INFERENTIAL STATISTICS PROJECT-CODED

DONE BY
R.SUKANYA

CONTENTS

LIST OF FIGURES

LIST OF TABLES

RUBRICS:

Criteria	Points
1.1 What is the probability that a randomly chosen player would suffer an injury?	1
1.2 What is the probability that a player is a forward or a winger?	1
1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?	2
1.4 What is the probability that a randomly chosen injured player is a striker?	2
2.1 What proportion of the gunny bags have a breaking strength less than 3.17 kg per sq cm?	1
2.2 What proportion of the gunny bags have a breaking strength at least 3.6 kg per sq cm.?	1
2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?	2
2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?	2
3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so? "- State the null and alternate hypotheses - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results Note: Consider the level of significance as 5%."	4
3.2 Is the mean hardness of the polished and unpolished stones the same? - State the null and alternate hypotheses. - Conduct the hypothesis test. - Write down conclusions from the test results. Note: Consider the level of significance as 5%.	4
4.1 How does the hardness of implants vary depending on dentists? "- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs,	10

Criteria	Points
identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."	
4.2 How does the hardness of implants vary depending on methods? "- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - In case the implant hardness differs, identify for which pairs it differs Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."	10
4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy? "- Create Interaction Plot - Inferences from the plot Note: Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys."	4
4.4 How does the hardness of implants vary depending on dentists and methods together? "- State the null and alternate hypotheses - Check the assumptions of the hypothesis test. - Conduct the hypothesis test and compute the p-value - Write down conclusions from the test results - Identify which dentists and methods combinations are different, and which interaction levels are different. Note: 1. Both types of alloys cannot be considered together. You must conduct the analysis separately for the two types of alloys. 2. Even if the assumptions of the test fail, kindly proceed with the test."	10
Quality of Business Report	6
Points	60

Problem Statement - AS Project

Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected.

	Striker	**Forward**	**Attacking Midfielder**	**Winger**	**Total**
Players Injured	45	56	24	20	145
Players not Injured	32	38	11	9	90
Total	**77**	**94**	**35**	**29**	**235**

Table 1

Based on the above data, answer the following questions.

1.1 What is the probability that a randomly chosen player would suffer an injury?

Ans: The probability that a randomly chosen player would suffer an injury is 0.617 or 61.7 %.

1.2 What is the probability that a player is a forward or a winger?

Ans: The probability that a player is a forward or a winger is 0.5234 or 52.34 %.

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

Ans: The probability that a randomly chosen player plays in a striker position and has a foot injury is 0.1915 or 19.15 %.

1.4 What is the probability that a randomly chosen injured player is a striker?

Ans: The probability that a randomly chosen injured player is a striker is 0.3103 or 31.03 %.

Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain.

Answer the questions below based on the given information.

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq. cm?

Step1: Import the important libraries like numpy, pandas, matplotlib, scipy.stats, seaborn, norm and pyplot.

Step 2: Plot the graph of the distribution using the plt.plot() function. Now, shade the area under the graph using the fill_between() function to show the breaking strength less than 3.17 kg/cm^2.

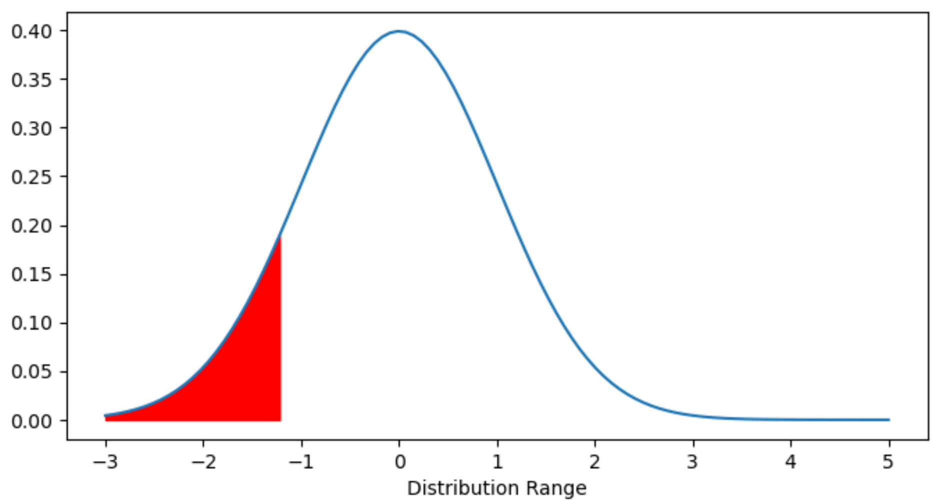


Fig 1 Proportion of gunny bags having a breaking strength of less than 3.17 kg per sq.cm

Step 3: Use the stats.norm.cdf() function to calculate the red coloured area under the bell curve.

Ans: The proportion of the gunny bags having a breaking strength of less than 3.17 kg per sq cm is 0.1112 or 11.12 %.

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq cm.?

Step 1: Plot the graph of the distribution using the plt.plot() function. Now, shade the area under the graph using the fill_between() function to show the breaking strength of at least 3.6 kg/cm^2.

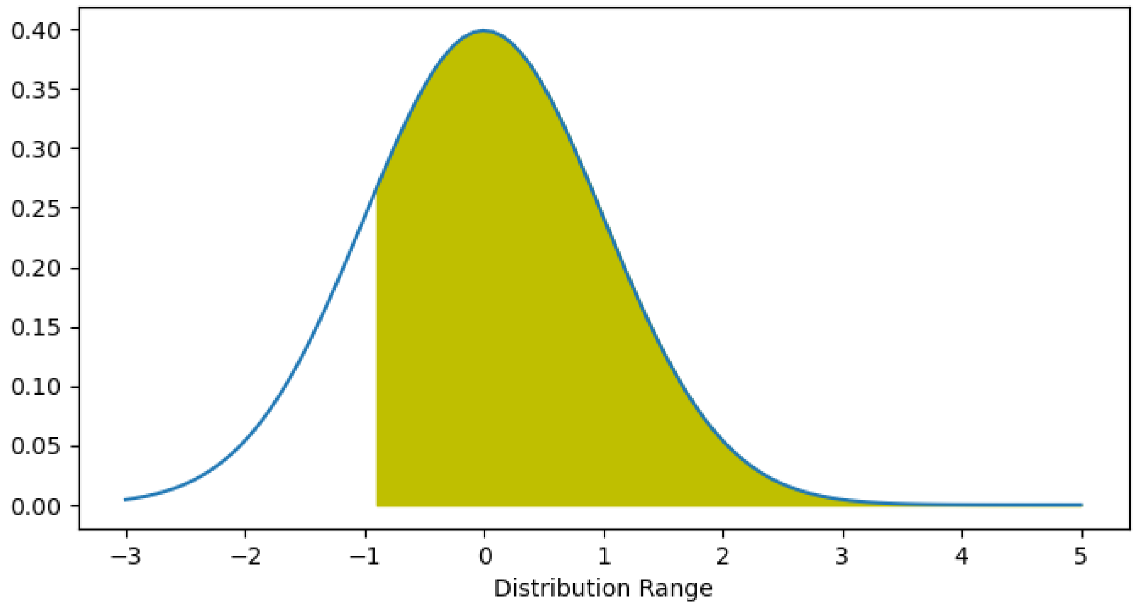


Fig 2 Proportion of gunny bags having a breaking strength of at least 3.6 kg sq.cm

Step 2: Use the 1- stats.norm.cdf() function to calculate the yellow-coloured area under the bell curve.

Ans: The proportion of the gunny bags having a breaking strength of at least 3.6 kg per sq cm is 0.8413 or 84.13 %.

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq cm.?

Step 1: Visual representation of the problem. Plot the graph using plt.plot() function and use the fill_between() function to shade the area under the curve to show the proportion of gunny bags having a breaking strength between 5 and 5.5 kg per sq. cm.

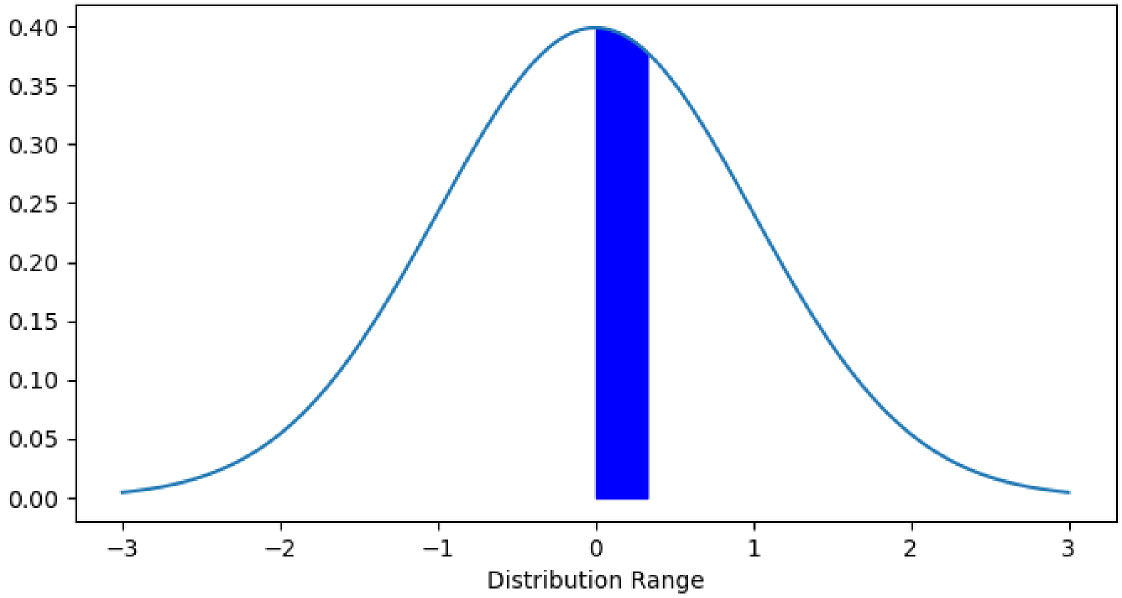


Fig 3: Proportion of gunny bags having a breaking strength between 5 and 5.5 kg sq.cm

Step 2: Find the norm.cdf() of 5.5 kg per sq cm.

Step 3: Find the norm.cdf() of 5 kg per sq.cm.

Step 4: Subtract Step 2 from Step 1. This will give you the area under the bell - curve pertaining to the proportion of the gunny bags in between 5 and 5.5 kg per sq cm.

Ans: The proportion of the gunny bags having a breaking strength between 5 and 5.5 kg per sq cm is 0.1306 or 13.06 %

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

Step 1: Visual representation of the problem. Plot the graph using plt.plot() function and use the fill_between() function twice to shade the area under the curve to show the proportion of gunny bags having a breaking strength NOT between 3 and 7.5 kg per sq. cm.

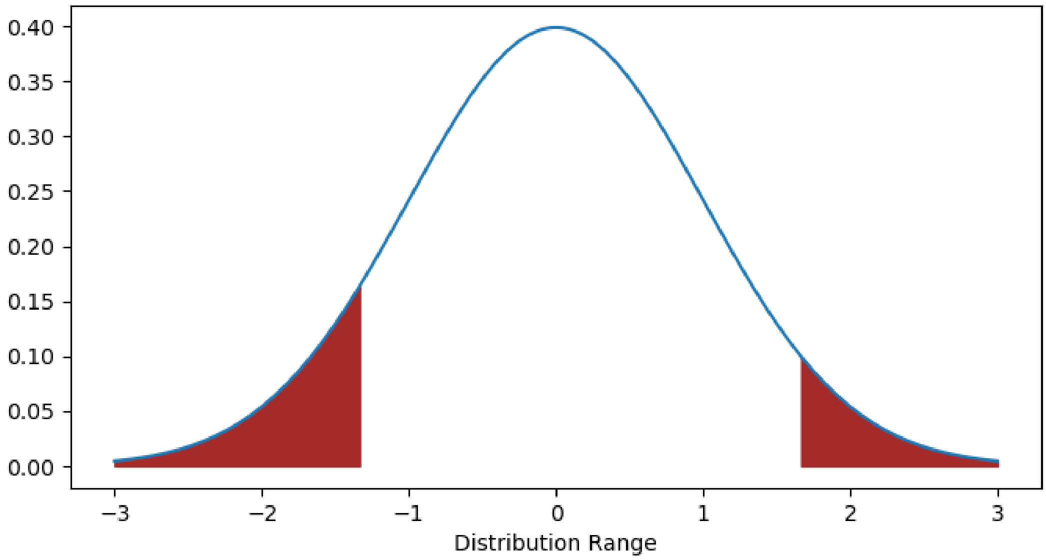


Fig 4: Proportion of gunny bags having a breaking strength NOT between 3 and 7.5 kg sq.cm

Step2: Find the norm.cdf() of 3 kg per sq cm.

Step 3: Find the 1 - norm.cdf() of 7.5 kg per sq.cm.

Step 4: Add Step 1 and Step 2. This will give you the area under the bell-curve pertaining to the proportion of the gunny bags not between 3 and 7.5 kg per sq cm.

Ans: The proportion of the gunny bags having a breaking strength NOT between 3 and 7.5 kg per sq cm is 0.139 or 13.9 %.

Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level).

Data provided: Zingaro_company.csv

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

3.1.1 State the null and alternate hypotheses.

H0: The null hypothesis is that the unpolished stones are suitable for printing.

HA: The alternative hypothesis is that the unpolished stones may not be suitable for printing.

3.1.2 Conduct the hypothesis test and compute the p-value.

a. We look at some of the rows of the dataset by using the head() function.

	Unpolished	Treated and Polished
0	164.481713	133.209393
1	154.307045	138.482771
2	129.861048	159.665201
3	159.096184	145.663528
4	135.256748	136.789227

Table 3.1

b. Use the len() of the database command to get the number of rows in the dataset. Sample size of the dataset is 75.

c. Identify the test statistic.

We have one sample for unpolished stones and we do not know the population standard deviation.

The sample size is n > 30. So we use 1-sample T-test.

d. We find the test_statistic and the p_value using the ttest_1samp() function. This gives us the test statistic and the p_value.

Output:

<<<<<<<<<<<<<<<

ONE-SAMPLED T-TEST

The t_statistic is -4.164629601426757
The p_value is 8.342573994839304e-05

alpha_level = 0.05

>>>>>>>>>>>>

3.1.3 Write down conclusions from the test results.

We have enough evidence to reject the null hypothesis in favour of alternative hypothesis. We conclude that the unpolished stones may not be suitable for printing.

Ans: Zingaro is justified in thinking that unpolished stones may not be suitable for printing. As the p_value is less than the level of significance, we can reject the null hypothesis.

3.2 Is the mean hardness of the polished and unpolished stones the same?

3.2.1: State the Null and the Alternate hypothesis.

The null hypothesis is that the mean hardness of the polished and the unpolished stones is the same.

$H0$: μP – μU = 0

or $\mu P = \mu U$

The alternative hypothesis is that the mean hardness of the polished and unpolished stones is not the same.

HA: μP - $\mu U \neq\neq 0$

or $\mu P \neq\neq \mu U$

alpha = 0.05

3.2.2 Conduct the hypothesis test.

a. Identify the test statistic.

* We have two samples and we do not know the population standard deviation.

* Sample sizes for both samples are the same.

* The sample size is n > 30. So, we perform the 2-Sample Independent T-test.

b. Calculate the test statistic and p_value:

Use the ttest_ind() function. This gives us the test statistic and the p_value.

Output:

<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

TWO-SAMPLED INDEPENDENT T-TEST

The t_statistic is -3.2422320501414053

The p_value is 0.0014655150194628353

>>>>>>>>>>>>>>>>>>>>>>>>>>>

3.2.3 Write down conclusions from the test results.

As p_value < alpha, we have enough evidence to reject the null hypothesis in favour of alternative hypothesis.

We conclude that the mean hardness of the polished stones and unpolished stones is not the same.

Ans: The mean hardness of polished and unpolished stones is not the same.

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favorite method. The response is the variable of interest.

4.1 How does the hardness of implants vary depending on dentists?

4.1.1 State the null and alternate hypotheses.

For Alloy1 and Alloy2:

$H0$: $\mu1 = \mu2 = \mu3 = \mu4 = \mu5$, that is the hardness of implants does not vary depending on the dentists.

Ha: At least one implant hardness varies depending on the dentists.

a. Loading the dataset:

Use the read_excel() function to read the Dental_implant dataset.

b. Checking the top 5 records of the dataset:

Use the head() function to see the first five records of the dataset.

	Dentist	Method	Alloy	Temp	Response
0	1	1	1	1500	813
1	1	1	1	1600	792
2	1	1	1	1700	792
3	1	1	2	1500	907
4	1	1	2	1600	792

Table 4.1.1

c. Checking for the shape and information about the dataset:

Use the shape() command to get the number of rows and columns in the dataset.

There are 90 rows and 5 columns.

On using the info() command, we get the following output:

#	Column	Non-Null Count	Dtype
0	Dentist	90 non-null	int64
1	Method	90 non-null	int64
2	Alloy	90 non-null	int64
3	Temp	90 non-null	int64
4	Response	90 non-null	int64

Table 4.1.2

The independent variables are Dentist, Method and Alloy which are all numerical data types. The dependent variable is Response which is numerical datatype.

d. Checking for the descriptive summary of the dataset:

Use the describe() function to get the summary of the variables in the dataset.

	count	mean	std	min	25%	50%	75%	max
Dentist	90.0	3.000000	1.422136	1.0	2.0	3.0	4.0	5.0
Method	90.0	2.000000	0.821071	1.0	1.0	2.0	3.0	3.0
Alloy	90.0	1.500000	0.502801	1.0	1.0	1.5	2.0	2.0
Temp	90.0	1600.000000	82.107083	1500.0	1500.0	1600.0	1700.0	1700.0
Response	90.0	741.777778	145.767845	289.0	698.0	767.0	824.0	1115.0

Table 4.1.3

We see that the mean response time of the dentists is 741. The minimum response time is 289 and the maximum is 1115.

e. Changing the independent variables into Categorical datatype:

The fields Dentist, Method and Alloy are not numerical types. They need to be changed to categorical variables.

On finding the datatypes of the variables now using the info() function,

0	Dentist	90 non-null	category
1	Method	90 non-null	category
2	Alloy	90 non-null	category
3	Temp	90 non-null	category
4	Response	90 non-null	int64

Table 4.1.4

We find that the independent variables are categorical in nature.

f. Separating the dataset into 2 datasets of Alloy1 and Alloy2:

Use the groupby() function to group the Dentist and Mean Response for Alloy1 and Alloy2.

4.1.2: Test for Assumptions of a ONE-WAY ANOVA

SHAPIRO-WILK'S TEST

Shapiro-wilk's test is performed to check for the normality of the distribution.

Null and Alternate Hypothesis for Shapiro Test:

$H0$: Response follows a normal distribution against

Ha: Response does not follow a normal distribution.

Plot the histogram for Response dependent variable for Alloy1 and Alloy2.

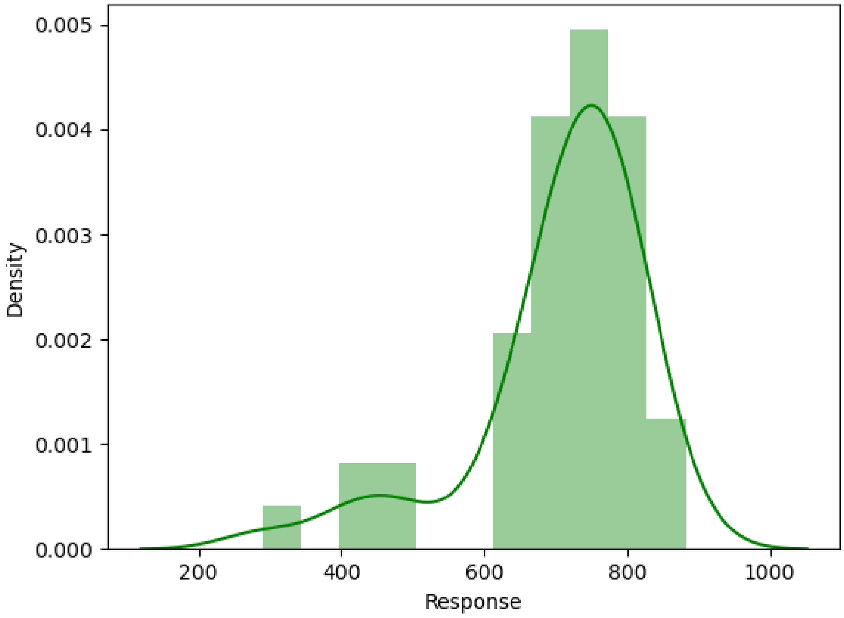


Fig 5: Histogram with kde plot for Response for Alloy1

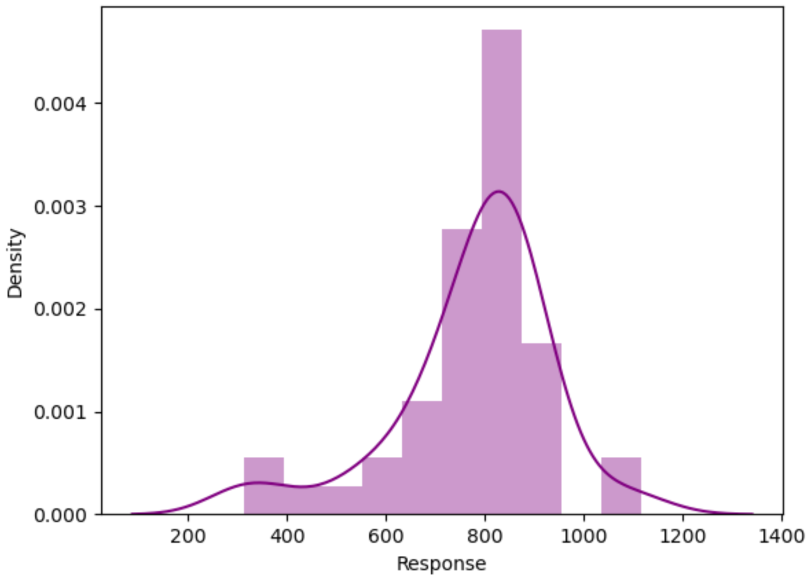


Fig 6: Histogram with kde plot for Response for Alloy2

Shapiro Test for Alloy1:

Output:

```
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
ShapiroResult(statistic=0.9113541841506958, pvalue=0.3254688084125519)
ShapiroResult(statistic=0.9642462134361267, pvalue=0.8415456414222717)
ShapiroResult(statistic=0.8721169233322144, pvalue=0.12953516840934753)
ShapiroResult(statistic=0.8368974328041077, pvalue=0.05333680287003517)
ShapiroResult(statistic=0.8534296751022339, pvalue=0.08127813786268234)
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

Since p-value of the test is very large for all the five dentists, that is, p_value > alpha, we fail to reject the null hypothesis. Hence the Response w.r.t Dentist follows Normal distribution for Alloy 1.

Shapiro Test for Alloy2:

Output:

```
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
ShapiroResult(statistic=0.9039731621742249, pvalue=0.27593979239463806)
ShapiroResult(statistic=0.9392004013061523, pvalue=0.5735077857971191)
ShapiroResult(statistic=0.9340971112251282, pvalue=0.5213080644607544)
ShapiroResult(statistic=0.7613219022750854, pvalue=0.00733268819749355)
ShapiroResult(statistic=0.9131584167480469, pvalue=0.33861100673675537)
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

Since p-value of the test is large for four dentists, that is, Dentist 1, 2, 3 and 5, p_value > alpha, we fail to reject the null hypothesis. H0 is true. Hence the Response w.r.t Dentist follows Normal distribution for Alloy 2.

LEVENE'S TEST:

Levene's test is performed to check for the equality of variances.

Null and Alternate Hypothesis:

$H0$: $\sigma 1 = \sigma 2 = \sigma 3 = \sigma 4 = \sigma 5$, that is , the variances of the responses of all the 5 dentists is the same.

Ha: At least one variance is different from the rest.

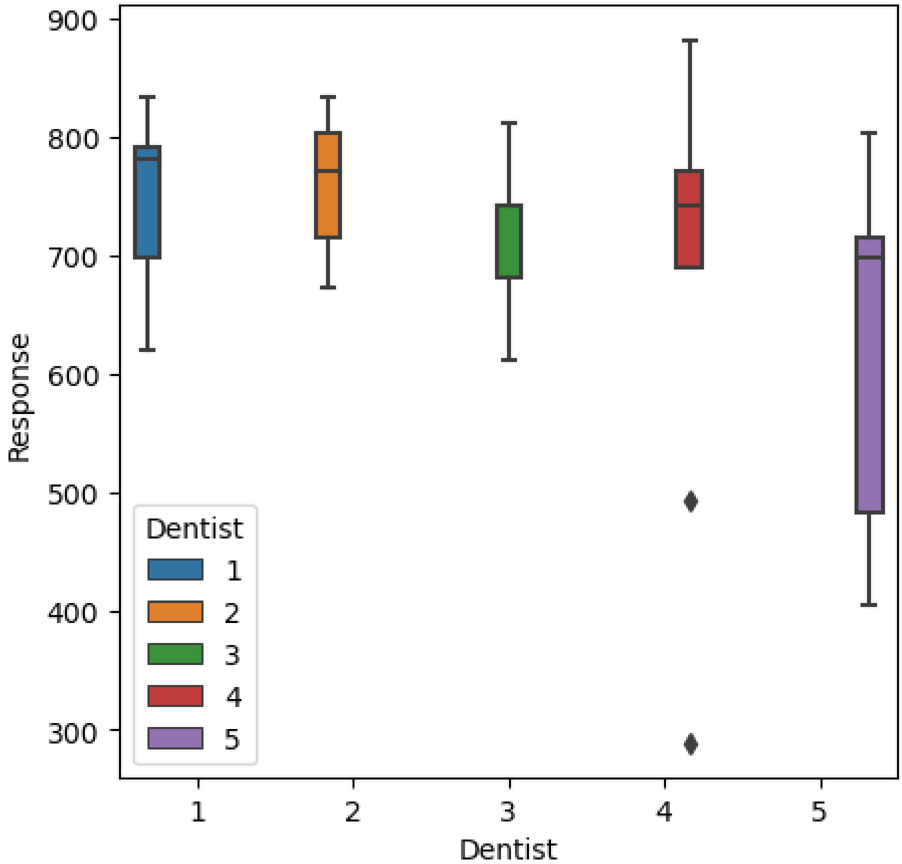


Fig 7: Boxplot for Dentist vs Response for Alloy1

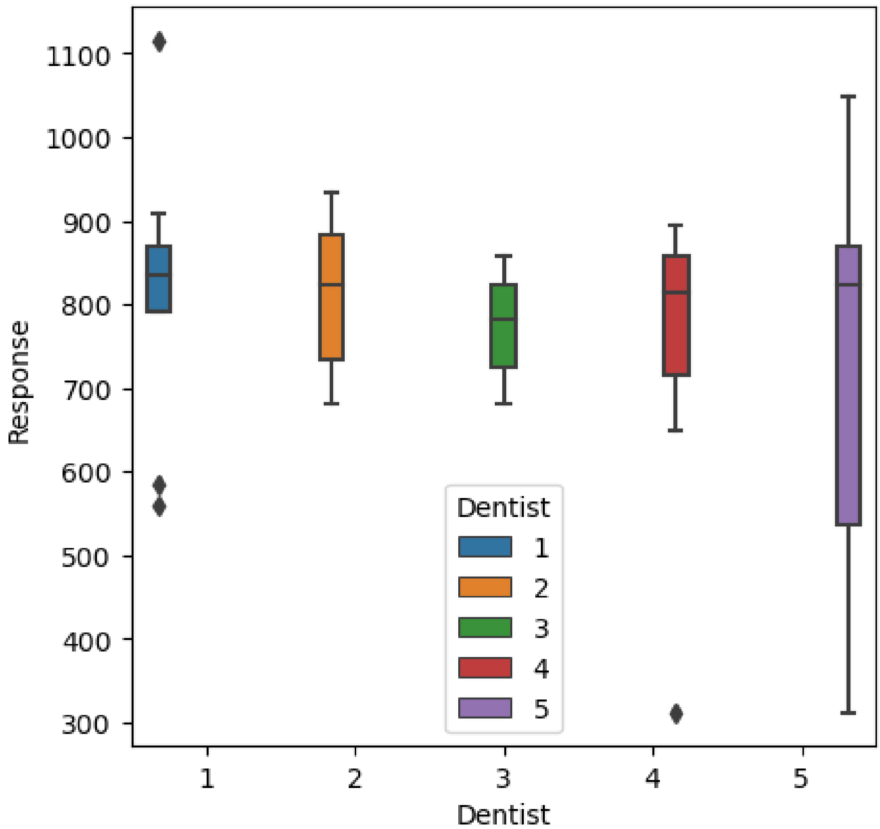


Fig 8: Boxplot for Dentist vs Response for Alloy2

Levene’s Test for Alloy1:

```
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

   statistic = 1.3847146992797106 p_value = 0.2565537418543795

>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

Levene’s Test for Alloy2:

```
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

   statistic = 1.4456166464566966 p_value = 0.23686777576324952

>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

Since the p-value is larger than significance level alpha (0.05) for both alloy1 and alloy2, we fail to reject the null hypothesis of homogeneity of variances. Null Hypothesis is true. Therefore, the variances of alloy1 and alloy2 are homogenous.

4.1.3 Conduct the hypothesis test and compute the p-value.

Applying One-Way ANOVA for Dentist vs Response:

Output of One-way ANOVA for Alloy1:

```
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

          df         sum_sq       mean_sq         F    PR(>F)
Dentist    4.0  106683.688889  26670.922222  1.977112  0.116567
Residual  40.0  539593.555556  13489.838889       NaN       NaN
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

Output of One-way ANOVA for Alloy2:

```
<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

df        sum_sq       mean_sq         F    PR(>F)
Dentist    4.0  5.679791e+04  14199.477778  0.524835  0.718031
Residual  40.0  1.082205e+06  27055.122222       NaN       NaN
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
```

4.1.4 Write down conclusions from the test results.

For Alloy1:

Based on the ANOVA test, as p-value > 0.05, we fail to reject the null hypothesis. F critical for (4,40) is approximately 2.08. As F $<$ Fcritical, H0 is true.

Therefore, the hardness of implants of alloy1 does not vary depending on the dentists.

For Alloy2:

Based on the ANOVA test, as p-value is more than the level of significance, we fail to reject the null hypothesis. Fcritical is approx. 2.606 for (4,40) degrees of freedom. As F $<$ Fcritical, Ho is true.

Therefore, the hardness of implants of alloy2 does not vary depending on the dentists.

Ans: For alloy1 and alloy2, the hardness of implants does not depend on the dentists.

4.2 How does the hardness of implants vary depending on methods?

4.2.1 State the null and alternate hypotheses.

$H0$: $\mu1 = \mu2 = \mu3$, that is the hardness of implants does not vary depending on the method.

Ha: At least one implant hardness varies depending on the method.

Find the count of each method for Alloy1 and Alloy2.

We see that each method has counts of 15 for both Alloy1 and Alloy2.

	Alloy1	Alloy2
Method 1	15	15
Method 2	15	15
Method 3	15	15

Table 4.2.1

4.2.2 Check the assumptions of the hypothesis test.

SHAPIRO-WILK'S TEST:

Null and Alternate Hypothesis for Shapiro Test:

$H0$: Response follows a normal distribution against

Ha: Response does not follow a normal distribution¶

Shapiro Test for Alloy1:

<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

```
ShapiroResult(statistic=0.9183822870254517, pvalue=0.18198540806770325)
ShapiroResult(statistic=0.9732585549354553, pvalue=0.9030335545539856)
ShapiroResult(statistic=0.9114548563957214, pvalue=0.14254699647426605)
```

>>

Since p-value of the test is very large for all the three methods, we fail to reject the null hypothesis. Hence the Response w.r.t Method follows Normal distribution for Alloy 1.

Shapiro Test for Alloy2:

<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

```
ShapiroResult(statistic=0.963810384273529, pvalue=0.7582374811172485)
ShapiroResult(statistic=0.755793035030365, pvalue=0.001051110913977027)
ShapiroResult(statistic=0.9021322131156921, pvalue=0.1025901660323143)
```

>>

The p-value of the test is very large as compared to alpha for Method1 and Method3 The p-value of the Shapiro test is lesser than alpha for Method3. In Shapiro Test, 50% normal and 50% not normal is acceptable. In this case, 66% is normal distribution, we fail to reject the null hypothesis. Hence the Response w.r.t Method follows Normal distribution for Alloy 2.

LEVENE'S TEST

Levene's test is conducted to test for the homogeneity of the variances of all the three methods in the two alloys.

Null and Alternate Hypotheses for Levene's Test:

$H0$: $\sigma 1 = \sigma 2 = \sigma 3$, that is the variances of all the methods are same for both the alloys.

Ha: At least one variance is different from the rest.

Boxplots for Alloy1 and Alloy2:

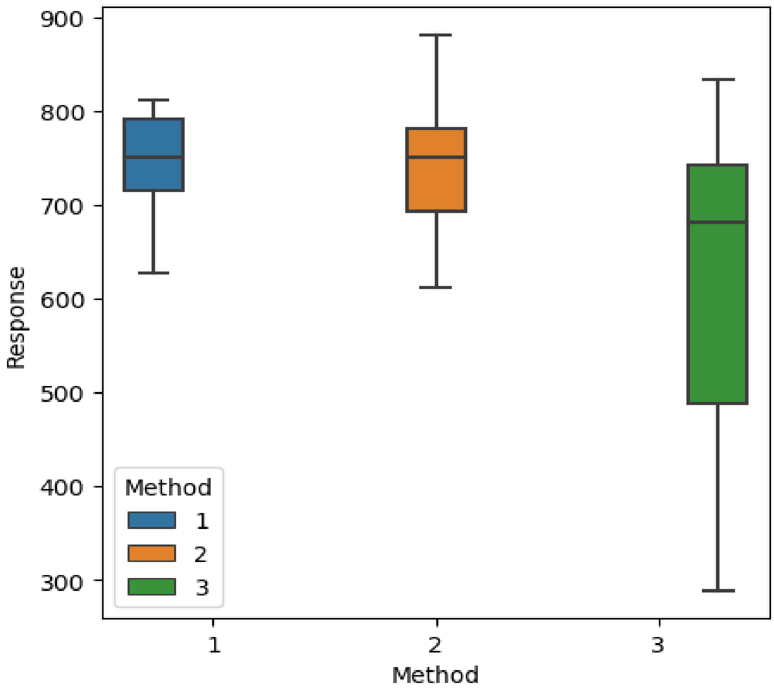


Fig 9: Boxplot for Method vs Response for Alloy1

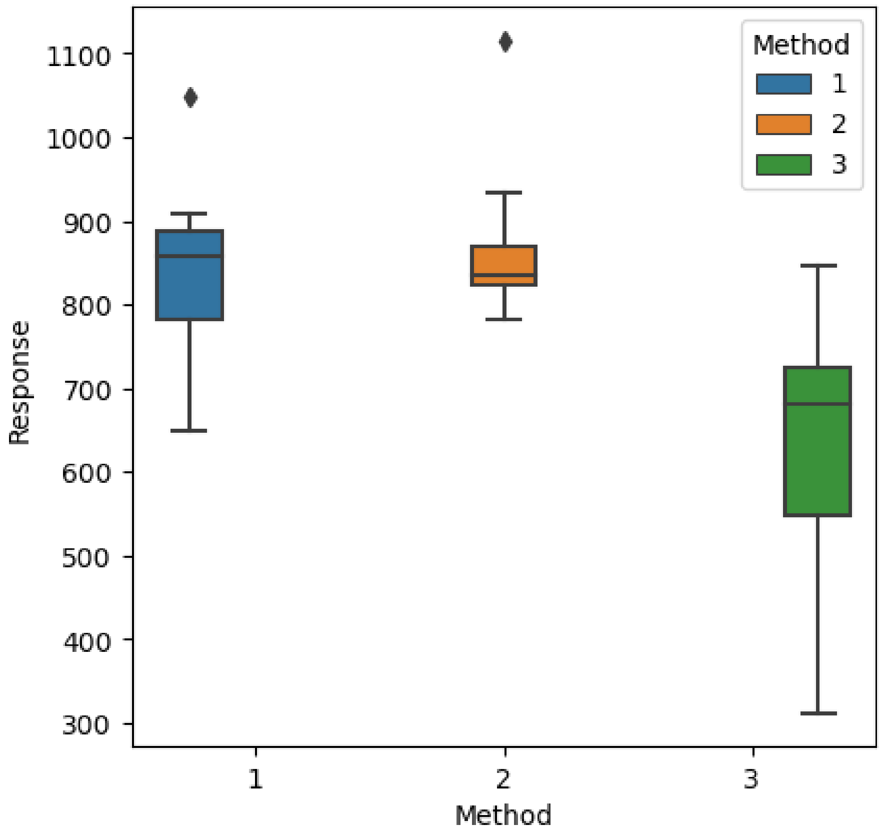


Fig 10: Boxplot for Method vs Response for Alloy2

Levene's Test for Alloy1:

<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

statistic = 6.52140454403598 p_value = 0.0034160381460233975

>>

Levene's Test for Alloy2:

<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<

statistic = 3.349707184158617 p_value = 0.04469269939158668

>>

Since the p-value is lesser than the significance level alpha (0.05) for both alloy1 and alloy2, we reject the null hypothesis of homogeneity of variances. Alternate hypothesis is true. At least one variance is different from the rest.

4.2.3 Conduct the hypothesis test and compute the p-value.

Applying One-Way ANOVA:

Alloy 1:

	df	sum_sq	mean_sq	F	PR(>F)
Method	2.0	148472.177778	74236.088889	6.263327	0.004163
Residual	42.0	497805.066667	11852.501587	NaN	NaN

Table 4.2.2

Alloy 2:

	df	sum_sq	mean_sq	F	PR(>F)
Method	2.0	499640.4	249820.200000	16.4108	0.000005
Residual	42.0	639362.4	15222.914286	NaN	NaN

Table 4.2.3

4.2.4 Write down conclusions from the test results.

For Alloy1:

Based on the ANOVA test,as p-value < 0.05, we reject the null hypothesis. Ha is true.
F critical for (2,42) is approximately 3.22. As F > Fcritical, H1 is true.

Therefore, the hardness of implants of alloy1 varies depending on the method. At least one implant hardness varies depending on the method.

For Alloy2:

Based on the ANOVA test, as p-value < 0.05, we reject the null hypothesis. Ha is true.
F critical for (2,42) is approximately 3.22. As F > Fcritical, H1 is true.

Therefore, the hardness of implants of alloy2 varies depending on the method. At least one implant hardness varies depending on the method.

4.2.5 In case the implant hardness differs, identify for which pairs it differs.

TukeyHSD() Test to determine which two methods are different for the two alloys:

In order to identify for which method mean Response is different from other groups, the hypotheses may be stated as:

$H0$: All pairs of group means are equal against

Ha: At least one group mean is different from the rest.

$H0$: $\mu1 = \mu2$ and $\mu1 = \mu3$ and $\mu2 = \mu3$ against

Ha: $\mu1 \neq \mu2$ or $\mu1 \neq \mu3$ or $\mu2 \neq \mu3$

where $\mu1$ represents mean response for method1, $\mu2$ represents mean response for method2 and $\mu3$ is the same for method3.

TukeyHSD() Test for Alloy1:

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	-6.1333	0.987	-102.714	90.4473	False
1	3	-124.8	0.0085	-221.3807	-28.2193	True
2	3	-118.6667	0.0128	-215.2473	-22.086	True

Table 4.2.4

Method1 and Method2: p_value > alpha, H0 is true. $\mu1 = \mu2$
Method1 and Method3: p_value < alpha, H1 is true. $\mu1 \neq \mu3$
Method2 and Method3: p_value < alpha, H1 is true. $\mu2 \neq \mu3$

P-value is significant for comparing response for the pair Method1 and Method3 and for Method2 and Method3, but not for Method1 and Method2. The null hypothesis of equality of all population means is rejected.

It is now clear that mean response for Method1 and Method2 are similar but response for Method1 and Method3 as well as Method2 and Method3 are significantly different from Method1 and Method2.

Visual Representation of Tukey-HSD() for Alloy1:

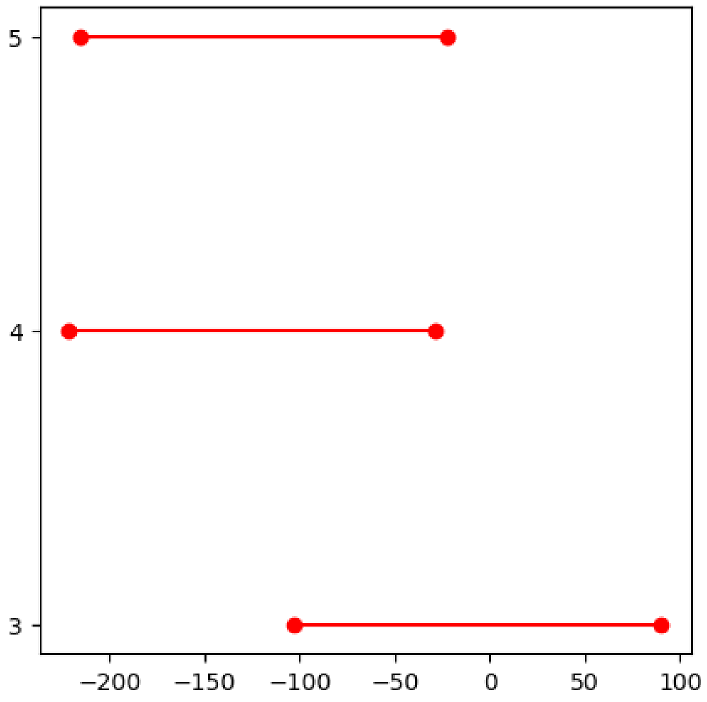


Fig 11: Tukey-HSD Test for Alloy1

The confidence intervals not containing 0 is for the difference between Method1 and Method3 and for Method2 and Method3.

This indicates that population means of these pairs of Methods are different.

TukeyHSD() Test for Alloy2:

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	27.0	0.8212	-82.4546	136.4546	False
1	3	-208.8	0.0001	-318.2546	-99.3454	True
2	3	-235.8	0.0	-345.2546	-126.3454	True

Table 4.2.5

Method1 and Method2: p_value > alpha, H0 is true. $\mu1 = \mu2$

Method1 and Method3: p_value < alpha, H1 is true. $\mu1 \neq \mu3$

Method2 and Method3: p_value < alpha, H1 is true. $\mu2 \neq \mu3$

P-value is significant for comparing response for the pair Method1 and Method3 and for Method2 and Method3, but not for Method1 and Method2. The null hypothesis of equality of all population means is rejected.

It is now clear that mean response for Method1 and Method2 are similar but response for Method1 and Method3 as well as Method2 and Method3 are significantly different from Method1 and Method2.

Visual Representation of Tukey-HSD() for Alloy2:

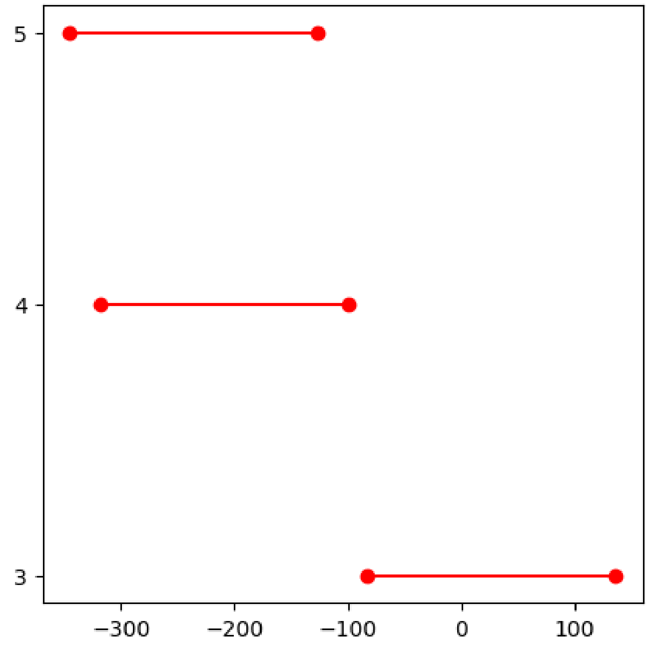


Fig 12: Tukey-HSD Test for Alloy2

The confidence intervals not containing 0 is for the difference between Method1 and Method3 and for Method2 and Method3.

This indicates that population means of these pairs of Methods are different.

Ans 4.2: For alloy1 and alloy2, the hardness of implants depend on the method.

The means of Method1 and Method3 as well as for Method2 and Method3 are significantly different.

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

4.3.1 Create Interaction Plot

Interaction plot for Dentist and Method for Alloy1:

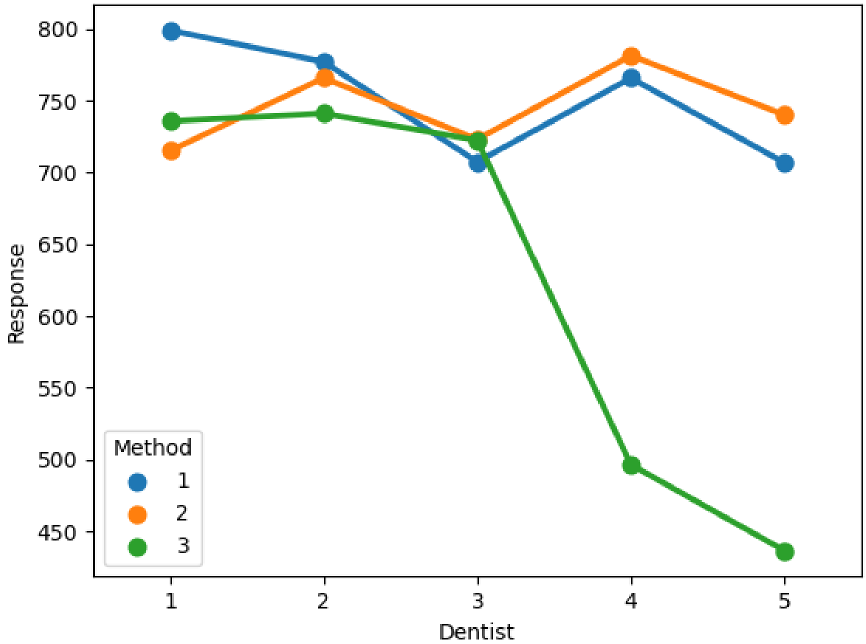


Fig 13: Interaction plot for Dentist and Method for Alloy1

Interaction plot for Dentist and Method for Alloy2:

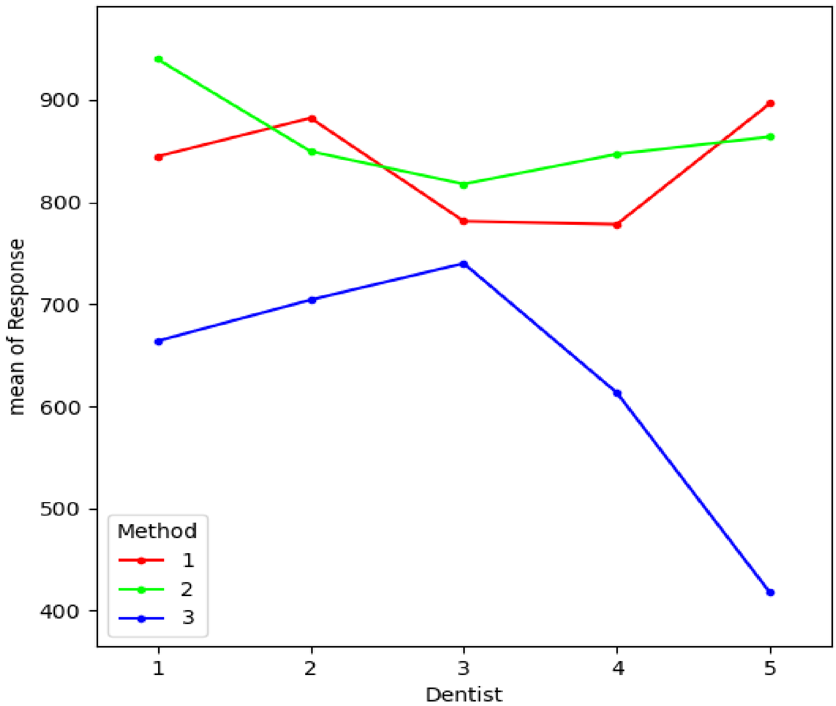


Fig 14: Interaction plot for Dentist and Method for Alloy2

4.3.2 Inference from the plot:

The interaction plot for Alloy1 clearly shows that there is interaction between the three methods, Method 1, Method 2 and Method 3 as the lines are not parallel. The mean response time for Dentist 4 and Dentist 5 is lowest in Method 3.

For Alloy2, Method 3 does not have any interaction with Method 1 and Method 2 as the lines do not intersect. Moreover the mean response time of Dentist 5 for Method 3 is the lowest in Alloy2.

4.4 How does the hardness of implants vary depending on dentists and methods together?

4.4.1 State the null and alternate hypotheses.

The Hypothesis for the Two-Way ANOVA including Interaction effect are:

Null Hypothesis: The hardness of implants does not vary depending on the dentists and methods together have equal effect.

Alternate Hypothesis: The hardness of implants is different for at least one dentist category or method category.

Boxplot for Dentist vs Response for Alloy1

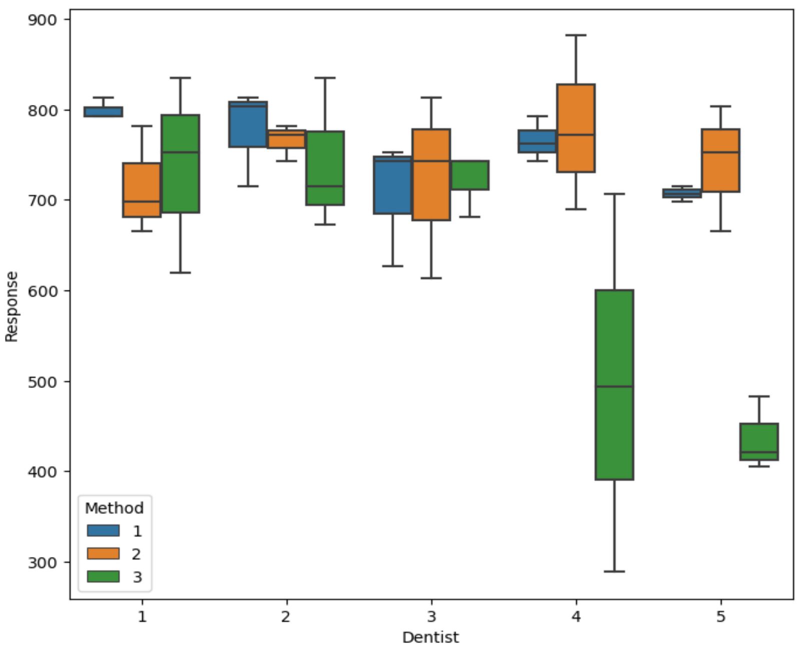


Fig 15: Boxplot for Dentist and Method factors for Alloy1

Boxplot for Dentist vs Response for Alloy2:

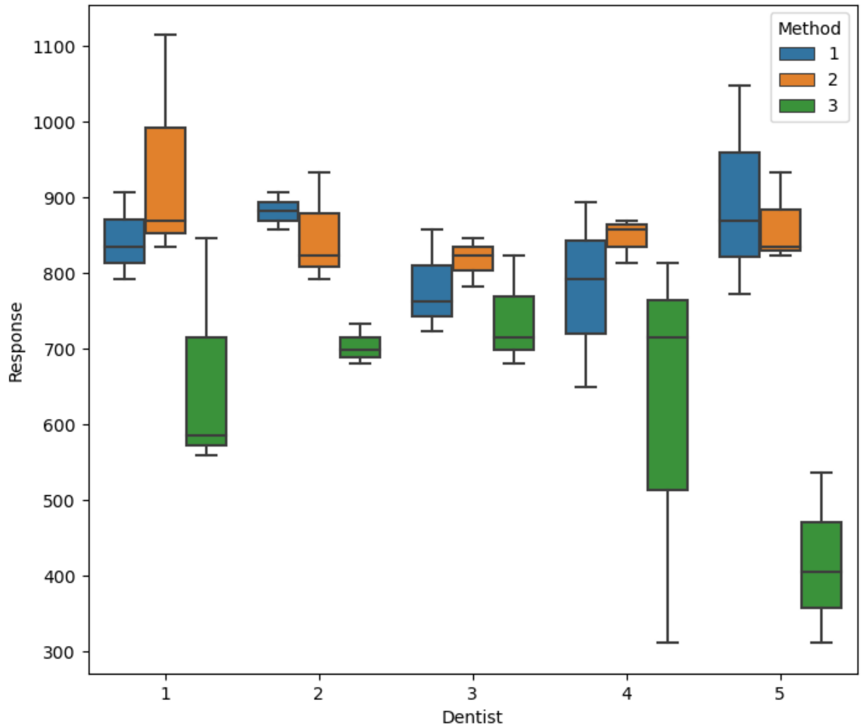


Fig 16: Boxplot for Dentist and Method factors for Alloy2

4.4.2 Check the assumptions of the hypothesis test.

SHAPIRO-WILK'S TEST

The normality distribution of Dentist and Method with respect to response has already been conducted in Problems 4.1 and 4.2. We had seen that the distribution for both is normal for both Alloy1 and Alloy2.

LEVENE'S TEST

Null and Alternate Hypothesis:

$H0$: $\sigma 1 = \sigma 2 \ldots \sigma 11 = \sigma 12$ against

Ha: At least one variance is different from the rest.

For Alloy 1

```
statistics = 1.22
p_value    = 0.31
```

For Alloy 2:

```
statistics = 0.67
p_value    = 0.78
```

As p_value is greater than alpha for both Alloy1 and Alloy2, we fail to reject the null hypothesis. Therefore, we conclude that the null hypothesis is true and the variances of the dentists and methods taken together are equal.

4.4.3 Conduct the hypothesis test and compute the p-value.

Two Way ANOVA without interaction of Dentist & Method types.

Null Hypothesis: The mean hardness of the implants is equal across all dentist and method types.

Alternate Hypothesis: The mean hardness of the implants is different for at least one dentist or method categories.

For Alloy1:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	2.591255	0.051875
C(Method)	2.0	148472.177778	74236.088889	7.212522	0.002211
Residual	38.0	391121.377778	10292.667836	NaN	NaN

Table 4.4.1

For Alloy2:

]:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	0.926215	0.458933
C(Method)	2.0	499640.400000	249820.200000	16.295479	0.000008
Residual	38.0	582564.488889	15330.644444	NaN	NaN

Table 4.4.2

p_value(Dentist) > alpha, hence we fail to reject the null hypothesis. Therefore, the hardness of the dental implants across all dentists is the same.

p_value(Methos) < alpha, hence we reject the null hypothesis. Therefore, the hardness of the dental implants for at least one of the methods is different.

Two-Way ANOVA with Interaction between Dentist and Method:

For Alloy1:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	106683.688889	26670.922222	3.899638	0.011484
C(Method)	2.0	148472.177778	74236.088889	10.854287	0.000284
C(Dentist):C(Method)	8.0	185941.377778	23242.672222	3.398383	0.006793
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Table 4.4.3

For Alloy2:

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist)	4.0	56797.911111	14199.477778	1.106152	0.371833
C(Method)	2.0	499640.400000	249820.200000	19.461218	0.000004
C(Dentist):C(Method)	8.0	197459.822222	24682.477778	1.922787	0.093234
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Table 4.4.4

4.4.4 Write down conclusions from the test results.

For both the alloys, Alloy1 and Alloy2,

p_value of interaction between Dentist and Method < alpha(0.5)

We see that the p-value of the interaction effect term of 'Dentist' and 'Method' suggests that the alternate hypothesis is accepted in this case as p_value is less than alpha.

At least one of the two, namely, dentist or method has an impact on the hardness of dental implants.

4.4.5 Identify which dentists and methods combinations are different, and which interaction levels are different.

TukeyHSD()

For Alloy1:

```
Multiple Comparison of Means - Tukey HSD, FWER=0.05
=========================================================
group1 group2  meandiff p-adj    lower     upper   reject
---------------------------------------------------------
   1:1    1:2     -84.0 0.9933 -332.8283  164.8283  False
   1:1    1:3  -63.3333 0.9996 -312.1617   185.495  False
   1:1    2:1     -22.0    1.0 -270.8283  226.8283  False
   1:1    2:2  -33.3333    1.0 -282.1617   215.495  False
   1:1    2:3     -58.0 0.9999 -306.8283  190.8283  False
   1:1    3:1  -91.6667 0.9853  -340.495  157.1617  False
   1:1    3:2     -76.0 0.9975 -324.8283  172.8283  False
   1:1    3:3  -76.6667 0.9972  -325.495  172.1617  False
   1:1    4:1  -33.3333    1.0 -282.1617   215.495  False
   1:1    4:2  -17.6667    1.0  -266.495  231.1617  False
   1:1    4:3 -302.6667  0.007  -551.495  -53.8383   True
   1:1    5:1  -92.3333 0.9844 -341.1617   156.495  False
   1:1    5:2     -59.0 0.9998 -307.8283  189.8283  False
   1:1    5:3 -362.6667 0.0007  -611.495 -113.8383   True
   1:2    1:3   20.6667    1.0 -228.1617   269.495  False
   1:2    2:1      62.0 0.9997 -186.8283  310.8283  False
   1:2    2:2   50.6667    1.0 -198.1617   299.495  False
   1:2    2:3      26.0    1.0 -222.8283  274.8283  False
```

1:2	3:1	-7.6667	1.0	-256.495	241.1617	False
1:2	3:2	8.0	1.0	-240.8283	256.8283	False
1:2	3:3	7.3333	1.0	-241.495	256.1617	False
1:2	4:1	50.6667	1.0	-198.1617	299.495	False
1:2	4:2	66.3333	0.9994	-182.495	315.1617	False
1:2	4:3	-218.6667	0.1324	-467.495	30.1617	False
1:2	5:1	-8.3333	1.0	-257.1617	240.495	False
1:2	5:2	25.0	1.0	-223.8283	273.8283	False
1:2	5:3	-278.6667	0.0173	-527.495	-29.8383	True
1:3	2:1	41.3333	1.0	-207.495	290.1617	False
1:3	2:2	30.0	1.0	-218.8283	278.8283	False
1:3	2:3	5.3333	1.0	-243.495	254.1617	False
1:3	3:1	-28.3333	1.0	-277.1617	220.495	False
1:3	3:2	-12.6667	1.0	-261.495	236.1617	False
1:3	3:3	-13.3333	1.0	-262.1617	235.495	False
1:3	4:1	30.0	1.0	-218.8283	278.8283	False
1:3	4:2	45.6667	1.0	-203.1617	294.495	False
1:3	4:3	-239.3333	0.0688	-488.1617	9.495	False
1:3	5:1	-29.0	1.0	-277.8283	219.8283	False
1:3	5:2	4.3333	1.0	-244.495	253.1617	False
1:3	5:3	-299.3333	0.0079	-548.1617	-50.505	True
2:1	2:2	-11.3333	1.0	-260.1617	237.495	False
2:1	2:3	-36.0	1.0	-284.8283	212.8283	False
2:1	3:1	-69.6667	0.999	-318.495	179.1617	False
2:1	3:2	-54.0	0.9999	-302.8283	194.8283	False
2:1	3:3	-54.6667	0.9999	-303.495	194.1617	False
2:1	4:1	-11.3333	1.0	-260.1617	237.495	False
2:1	4:2	4.3333	1.0	-244.495	253.1617	False
2:1	4:3	-280.6667	0.016	-529.495	-31.8383	True
2:1	5:1	-70.3333	0.9989	-319.1617	178.495	False
2:1	5:2	-37.0	1.0	-285.8283	211.8283	False
2:1	5:3	-340.6667	0.0016	-589.495	-91.8383	True
2:2	2:3	-24.6667	1.0	-273.495	224.1617	False
2:2	3:1	-58.3333	0.9999	-307.1617	190.495	False
2:2	3:2	-42.6667	1.0	-291.495	206.1617	False
2:2	3:3	-43.3333	1.0	-292.1617	205.495	False
2:2	4:1	0.0	1.0	-248.8283	248.8283	False
2:2	4:2	15.6667	1.0	-233.1617	264.495	False
2:2	4:3	-269.3333	0.0243	-518.1617	-20.505	True
2:2	5:1	-59.0	0.9998	-307.8283	189.8283	False
2:2	5:2	-25.6667	1.0	-274.495	223.1617	False
2:2	5:3	-329.3333	0.0025	-578.1617	-80.505	True
2:3	3:1	-33.6667	1.0	-282.495	215.1617	False
2:3	3:2	-18.0	1.0	-266.8283	230.8283	False
2:3	3:3	-18.6667	1.0	-267.495	230.1617	False
2:3	4:1	24.6667	1.0	-224.1617	273.495	False
2:3	4:2	40.3333	1.0	-208.495	289.1617	False

2:3	4:3	-244.6667	0.0576	-493.495	4.1617	False
2:3	5:1	-34.3333	1.0	-283.1617	214.495	False
2:3	5:2	-1.0	1.0	-249.8283	247.8283	False
2:3	5:3	-304.6667	0.0065	-553.495	-55.8383	True
3:1	3:2	15.6667	1.0	-233.1617	264.495	False
3:1	3:3	15.0	1.0	-233.8283	263.8283	False
3:1	4:1	58.3333	0.9999	-190.495	307.1617	False
3:1	4:2	74.0	0.9981	-174.8283	322.8283	False
3:1	4:3	-211.0	0.166	-459.8283	37.8283	False
3:1	5:1	-0.6667	1.0	-249.495	248.1617	False
3:1	5:2	32.6667	1.0	-216.1617	281.495	False
3:1	5:3	-271.0	0.0229	-519.8283	-22.1717	True
3:2	3:3	-0.6667	1.0	-249.495	248.1617	False
3:2	4:1	42.6667	1.0	-206.1617	291.495	False
3:2	4:2	58.3333	0.9999	-190.495	307.1617	False
3:2	4:3	-226.6667	0.1035	-475.495	22.1617	False
3:2	5:1	-16.3333	1.0	-265.1617	232.495	False
3:2	5:2	17.0	1.0	-231.8283	265.8283	False
3:2	5:3	-286.6667	0.0128	-535.495	-37.8383	True
3:3	4:1	43.3333	1.0	-205.495	292.1617	False
3:3	4:2	59.0	0.9998	-189.8283	307.8283	False
3:3	4:3	-226.0	0.1057	-474.8283	22.8283	False
3:3	5:1	-15.6667	1.0	-264.495	233.1617	False
3:3	5:2	17.6667	1.0	-231.1617	266.495	False
3:3	5:3	-286.0	0.0131	-534.8283	-37.1717	True
4:1	4:2	15.6667	1.0	-233.1617	264.495	False
4:1	4:3	-269.3333	0.0243	-518.1617	-20.505	True
4:1	5:1	-59.0	0.9998	-307.8283	189.8283	False
4:1	5:2	-25.6667	1.0	-274.495	223.1617	False
4:1	5:3	-329.3333	0.0025	-578.1617	-80.505	True
4:2	4:3	-285.0	0.0137	-533.8283	-36.1717	True
4:2	5:1	-74.6667	0.9979	-323.495	174.1617	False
4:2	5:2	-41.3333	1.0	-290.1617	207.495	False
4:2	5:3	-345.0	0.0013	-593.8283	-96.1717	True
4:3	5:1	210.3333	0.1692	-38.495	459.1617	False
4:3	5:2	243.6667	0.0596	-5.1617	492.495	False
4:3	5:3	-60.0	0.9998	-308.8283	188.8283	False
5:1	5:2	33.3333	1.0	-215.495	282.1617	False
5:1	5:3	-270.3333	0.0234	-519.1617	-21.505	True
5:2	5:3	-303.6667	0.0067	-552.495	-54.8383	True

Visual Representation for comparison of the interaction of different methods and dentists for Alloy1:

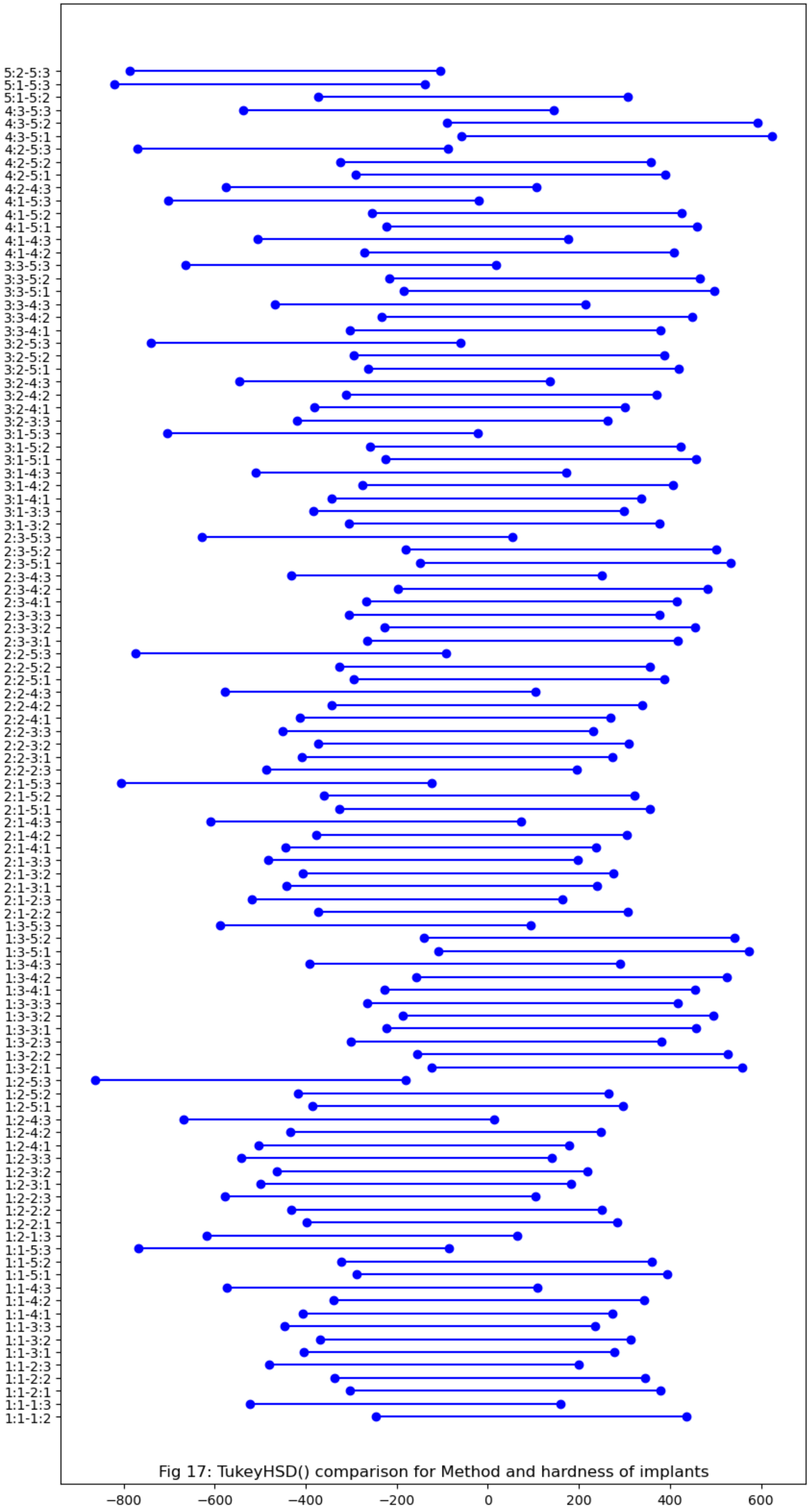


Fig 17: TukeyHSD() comparison for Method and hardness of implants

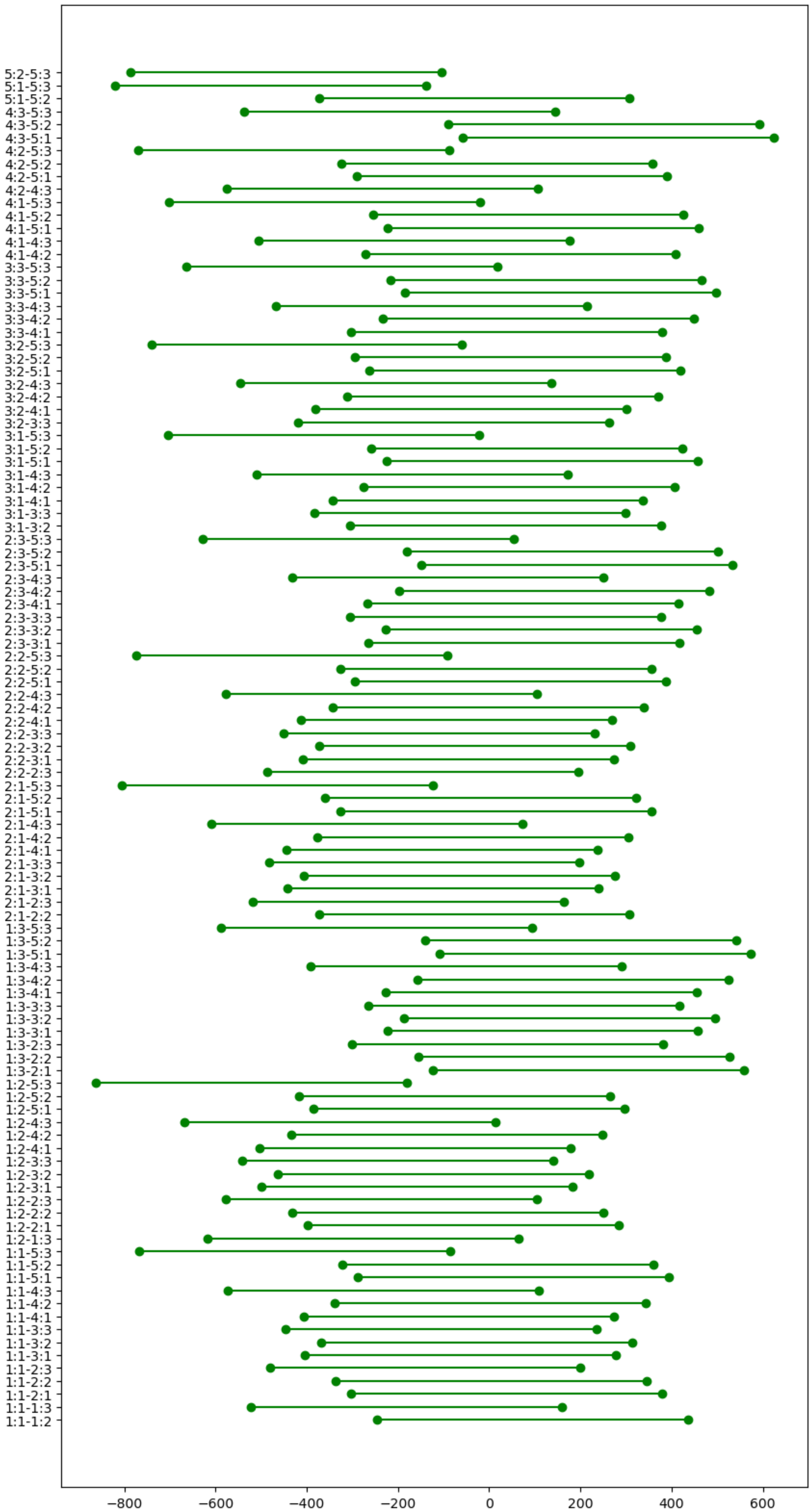
For Alloy2:

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1:1	1:2	95.3333	0.999	-245.5625	436.2292	False
1:1	1:3	-180.6667	0.8085	-521.5625	160.2292	False
1:1	2:1	37.6667	1.0	-303.2292	378.5625	False
1:1	2:2	5.0	1.0	-335.8958	345.8958	False
1:1	2:3	-140.3333	0.9635	-481.2292	200.5625	False
1:1	3:1	-63.3333	1.0	-404.2292	277.5625	False
1:1	3:2	-27.0	1.0	-367.8958	313.8958	False
1:1	3:3	-104.6667	0.9973	-445.5625	236.2292	False
1:1	4:1	-66.3333	1.0	-407.2292	274.5625	False
1:1	4:2	2.3333	1.0	-338.5625	343.2292	False
1:1	4:3	-231.3333	0.4686	-572.2292	109.5625	False
1:1	5:1	52.0	1.0	-288.8958	392.8958	False
1:1	5:2	19.3333	1.0	-321.5625	360.2292	False
1:1	5:3	-427.0	0.0049	-767.8958	-86.1042	True
1:2	1:3	-276.0	0.2169	-616.8958	64.8958	False
1:2	2:1	-57.6667	1.0	-398.5625	283.2292	False
1:2	2:2	-90.3333	0.9994	-431.2292	250.5625	False
1:2	2:3	-235.6667	0.4396	-576.5625	105.2292	False
1:2	3:1	-158.6667	0.912	-499.5625	182.2292	False
1:2	3:2	-122.3333	0.9884	-463.2292	218.5625	False
1:2	3:3	-200.0	0.6868	-540.8958	140.8958	False
1:2	4:1	-161.6667	0.9005	-502.5625	179.2292	False
1:2	4:2	-93.0	0.9992	-433.8958	247.8958	False
1:2	4:3	-326.6667	0.0709	-667.5625	14.2292	False
1:2	5:1	-43.3333	1.0	-384.2292	297.5625	False
1:2	5:2	-76.0	0.9999	-416.8958	264.8958	False
1:2	5:3	-522.3333	0.0003	-863.2292	-181.4375	True
1:3	2:1	218.3333	0.5587	-122.5625	559.2292	False
1:3	2:2	185.6667	0.7793	-155.2292	526.5625	False
1:3	2:3	40.3333	1.0	-300.5625	381.2292	False
1:3	3:1	117.3333	0.992	-223.5625	458.2292	False
1:3	3:2	153.6667	0.9291	-187.2292	494.5625	False
1:3	3:3	76.0	0.9999	-264.8958	416.8958	False
1:3	4:1	114.3333	0.9937	-226.5625	455.2292	False
1:3	4:2	183.0	0.7951	-157.8958	523.8958	False
1:3	4:3	-50.6667	1.0	-391.5625	290.2292	False
1:3	5:1	232.6667	0.4596	-108.2292	573.5625	False
1:3	5:2	200.0	0.6868	-140.8958	540.8958	False
1:3	5:3	-246.3333	0.3717	-587.2292	94.5625	False
2:1	2:2	-32.6667	1.0	-373.5625	308.2292	False
2:1	2:3	-178.0	0.8234	-518.8958	162.8958	False
2:1	3:1	-101.0	0.9981	-441.8958	239.8958	False
2:1	3:2	-64.6667	1.0	-405.5625	276.2292	False
2:1	3:3	-142.3333	0.9594	-483.2292	198.5625	False
2:1	4:1	-104.0	0.9975	-444.8958	236.8958	False
2:1	4:2	-35.3333	1.0	-376.2292	305.5625	False
2:1	4:3	-269.0	0.2485	-609.8958	71.8958	False
2:1	5:1	14.3333	1.0	-326.5625	355.2292	False
2:1	5:2	-18.3333	1.0	-359.2292	322.5625	False
2:1	5:3	-464.6667	0.0017	-805.5625	-123.7708	True
2:2	2:3	-145.3333	0.9525	-486.2292	195.5625	False
2:2	3:1	-68.3333	1.0	-409.2292	272.5625	False

2:2	3:2	-32.0	1.0	-372.8958	308.8958	False
2:2	3:3	-109.6667	0.9958	-450.5625	231.2292	False
2:2	4:1	-71.3333	1.0	-412.2292	269.5625	False
2:2	4:2	-2.6667	1.0	-343.5625	338.2292	False
2:2	4:3	-236.3333	0.4352	-577.2292	104.5625	False
2:2	5:1	47.0	1.0	-293.8958	387.8958	False
2:2	5:2	14.3333	1.0	-326.5625	355.2292	False
2:2	5:3	-432.0	0.0043	-772.8958	-91.1042	True
2:3	3:1	77.0	0.9999	-263.8958	417.8958	False
2:3	3:2	113.3333	0.9942	-227.5625	454.2292	False
2:3	3:3	35.6667	1.0	-305.2292	376.5625	False
2:3	4:1	74.0	0.9999	-266.8958	414.8958	False
2:3	4:2	142.6667	0.9586	-198.2292	483.5625	False
2:3	4:3	-91.0	0.9994	-431.8958	249.8958	False
2:3	5:1	192.3333	0.7376	-148.5625	533.2292	False
2:3	5:2	159.6667	0.9083	-181.2292	500.5625	False
2:3	5:3	-286.6667	0.1746	-627.5625	54.2292	False
3:1	3:2	36.3333	1.0	-304.5625	377.2292	False
3:1	3:3	-41.3333	1.0	-382.2292	299.5625	False
3:1	4:1	-3.0	1.0	-343.8958	337.8958	False
3:1	4:2	65.6667	1.0	-275.2292	406.5625	False
3:1	4:3	-168.0	0.8735	-508.8958	172.8958	False
3:1	5:1	115.3333	0.9932	-225.5625	456.2292	False
3:1	5:2	82.6667	0.9998	-258.2292	423.5625	False
3:1	5:3	-363.6667	0.0279	-704.5625	-22.7708	True
3:2	3:3	-77.6667	0.9999	-418.5625	263.2292	False
3:2	4:1	-39.3333	1.0	-380.2292	301.5625	False
3:2	4:2	29.3333	1.0	-311.5625	370.2292	False
3:2	4:3	-204.3333	0.657	-545.2292	136.5625	False
3:2	5:1	79.0	0.9999	-261.8958	419.8958	False
3:2	5:2	46.3333	1.0	-294.5625	387.2292	False
3:2	5:3	-400.0	0.0105	-740.8958	-59.1042	True
3:3	4:1	38.3333	1.0	-302.5625	379.2292	False
3:3	4:2	107.0	0.9967	-233.8958	447.8958	False
3:3	4:3	-126.6667	0.9842	-467.5625	214.2292	False
3:3	5:1	156.6667	0.9191	-184.2292	497.5625	False
3:3	5:2	124.0	0.9869	-216.8958	464.8958	False
3:3	5:3	-322.3333	0.0786	-663.2292	18.5625	False
4:1	4:2	68.6667	1.0	-272.2292	409.5625	False
4:1	4:3	-165.0	0.8868	-505.8958	175.8958	False
4:1	5:1	118.3333	0.9914	-222.5625	459.2292	False
4:1	5:2	85.6667	0.9997	-255.2292	426.5625	False
4:1	5:3	-360.6667	0.0302	-701.5625	-19.7708	True
4:2	4:3	-233.6667	0.4529	-574.5625	107.2292	False
4:2	5:1	49.6667	1.0	-291.2292	390.5625	False
4:2	5:2	17.0	1.0	-323.8958	357.8958	False
4:2	5:3	-429.3333	0.0046	-770.2292	-88.4375	True
4:3	5:1	283.3333	0.1871	-57.5625	624.2292	False
4:3	5:2	250.6667	0.3458	-90.2292	591.5625	False
4:3	5:3	-195.6667	0.7158	-536.5625	145.2292	False
5:1	5:2	-32.6667	1.0	-373.5625	308.2292	False
5:1	5:3	-479.0	0.0011	-819.8958	-138.1042	True
5:2	5:3	-446.3333	0.0028	-787.2292	-105.4375	True

Visual Representation for comparison of the interaction of different methods and dentists for Alloy2:


5:2-5:3
5:1-5:3
5:1-5:2
4:3-5:3
4:3-5:2
4:3-5:1
4:2-5:3
4:2-5:2
4:2-5:1
4:2-4:3
4:1-5:3
4:1-5:2
4:1-5:1
4:1-4:3
4:1-4:2
3:3-5:3
3:3-5:2
3:3-5:1
3:3-4:3
3:3-4:2
3:3-4:1
3:2-5:3
3:2-5:2
3:2-5:1
3:2-4:3
3:2-4:2
3:2-4:1
3:2-3:3
3:1-5:3
3:1-5:2
3:1-5:1
3:1-4:3
3:1-4:2
3:1-4:1
3:1-3:3
3:1-3:2
2:3-5:3
2:3-5:2
2:3-5:1
2:3-4:3
2:3-4:2
2:3-4:1
2:3-3:3
2:3-3:2
2:3-3:1
2:2-5:3
2:2-5:2
2:2-5:1
2:2-4:3
2:2-4:2
2:2-4:1
2:2-3:3
2:2-3:2
2:2-3:1
2:2-2:3
2:1-5:3
2:1-5:2
2:1-5:1
2:1-4:3
2:1-4:2
2:1-4:1
2:1-3:3
2:1-3:2
2:1-3:1
2:1-2:3
2:1-2:2
1:3-5:3
1:3-5:2
1:3-5:1
1:3-4:3
1:3-4:2
1:3-4:1
1:3-3:3
1:3-3:2
1:3-3:1
1:3-2:3
1:3-2:2
1:3-2:1
1:2-5:3
1:2-5:2
1:2-5:1
1:2-4:3
1:2-4:2
1:2-4:1
1:2-3:3
1:2-3:2
1:2-3:1
1:2-2:3
1:2-2:2
1:2-2:1
1:2-1:3
1:1-5:3
1:1-5:2
1:1-5:1
1:1-4:3
1:1-4:2
1:1-4:1
1:1-3:3
1:1-3:2
1:1-3:1
1:1-2:3
1:1-2:2
1:1-2:1
1:1-1:3
1:1-1:2
−800
−600
−400
−200
0
200
400
600

Inferences:

From the interaction plots of Alloy1:

The pairs which have the reject status as True or the pairs with p-adj value less than 0.05 are the pairs which differ in implant hardness from the rest of the pairs.

These Dentist:Method interactions with each other have significantly different means from each other:

Dentist: Method	Dentist: Method
1:1	4:3
1:1	5:3
1:2	5:3
1:3	5:3
2:1	4:3
2:1	5:3
2:2	4:3
2:2	5:3
2:3	5:3
3:1	5:3
3:2	5:3
3:3	5:3
4:1	4:3
4:1	5:3
4:2	4:3
4:2	5:3
5:1	5:3
5:2	5:3

Table 4.4.5

From the interaction plots of Alloy 2:

The pairs which have the reject status as True or the pairs with p-adj value less than 0.05 are the pairs which differ in implant hardness from rest of the pairs.

These Dentist:Method interactions with each other have significantly different means from each other:

Dentist: Method	Dentist: Method
1:1	5:3
1:2	5:3
2:1	5:3
2:2	5:3
3:1	5:3
3:2	5:3
4:1	5:3
4:2	5:3
5:1	5:3
5:2	5:3

Table 4.4.6