26 February 2021 10:38 PM

How Long Is a Piece of String? by Eastaway, Rob

#1

“Quiz: How big is the moon? Which of the following circular objects when held at arm’s length is about the same size as the moon?   (a) a pea (b) a penny (c) a ping-pong ball (d) an orange The answer is (a), a pea – a petit pois, in fact. For something that so dominates the night sky, this is surprisingly small. Thanks to the workings of the human brain, we perceive it to be much larger.”

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How Long Is a Piece of String? by Eastaway, Rob

#2

“12: an abundant number All whole numbers have factors – that is, smaller whole numbers that divide exactly into them (except for 1, of course). The factors of 12 are 6, 4, 3, 2 and 1, which add up to 16. A number whose factors add up to more than the number itself is called ‘abundant’ and 12 is the smallest such number. Abundant numbers are in fact extremely common. Looking for something to get their teeth into, mathematicians therefore began to look not just at whether numbers are abundant or not, but how abundant they are. For example, 12’s abundancy is 16/12 or 1.33. It is beaten by 24 (36/24 = 1.5) and this abundancy ratio increases with every multiple of 12 up to 60, whose factors add up to 108. (108/60 scores a whopping 1.8). The number 60 is highly abundant because so many numbers divide into it, and this Is why it became a popular base for counting. There seems to be no upper limit to how abundant a number can get, If you go high enough. Does abundancy matter? Apart from enriching the understanding of numbers, no it doesn’t. But ancient Greeks, particularly Pythagoras and his cronies, were always looking for evidence to support their view that numbers controlled the entire universe. Any obscure property of numbers took on a significance that might now be regarded as excessive.”

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How Long Is a Piece of String? by Eastaway, Rob

#3

“28: a perfect number The factors of 28 are 1,2, 4, 7 and 14 – and it so happens that these add up to 28. The Greeks spotted this coincidence, and labelled this number, and others with the same property, as ‘perfect numbers’. Rather an arbitrary definition of perfection, you might think. Perfect numbers are rare. The Greeks discovered only four of them: 6, 28, 496 and 8128, and as far as we know they didn’t find any others. This is hardly surprising since the next smallest perfect number is 33, 550, 336. It is believed that all perfect numbers end in 6 or 8, but it isn’t known if there are infinitely many of them. The belief that numbers with curious mathematical properties were in some way mystical would certainly have helped to establish those numbers as part of the culture. The perfect number 28 and the abundant number 12 were two of the main beneficiaries.   This second occurrence of the number seven in the skies is a fluke, of course - the umber of days in the moon cycle has nothing to do with there being seven ‘planets’ – but, given man – kind’s natural tendency to read meaning into any coincidence, it is hardly surprising that early civilisations believed that the number seven had a fundamental link with the heavens. For convenience and for ritual, the moon cycle was therefore divided into partitions of four lots of seven days. ‘In the beginning’, according to the Bible, God took seven days to create the earth. Is this an even more fundamental reason for the mystical importance of the number seven? Or did the people who wrote this story choose a number that already carried significance for them because it divided up the lunar month and represented the number of the planets? It doesn’t really matter. The fact is, this story firmly established the idea of a seven-day week in Jewish culture with the seventh day being named the rest day, or Sabbath. The Jews spread the idea across the Middle East, and the Romans adopted it, too, despite the fact that they already had a separate eight-day ‘market’ week of their own whose origin is o”

How Long Is a Piece of String? by Eastaway, Rob

#4

“The trick of the operational researcher is very often to develop sets of rules or algorithms that are guaranteed to lead to a solution that is within a certain target – say 10 per cent – of the best possible. Think of those occasions where you are moving house. You know that the volume of items you need to shift is enough to fill about twenty identical boxes – except that it would require far too much time and planning to work out what to put where. Instead, you go for the easiest method of packing, which is to put items at random into the first box, and, when the next item you pick up doesn’t fit, you seal up the box you have been filling and start on a new one. This is known as a ‘first-fit strategy’. How efficient is it? It turns out that, however unlucky you are with the order in which things come to hand, the number of boxes you need will always be within 70 per cent of what could be achieved with perfect allocation. So if your best possible is 20 boxes, you can reassure yourself that even the lazy first-fit method strategy should require at most 34 boxes. If this isn’t good enough – and, let’s be honest, 70 per cent is a bit of a waste, though this is the worst-case scenario – a strategy of packing the biggest things first and the smallest last always turns out to be within 22 per cent of the very best solution. This means a worst case of 25 boxes, instead of the optimal 20. And, since many of us tend to use a biggest-first strategy, especially when filling the car boot, this shows that when it comes to packing, common sense is a good substitute for deep mathe -matical thinking.”

28 February 2021 6:24 PM

How Long Is a Piece of String? by Eastaway, Rob

#5

“Imagine if your school timetable had offered the following optional topics. Monday: How to avoid being ripped off Tuesday: Thinking games Wednesday: Tips for highly paid jobs Thursday: Patterns in the real world Friday: When to take a chance No doubt you would have chosen at least one of these options, and maybe all of them. And yet, without stretching reality too far, that is exactly how your timetable could have looked. It’s just that some administrators decided to call each of these topics mathematics. Then, to be certain that all the fun was squeezed out, they made as much of the subject as abstract and detached from the real world as they could”

1 March 2021 9:09 PM

How Long Is a Piece of String? by Eastaway, Rob

#6

“The lateral-thinking approach There has been an underlying assumption in this chapter that the waiting time for lifts needs to be reduced. However, this is true only because people become frustrated when waiting. If people didn’t get bored while waiting, then they would be less concerned about how long the lift took to arrive. According to office legend, one company with slow lifts got around the problem by putting mirrors outside the lifts. This didn’t alter the speed of service, but customers spent the waiting time combing their hair and otherwise grooming themselves. The level of customer satisfaction rocketed. If this story is true, then the building manager deserves a medal for saving a fortune in lift engineering fees.”

1 March 2021 9:39 PM

How Long Is a Piece of String? by Eastaway, Rob

#7

“It has often been said that, while most countries have climates, Britain just has weather. If there was an index that measured the degree of fluctuation from rain to sunshine, windy to still, and warm to cold over short periods of time, the British Isles would surely top the league”

1 March 2021 9:41 PM

How Long Is a Piece of String? by Eastaway, Rob

#8

“It seems fair to describe the spreading out of the coloured balls at the start of a pool game as chaotic, and in fact chaos is exactly the term that mathematicians would use to describe it. Because it is a relatively new science, mathematicians are still a little bit vague about the precise definition of chaos, though definitions do exist, some of them extremely complex. However, there is one underlying theme to chaos that most mathematicians agree on. Something is chaotic if the tiniest of changes to the initial input can lead to a completely different, and unpredictable, outcome. This effect of small errors having huge consequences has been known for a long time. Benjamin Franklin, one of the founding fathers of the USA, was responsible for this well-known quotation: For the want of a nail, the shoe was lost; for the want of a shoe the horse was lost; and for the want of a horse the rider was lost, being overtaken and slain by the enemy, all for the want of care about a horseshoe nail.”

1 March 2021 9:48 PM

How Long Is a Piece of String? by Eastaway, Rob

#9

“Randomness: how can a computer simulate dice? Most games programs on computers require the computer to do ‘random’ things. Any computer, therefore, needs to be able to produce on command numbers that are as unpredictable as the outcome of rolling dice. This is not as easy as it may sound, because the whole point of computers, of course, is that they are there to be predictable by following rules. Although they are not capable of generating truly random numbers, all computers contain a formula that will produce ‘pseudo-random’ numbers – numbers that appear random even though they are generated by a precise sequence of calculations. Hundreds of different methods exist for generating such sequences, many of them requiring an initial seed number to start them off. This seed can either be entered by the user, or else taken from the computer’s clock (for example the number of seconds past the minute at the instant when the keyboard is struck). Randomness is quite tricky to define, but a common way of testing for it is to check that (a) all numbers in the sequence appear roughly the same number of times as each other, and (b) the numbers do not follow any predictable pattern. The sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 passes the first test of randomness, but fails the second. 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0 appears, at first glance anyway, to pass both tests, though it is only pseudo-random because it was in fact generated using a simple rule -can you work out the rule?”

2 March 2021 3:03 AM

How Long Is a Piece of String? by Eastaway, Rob

#10

"Where 'e' comes from Imagine you have £1, and you put it into a bank account that offers 100 per cent interest per year. If the bank pays you the 100 per cent interest at the end of the year, you end the year with £2. If, instead, it pays 50 per cent every six months, then you have £1.50 after six months, and 50 per cent more than that at the end of the year, or £2.25. What about four lots of 25 per cent at three-month intervals? This works out at even more, a final amount of £2.44. As the periods between interest payments get shorter, you get closer and closer to continuous growth of your investment, but the sum of money at the end of the year tends towards a maximum figure. That maximum is about £2.72. The actual number begins 2.71828… and is known as Euler's number, 'e'. It is the number at the root of all natural population growth and a fundamental player in many other areas of maths, too. Expressed as the formula (1 + 1/n)n, the larger the value of 'n'becomes, the nearer you get to e. Like the bank that adds interest continuously, infectious diseases constantly spread themselves – or, at least, that is a pretty close approximation. The spread factor, S, is the number of new cases created by an infected person, and the number of people infected at the start of the outbreak is I. If infection only happened as a sudden event at the end of one infectious period, the number of newly infected people at the end of the period would be:"

2 March 2021 3:44 AM

How Long Is a Piece of String? by Eastaway, Rob

#11

“The basic formula The basic principle behind a taxi fare is simple enough. If you make a long journey, you should expect to pay more than you do on a short journey, and the taximeter, a device invented by Wilhelm Bruhn in 1896, charges you a rate for distance travelled. But what about heavy traffic or unexpected delays due to an accident? As far as taxis are concerned, a ‘long’ journey means a long time, as well as a long distance. To cover the driver for his work time spent sitting in a traffic jam or crawling through the rush hour, the taximeter also has a rate for the time spent on the journey”

2 March 2021 3:51 AM

How Long Is a Piece of String? by Eastaway, Rob

#12

“How to calculate a city’s average speed If you ever want to know the average speed of traffic in a city, check the taxi fares. The fare will be quoted as X pence per Y distance and X pence per Z time. Divide Y by Z and you have a good estimate of the average speed. For example, in London Y and Z are 189.3 metres per 40.8 seconds; 189.3 divided by 40.8 is 4.6 metres per second, or just over 10 m.p.h. In New York, Y/Z works out as 1⁄2-mile/90 seconds, or 20 m.p.h. Why does this trick work? Because taxi rates are set by modelling what the expected income will be on an average day in average traffic.”

2 March 2021 4:10 AM

How Long Is a Piece of String? by Eastaway, Rob

#13

“The moral from this is that graphs with sudden jumps can lead to corruption. That is why the smooth line of the graph of taxi fare against speed is a good one. It removes the incentive for a cabby to travel at a particular speed to receive a jump in incom”

2 March 2021 4:23 AM

How Long Is a Piece of String? by Eastaway, Rob

#14

“How commitment-phobic are you? Play the blind-date game. Pick out ten cards from a pack that are numbered ace to ten, with ace scoring low. These cards represent your blind dates, and the aim of the game is to end up with the highest-scoring card. Shuffle the cards, and deal them out face down: Starting at the left, you can choose how many of the cards you want to date, but not get serious with. This is you, ‘playing the field’ without making a commitment. The lowest playing-the-field (or PF) number is zero – in other words, you’ll take the first card that comes along. The highest PF number is nine, which means your commitment is at a minimum but you are forced to commit to the last card you turn up, the tenth. Once you have chosen your PF number, turn over that many cards, and note the highest score. This is your benchmark. Now start turning over your potential partners. The first card that is higher than the benchmark is your partner. If none scores higher, the tenth card is your partner regardless of its value. On average, PF numbers of 0 or 9 give you the worst out comes. The best results come from choosing a PF number of 3. This will give the best available partner about one time in three.”

2 March 2021 5:29 AM

How Long Is a Piece of String? by Eastaway, Rob

#15

“Early in the 1990s, Benford’s Law made its famous entry into the world of fraud detection. Mark Nigrini, a lecturer in accountancy, asked his students to look at the accounts of a business they knew, in order to demonstrate to themselves the predictable distribution of first digits. One student decided to look at the books of his brother-in-law, who ran a hardware shop. To his surprise, the numbers didn’t resemble the Benford distribution at all. In fact 93 per cent of them began with the digit 1, instead of the predicted 30 per cent. The remainder began with 8 or 9. The discrepancy was so huge, that it suggested that something must be wrong with the figures. In fact, rather embarrassingly for all concerned, the student had inadvertently”

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How Long Is a Piece of String? by Eastaway, Rob

#16

“inadvertently discovered that his relation had been fraudulently cooking the books.”

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How Long Is a Piece of String? by Eastaway, Rob

#17

“How chi-squared won the girl One statistical test used by literary analysts, as well as by scientists and many others, is known as the chi-squared test. In this test the observed frequencies in the sample, such as the number of occurrences of words such as ‘bible’ and ‘discontent’, are compared with the expected number. The conclusion of this test is expressed as a percentage chance: for example, ‘Fewer than 5 per cent of Shakespeare’s documents might be expected to have produced the patterns we see here.’ In one of its more unusual applications, back in the 1980s a student used a chi-squared test to demonstrate to manufacturer Rowntree’s that the letters inside the caps of packets of Smarties chocolates were not randomly distributed. He had been trying in vain to collect the letters of his Valentine’s name. He won the argument, received some free packets – and got the girl, too.”

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How Long Is a Piece of String? by Eastaway, Rob

#18

“There is one other major factor that can benefit an underdog, and that is the freakish accident or fluke. In 1967, a horse called Foinavon won the Grand National, despite being the rank outsider at 100-1 against. Had the horses all completed the course, Foinavon wouldn’t have had a chance. On this occasion, however, about twenty horses fell or refused at one fence. Foinavon was so far behind the rest that he was able to avoid the melee and had a free run for victory. In this case being hopeless turned out to be a huge advantage to the underdog. Motor racing is also prone to accidents or car problems that randomly hit the strong as much as the weak. Almost always the reason why a non-favourite wins a grand prix is because something other than driver skill disabled the frontrunners.”

3 March 2021 3:04 AM

How Long Is a Piece of String? by Eastaway, Rob

#19

“Precision versus approximation How can you tell the difference between a mathematician and an engineer? Ask them what pi is. Mathematician: ‘It is a ratio describing the circumference of a circle to its diameter, a transcendental number which begins 3.14 and continues for an infinite number of digits.’ Engineer: ‘It’s about 3, but let’s call it 10 just to be on the safe side.’”

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How Long Is a Piece of String? by Eastaway, Rob

#20

“This type of proof can be used for an old sock problem, too. To get around the problem of lots of odd socks, one of the authors has adopted a simple strategy. All he ever buys are identical black socks and identical blue socks. That way, he always has lots of pairs, even if he loses the occasional sock. But, on dark winter mornings, black and blue socks look remarkably alike. If he has ten black and ten blue socks in a drawer, how many socks does he have to take out to be absolutely certain that he has a pair of socks in his hand? To some people, the answer is obvious. Only three socks need to be taken from the drawer. However, others argue that, to be certain of ending up with a pair, eleven or even nineteen socks have to be taken out. One way to prove how many socks are needed for a pair would be to look at every single possible order in which socks can be removed from the drawer: black, black, blue, black; blue, black, black, blue… and so on. This would, however, take rather a long time, since there are nearly 200,000 different combinations. A simpler proof requires far fewer combinations, by using a logical short cut. Imagine taking any two socks from the drawer. If they make a pair, then the problem is solved in just two socks. If they don’t make a pair, they must be odd, i.e. one black and one blue. Since the next sock out of the drawer is going to be one of these two colours, the three socks must certainly contain a pair. Hence three socks is the maximum needed. Surprisingly, the sock problem has a lot in common with the four-colour theorem. An exhaustive checking of a huge number of options finally proved the latter, but, as with the odd sock, there were short cuts to reduce the range of possibilities that needed to be tested. Exhaustive testing is all right as a way of finding a proof, but if it looks as if it is going to take a long time, it’s worth thinking about how the search could be simplified. And, besides, short cuts are more fun.”

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How Long Is a Piece of String? by Eastaway, Rob

#21

“All right, that was not an earth-shattering example, but what matters here is the principle of imagining an answer and seeing where it takes us. This approach of making an assumption and then following it through until it leads to a conclusion that we know is false carries the formal Latin name of reductio ad absurdum, literally ‘reduction to the absurd’. Informally, this technique is used in conversation all the time. Barristers and politicians are particularly fond of it as a means of demonstrating the weakness of an opponent’s argument. Hansard, the written record of parliamentary debates, is no doubt full of arguments along the lines of: ‘The Right Honourable Gentleman claims that he will increase public spending. The only way in which he can achieve this is by increasing taxes – which he has already ruled out. I therefore pronounce his argument to be in tatters.’”

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How Long Is a Piece of String? by Eastaway, Rob

#22

“Three women live alone in a row of three isolated houses on a hill. Here are three facts that refer only to them. (1) Maureen does not live in the middle house. (2) Debbie shares her lawnmower with the architect. (3) Jan lives two doors up from the artist. Who lives where? The standard way into these puzzles is to make a guess and test it out. For example, let’s suppose that Debbie is the artist. From fact (3), this means Debbie must live two doors from Jan, which we can represent like this: However, this would mean that Maureen occupies the middle house. This is a contradiction of fact (1) which says she doesn’t live there. Hence our initial assumption is false, so Debbie is not the artist. From here it can quickly be concluded that Debbie must live in the middle house, Jan in the top house and Maureen in the bottom one. Phew, that’s over.”

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How Long Is a Piece of String? by Eastaway, Rob

#23

“Starting small and working upwards Another helpful approach in tackling life’s problems is to start by thinking about the question in its simplest form, and work up from there. After all, a lot of problem-solving goes wrong simply because the issues are too complicated to take in all at once. A similar principle of starting with simple examples often helps in mathematical problem solving too. And it can help in some proofs. Here is a nice example. Everyone knows that 32 – 22 = 9 – 4 = 5. But what is the answer to this sum: 222,222,222,222,222,222,2222 – 222,222,222,222,222,222,2212 Most calculators won’t help here, since they won’t handle numbers this big. And even desktop computers are likely to get it wrong. One computer gave the answer zero, which is patently nonsense since the difference between these two square numbers must be a huge number itself.”

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How Long Is a Piece of String? by Eastaway, Rob

#24

“The pigeonhole proof There is one type of short cut that carries the name pigeonhole proof. At the peak of the National Lottery’s popularity, over 15 million people bought tickets in a single week, and somebody in the press wondered if everyone had picked a different combi nation. There are, of course, millions of different combinations of numbers that could be chosen, but how can we be sure whether 15 million people chose different combinations or not? Here is how. The total number of different combinations that you can choose for a lottery ticket is 13,983,816. Let’s take the extreme case. Suppose that, for the first 13,983,816 tickets sold, every person chose a different combination. Every single possible combination had now been used, and without a single duplicate. What about the 13,983,817th person? Since every combination had already been entered, he had no choice but to choose a combination that somebody else had already chosen. Hence it is certain that, if 15 million people buy lottery tickets, at , least two of them choose the same numbers. Of course we don’t know what that coincidental combination is, but that isn’t what is being proved. All that has been proved to be universally true is that there was at least one combination that appeared at least twice. This is a pigeonhole proof. You can imagine creating a pigeonhole for every possible choice of lottery numbers. You then try to place each selection into a different pigeonhole. When you run out of empty pigeonholes you have no choice but to put a second entry into one of them. Exactly this principle can be used to prove that, in Manchester United’s last home game, there were certainly at least two people who were born on the same day of the same year. How can we know this? Let’s guess conservatively that there were only 50,000 at the game (there were probably more like 70,000), and that everyone in the crowd was aged between 0 and 100 (another very conservative estimate – the age range was probably far narrower than that). All of those people were born in the last 100 years. Could they all have been born on different days?”

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How Long Is a Piece of String? by Eastaway, Rob

#25

“Conveniently ignoring inflation is probably one of the most common sleights of hand practised by spin doctors, and passed on to the public without challenge by the media. Everybody likes to see teachers’ salaries, money spent on hospitals and the value of possessions rise every year, and because of the normal process of inflation they usually do. It all sounds like good news, but in itself these increases are meaningless. ‘More’ does not necessarily mean ‘better’. Nor, of course, does it necessarily mean worse. By exactly the same argument, electricity bills, beer prices and the amount of tax raised by the government are all likely to rise each year (each one a ‘shockhorror’ story) and yet, because of pay rises, these increases may have no effect on people’s standard of living.”

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How Long Is a Piece of String? by Eastaway, Rob

#26

“Making something smaller and bigger at the same time Percentages are a particularly good prop for performing the spin doctor’s magic. Take, for example, the miracle of vanishing exports.”

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How Long Is a Piece of String? by Eastaway, Rob

#27

“I won’t deny that these have been tough times for the company,’ said the spokesman. ‘Last year, due to the strength of the currency, our exports fell by 40 per cent, but I’m delighted to announce that, thanks to the outstanding efforts of our marketing team, this year has seen us bounce back with a sensational 50 per cent increase.’ The shareholders are impressed – 40 per cent down followed by 50 per cent up. Sounds like a net increase of 10 per cent. This is another classic piece of misdirection by the number magicians. Here are the real figures: So last year, the exports dropped by 40,000 from their previous 100,000 level. That is indeed a 40 per cent decrease. This year, we are told, has seen a 50 per cent increase over last year. Last year’s exports were 60,000, and 50 per cent of this is 30,000, so after the 50 per cent increase we now have: Hang on a second, that isn’t a net increase of 10 per cent over two years ago. A drop of 40 per cent followed by an increase of 50 per cent led to a net decrease of 10 per cent. Incredible! How’s it done?”

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#28

“done? There’s nothing hidden, that’s just the way that percentages work. The spokesman compared the 40 per cent and the 50 per cent as if they were the same thing, but, since they were based on different starting figures, it was like comparing apples with pears.”