1.5 Day 5

1.5.1 Recognizing multiples of 3

Sometimes, it is required to detect if a given number is divisible by an integer k. Let us design a circuit that detects if a number is diviible by 3. Remember that when an integer is divided by 3, the remainder can be 0, 1, or 2. Let us create a state machine with three states, one corresponding to each of these remainders. We will call them S_0, S_1, S_2.

If a sequence B has already been received and the circuit is in state S_0, then $B = 3p$ for some p. Now two things can happen: a 0 can follow B or a 1 can follow B.

1. In the former case, the number is of the form $2B$. Thus the number must be of the form $B = 6p$ or $3p'$ and we continue to remain in S_0,

2. In the latter case, the number is of the form $2B + 1$ or $6p + 1$ or $3p' + 1$. Thus we move to State S_1.

Using the above idea, we can build the state diagram as shown in Figure 1.2.

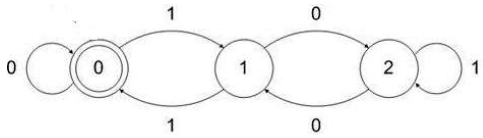

Figure 1.2: Machine to check divisibility by 3

1.5.2 Assignment

Using the idea given in the preceding section, build a circuit that recognizes numbers that are multiples of 5. How many states do you need? Indicate the number of states by n. How many flip-flops do you need to build this circuit? Note that you can use $p = \lceil \log_2 n \rceil$ bits to encode the states. How can we implement the FSM using a ROM? Assume that you have a ROM which as $p + 1$ address inputs. Sketch the circuit.