

Analytical Modeling of Parallel Programs.

Q- Sources of Overhead in Parallel programs.

- sequential algorithm \rightarrow its runtime (evaluated by)
- parallel runtime \rightarrow depends on input size
 - \rightarrow parameters of machine (communication)
 - \rightarrow ~~depends on input~~ no. of processors.
- execution time of parallel algo \rightarrow input size + no. of processing elements
- combination of parallel algo & parallel architecture.

* Diff overhead-

• Interprocessor communication-

- ① Processor working on non-trivial parallel problem will need to talk.
- ② source of parallel processing overhead \rightarrow time spent to interact & commⁿ data b/w processing elements.
- ③ data dependancy exists in problem & no. of processing element working on same data.
- ④ Two processing element are independent they can't work in parallel.

2) Idling (Load Balance (im)) -

- process may become idle because of many reason.
- Diff. factors that contribute to idling of processing elements like load imbalance, synchronization, presence of serial component in dynamic subtask generated.
- Dynamic task generation application; it is very difficult to predict size of subtasks assigned to processor in advance.
- To maintain uniform workload among processors, dynamic subtasks generated.
- dynamic task generation applⁿ \rightarrow subtasks assigned to processor in advance.
- maintain uniform workload among processors. Can't divide problem statistically b/w processing elements.

3) Excess Computation -

- computation \rightarrow not by serial version.
- serial algo \rightarrow difficult to parallelize or are repeated across processors to minimize communication.
- excess = parallel - Best serial
- Best serial program \rightarrow diff factors parallel code \rightarrow excess comp

* Performance measure & Analysis -

Metrics \rightarrow 1) Efficiency 2) Speedup 3) Cost 4) Parallel Overhead
5) Execution time

- speedup - captures relative benefit of solving a problem in parallel.

$$S = \text{speedup} = \frac{T_0}{T_p} \quad \left[\begin{array}{l} \text{time taken by best sequential algorithm / by single processor} \\ \text{time taken by } p \text{ processors} \end{array} \right]$$

1) * Execution Time \rightarrow time to complete a task & denoted by T_{exec} .

i) Sequential or serial runtime (T_s) - time elapse \rightarrow b/w start of execution, end of execution \rightarrow sequential computer serial runtime.

ii) Parallel Runtime (T_p) - time that elapse from moment first processor starts to moment last processor finishes execution.

2) * Total Overhead -

- denoted by T_o .

• time spent by parallel program - time spent by serial programs -

$$S = \frac{T_s}{T_p}$$

$$T_o = P T_p - T_s \quad (P \text{ is processing element})$$

$T_s \rightarrow$ execution time on a single processor

$T_p \rightarrow$ execution time using a parallel processors.

• Speedup \rightarrow S given by $S = O(n/\log n)$

3) Speed Up -

- faster the code runs \rightarrow parallel over serial execution.
- evaluates \rightarrow benefit of solving problem in parallel.
- $S = \frac{T_s}{T_p}$ (execution time on single processor) / (speedup execution time using a parallel processor).

$$\text{speedup } S = O(n/\log n)$$

4) Efficiency -

- measure of fraction \rightarrow time \rightarrow processing element (processor) is engaged.
- Ratio of speedup to no. of processing elements

$$E = \frac{\text{speedup}}{\text{processing elements}} = \frac{S}{P} \quad E = O(1/\log n)$$

5) Cost -

- cost of parallel algo \rightarrow or program cost = parallel running time / no. of processors.
- cost reflects sum of time \rightarrow spends solving problem.
- cost is referred as work or processor time product.
- cost of system $P T_p = n \log n$.

* Amdahl's Law -

- consider a fixed computational workload w & a number of processors p .
- let execution rate of i processor be R_i so far sequential processors
($i=1$) $R_1=1$ & $R_n=n$.
- Assume sequential workload is αw ($0 \leq \alpha \leq 1$) & parallel workload is $(1-\alpha)w$.
- Used for fixed workload parallel system.
- no. of processing elements or processors increase, time required for execution must reduce.
- speedup for n processors according to execution must reduce.

amdaahl's law can given as-

$$S(n) = \frac{t_s}{f t_s + (1-f) t_s / n}$$

t_s = total time required on sequential system.

f = fraction of code.

$1-f$ = parallelizable part.

2) Gustafson's Law -

- overcome the drawbacks of Amdahl's law.
- relaxed the problem size from being fixed to be of any size.
- instead of having fixed problem size or fixed workload we must assume \rightarrow fixed execution time.
- case of huge problem size \rightarrow increase our system size.

$$s'(n) = \frac{n + s(1-n)}{1}$$

$$s'(n) = n + s(1-n).$$

eqn \rightarrow scaled speedup factor of Gustafson's law.

* Effect of Granularity on Performance -

- No. of task divisions \propto Granularity.
- Fine Granularity \rightarrow Decomposition into large no. of tasks.
- Coarse Granularity \rightarrow decomposition into small no. of tasks.
- Degree of concurrency \propto Granularity.
- practice we do fine granularity.
- less processing elements than available is called scaling down.
- n inputs & p processing elements & $p < n$ we can design as parallel algo works for n processing elements by using n/p virtual processing elements by n/p workload at each processing unit increases by a factor of n/p .

Granularity impacts -

- affects performance of parallel computers.
- fine granularity increases speedup.
- However synchronization overhead ~~negatively~~ negatively affects ~~performance~~.

* Scalability of parallel systems -

- capacity of increase speedup in proportion to no. of processing elements.
- reflects parallel system's ability to utilize increasing processing resources effectively.
- efficiency is given as - $E = \frac{S}{P} = \frac{T_s}{PT_p} = \frac{1}{1 + \frac{T_o}{T_s}}$

- total overhead to increasing f^n of p .
- every parallel program always has a sequential that runs for time t_s .
- overload increases superlinearly due to communication overhead dealing, excess computation.
- As $p \uparrow$, $E \downarrow$ goes down $E \propto 1/p$.
- scale more than efficiency shall decrease.
- efficiency may be more than efficiency shall decrease.
- efficiency may increase if problem size is increased keeping no. of processors constant.

* Minimum execution time & min cost -

- no. of processors increase parallel runtime continues to decrease & asymptotically approaches a minimum values, goes on increasing after a min val.
- give min execution time as -

$$\frac{d}{dp} T_p^{\min} = 0$$

- gives no. of processing elements for which T_p is min.
- max. no. of processing elements is bounded by degree of concurrency,

$$T_p^{\min} = \frac{w + T_o(w, c[w])}{c[w]}$$

* Asymptotic Analysis of parallel programs -

Example of adding n nos. with n processing elements -

$$T_p = O(\log n) \cdot S = O(n / \log n)$$

* ~~Matrix Vector Multiplication~~

- evaluates behaviour of parallel algo \rightarrow input size approaches infinity
- characterizing algo complexity & performance scaling.
- examines how algorithmic efficiency changes with increasing problem
- common asymptotic notation \rightarrow Big O, Theta & Big Omega.
- provides insights into scalability & efficiency of parallel algo.
- identify dominant computational & costs.
- guides algo selection & optimization.
- with lower asymptotic complexities are preferred for large scale computation.
- facilitates comparison b/w diff parallel algo & system config.

* Matrix Vector Multiplication -

- computes product of a matrix & vector, resulting in new vector
- fundamental operation in linear algebra & computational science.
- can be parallelized to exploit concurrency & improve performance.
- algo for include row-wise, columnwise & block wise approaches.
- parallelized aim to distribute computational workload across multiple processing units efficiently.
- optimize data locality & minimize comm'n overhead.
- key building block in scientific & engineering applications & big processing.
- essential for accelerating computations in large scale numerical simulations.
- using specialized computing devices like GPUs enhances performance.
- optimization techniques like loop unrolling, data blocking & vectorization further improve efficiency of parallel matrix vector multiplication algo.

* Matrix Matrix Multiplication Algorithm -

- computes product of two matrices using parallel processing.
- divides matrices using parallel processing into smaller blocks & distributes among multiple processing.
- each process computes a portion of resulting matrix independently.
- reduces overall time & leverage concurrent processing.
- utilizes techniques like data processing partitioning & commⁿ among processes.
- scales well with no. of available processing units, improving performance.
- implemented using parallel algo like Cannon's algo.
- optimizes resource utilization by distributing workload across multiple processors.
- overheads minimized by efficiently exchanging data b/w processors.

* Execution Rate & Redundancy -

measures increase in required computation when using more processing units.

to defined on per component basis & assigned a sample time based on sample times of components it is connected to.