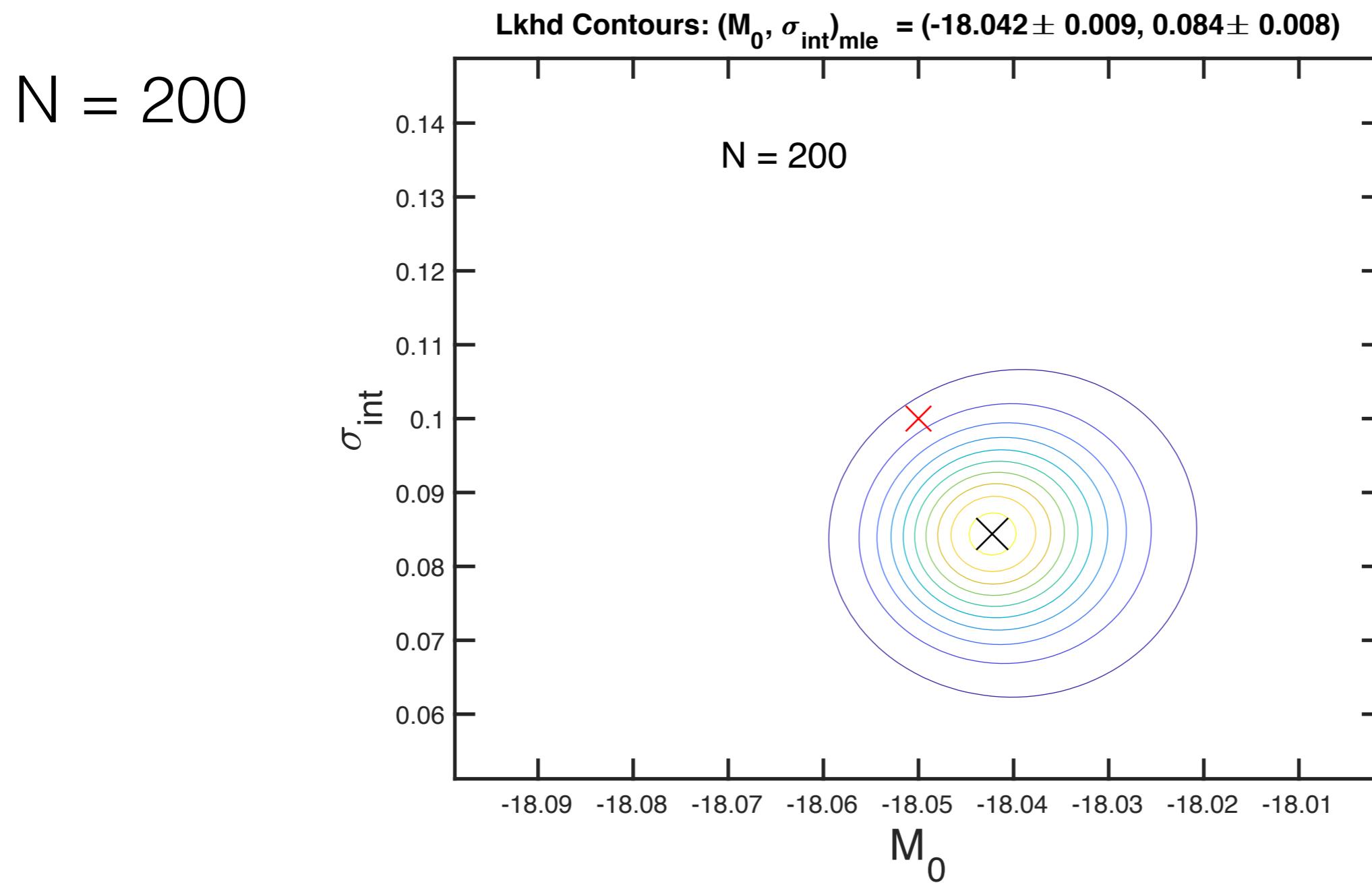
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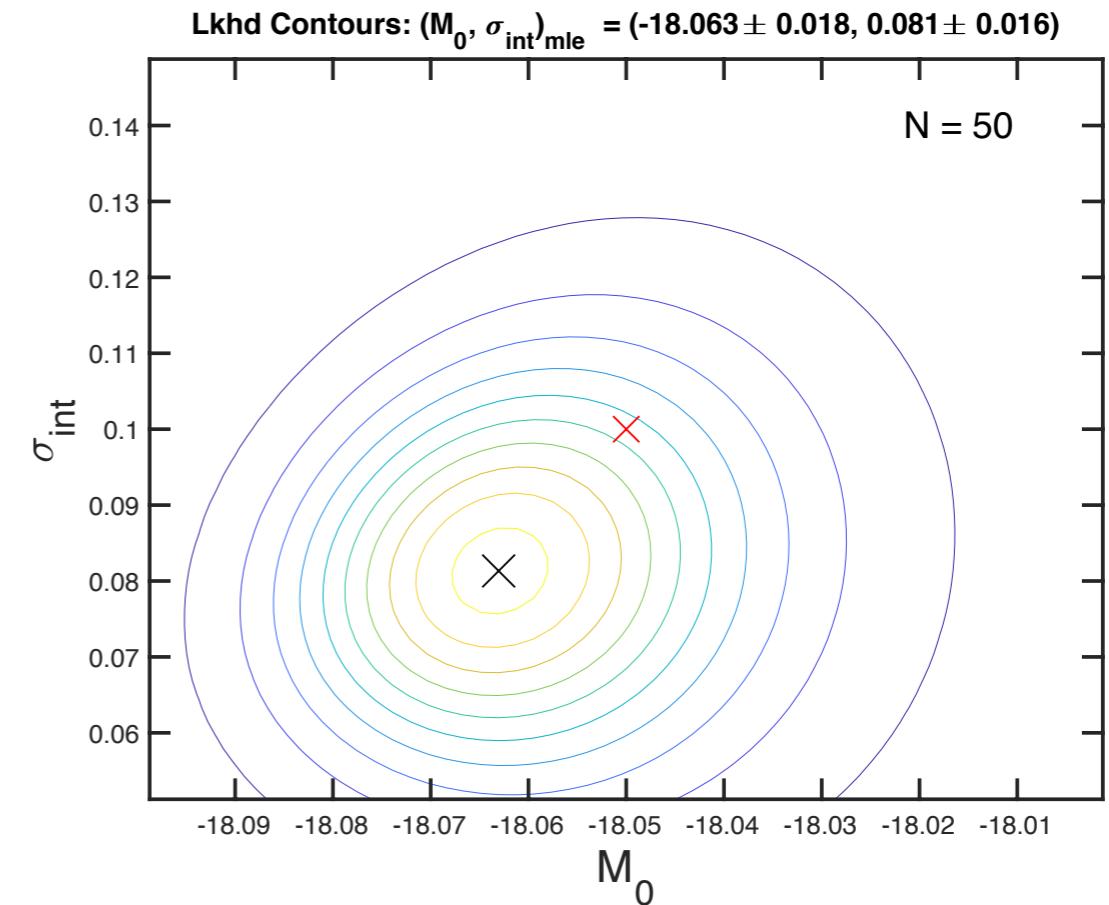
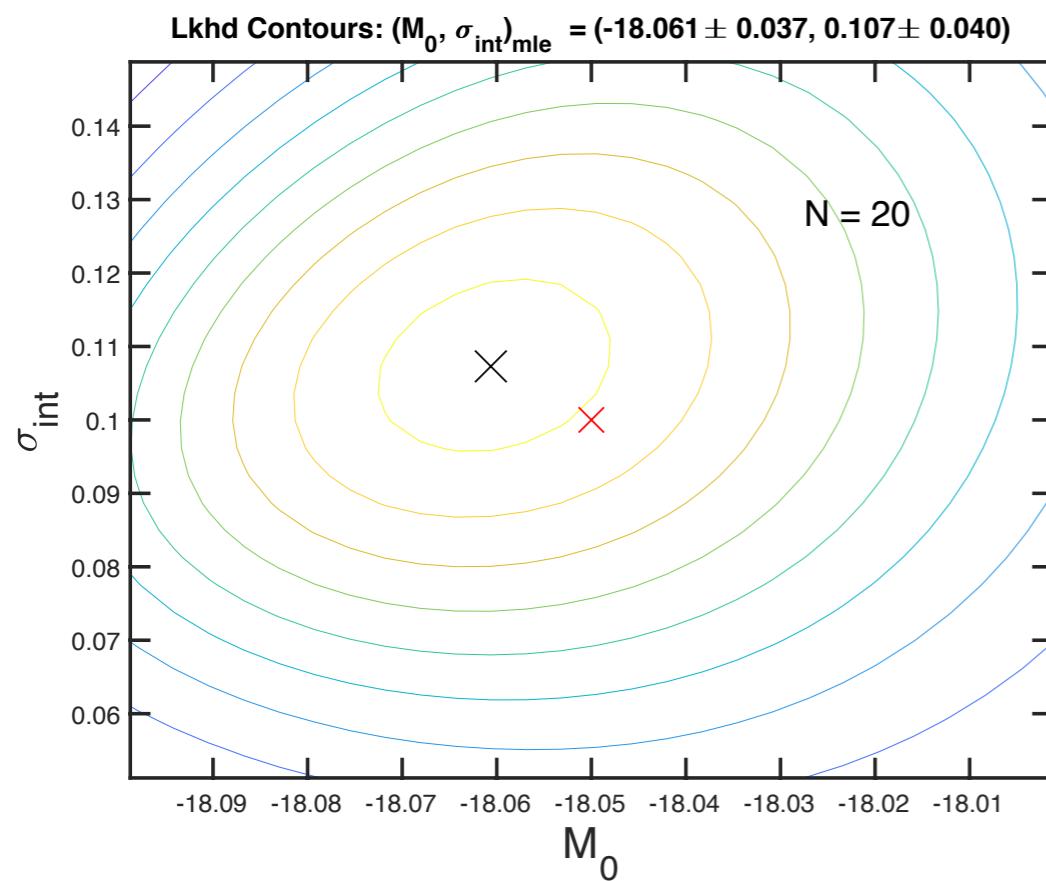
$N = 200$



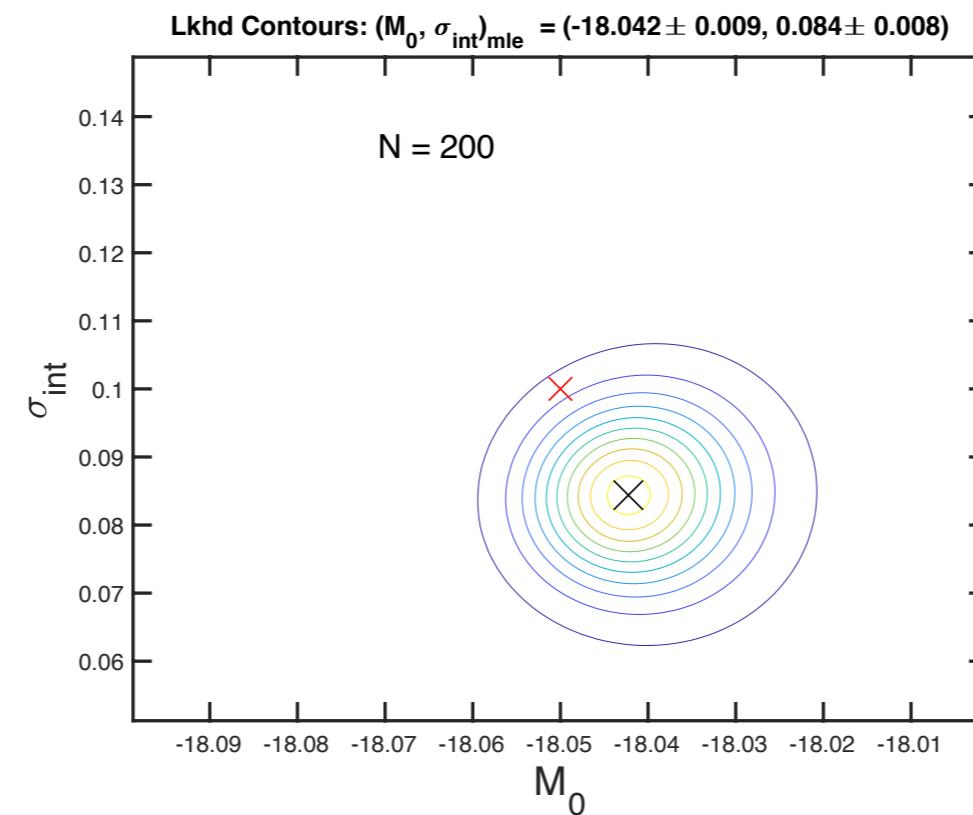
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# Constraints vs. sample size



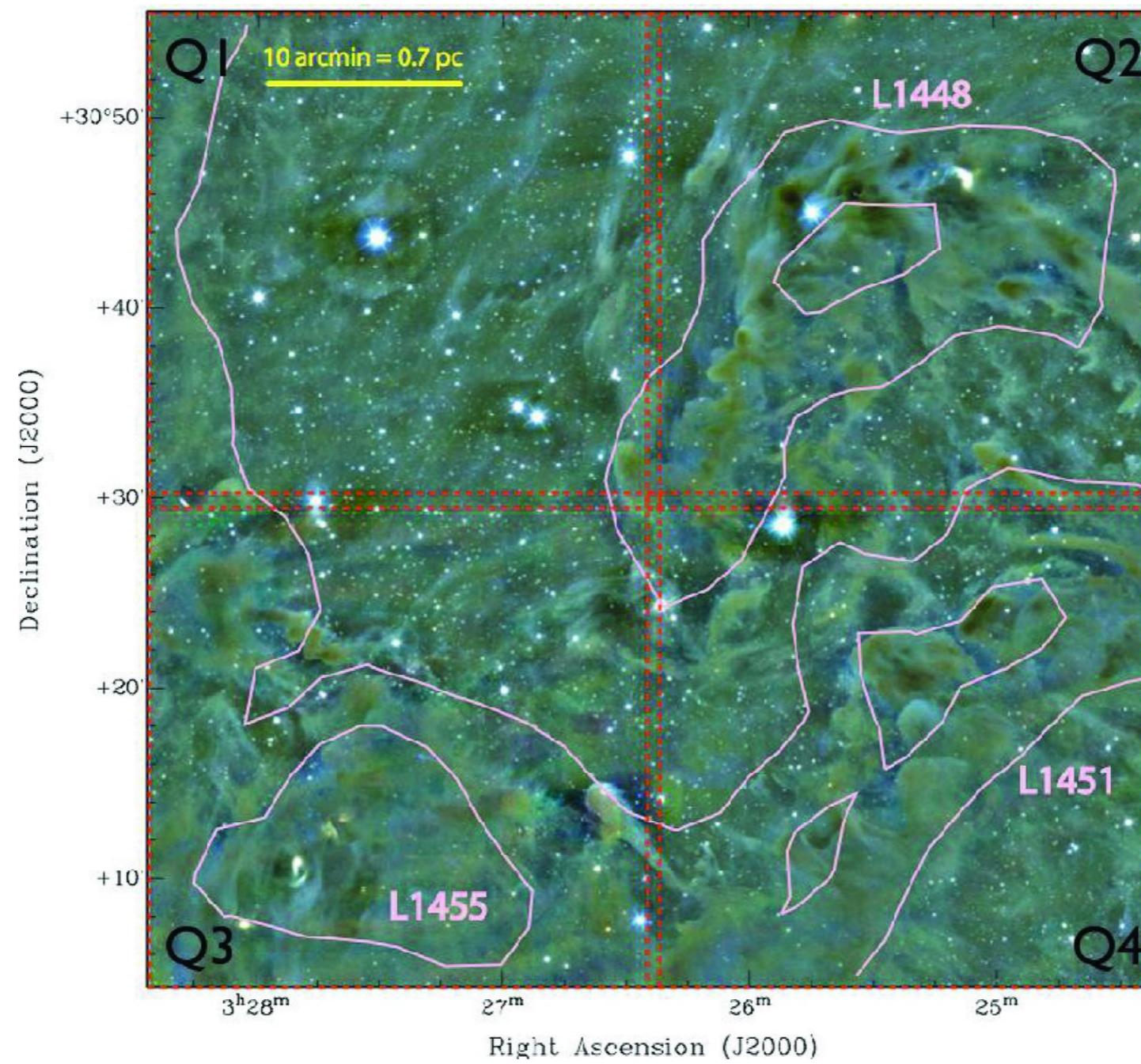
$N=20$



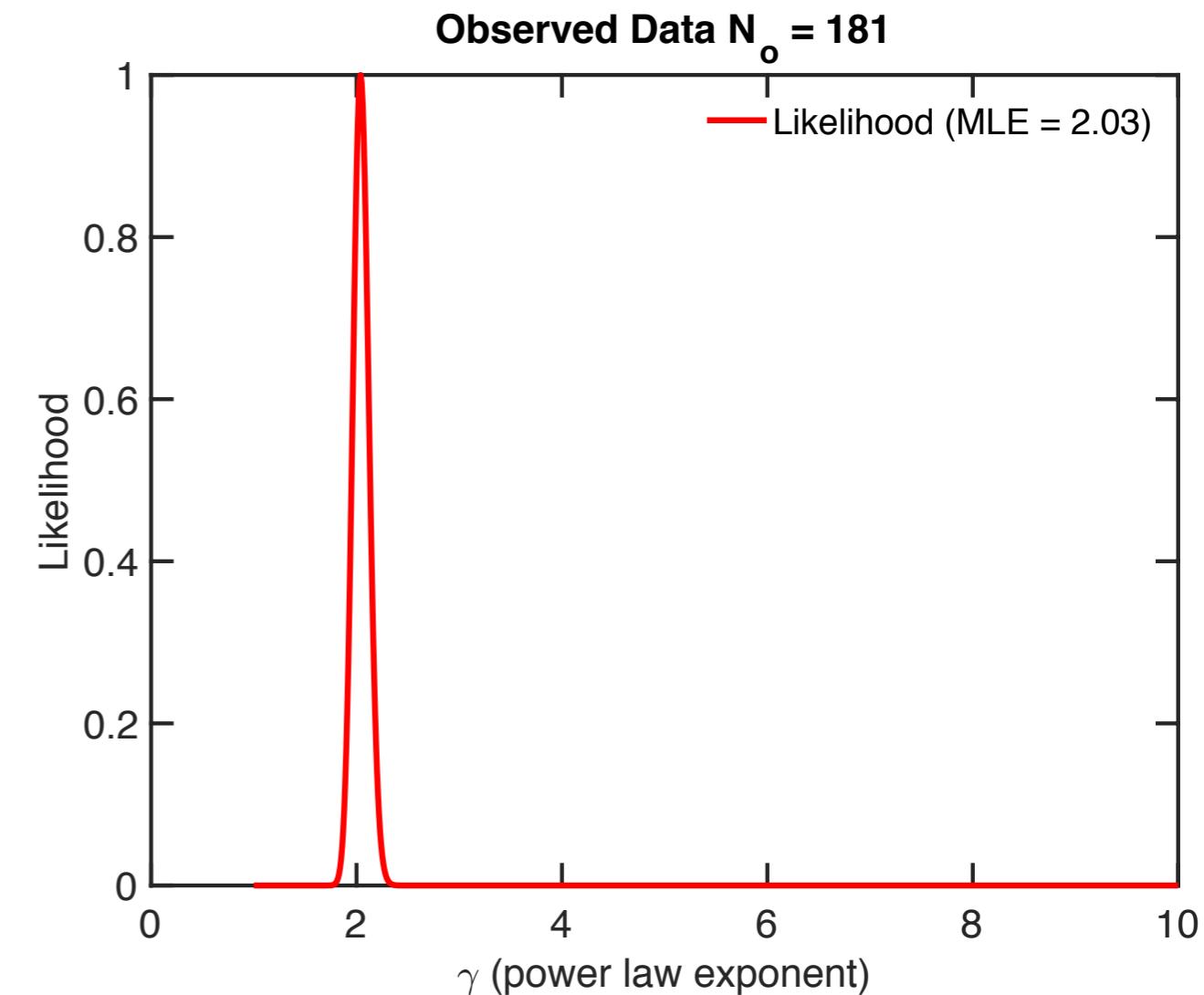
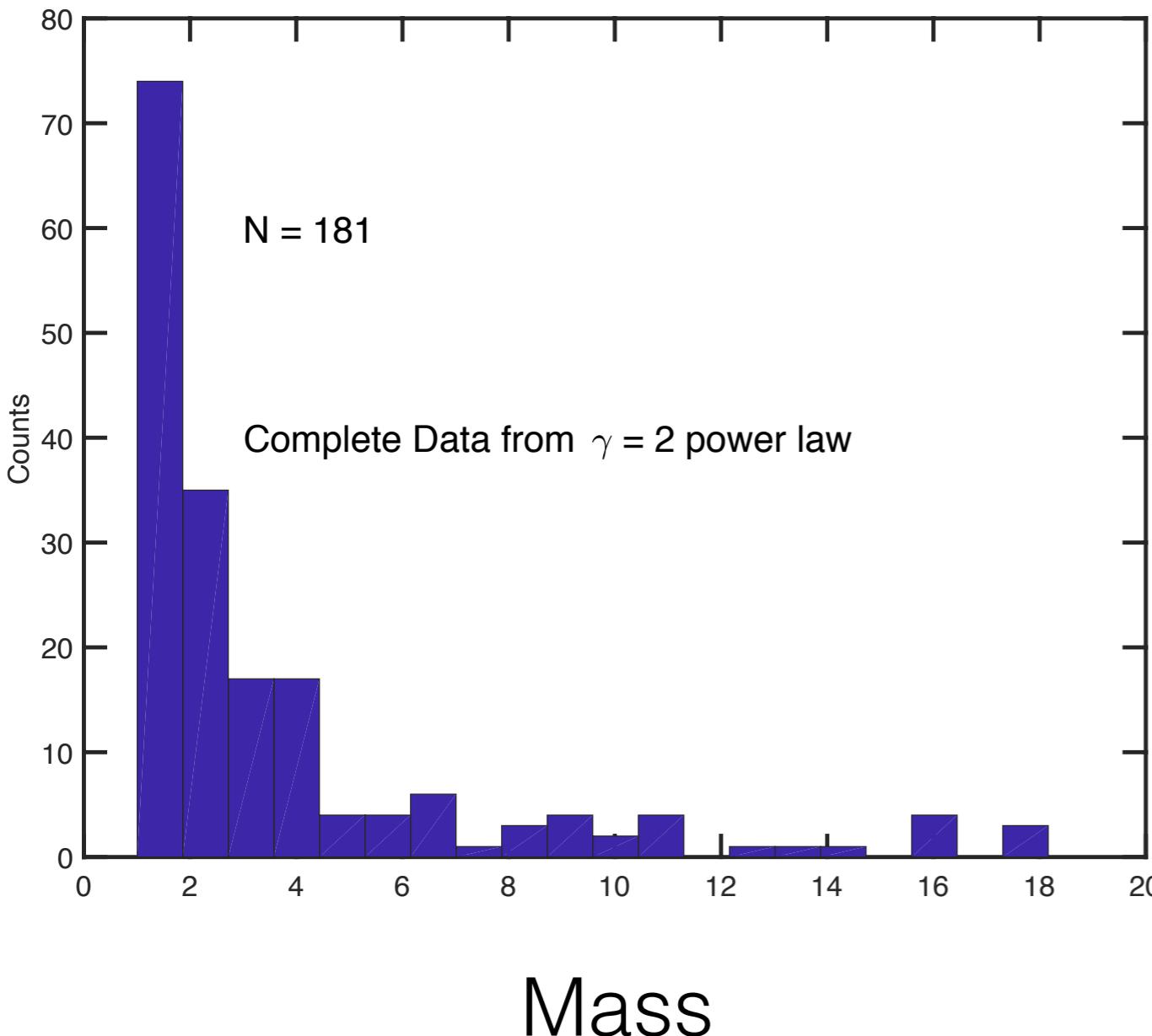
$N=50$

$N=200$

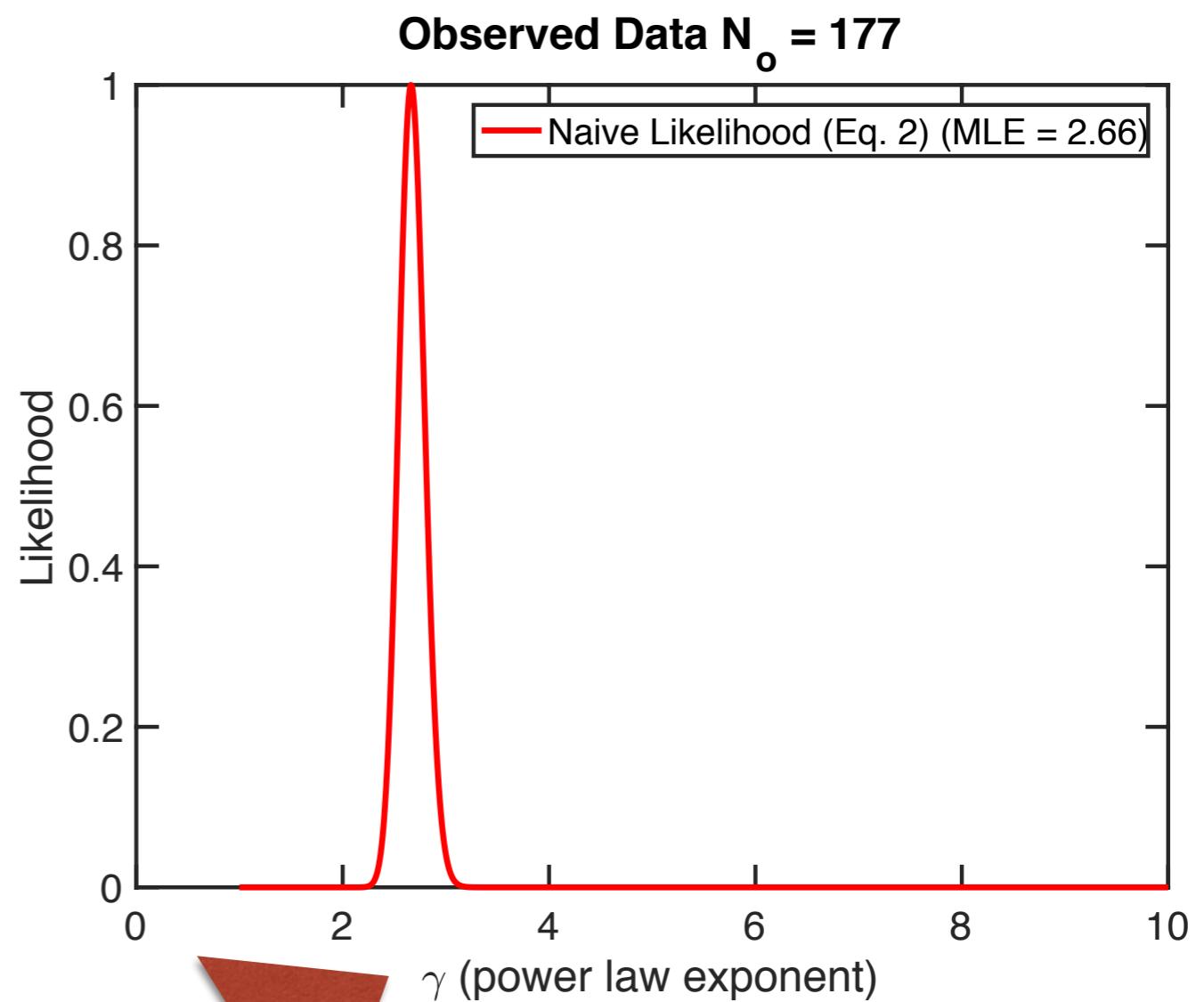
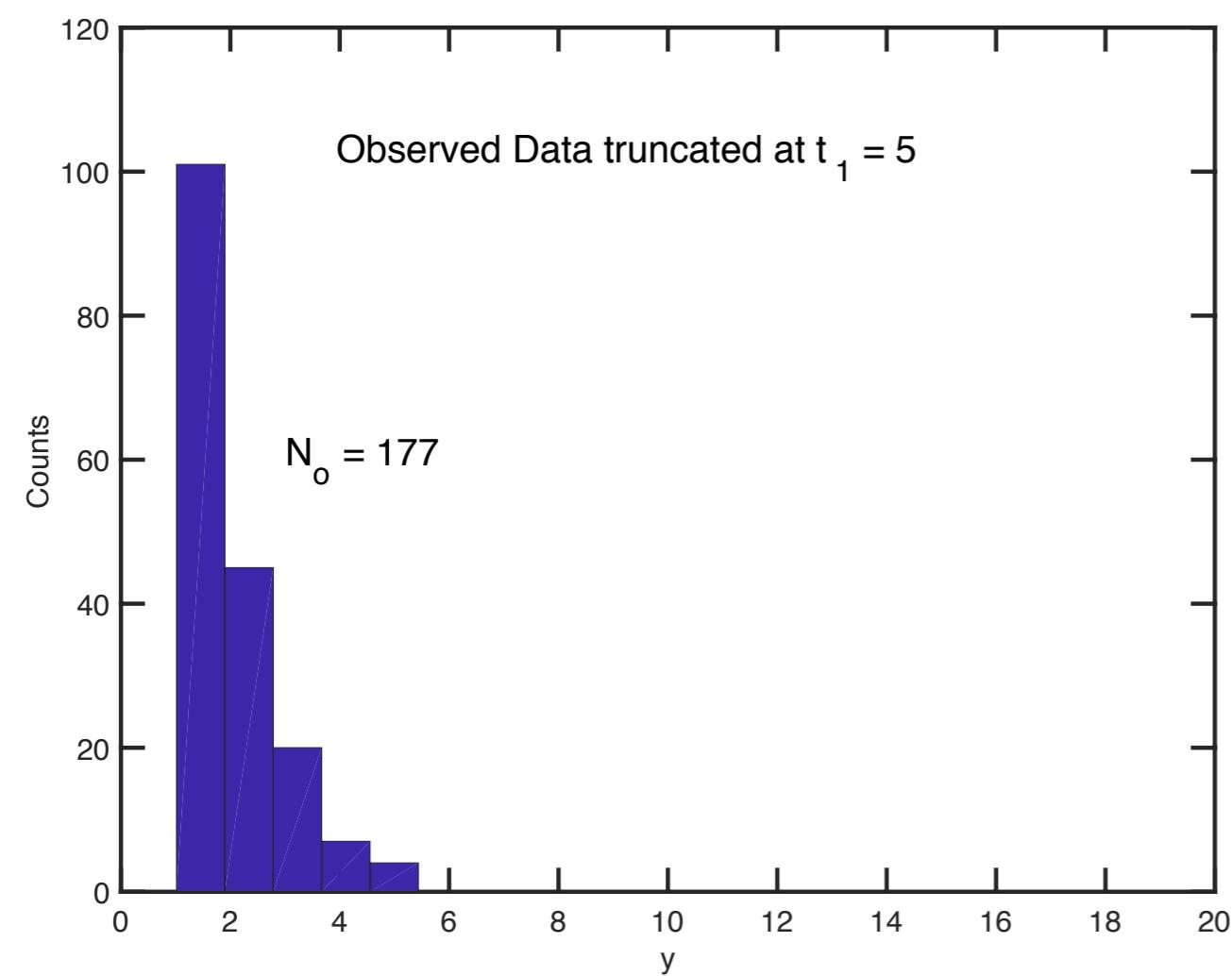
# Star Formation in Perseus



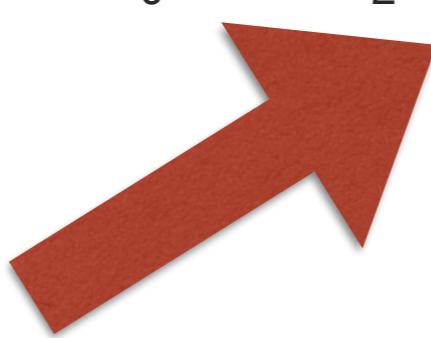
# Inference of Stellar Mass Function

$$P(M) \propto M^{-\gamma}$$


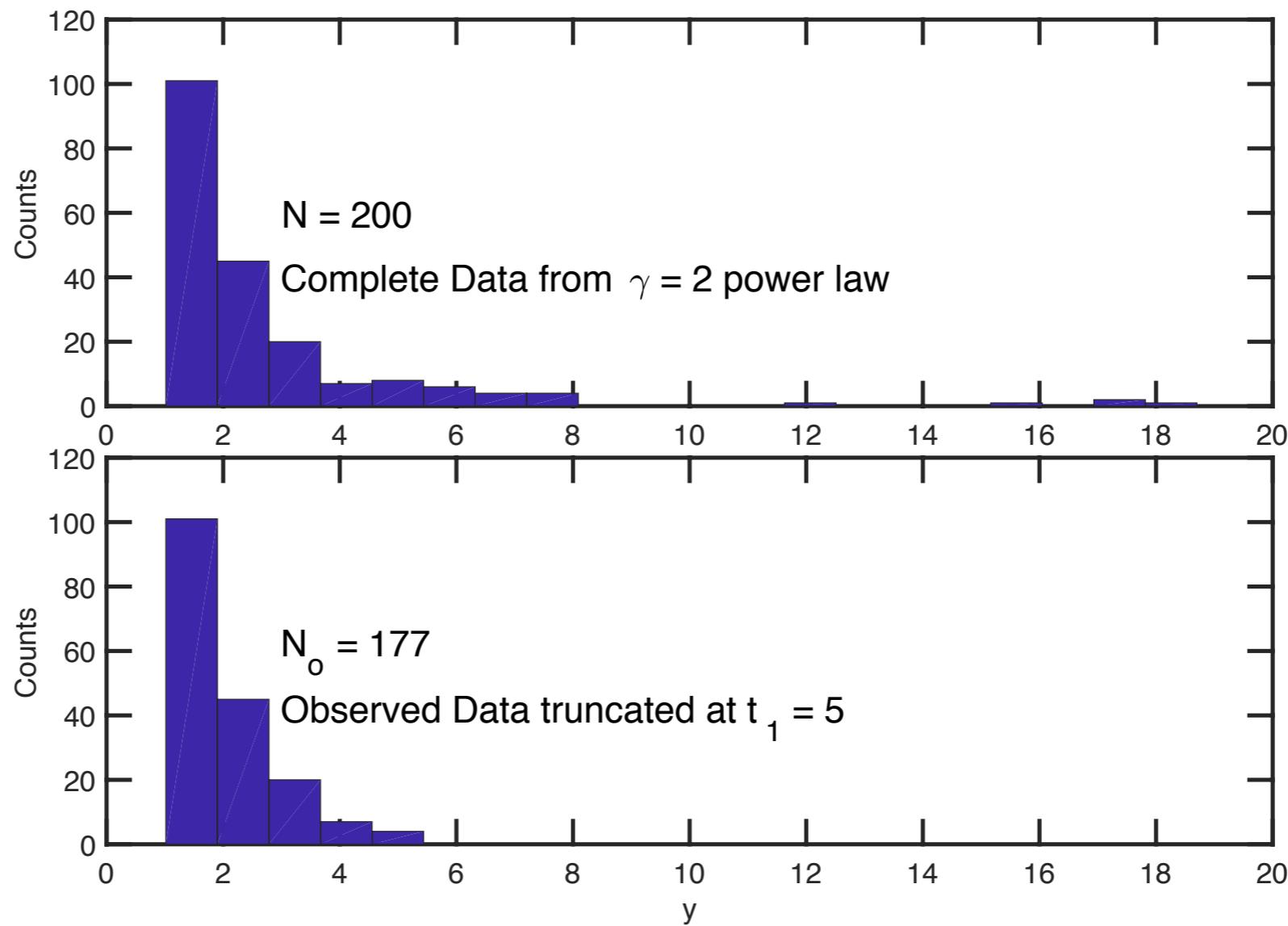
# Selection Effect (Truncation)



MLE Biased!



# Stellar Mass Function Inference with Selection Effect: Complete Data vs. Truncated Data



# Stellar Mass Function Inference with Selection Effect: Modified Likelihood Function

