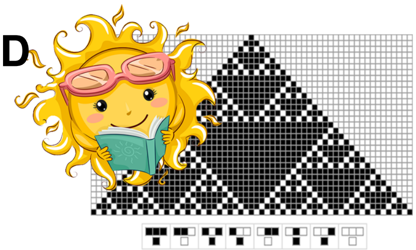
Finding the Inverse Cellular Automata of a given 1-D Reversible CA



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Reversibility is one of the important characteristic in the domain of cellular automata since it gives the notion of preserving the information during the evolution. This property leads to give wide aspect of applications in many real life scenarios. The main objective of this project work is to analyze the bijectivity property of the reversible CA with the help of state transition diagram (STD). This report also describes the modeling approach by mapping of bits of transition states with the rule minterm (RMT) sequences to obtain the inverse function for the reversible and semi-reversible CAs. The reversibility property of the cellular automata describes the preservation of information during the evolution of the cellular automata. The reversibility property also signifies that the each configuration has an preimage or predecessor. Due to the inherent characteristics the reversible CAs are widely used in pattern generation, cryptography, pseudo-random number generation , language recognition.

In our project work, we consider the nature of transition of states of the reversible cellular automata with the help of state transition diagrams and establish the relationship between the different rules of ECA and their inverse functions. We also map the transition of the states in order to analyze the injective and surjective properties of the reversible cellular automata. Figure 1 describes the state transition diagram and the mapping of states to show the bijective property that the CA with rule number 15 which is a reversible CA for a 4-cell under periodic boundary condition.

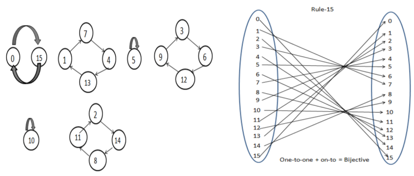

Figure 1: (a)State Transition Diagram (b)Mapping Function

In the next phase of our project work we propose an approach to find the inverse function of the reversible CA for a 4-cell perodic boundary configuration by considering the transition states in a reverse order. In the mapping we consider a window of size three for 3-neighborhood structure for two successive configurations and then we trace the change in the corresponding bit of the window for a map of that changed bit in the RMT sequences.After mapping of all bits in RMT sequences we observe the inverse function rule by finding the decimal equivalent of the RMT bits arranged from left to right with MSB in the left and the LSB in the right. Figure. 2 describes the mapping process for finding the inverse function.

As in the initial phase, we consider the CA under periodic boundary condition for cell size of 4. Then in successive cases we investigate the inverse rule finding approach for cell sizes of $1, 2, 3, 5, 6$ and 7 respectively. Therfore we propose a generalized approach for the finding of inverse rule for reversible CA of length n.

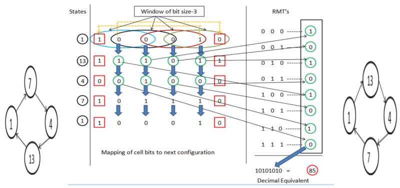

Figure 2: Rule mapping for inverse function(Rule-15)

We propose a generalized appraoch for find the inverse rule for a reversible CA for length of size n. From the observations and work analysis of all the ECA rules, the reversible CAs of length n under periodic boundary conditions are listed in Table. 1.

Table 1: List of Reversible ECAs

Reversible ECAs		
Rule No.	Inverse Function Rule	Relationship
15	85	Both rules are inverse of each other
170	240	Both rules are inverse of each other
51	51	Inverse functioned rule of itself
204	204	Inverse functioned rule of itself

The previous paragraph describes the finding of inverse rule for reversible CA for ECA rules of cell size n under null boundary conditions and in this section we extend our analysis further to

find the inverse rule for the semi-reversible CA.

In our analysis, we consider different non-trivial semi-reversible CA rules like rule-45, rule 154 and rule 105. The non-trivial rule 45 and rule-154 are reversible for odd values of n that means, for $n = 1, 3, 5, 7, \cdots$ where as rule-105 is a reversible CA for n values like $n = 1, 2, 4, 5, 7, 8, \cdots$ and $n \neq 3k, \forall k \in N$. We try to consider the same mapping processes discussed in previous section to

nd the inverse rule for the above mentioned semi-reversible cases. From the work analysis we observe some salient features those are listed below:

- The algorithmic approach for reversible CA is true for semi-reversible CA for $n = 1$ and $n = 3$ and the inverse rule is rule-101 with a three neighborhood mapping process.
- imilarly for rule-154, the inverse rule mapping approach is true for $n = 1$ and $n = 3$ and the inverse rule is rule-180 with a three neighborhood mapping process.