CptS 575: Data Science, Fall 2018

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```
                         Assignment 4: Linear Regression
```

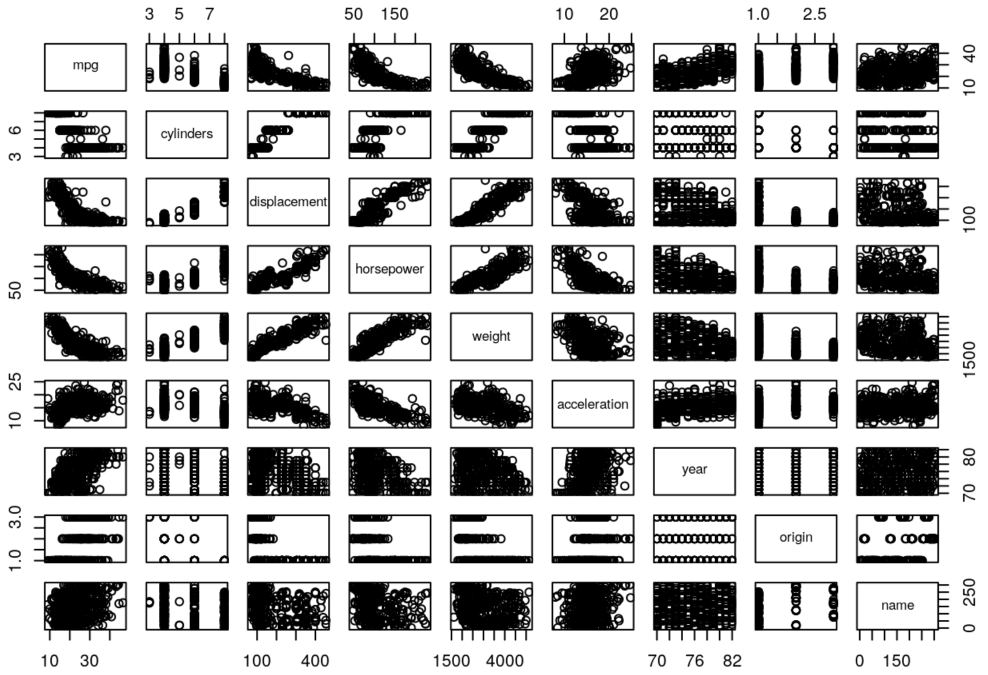
Problem 1

This question involves the use of multiple linear regression on the Auto data set from the course webpage (https://scads.eecs.wsu.edu/index.php/datasets/). Ensure that you remove missing values from the dataframe, and that values are represented in the appropriate types (num or int for quantitative variables, factor, logi or str for qualitative).

```
Auto = read.csv("Auto.csv", header = T, na.strings = "?")
```

a).Produce a scatterplot matrix which includes all of the variables in the data set.

```
pairs(Auto)
```



```
Auto <- na.omit(Auto)
```

b).Compute the matrix of correlations between the variables using the function cor(). You will need to exclude the name variable, which is qualitative.

```
cor(Auto[sapply(Auto, function(x) !is.factor(x))])
```

```
##                     mpg  cylinders displacement horsepower     weight
## mpg           1.0000000 -0.7776175   -0.8051269 -0.7784268 -0.8322442
## cylinders    -0.7776175  1.0000000    0.9508233  0.8429834  0.8975273
## displacement -0.8051269  0.9508233    1.0000000  0.8972570  0.9329944
## horsepower   -0.7784268  0.8429834    0.8972570  1.0000000  0.8645377
## weight       -0.8322442  0.8975273    0.9329944  0.8645377  1.0000000
## acceleration  0.4233285 -0.5046834   -0.5438005 -0.6891955 -0.4168392
## year          0.5805410 -0.3456474   -0.3698552 -0.4163615 -0.3091199
## origin        0.5652088 -0.5689316   -0.6145351 -0.4551715 -0.5850054
##              acceleration       year     origin
## mpg             0.4233285  0.5805410  0.5652088
## cylinders      -0.5046834 -0.3456474 -0.5689316
## displacement   -0.5438005 -0.3698552 -0.6145351
## horsepower     -0.6891955 -0.4163615 -0.4551715
## weight         -0.4168392 -0.3091199 -0.5850054
## acceleration    1.0000000  0.2903161  0.2127458
## year            0.2903161  1.0000000  0.1815277
## origin          0.2127458  0.1815277  1.0000000
```

C).Use the lm() function to perform a multiple linear regression with mpg as the response and all other variables except name as the predictors. Use the summary() function to print the results. Comment on the output:

```
model = lm(mpg ~. -name, data = Auto)
summary(model)
```

```
##
## Call:
## lm(formula = mpg ~ . - name, data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5903 -2.1565 -0.1169  1.8690 13.0604
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.218435   4.644294  -3.707  0.00024 ***
## cylinders     -0.493376   0.323282  -1.526  0.12780
## displacement   0.019896   0.007515   2.647  0.00844 **
## horsepower    -0.016951   0.013787  -1.230  0.21963
## weight        -0.006474   0.000652  -9.929  < 2e-16 ***
## acceleration   0.080576   0.098845   0.815  0.41548
## year           0.750773   0.050973  14.729  < 2e-16 ***
## origin         1.426141   0.278136   5.127 4.67e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.328 on 384 degrees of freedom
## Multiple R-squared:  0.8215, Adjusted R-squared:  0.8182
## F-statistic: 252.4 on 7 and 384 DF,  p-value: < 2.2e-16
```

i).Which predictors appear to have a statistically significant relationship to the response, and how do you determine this?

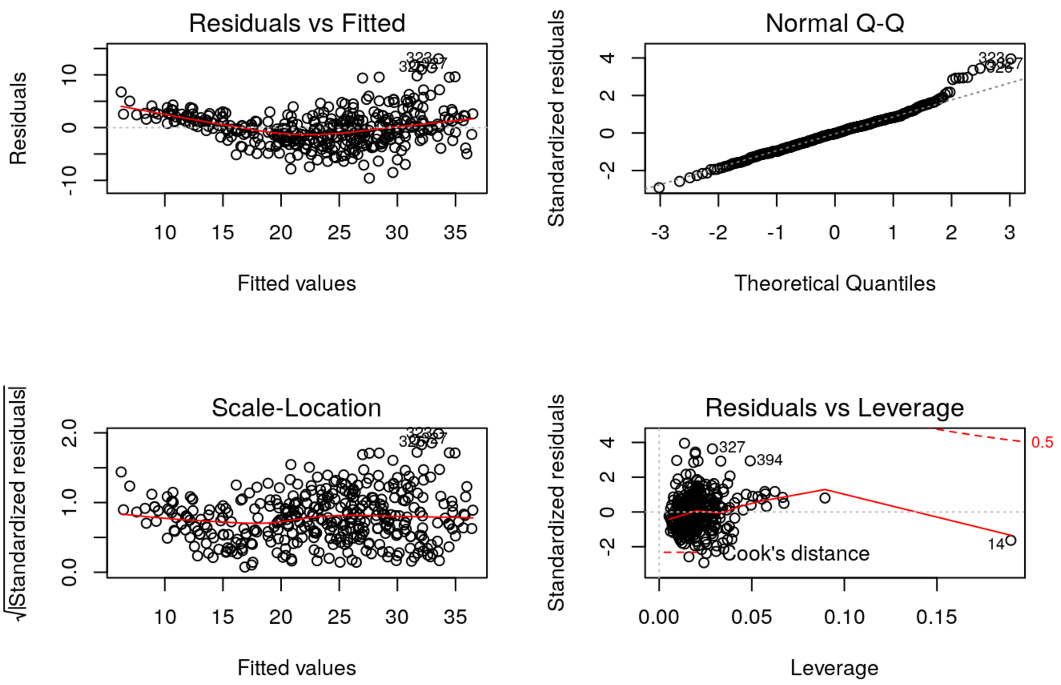
```
The low p-values for displacement, weight, year, and origin indicate a statistically significant relationshi
p to mpg.The predictors that have the highest statstical sginficance are: weight, year and origin. This make
s intuitive sense, as lighter cars would logically have better mpg and more modern cars employ better gas-sa
ving technology. displacement and horsepower are also sgificant at the .05 level.
```

ii).What does the coefficient for the cylinders variable suggest, in simple terms?

```
The coefficient ot the “cylinder” variable suggests that the average effect of an increase of 1 year is an d
ecrease of 0.493376 in “mpg” (all other predictors remaining constant).
```

d).Use the plot() function to produce diagnostic plots of the linear regression fit. Comment on any problems you see with the fit. Do the residual plots suggest any unusually large outliers? Does the leverage plot identify any observations with unusually high leverage?

```
par(mfrow=c(2,2))
plot(model)
```



I have come up with diifferent comments based on these graphs.

```
1). On-linearity of response-predictors values
    There does not seems to be any pattern for Residuals vs Fitted graph, so it points no strong evidence of
non-linearity.

2). Non-constant Variance of Error Terms
    There is a bit of funnel shape(assume) for the Residuals vs Fitted graph, so it presents a bit of hetero
scedasticity.

3).High Leverage Points
    Specifically the observation 14 is a highly leverage point as shown in Residuals vc Leverage graph.
```

e).Use the ‘*’ and : symbols to fit linear regression models with interaction effects. Do any interactions appear to be statistically significant?

```
autolm3 = lm(mpg ~ (.-name)*(.-name), data = Auto)
summary(autolm3)
```

```
##
## Call:
## lm(formula = mpg ~ (. - name) * (. - name), data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.6303 -1.4481  0.0596  1.2739 11.1386
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)                3.548e+01  5.314e+01   0.668  0.50475
## cylinders                  6.989e+00  8.248e+00   0.847  0.39738
## displacement              -4.785e-01  1.894e-01  -2.527  0.01192 *
## horsepower                 5.034e-01  3.470e-01   1.451  0.14769
## weight                     4.133e-03  1.759e-02   0.235  0.81442
## acceleration              -5.859e+00  2.174e+00  -2.696  0.00735 **
## year                       6.974e-01  6.097e-01   1.144  0.25340
## origin                    -2.090e+01  7.097e+00  -2.944  0.00345 **
## cylinders:displacement    -3.383e-03  6.455e-03  -0.524  0.60051
## cylinders:horsepower       1.161e-02  2.420e-02   0.480  0.63157
## cylinders:weight           3.575e-04  8.955e-04   0.399  0.69000
## cylinders:acceleration     2.779e-01  1.664e-01   1.670  0.09584 .
## cylinders:year            -1.741e-01  9.714e-02  -1.793  0.07389 .
## cylinders:origin           4.022e-01  4.926e-01   0.816  0.41482
## displacement:horsepower   -8.491e-05  2.885e-04  -0.294  0.76867
## displacement:weight        2.472e-05  1.470e-05   1.682  0.09342 .
## displacement:acceleration -3.479e-03  3.342e-03  -1.041  0.29853
## displacement:year          5.934e-03  2.391e-03   2.482  0.01352 *
## displacement:origin        2.398e-02  1.947e-02   1.232  0.21875
## horsepower:weight         -1.968e-05  2.924e-05  -0.673  0.50124
## horsepower:acceleration   -7.213e-03  3.719e-03  -1.939  0.05325 .
## horsepower:year           -5.838e-03  3.938e-03  -1.482  0.13916
## horsepower:origin          2.233e-03  2.930e-02   0.076  0.93931
## weight:acceleration        2.346e-04  2.289e-04   1.025  0.30596
## weight:year               -2.245e-04  2.127e-04  -1.056  0.29182
## weight:origin             -5.789e-04  1.591e-03  -0.364  0.71623
## acceleration:year          5.562e-02  2.558e-02   2.174  0.03033 *
## acceleration:origin        4.583e-01  1.567e-01   2.926  0.00365 **
## year:origin                1.393e-01  7.399e-02   1.882  0.06062 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.695 on 363 degrees of freedom
## Multiple R-squared:  0.8893, Adjusted R-squared:  0.8808
## F-statistic: 104.2 on 28 and 363 DF,  p-value: < 2.2e-16
```

If we look into the model then we had an improvement in R2 from 0.82 to almost 0.89, maybe it can be overfitting, though the interactive term most significant was acceleration:origin with a good coefficient in comparison with the main terms and a small p-value, validating thecoefficient.If we check the interactions between displacement and year, acceleration and year, and acceleration and origin all have low p values that indicate significance.

f).Try transformations of the variables with X3 and log(X). Comment on your findings.

```
par(mfrow = c(2, 2))
autolmx2 <- lm(mpg ~ (horsepower)^3 + (weight)^3 + (acceleration)^3, data = Auto)
summary(autolmx2)
```

```
## 
## Call:
## lm(formula = mpg ~ (horsepower)^3 + (weight)^3 + (acceleration)^3, 
##     data = Auto)
## 
## Residuals:
##     Min      1Q  Median      3Q     Max 
## -11.079  -2.736  -0.331   2.170  16.262 
## 
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)    
## (Intercept)  45.6782929  2.4085431  18.965  < 2e-16 ***
## horsepower   -0.0474956  0.0159891  -2.970  0.00316 ** 
## weight       -0.0057894  0.0005776 -10.024  < 2e-16 ***
## acceleration -0.0020657  0.1233378  -0.017  0.98665    
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 4.246 on 388 degrees of freedom
## Multiple R-squared:  0.7064, Adjusted R-squared:  0.7041 
## F-statistic: 311.1 on 3 and 388 DF,  p-value: < 2.2e-16
```

```
autolmlog <- lm(mpg ~ log(horsepower) + log(weight) + log(acceleration), data = Auto)
summary(autolmlog)
```

```
## 
## Call:
## lm(formula = mpg ~ log(horsepower) + log(weight) + log(acceleration), 
##     data = Auto)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -10.8237  -2.5240  -0.2389   2.0105  15.3681 
## 
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)    
## (Intercept)        190.152      8.255  23.035  < 2e-16 ***
## log(horsepower)    -11.799      1.933  -6.103 2.53e-09 ***
## log(weight)        -12.306      1.820  -6.762 5.03e-11 ***
## log(acceleration)   -5.363      1.970  -2.723  0.00677 ** 
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## Residual standard error: 3.961 on 388 degrees of freedom
## Multiple R-squared:  0.7445, Adjusted R-squared:  0.7425 
## F-statistic: 376.8 on 3 and 388 DF,  p-value: < 2.2e-16
```

```
After applying the log function to each of the variables which resulted into the highest R2 value and F-stat
istic. It also provided the lowest individual p-values for horsepower and acceleration while squaring the we
ight variable resulted in the lowest p-value.
```

Problem 2

This problem involves the Boston data set, which we saw in the lab. We will now try to predict per capita crime rate using the other variables in this data set. In other words, per capita crime rate is the response, and the other variables are the predictors.

```
library(MASS)
attach(Boston)
summary(Boston)
```

```
##       crim                zn             indus            chas        
##  Min.   : 0.00632   Min.   :  0.00   Min.   : 0.46   Min.   :0.00000  
##  1st Qu.: 0.08204   1st Qu.:  0.00   1st Qu.: 5.19   1st Qu.:0.00000  
##  Median : 0.25651   Median :  0.00   Median : 9.69   Median :0.00000  
##  Mean   : 3.61352   Mean   : 11.36   Mean   :11.14   Mean   :0.06917  
##  3rd Qu.: 3.67708   3rd Qu.: 12.50   3rd Qu.:18.10   3rd Qu.:0.00000  
##  Max.   :88.97620   Max.   :100.00   Max.   :27.74   Max.   :1.00000  
##       nox               rm             age              dis        
##  Min.   :0.3850   Min.   :3.561   Min.   :  2.90   Min.   : 1.130  
##  1st Qu.:0.4490   1st Qu.:5.886   1st Qu.: 45.02   1st Qu.: 2.100  
##  Median :0.5380   Median :6.208   Median : 77.50   Median : 3.207  
##  Mean   :0.5547   Mean   :6.285   Mean   : 68.57   Mean   : 3.795  
##  3rd Qu.:0.6240   3rd Qu.:6.623   3rd Qu.: 94.08   3rd Qu.: 5.188  
##  Max.   :0.8710   Max.   :8.780   Max.   :100.00   Max.   :12.127  
##       rad              tax           ptratio          black       
##  Min.   : 1.000   Min.   :187.0   Min.   :12.60   Min.   :  0.32  
##  1st Qu.: 4.000   1st Qu.:279.0   1st Qu.:17.40   1st Qu.:375.38  
##  Median : 5.000   Median :330.0   Median :19.05   Median :391.44  
##  Mean   : 9.549   Mean   :408.2   Mean   :18.46   Mean   :356.67  
##  3rd Qu.:24.000   3rd Qu.:666.0   3rd Qu.:20.20   3rd Qu.:396.23  
##  Max.   :24.000   Max.   :711.0   Max.   :22.00   Max.   :396.90  
##      lstat            medv      
##  Min.   : 1.73   Min.   : 5.00  
##  1st Qu.: 6.95   1st Qu.:17.02  
##  Median :11.36   Median :21.20  
##  Mean   :12.65   Mean   :22.53  
##  3rd Qu.:16.95   3rd Qu.:25.00  
##  Max.   :37.97   Max.   :50.00  
```

a). For each predictor, fit a simple linear regression model to predict the response. Include the code, but not the output for all models in your solution.

Include the code for all the model

```r
#First Model
fitresultzn <- lm(crim ~ zn)
#Second model
fitresultindus <- lm(crim ~ indus)
#Third Model
chas <- as.factor(chas)
#Fourth Model
fitresultchas <- lm(crim ~ chas)
#Fifth Model
fitresultnox <- lm(crim ~ nox)
#6th Model
fitresultrm <- lm(crim ~ rm)
#7th model
fitresultage <- lm(crim ~ age)
#8th Model
fitresultdis <- lm(crim ~ dis)
#9th Model
fitresultrad <- lm(crim ~ rad)
#10th Model
fitresulttax <- lm(crim ~ tax)
#11th Model
fitresultmedv <- lm(crim ~ medv)
#12thmodel
fitresultptratio = lm(crim ~ ptratio)
#13th model
fitresultblack = lm(crim ~ black)
#14th model
fitresultlstat = lm(crim ~ lstat)
```

In which of the models is there a statistically significant association between the predictor and the response?

```
To find which model has significant association between the predictor and the response, we have to test H0:β
1=0. All predictors have a p-value less than 0.05 except “chas”, so we may conclude that there is a statisti
cally significant association between each predictor and the response except for the “chas” predictor.
```

Considering the meaning of each variable, discuss the relationship between crim and nox, chas, medv and dis in particular. How do these relationships differ?

I have considered following relationships when I saw the dataset and run the linear model to check the relationships.

```
1).We can see that there is a strong correlation between the predictor and the response for every variable a
part from the Charles River Dummy.
2). Linear regression with the response variables vs crime in simple scatter-plots gives us a better predict
ion of crime than just using the mean of crime.
3).The low R(squared) indicates that the level of the variation in the response described by these predictor
s is also very low.
4).When looking at the response variables and crime in simple scatter plots, one can see how a general linea
r regression with these variables would allow for a better prediction of crime than simply using the mean of
crime. That is, the data seems to have some slight shape sloping up or down, and isn’t a random cloud of dat
a. That being said, while almost every variable is statistically significant, R-squared is very low, and so
these predictors only describe a small amount of the variation in the response.
```

```
fitresultnox <- lm(crim ~ nox)
summary(fitresultnox)
```

```
##
## Call:
## lm(formula = crim ~ nox)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -12.371  -2.738  -0.974   0.559  81.728
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -13.720      1.699  -8.073 5.08e-15 ***
## nox           31.249      2.999  10.419  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.81 on 504 degrees of freedom
## Multiple R-squared:  0.1772, Adjusted R-squared:  0.1756
## F-statistic: 108.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
chas <- as.factor(chas)
fitresultchas <- lm(crim ~ chas)
summary(fitresultchas)
```

```
##
## Call:
## lm(formula = crim ~ chas)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -3.738 -3.661 -3.435  0.018 85.232
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   3.7444     0.3961   9.453   <2e-16 ***
## chas1        -1.8928     1.5061  -1.257    0.209
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.597 on 504 degrees of freedom
## Multiple R-squared:  0.003124,   Adjusted R-squared:  0.001146
## F-statistic: 1.579 on 1 and 504 DF,  p-value: 0.2094
```

```
fit.medv <- lm(crim ~ medv)
summary(fitresultmedv)
```

```
##
## Call:
## lm(formula = crim ~ medv)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -9.071 -4.022 -2.343  1.298 80.957
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 11.79654    0.93419   12.63   <2e-16 ***
## medv        -0.36316    0.03839   -9.46   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.934 on 504 degrees of freedom
## Multiple R-squared:  0.1508, Adjusted R-squared:  0.1491
## F-statistic: 89.49 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
fitresultdis <- lm(crim ~ dis)
summary(fitresultdis)
```

```
##
## Call:
## lm(formula = crim ~ dis)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -6.708 -4.134 -1.527  1.516 81.674
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   9.4993     0.7304  13.006   <2e-16 ***
## dis          -1.5509     0.1683  -9.213   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 7.965 on 504 degrees of freedom
## Multiple R-squared:  0.1441, Adjusted R-squared:  0.1425
## F-statistic: 84.89 on 1 and 504 DF,  p-value: < 2.2e-16
```

b).Fit a multiple regression model to predict the response using all of the predictors. Describe your results. For which predictors can we reject the null hypothesis H0 : βj = 0?

```
fit.all <- lm(crim ~ ., data = Boston)
summary(fit.all)
```

```
##
## Call:
## lm(formula = crim ~ ., data = Boston)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -9.924 -2.120 -0.353  1.019 75.051
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  17.033228   7.234903   2.354 0.018949 *
## zn            0.044855   0.018734   2.394 0.017025 *
## indus        -0.063855   0.083407  -0.766 0.444294
## chas         -0.749134   1.180147  -0.635 0.525867
## nox         -10.313535   5.275536  -1.955 0.051152 .
## rm            0.430131   0.612830   0.702 0.483089
## age           0.001452   0.017925   0.081 0.935488
## dis          -0.987176   0.281817  -3.503 0.000502 ***
## rad           0.588209   0.088049   6.680 6.46e-11 ***
## tax          -0.003780   0.005156  -0.733 0.463793
## ptratio      -0.271081   0.186450  -1.454 0.146611
## black        -0.007538   0.003673  -2.052 0.040702 *
## lstat         0.126211   0.075725   1.667 0.096208 .
## medv         -0.198887   0.060516  -3.287 0.001087 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.439 on 492 degrees of freedom
## Multiple R-squared:  0.454,  Adjusted R-squared:  0.4396
## F-statistic: 31.47 on 13 and 492 DF,  p-value: < 2.2e-16
```

```
As we fit the multiple regression model, very few variables appear to be statistically significant at the following levels:
dis- .001, rad- .001, medv - .01, black - .05 and zn -.05. In this case R squared is significantly higher than either of the predictors.
For every other variable, The Null Hypothesis cannot be rejected for all other variables. R-squared is also much higher using a multiple  regression model than any of the  predictors on their own, meaning we better explain more of the variance in the outcome.
```

c).How do your results from (a) compare to your results from (b)? Create a plot displaying the univariate regression coefficients from (a) on the x-axis, and the multiple regression coefficients from (b) on the y-axis. That is, each predictor is displayed as a single point in the plot. Its coefficient in a simple linear regression model is shown on the x-axis, and its coefficient estimate in the multiple linear regression model is shown on the y-axis. What does this plot tell you about the various predictors?

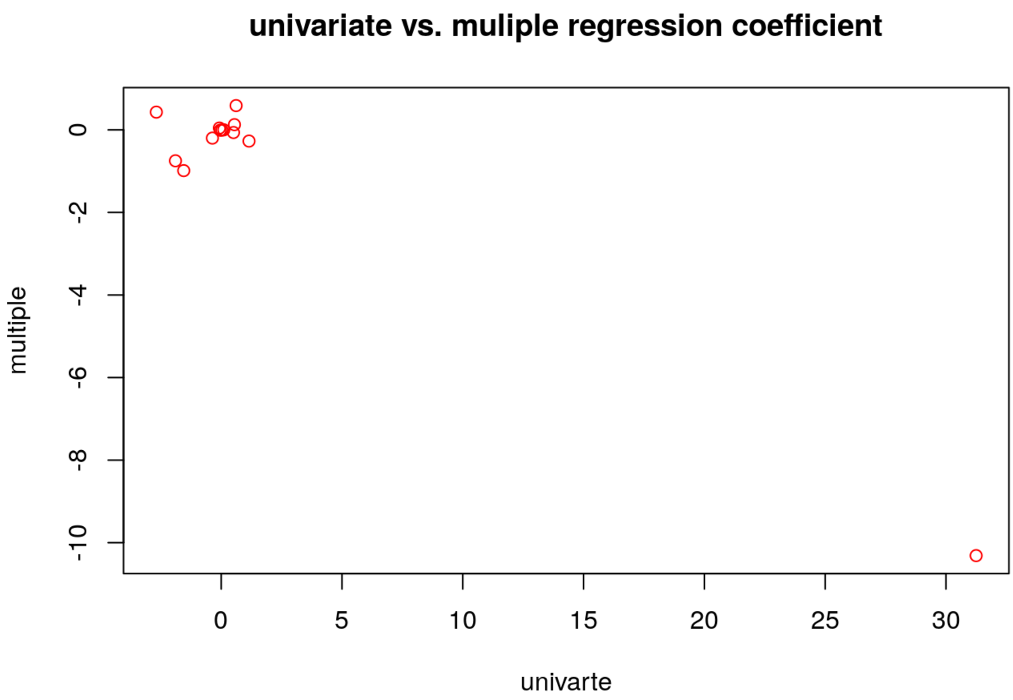
```
univcoof=c(coefficients(fitresultzn)[2],
      coefficients(fitresultindus)[2],
      coefficients(fitresultchas)[2],
      coefficients(fitresultnox)[2],
      coefficients(fitresultrm)[2],
      coefficients(fitresultage)[2],
      coefficients(fitresultdis)[2],
      coefficients(fitresultrad)[2],
      coefficients(fitresulttax)[2],
      coefficients(fitresultptratio)[2],
      coefficients(fitresultblack)[2],
      coefficients(fitresultlstat)[2],
      coefficients(fitresultmedv)[2])

fooBoston <- (lm(crim ~., data = Boston))

fooBoston$coefficients[2:14]
```

```
##            zn         indus          chas           nox            rm
##   0.044855215  -0.063854824  -0.749133611 -10.313534912   0.430130506
##           age           dis           rad           tax       ptratio
##   0.001451643  -0.987175726   0.588208591  -0.003780016  -0.271080558
##         black         lstat          medv
##  -0.007537505   0.126211376  -0.198886821
```

```
plot(univcoof,fooBoston$coefficients[2:14],main = "univariate vs. muliple regression coefficient",xlab = "un
ivarte",ylab = "multiple",col="red")
```



```
If we look into the plots, then there is a difference between the simple and multiple regression coefficient
s. This difference is due to the fact that in the simple regression case, the slope term represents the aver
age effect of an increase in the predictor, ignoring other predictors.

In contrast, in the multiple regression case, the slope term represents the average effect of an increase in
the predictor, while holding other predictors fixed. It does make sense for the multiple regression to sugge
st no relationship between the response and some of the predictors while the simple linear regression implie
s the opposite because the correlation between the predictors show some strong relationships between some of
the predictors.
```

```
cor(Boston[-c(1, 4)])
```

```
##                 zn      indus        nox         rm        age        dis
## zn       1.0000000 -0.5338282 -0.5166037  0.3119906 -0.5695373  0.6644082
## indus   -0.5338282  1.0000000  0.7636514 -0.3916759  0.6447785 -0.7080270
## nox     -0.5166037  0.7636514  1.0000000 -0.3021882  0.7314701 -0.7692301
## rm       0.3119906 -0.3916759 -0.3021882  1.0000000 -0.2402649  0.2052462
## age     -0.5695373  0.6447785  0.7314701 -0.2402649  1.0000000 -0.7478805
## dis      0.6644082 -0.7080270 -0.7692301  0.2052462 -0.7478805  1.0000000
## rad     -0.3119478  0.5951293  0.6114406 -0.2098467  0.4560225 -0.4945879
## tax     -0.3145633  0.7207602  0.6680232 -0.2920478  0.5064556 -0.5344316
## ptratio -0.3916785  0.3832476  0.1889327 -0.3555015  0.2615150 -0.2324705
## black    0.1755203 -0.3569765 -0.3800506  0.1280686 -0.2735340  0.2915117
## lstat   -0.4129946  0.6037997  0.5908789 -0.6138083  0.6023385 -0.4969958
## medv     0.3604453 -0.4837252 -0.4273208  0.6953599 -0.3769546  0.2499287
##                rad        tax    ptratio      black      lstat       medv
## zn      -0.3119478 -0.3145633 -0.3916785  0.1755203 -0.4129946  0.3604453
## indus    0.5951293  0.7207602  0.3832476 -0.3569765  0.6037997 -0.4837252
## nox      0.6114406  0.6680232  0.1889327 -0.3800506  0.5908789 -0.4273208
## rm      -0.2098467 -0.2920478 -0.3555015  0.1280686 -0.6138083  0.6953599
## age      0.4560225  0.5064556  0.2615150 -0.2735340  0.6023385 -0.3769546
## dis     -0.4945879 -0.5344316 -0.2324705  0.2915117 -0.4969958  0.2499287
## rad      1.0000000  0.9102282  0.4647412 -0.4444128  0.4886763 -0.3816262
## tax      0.9102282  1.0000000  0.4608530 -0.4418080  0.5439934 -0.4685359
## ptratio  0.4647412  0.4608530  1.0000000 -0.1773833  0.3740443 -0.5077867
## black   -0.4444128 -0.4418080 -0.1773833  1.0000000 -0.3660869  0.3334608
## lstat    0.4886763  0.5439934  0.3740443 -0.3660869  1.0000000 -0.7376627
## medv    -0.3816262 -0.4685359 -0.5077867  0.3334608 -0.7376627  1.0000000
```

For example,

```
when “age” is high there is a tendency in “dis” to be low, hence we can say in simple linear regression whic
h only examines “crim” versus “age”, we can observe that there is higher values of “age” are associated with
higher values of “crim”, even though “age” does not actually affect “crim”.So “age” is a surrogate for “dis”
; “age” gets credit for the effect of “dis” on “crim”.
```

d).Is there evidence of non-linear association between any of the predictors and the response? To answer this question, for each predictor X, fit a model of the form Y = β0 + β1X + β2X2 + β3X3+ ε

```
y=lm(crim ~ zn + I(zn^2) + I(zn^3), data = Boston)
y=lm(crim ~ indus + I(indus^2) + I(indus^3), data = Boston)
y=lm(crim ~ chas + I(chas^2) + I(chas^3), data = Boston)
y=lm(crim ~ nox + I(nox^2) + I(nox^3), data = Boston)
y=lm(crim ~ rm + I(rm^2) + I(rm^3), data = Boston)
y=lm(crim ~ age + I(age^2) + I(age^3), data = Boston)
y=lm(crim ~ dis + I(dis^2) + I(dis^3), data = Boston)
y=lm(crim ~ rad + I(rad^2) + I(rad^3), data = Boston)
y=lm(crim ~ tax + I(tax^2) + I(tax^3), data = Boston)
y=lm(crim ~ ptratio + I(ptratio^2) + I(ptratio^3), data = Boston)
y=lm(crim ~ black + I(black^2) + I(black^3), data = Boston)
y=lm(crim ~ lstat + I(lstat^2) + I(lstat^3), data = Boston)
y=lm(crim ~ medv + I(medv^2) + I(medv^3), data = Boston)
```

Observing the results,

```
The first thing to note is the chas variable, we get NA values for the squared and cubed term. This makes se
nse as chas is a dummy variable, composed of only 0s and 1s, and these values will not change if they are sq
uared or cubed.I come up with some analysis results such as For “zn”, “rm”, “rad”, “tax” and “lstat” as pred
ictor, the p-values suggest that the cubic coefficient is not statistically significant; however,for “indus”
, “nox”, “age”, “dis”, “ptratio” and “medv” as predictor, the p-values suggest the adequacy of the cubic fit
;Similarly, for “black” as predictor, the p-values suggest that the quandratic and cubic coefficients are no
t statistically significant, so in this latter case no non-linear effect is visible.
```

Problem 3

An important assumption of the linear regression model is that the error terms are uncorrelated (independent). But error terms can sometimes be correlated, especially in time-series data.

a).What are the issues that could arise in using linear regression (via least squares estimates) when error terms are correlated? Comment in particular with respect to i) regression coefficients ii) the standard error of regression coefficients iii) confidence intervals

Solution:

(i) Regression Coefficients

```
Regression coefficients represent the mean change in the response variable for one unit of change in the pre
dictor variable while holding other predictors in the model constant. This statistical control that regressi
on provides is important because it isolates the role of one variable from all of the others in the model

Multicolinearity is often at the source of the problem when a positive simple correlation with the dependent
variable leads to a negative regression coefficient in multiple regression. Some regression techniques may h
elp there : ridge regression, partial least square regression. Start by finding out which variable(s) are ca
using the colinearity (i.e with the inflation or the distortion factor).  Remove them or attenuate the corre
lation with the ridge coefficient.
```

(ii) Standard error of regression coefficients

```
The sample standard deviation of the errors is a downward-biased estimate of the size of the true unexplaine
d deviations in Y because it does not adjust for the additional "degree of freedom" used up by estimating th
e slope coefficient. An unbiased estimate of the standard deviation of the true errors is given by the stand
ard error of the regression, denoted by s. In the special case of a simple regression model, it is:
```

Standard error of regression coefficient = STDEV.S(errors) x SQRT((n-1)/(n-2))

```
The sum of squared errors is divided by n-2 in this calculation rather than n-1 because an additional degree of freedom for error has been used up by estimating two parameters (a slope and an intercept) rather than only one (the mean) in fitting the model to the data. The standard error of the regression is an unbiased estimate of the standard deviation of the noise in the data, i.e., the variations in Y that are not explained by the model.
```

For Example :

```
When multicollinearity occurs, the least-squares estimates are still unbiased and efficient. The problem is that the estimated standard errors of the coefficients tend to be inflated.  That is, the standard error tends to be larger than it would be in the absence of multicollinearity because the estimates are very sensitive to changes in the sample observations or in the model specification. In other words, including or excluding a particular variable or certain observations may greatly change the estimated coefficients.
```

(iii) Confidence Intervals

```
Confidence intervals for the mean and for the forecast are equal to the point estimate plus-or-minus the appropriate standard error multiplied by the appropriate 2-tailed critical value of the t distribution. The critical value that should be used depends on the number of degrees of freedom for error (the number data points minus number of parameters estimated, which is n-1 for this model) and the desired level of confidence. So, for example, a 95% confidence interval for the forecast is given by

Bo=(plus-or-minus) SEfcst + T.INV.2T(0.05,n-1)

In general, T.INV.2T(0.05, n-1) is fairly close to 2 except for very small samples, i.e., a 95% confidence interval for the forecast is roughly equal to the forecast plus-or-minus two standard errors.
```

For Example:

```
The issue that depends on the correctness of the model and the representativeness of the data set, particularly in the case of time series data.  If the model is not correct or there are unusual patterns in the data, then if the confidence interval for one period's forecast fails to cover the true value, it is relatively more likely that the confidence interval for a neighboring period's forecast will also fail to cover the true value, because the model may have a tendency to make the same error for several periods in a row.
```

b).What methods can be applied to deal with correlated errors? Mention at least one method.

As I mentioned if there’s multicollinearity problem occurs in linear regression model or VIF for a factor is near or above 5.We can apply following methods.

```
1). Remove highly correlated predictors from the model- If we have two or more factor with a high variance inflation factor, remove one of the model.Because they supply redundant information, removing one of the correlated factors usually doesn't drastically reduce the R-squared.  We can consider using stepwise regression, best subsets regression, or specialized knowledge of the data set to remove these variables.We can select the model that has the highest R-squared value.

2).We can use Partial Least Squares Regression (PLS) or Principal Components Analysis (PCA), regression methods that cut the number of predictors to a smaller set of uncorrelated components.
```