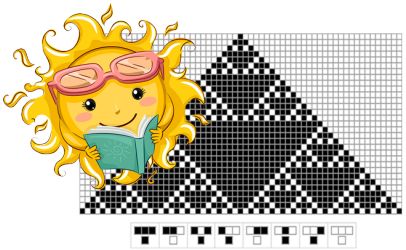
Characterization of 2D Linear Cellular Automata by using Linear Algebra



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In this project we studied the behavior of $2D$ linear Automaton generated by linear rule under the null boundary condition over the field $\mathbb{Z}^2$ transition matrix or rule matrix of this cellular Automata by represent each cell of block into linear form $1, 0$ according to dependency and independency of each cell respectively determined by applied rule .viz,for 2×2 block there is 4 cell, we identified each cell by the number $1, 2, 3, 4$ and for each cell like cell number 1,we put 1,0 if it is dependent or independent to particular that cell in appropriate order respectively, corresponding to each cell we get one representation and ultimately we obtain a 4×4 matrix, for 3×3 matrix we obtain 9×9 order matrix. We are characterize this Automata by using matrix rule and able to Identify Reversibility of this Automata by calculating the Determinant of the Rule matrix. Lastly, we checked Reversibility of 2×2 block and 3×3 block for all 32 rules over $\mathbb{Z}^2$ field.

Table 1: Results for 2×2 Block

Rule	Reversible/Irreversible
Rule 0	Irreversible
Rule 1	Reversible
Rule 2	Irreversible
Rule 3	Irreversible
Rule 4	Irreversible
Rule 5	Irreversible
Rule 6.	Irreversible
Rule 7.	Reversible
Rule 8.	Irreversible
Rule 9.	Reversible
Rule 10.	Irreversible
Rule 11.	Reversible
Rule 12.	Reversible
Rule 13.	Irreversible
Rule 14.	Reversible
Rule 15.	Reversible
Rule 16.	Irreversible
Rule 17.	Reversible
Rule 18.	Reversible
Rule 19.	Reversible
Rule 20.	Irreversible
Rule 21.	Reversible
Rule 22.	Irreversible
Rule 23.	Irreversible
Rule 24.	Irreversible
Rule 25.	Irreversible
Rule 26.	Irreversible
Rule 27.	Irreversible
Rule 28.	Reversible
Rule 29.	Reversible
Rule 30.	Reversible
Rule 31.	Reversible.

Working procedure: Rule 31 :This is an example of a Reversible Automata

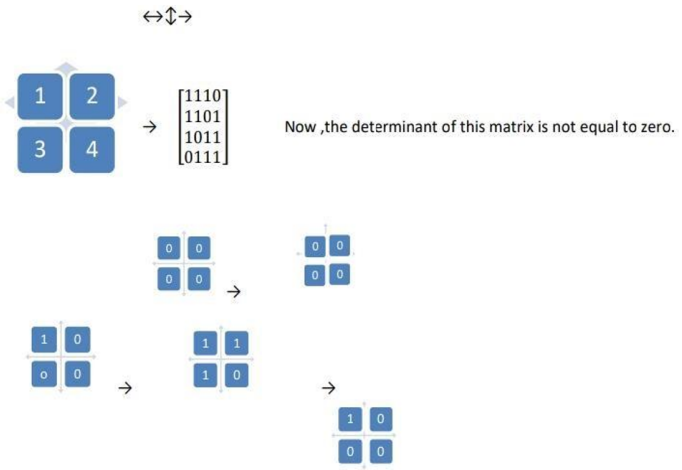

Figure 1: Working procedure of Rule 31; This is an example of a Reversible Automata ;