PERT: Time Estimates

Chapter 5

Programme Evaluation and Review Technique

Time Estimates

PERT (Program Evaluation and Review Technique) is a project management tool that is used to estimate the duration of a project. It is a statistical tool that helps to determine the expected time required to complete a project. PERT estimates are based on three types of time estimates:

- Optimistic Time
- Pessimistic Time
- Most Likely Time.

Optimistic Time

The optimistic time is the shortest time in which a task can be completed, assuming everything goes perfectly. The pessimistic time is the longest time in which a task can be completed, assuming everything goes wrong. The most likely time is the most probable time in which a task can be completed, assuming normal conditions.

Assume that building a house has a number of tasks or activities, such as laying the foundation, framing the walls, installing the plumbing, electrical work, etc. Each activity has an optimistic, pessimistic, and most likely time estimate.

Let's take the activity of laying the foundation. The optimistic time estimate for this activity might be 3 weeks. This means that if everything goes perfectly, with no delays or obstacles, it could take as little as 3 weeks to complete the activity.

However, it is important to note that the optimistic time estimate is not a guarantee that the activity will be completed in 3 weeks. It is simply the best-case scenario estimate, assuming that everything goes according to plan.

In PERT, the optimistic time estimate is used along with the pessimistic and most likely time estimates to calculate the expected time for the activity. This helps to provide a more accurate estimate of how long the activity is likely to take, given the range of possible scenarios.

Pessimistic Time

In PERT (Program Evaluation and Review Technique), pessimistic time refers to the longest estimated time required to complete a specific activity, assuming unfavourable conditions or delays.

The pessimistic time estimate is an essential component of PERT, as it helps project managers plan for potential delays or risks. By considering the pessimistic estimate, project managers can allocate resources and plan contingencies to avoid or mitigate the negative impact of any delays or obstacles that may arise during the project.

Most Likely Time Estimates

In PERT (Program Evaluation and Review Technique), most likely time refers to the estimated time required to complete a specific activity, assuming normal or expected conditions.

PERT uses three time estimates for each activity: optimistic time, pessimistic time, and most likely time. The most likely time estimate represents the best estimate of how long the activity will take to complete, given normal or expected conditions.

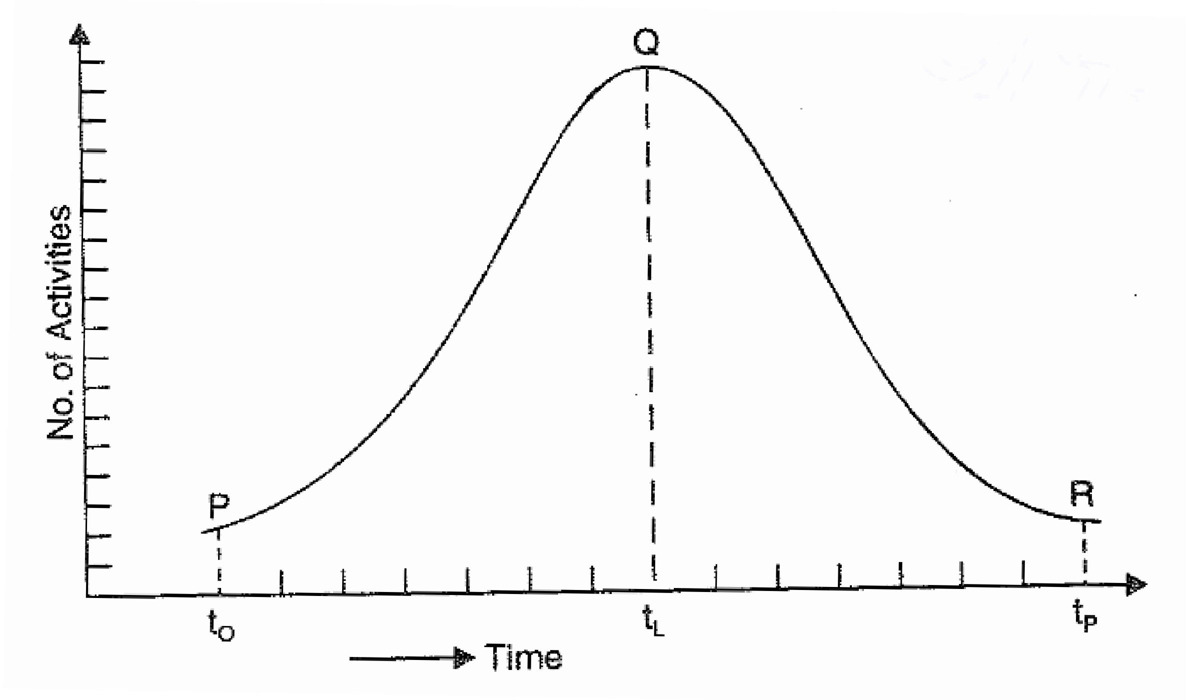
The most likely time estimate is used in the PERT formula to calculate the expected time for an activity. It is given more weight than the optimistic and pessimistic time estimates because it is the most probable estimate of how long the activity will take to complete.

By using the most likely time estimate along with the optimistic and pessimistic time estimates, project managers can create a more accurate estimate of how long the activity is likely to take, given a range of possible scenarios. This helps them to plan and allocate resources more effectively to ensure the project is completed on time and within budget.

Frequency Distribution Curves

In PERT (Program Evaluation and Review Technique), a frequency distribution curve is a graphical representation of the expected time for completing a project or activity, based on the three time estimates (optimistic, pessimistic, and most likely) used in PERT analysis.

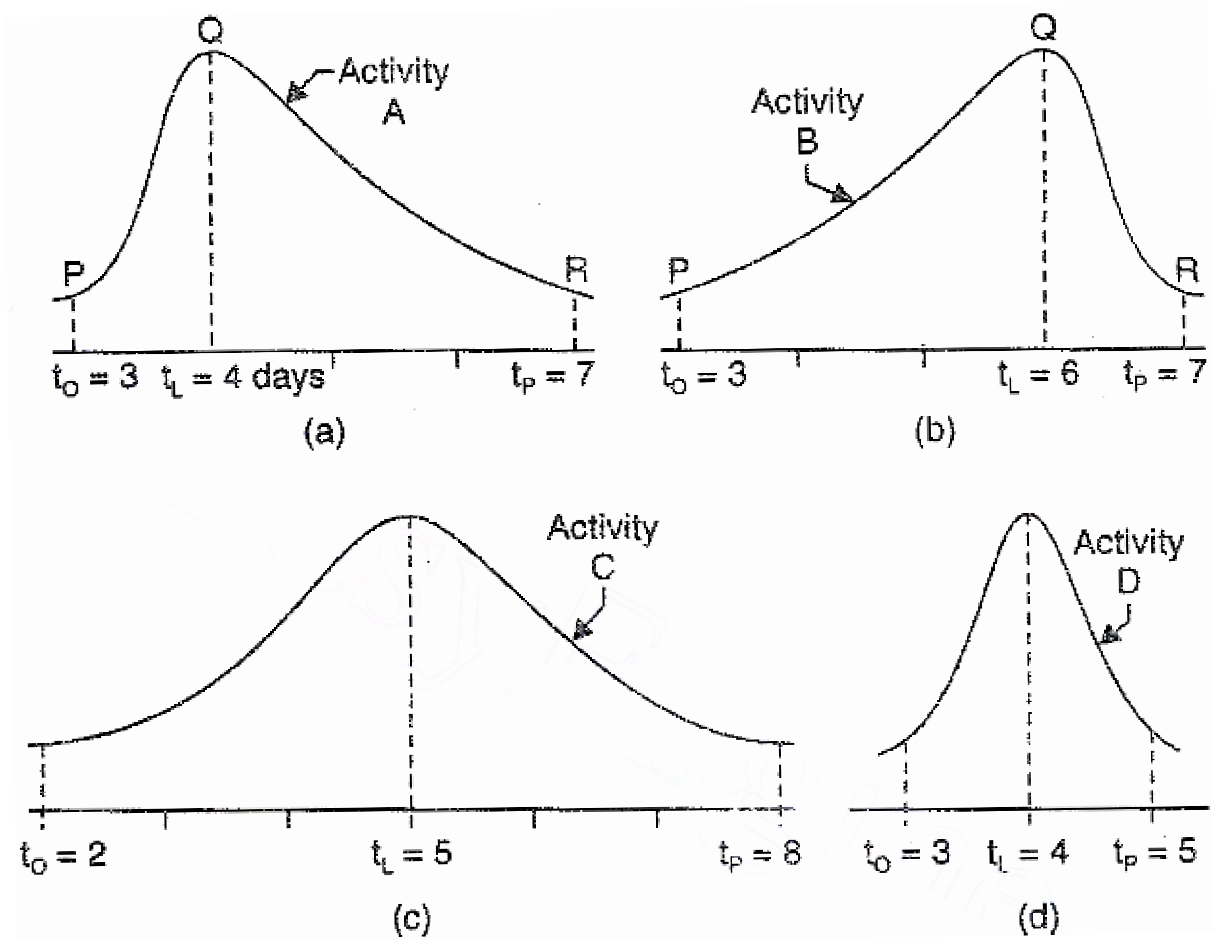
The frequency distribution curve is a bell-shaped curve that shows the range of possible durations for a project or activity, along with the likelihood of each duration. The curve is based on the PERT formula, which uses the optimistic, pessimistic, and most likely time estimates to calculate the expected time for an activity.



Frequency Distribution Curve (Normal Curve)

The peak of the curve represents the most likely duration for the activity, while the width of the curve represents the range of possible durations. The curve also shows the probability of completing the activity within a certain duration, based on statistical analysis of the three time estimates.

The frequency distribution curve is a useful tool for project managers to visualize the expected time for completing a project or activity, along with the risks and uncertainties associated with the project. By analyzing the curve, project managers can identify potential bottlenecks or areas of the project that may require additional resources or attention to ensure timely completion.



Note: It is not necessary that a frequency distribution curve may be a normal; it may have *skew* due to which it is not symmetrical about the peak Q

To conclude a wide range in time estimates represents, greater uncertainty and hence less confidence in our ability to correctly anticipated the actual time that the activity will require.

Example: *In a certain project, the times required for digging 54 trenches of fixed dimensions are recorded below. The trenches were excavated by different parties, each consisting of the same number of persons. Plot the frequency distribution curve.*

Table 1

TIMES OF COMPLETION OF TRENCH (DAYS)

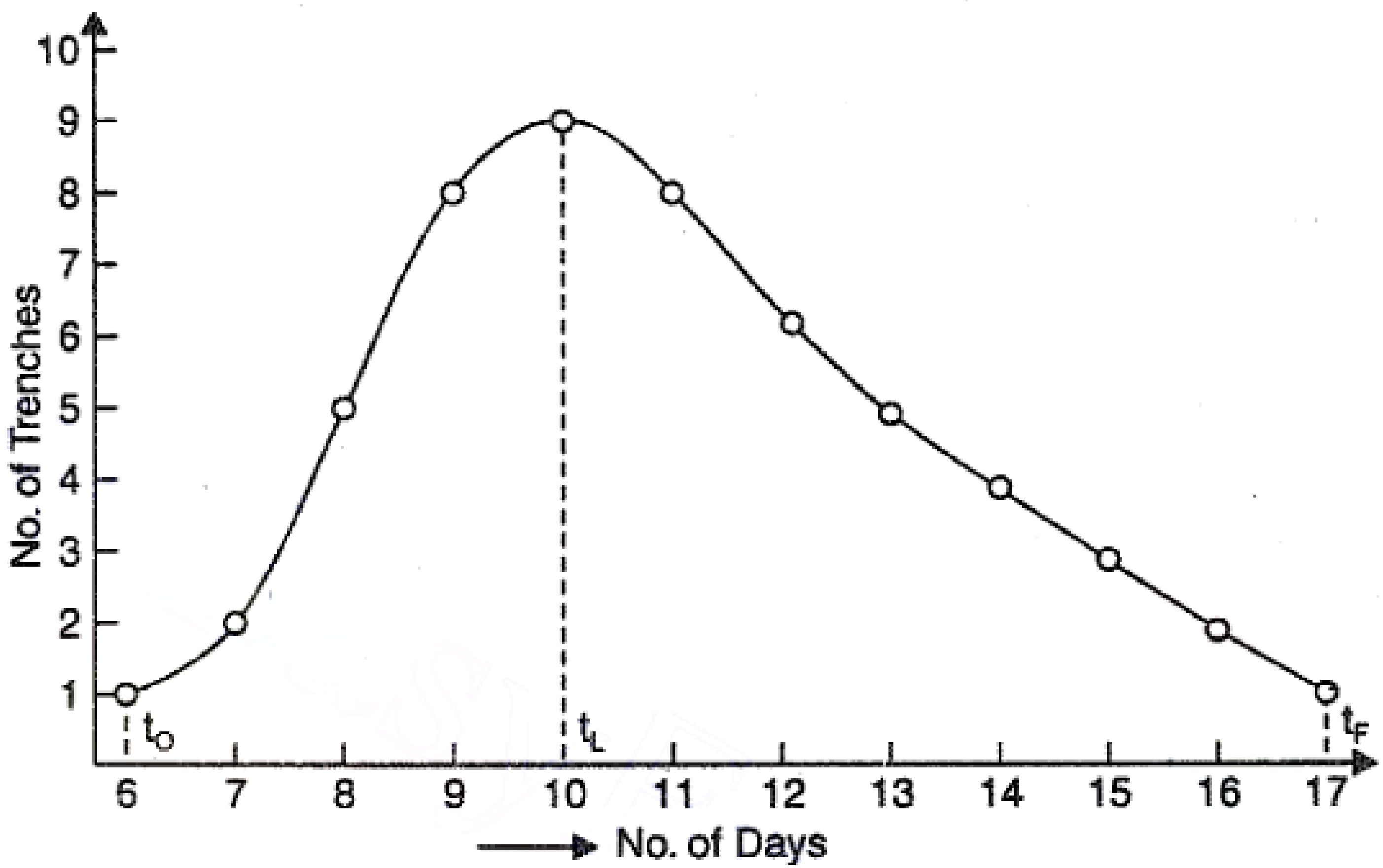
8	11	14	9	10	8
10	9	12	11	9	10
12	8	7	13	11	9
6	10	9	10	10	11
9	14	13	14	7	
11	16	10	9	13	
10	12	8	12	11	
13	16	11	15	8	
15	15	17	14	12	
12	10	13	9	11	

Solution. From Table 1, we find that the minimum time taken for completion of trench is 6 days which corresponds to the optimistic time (t_O), while the maximum time taken is 17 days which corresponds to the pessimistic time (t_P). The time varies between 6 days to 17 days. Table 2 gives the No. of trenches completed in 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 and 17 days respectively.

Table 2

Days of completion	*No. of trenches completed during these days*	*Days of completion*	*No. of trenches completed during these days*
6	1	12	6
7	2	13	5
8	5	14	4
9	8	15	3
10	9	16	2
11	8	17	1

The data of Table .2 can now be plotted to get the frequency distribution curve between No. of days of completion and No. of trenches completed during this period, as shown in **Figure.** From the curve, the most likely time (t_L), corresponding to the peak of the curve, comes out to be 10 days. The frequency curve so obtained is not symmetrical ; it has skew to the left.



Mean Time or Average Time

In the context of PERT (Program Evaluation and Review Technique), the mean time is typically represented by the statistical average of the three estimated times for an activity: optimistic time (O), pessimistic time (P), and most likely time (M).

$$t_m = \frac{\Sigma t}{n}$$

Deviation

In general, the deviation response time value is used in reports to provide greater depth of analysis. It **shows how much variation there is from the average, or mean**.

In the context of PERT (Program Evaluation and Review Technique), the deviation in a frequency distribution curve refers to the degree of uncertainty or variability associated with the estimated completion time of an activity.

The deviation in a frequency distribution curve is typically represented by the width of the curve. A narrower curve indicates less variability or uncertainty in the estimated completion time, while a wider curve indicates more variability or uncertainty.

$$\delta = t - t_m$$

δ= deviation of any time from the mean

t = time under consideration, for which deviation is being found

Variance

In general , A time variance is **the difference between the standard hours and actual hours assigned to a job**.

In the context of PERT (Program Evaluation and Review Technique), the variance in a frequency distribution curve refers to the measure of the spread or dispersion of the estimated completion times for an activity.

The variance in a frequency distribution curve is calculated as the square of the standard deviation, which is a measure of the degree of uncertainty or variability associated with the estimated completion time of an activity.

Thus,
$$\sigma^2 = \frac{\Sigma\delta^2}{n} = \frac{\Sigma(t - t_m)^2}{n}$$

Variance is calculated in the following steps :

(*i*) Obtain the *mean* of the distribution

(*ii*) Determine the deviation of each time from the mean.

(*iii*) Find square of these individual deviations.

(*iv*) Find the mean of the squared deviations.

Standard Deviation

In the context of PERT (Program Evaluation and Review Technique), the standard deviation in a frequency distribution curve represents the degree of uncertainty or variability associated with the estimated completion time of an activity.

The standard deviation is a measure of how much the estimated completion times vary from the mean completion time. A larger standard deviation indicates that the estimated completion times are spread out over a wider range, while a smaller standard deviation indicates that the estimated completion times are clustered more tightly around the mean.

Standard deviation. It is simply the square root of the variance. Standard deviation is denoted by symbol σ.

Thus, $$\sigma = \sqrt{\frac{\Sigma(t - t_m)^2}{n}}$$

Probability Distribution

Probability is connected with *chance* and *uncertainty*. The three time estimates that the estimator selects either from his experience or from the *frequency distribution* has inherent uncertainties. In probability analysis, and in consequent probability distribution, we try to associate numbers with uncertainties. In the frequency distribution one studies the group behaviour, while in the probability distribution, we have the distribution of probability values for all possible outcomes. The probability measures are always between 0 to 1. If an event has probability of 1, it is certain to occur, while if the probability is zero it will not occur. Closer the probability value is to 1, more certain is the occurrence of the event.

Let us take an example of manufacture of steel trusses by a factory. Let us assume that the factory manufactures 50 trusses in all, under varying circumstances, and the duration of time taken are as follows :

5 trusses in 12 days each

12 trusses in 14 days each

13 trusses in 15 days each

8 trusses in 16 days each

12 trusses in 18 days each.

Let us now find the probability of manufacturing a truss in 12 days. This is evidently equal to the ratio of number of trusses manufactured in 12 days each to the total number of trusses manufactured. Thus, probability

$$= \frac{5}{50} = 0.1 \text{ or } 10\%.$$

Similarly, the probability of manufacturing the truss in 15 days

$$= \frac{5 + 12 + 13}{50} = 0.6 \text{ or } 60\%.$$

Thus, probability number can always be assigned to the estimated time, if sufficient data is available. Generally, the available data (frequency distribution) is used to plot *probability distribution.*

Probability Distribution Curve

The probability distribution curve is useful for estimating the probability of completing an activity within a given time frame. The area under the curve between two points represents the probability of completing the activity within that time frame. This information is important for project managers to identify potential delays and manage project timelines effectively.

Probability distribution is the curve, with its height so standardised that the area under the curve is equal to unity. The height or the ordinate of the curve at any point x, is denoted by function $f(x)$, usually called the *probability density function.*

Thus $$\int_{-\infty}^{+\infty} f(x)\, dx = 1$$

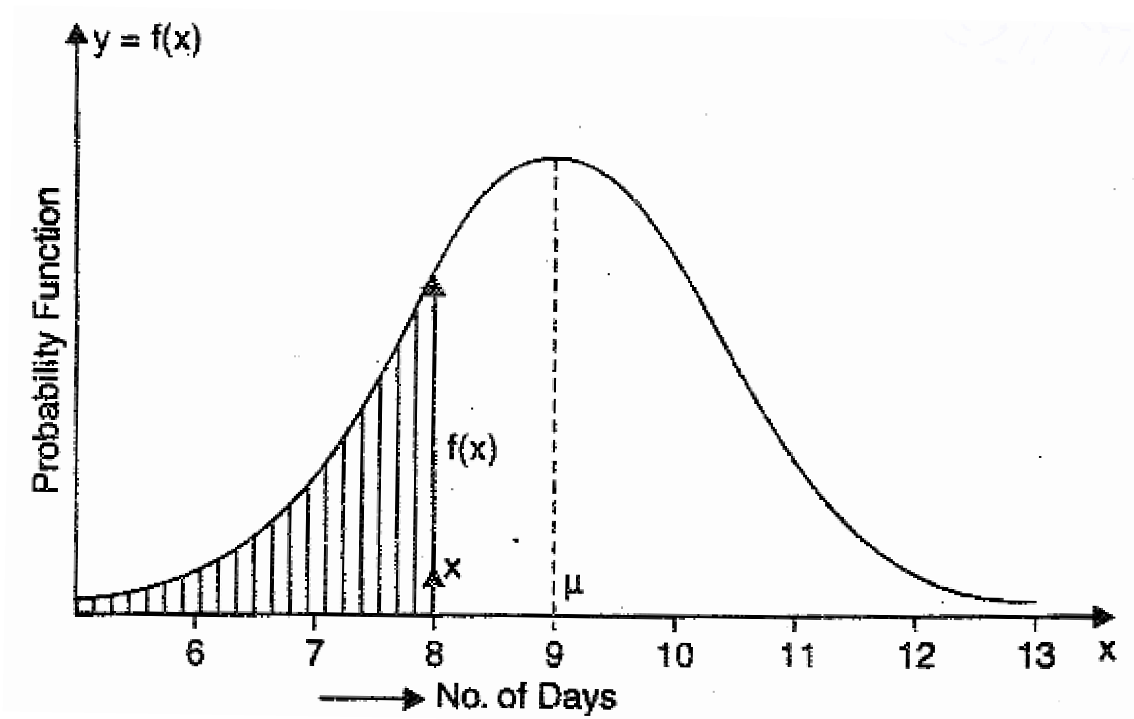


Fig. shows the probability curve. It is to be noted that the ordinate $f(x)$ to the curve at any point x does not give the probability. The probability of completion of work in 8 days is equal to the ratio of the shaded area (area to the left of 8 days) to the total area of the curve. Since the total area of the curve is equal to unity, the probability of completion of the job in 8 days is equal to the shaded area itself.

Normal Probability Distribution

The probability curve is not necessarily symmetrical about its apex. If the curve is symmetrical, then it is known to have *normal* or Gaussian distribution, shown in Fig.

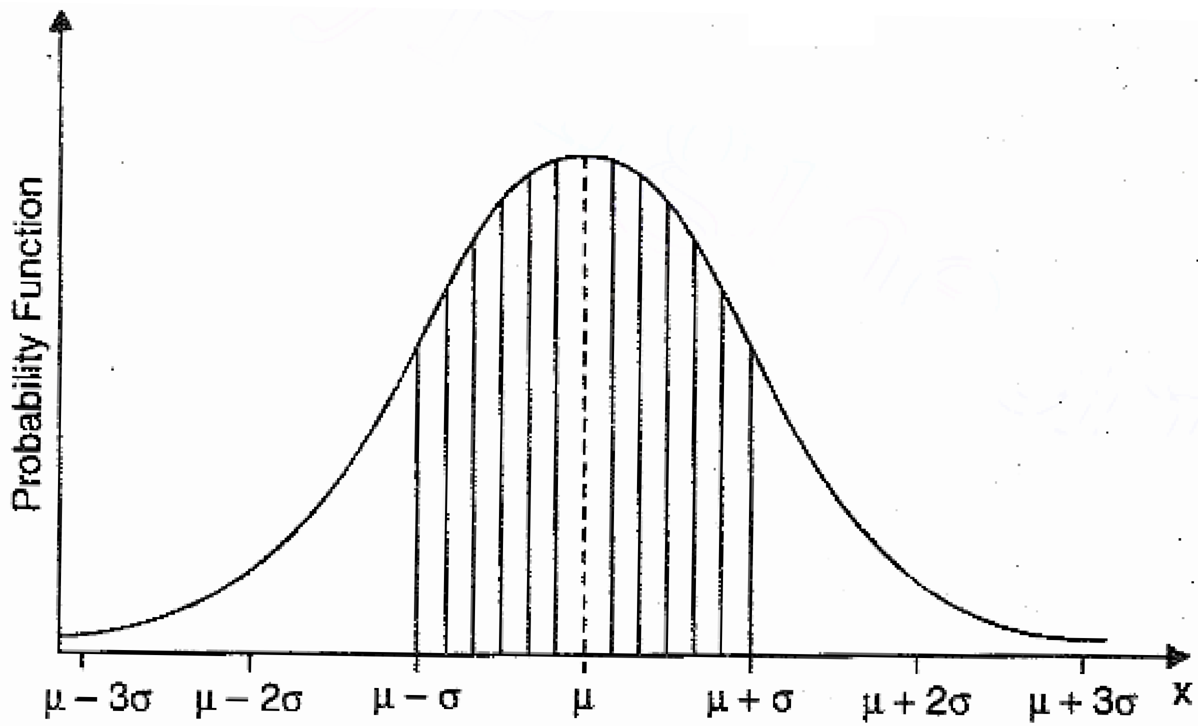


FIG. NORMAL PROBABILITY DISTRIBUTION.

The mean of the *normal probability distribution* is denoted by μ (*i.e.* $x = \mu$). It can be proved that :

(*a*) Approximately 68% of the values of the normal distribution lie within $\pm\ \sigma$ from the average, where σ is the standard deviation. This means that the shaded area of the curve (Fig. between $x = \mu - \sigma$ to $x = \mu + \sigma$ is 68% of the total area.

(*b*) Approximately 95% of all the values lie within $\pm\ 2\sigma$ from the average. This means that the area of the curve between $x = \mu - 2\sigma$ to $x = \mu + 2\sigma$ is 95% of the total area.

(*c*) Approximately 99.7% of all the values lie within $\pm\ 3\sigma$ from the average. This means that the area of the curve between $x = \mu - 3\sigma$ to $x = \mu + 3\sigma$ is 99.7%.

The last property (*c*) can be used to calculate the *standard deviation* directly if the minimum time (t_O) and maximum time (t_P) are known. Let us say that the minimum time is 6 days and maximum time is 18 days for the completion of a job. If 99.7% of all the values (*i.e.* possible completion times) are assumed to lie between 6 and 18 days then the distance between the extreme left value (6 days) and extreme right value (18 days) should be equal to $\pm 3\sigma$ or 6σ in total. The standard deviation

$$= \frac{18 - 6}{6} = 2 \text{ days.}$$

Hence we conclude, in general, that standard deviation is given by

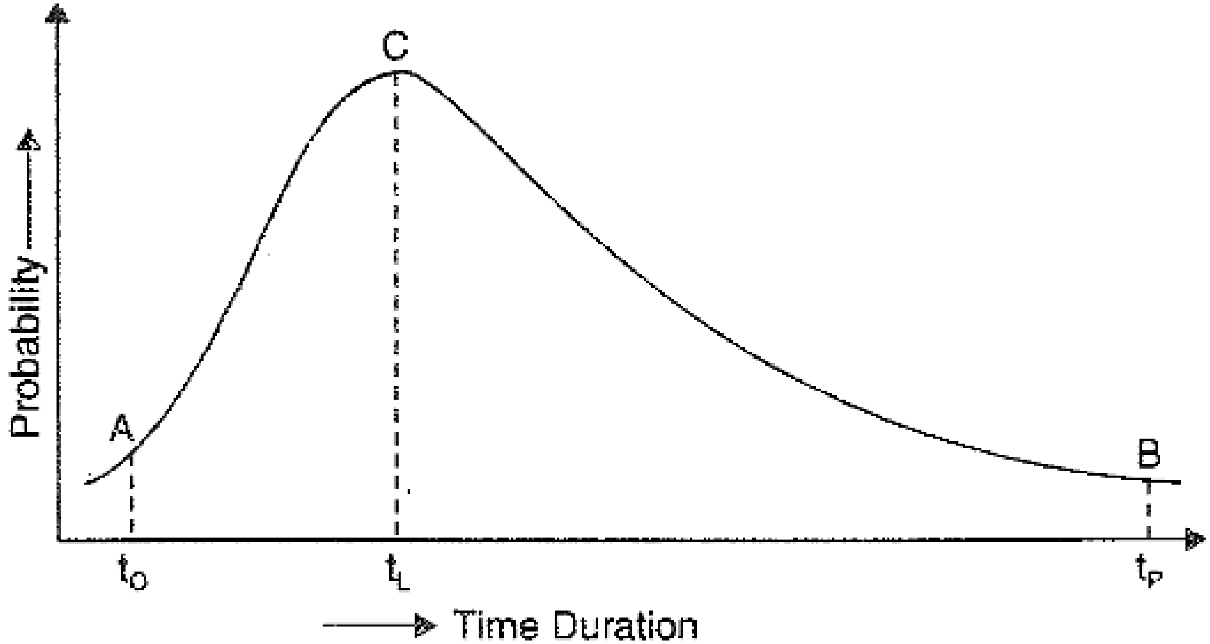
$$\sigma = \frac{t_P - t_O}{6}$$

or variance $$\sigma^2 = \left(\frac{t_P - t_O}{6}\right)^2$$

It is seen that the standard deviation is affected by the relative distance from the most optimistic estimate to the most pessimistic estimate. It is not influenced by the most likely estimate (t_L).

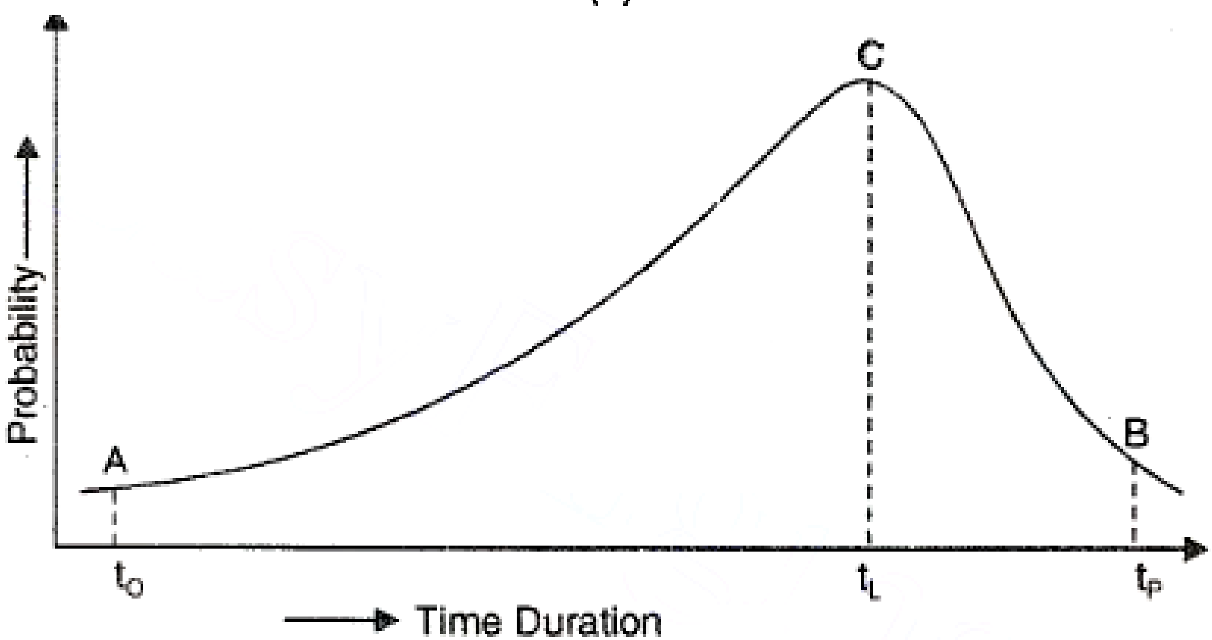
Beta Distribution

The beta distribution is a typical type of probability distribution, which fits well for PERT analysis. A beta distribution is the one which is not symmetrical about its apex. Fig. shows two beta distributions, one having skew to the left (*beta distribution for optimistic estimator*) and the other having skew to the right (*beta distribution for the pessimistic estimator*).



C
Probability
A
B
t_O
t_L
t_P
Time Duration

(a)



C
Probability
A
B
t_O
t_L
t_P
Time Duration

(b)

The originators of PERT were interested in finding that type of probability distribution which satisfies the following conditions :

1. The distribution should have a small probability of reaching the most optimistic time (shortest time).

2. The distribution should have a small probability of reaching the most pessimistic time (longest time).

3. The distribution should have one and only one most likely time (*i.e.* unimodal) which would be free to move between the two extremes mentioned in 1 and 2 above.

4. The distribution should be such that the amount of uncertainty in the estimating can be measured easily.

The above mentioned four requirements are met with beta distribution. Hence this distribution is used in PERT analysis.

It can be shown that for Beta distribution, the standard deviation is given by

$$\sigma = \frac{t_P - t_O}{6}$$

The variance $\sigma^2 = \left(\frac{t_P - t_O}{6} \right)^2$.

We have already seen that *variance* is the measure of *uncertainty*. Greater the variance, greater will be the uncertainty.

Expected Time

The three time estimates t_O (optimistic time), t_P (pessimistic time) and t_L (most likely time) are identified on the Beta-distribution. The variance and standard deviation can be computed using t_O and t_P. However, one must combine the three time estimates into one single time—the average time taken for the completion of the activity or job. This average time or single workable time is commonly called the *expected time* and is denoted by t_E. If the exact shape of the probability distribution curve is known, the average time or expected time could be accurately calculated. However, since the precise curves are never available (specially for non-repetitive jobs) we must use *approximation*. This is done algebraically, using a weighted average derived by statisticians. In computing the expected time, a weightage of 1 is given to the optimistic time t_O, weightage of 4 to the most likely time (t_L) and weightage of 1 to the most pessimistic time (t_P).

Thus, $$t_E = \frac{t_O + 4t_L + t_P}{6}$$

The above expression for t_E, based on weighted average method, is reasonable since the chance of completion of the job in t_O or t_P is much less than the most likely time (t_L).

Let us take examples of estimated times of completion of two jobs A and B, as under.

	t_O	t_L	t_P (days)
Job A	4	6	11
Job B	5	10	12

The expected time for these jobs are

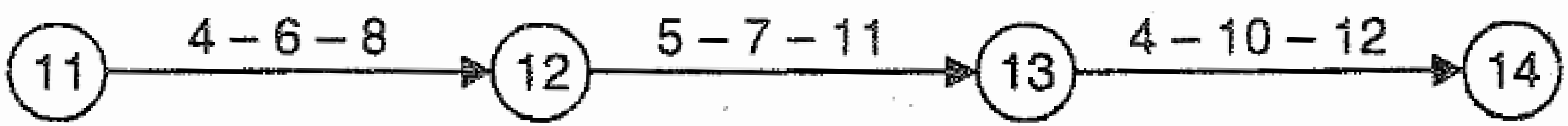
$$(t_E)_A = \frac{t_O + 4t_L + t_P}{6} = \frac{4 + (4 \times 6) + 11}{6} = 6.5 \text{ days}$$

$$(t_E)_B = \frac{t_O + 4t_L + t_P}{6} = \frac{5 + (4 \times 10) + 12}{6} = 9.5 \text{ days}$$

Thus, for job A, the expected time falls to the right of the most likely time, though the curve has skew to the left For job B, the expected time t_E falls to the left of the most likely time, though the curve has skew to the right.

Expected Time for Activities in Series

When a number of activities are in series, the expected time for the path, along the activities, can be found by first finding the t_E for each activity, and then taking their sum. Alternatively, the optimistic time (t_O), the most likely time (t_L) and the pessimistic times (t_P) of the path can be calculated first by taking the sum of all t_O, t_L and t_P respectively and then t_E can be computed.



For example, consider the three activities. 11—12, 12—13 and 13—14 shown in Fig. 5.8 with their individual time estimates (t_O, t_L and t_P) marked.

$$t_E \text{ for the path} = \Sigma t_E$$

Activity	t_O	t_L	t_P	t_E
11—12	4	6	8	6
12—13	5	7	11	7.333
13—14	4	10	12	9.333
				$\Sigma t_E = 22.666$

Alternatively,

$$\Sigma t_O = 4 + 5 + 4 = 13$$

$$\Sigma t_L = 6 + 7 + 10 = 23$$

$$\Sigma t_P = 8 + 11 + 12 = 31$$

$$\therefore \quad \Sigma t_E = \frac{\Sigma t_O + 4\Sigma t_L + \Sigma t_P}{6}$$

$$= \frac{13 + 4 \times 23 + 31}{6} = 22.67$$

A similar approach can be made for a network consisting of several paths, each path with a number of activities in series. When t_E for path in a network is known, the *critical path* can be chosen easily. A *critical path* is the one which consumes maximum of time resources. This is illustrated in example

Example 1: *For a particular activity of a project, time estimates received from two engineers X and Y are as follows :*

	Optimistic time	Most likely time	Pessimistic time
Engineer X	4	6	8
Engineer Y	3	5	8

State who is more certain about the time of completion of the job.

Example 2: *The network for a certain project is shown in Fig. 5.9. Determine the expected time for each of the path. Which path is critical ?*