Logic Representation

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How to represent the knowledge into logical form?

- There are two ways to represents the knowledge:
- Propositional Logic
- Predicate Logic

Propositional logic

- Propositional logic (PL) is the simplest form of logic where all the statements are made by propositions.
- A proposition is a declarative statement which is either true or false.
- It is a technique of knowledge representation in logical and mathematical form.

- Example:
  a) It is Sunday.
  b) The Sun rises from West (False proposition)
  c) 3+3= 7(False proposition)
  d) 5 is a prime number.

Kinds of Proposition

- **Atomic Propositions:**
- Atomic propositions are the simple propositions. It consists of a single proposition symbol. These are the sentences which must be either true or false.
  - 2+2 is 4, it is an atomic proposition as it is a true fact.
  - "The Sun is cold" is also a proposition as it is a false fact.
- **Compound propositions**
- Compound propositions are constructed by combining simpler or atomic propositions, using parenthesis and logical connectives.
  - "It is raining today, and street is wet."
  - "Ankit is a doctor, and his clinic is in Mumbai."

Commonly used connectives

Name	Connective	Symbol
Conjunction	AND	&
Inclusive Disjunction	OR	V
Negation	NOT	~
Material Condition	IMPLIES	$\rightarrow$
Material Biconditional	IFF	$\leftrightarrow$
Exclusive Disjunction	XOR	+
Joint Denial	NAND	|
Disjoint Denial	NOR	$\downarrow$

Syntax/Rules of Propositional Logic

- Atomic proposition are represented as A…Z and are known as well form atomic proposition.
- If A and B are well form proposition then, ~A, (A&B), (AVB), (A->B), (A<->B), (A+B), (A|B), (A↓B) are also well form proposition.
- Nothing else is a well form proposition.

Truth table

Table 5.2 Truth tables for logical connectives

A	B	⁻A	⁻B	AVB	A&B	A→B	A↔B	A⊕B	A : B	A : B
T	T	F	F	T	T	T	T	F	F	F
F	T	T	F	T	F	T	F	T	T	F
T	F	F	T	T	F	F	F	T	T	F
F	F	T	T	F	F	T	T	F	T	T

Logical Equivalence

- Two propositions are logically equivalence if they have the same truth table for all combinations.
- (A -> B) ⇔ (~A V B)

Table 5.3 Some commonly used logical equivalences

1.	A	is logically equivalent to	(~~A)
		is logically equivalent to	(A & A)
2.	(A & B)	is logically equivalent to	(B & A)
3.	(A V B)	is logically equivalent to	(B V A)
4.	(A & (B & C))	is logically equivalent to	((A & B) & C)
	(AV(BVC))	is logically equivalent to	((AVB)VC)
5.	(A & (BVC))	is logically equivalent to	((A & B)V(A & C))
	(AV(B & C))	is logically equivalent to	((AVB) & (AVC))
6.	~(A & B)	is logically equivalent to	(~A V ~B)
	~(AVB)	is logically equivalent to	(~A & ~B)
7.	(A→B)	is logically equivalent to	(~A V B)
	(A→B)	is logically equivalent to	~(A & ~B)
	(A→B)	is logically equivalent to	(~B → ~A)
8.	(A→(B→C))	is logically equivalent to	((A→B)→C)
9.	(A ↔ B)	is logically equivalent to	(A & B)V(~A & ~B)
	(A ↔ B)	is logically equivalent to	(A→B) & (B→A)

Tautology and Contradiction

- If the result of a proposition is TRUE for all possible combination, it is known as tautology.
- The dog is either brown, or the dog is not brown.
- $(X \rightarrow Y) \vee (Y \rightarrow X)$
- If the result of a proposition is FALSE for all possible combination, it is known as contradiction.
- $\sim(X \rightarrow Y) \vee (Y \rightarrow X)$

Properties of Operators

- Commutativity:
  - $P \wedge Q = Q \wedge P$, or
  - $P \vee Q = Q \vee P$.
- Associativity:
  - $(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$,
  - $(P \vee Q) \vee R = P \vee (Q \vee R)$
- Identity element:
  - $P \wedge True = P$,
  - $P \vee True = True$.
- Distributive:
  - $P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$.
  - $P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$.
- DE Morgan's Law:
  - $\neg (P \wedge Q) = (\neg P) \vee (\neg Q)$
  - $\neg (P \vee Q) = (\neg P) \wedge (\neg Q)$.
- Double-negation elimination:
  - $\neg (\neg P) = P$.

Limitations of Propositional logic

- We cannot represent relations like ALL, some, or none with propositional logic. Example:
  - All the girls are intelligent.
  - Some apples are sweet.
- Propositional logic has limited expressive power.
- In propositional logic, we cannot describe statements in terms of their properties or logical relationships.

First-Order Logic or Predicate Logic

- A predicate is defined as a relation that binds two atom together.
- E.g.: Bhaskar like airplane can be written as: **LIKES(Bhaskar, airplane)**
- Here, LIKES is a **predicate** and Bhaskar and airplane are **atoms**.
- So the general form can be written as: **LIKES(x,y)**
- E.g.: Ravi's father is Rohits's father**.** **FATHER(father(Ravi), Rohit)**
- Terms: These are the arguments in a predicate.

- Terms can be defined as:
  - A constant is a term.
  - A variable is a term.
  - A function f defined as f(x1, x2…) is also a term.
- It assumes the following things in the world:
  - Objects: A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ...
  - Relations: It can be unary relation such as: red, round, is adjacent, or n-any relation such as: the sister of, brother of, has color, comes between
  - Function: Father of, best friend, third inning of, end of, ...

Basic Elements of First-order logic

Constant	1, 2, A, John, Mumbai, cat,....
Variables	x, y, z, a, b,....
Predicates	Brother, Father, >,....
Function	sqrt, LeftLegOf, ...
Connectives	$\wedge$, $\vee$, $\neg$, $\Rightarrow$, $\Leftrightarrow$
Equality	==
Quantifier	$\forall$, $\exists$

Quantifiers

- It is a symbol that permits to declare or identify the range or scope of the variable in a logical expression.
- There are 2 quantifiers:
- Universal Quantifier (∀): a) All a b) Each a c) Every a
- Existential Quantifier (∃): a) Some b b) at least one b c) exists a b

All man drink coffee

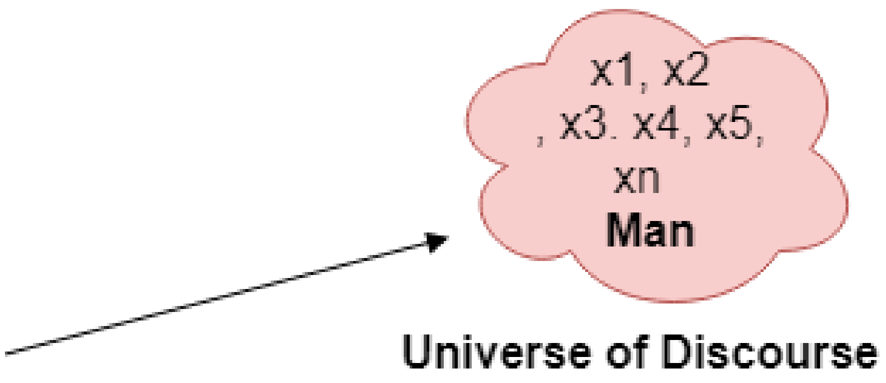
- x1 drinks coffee

  ∧
- x2 drinks

  ∧
- x3 drinks milk

  ∧
- .
- .

  ∧
- xn drinks milk



So in shorthand notation, we can write it as :

$\forall x \ man(x) \rightarrow drink\ (x, coffee).$

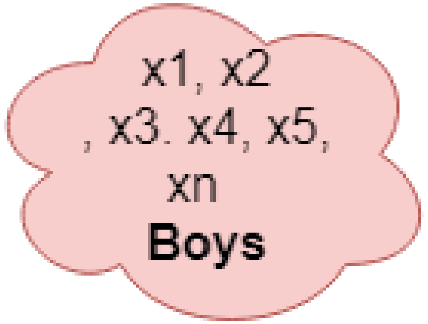
Some boys are intelligent

- x1 is intelligent

  ∨
- x2 is intelligent∨
- x3 is intelligent

  ∨
- .
- .

  ∨
- xn is intelligent



Universe of Discourse

So in short-hand notation, we can write it as:

$\exists x$: **boys(x) $\wedge$ intelligent(x)**

Points to remember

- The main connective for universal quantifier $\forall$ is implication $\rightarrow$.
- The main connective for existential quantifier $\exists$ is and $\wedge$.
- In universal quantifier, $\forall x \forall y$ is similar to $\forall y \forall x$.
- In Existential quantifier, $\exists x \exists y$ is similar to $\exists y \exists x$.
- $\exists x \forall y$ is not similar to $\forall y \exists x$.

Free and Bound Variable

- Free: If its occurrence is outside the scope of quantifier
- Bound: If its occurrence is within the scope of quantifier
- E.g.: ∀x (A(x) -> B(x)), change in the x will also effect the values of A(x) and B(x), hence the variable x is a bound variable.
- E.g.: ∀x (A(x, y) -> B(x, y)), change in the x will not effect the values of y, hence the variable y is a free variable but x is bound variable.

Atomic sentences

- Atomic sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as Predicate (term1, term2, ..., term n).
- Example: Rohit and Rahul are brothers: => Brothers(Ravi, Ajay).

  Rita is a cat: => cat (Rita).

Complex Sentences

- Complex sentences are made by combining atomic sentences using connectives.
- First-order logic statements can be divided into two parts:
  - Subject: Subject is the main part of the statement.
  - Predicate: A predicate can be defined as a relation, which binds two atoms together in a statement.
- Consider the statement: "x is an integer.", it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate.

Some Examples

- All birds fly.
- $\forall x$ bird(x) $\rightarrow$fly(x).
- Every man respects his parent.
- $\forall x$ man(x) $\rightarrow$ respects (x, parent).
- Some boys play cricket.
- $\exists x$ boys(x) $\rightarrow$ play(x, cricket).
- Not all students like both Mathematics and Science.
- $\neg\forall$ (x) [student(x) $\rightarrow$ like(x, Mathematics) $\wedge$ like(x, Science)].
- Only one student failed in Mathematics.
- $\exists$(x) [student(x) $\rightarrow$ failed (x, Mathematics) $\wedge\forall$ (y) [$\neg$(x==y) $\wedge$ student(y) $\rightarrow$ $\neg$failed (x, Mathematics)].

Thank You !

Any Queries ?