Time Computations

Chapter 6: Programme Evaluation and Review techniques

Introduction

For simple networks, expected time or average time of completion of activities enables us to find the critical path, but in complex networks, it is necessary to follow a systematic method to determine the critical path. This is achieved by first computing each event and the following two time estimates:

a) Earliest expected time (T_E)
b) Latest allowable occurrence time (T_L)

The three times estimates t_O, t_L and t_P, as well as the expected or average time t_E, which refer to an activity or job are designated by small t while the above two times (*i.e.* earliest expected time T_E and latest allowable occurrence time T_L) which refer to an event are symbolised by capital T.

Earliest Expected Time

To calculate the earliest expected time for an activity in PERT, the project manager needs to consider the duration estimates and the logical sequence of activities in the network diagram. The earliest expected time for an activity is determined by calculating the earliest start time (EST) and the earliest finish time (EFT) for that activity.

The earliest start time (EST) is the earliest point in time at which the activity can start based on the completion of its predecessor activities. The earliest finish time (EFT) is the earliest point in time at which the activity can be completed based on the EST and the duration estimate.

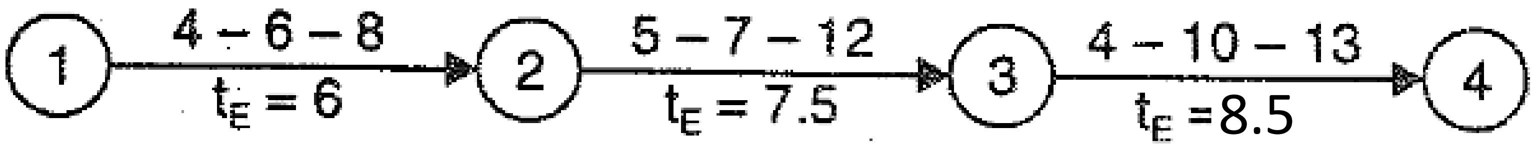
The formula for calculating the earliest expected time for an activity in PERT is as follows:

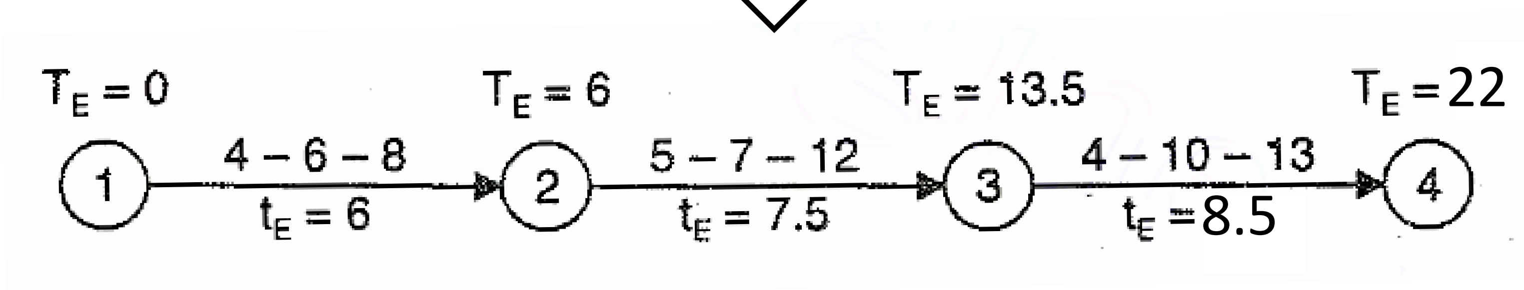
Earliest Expected Time (EET) = EST + Duration

Where, EST = Earliest Start Time Duration = Estimated duration of the activity

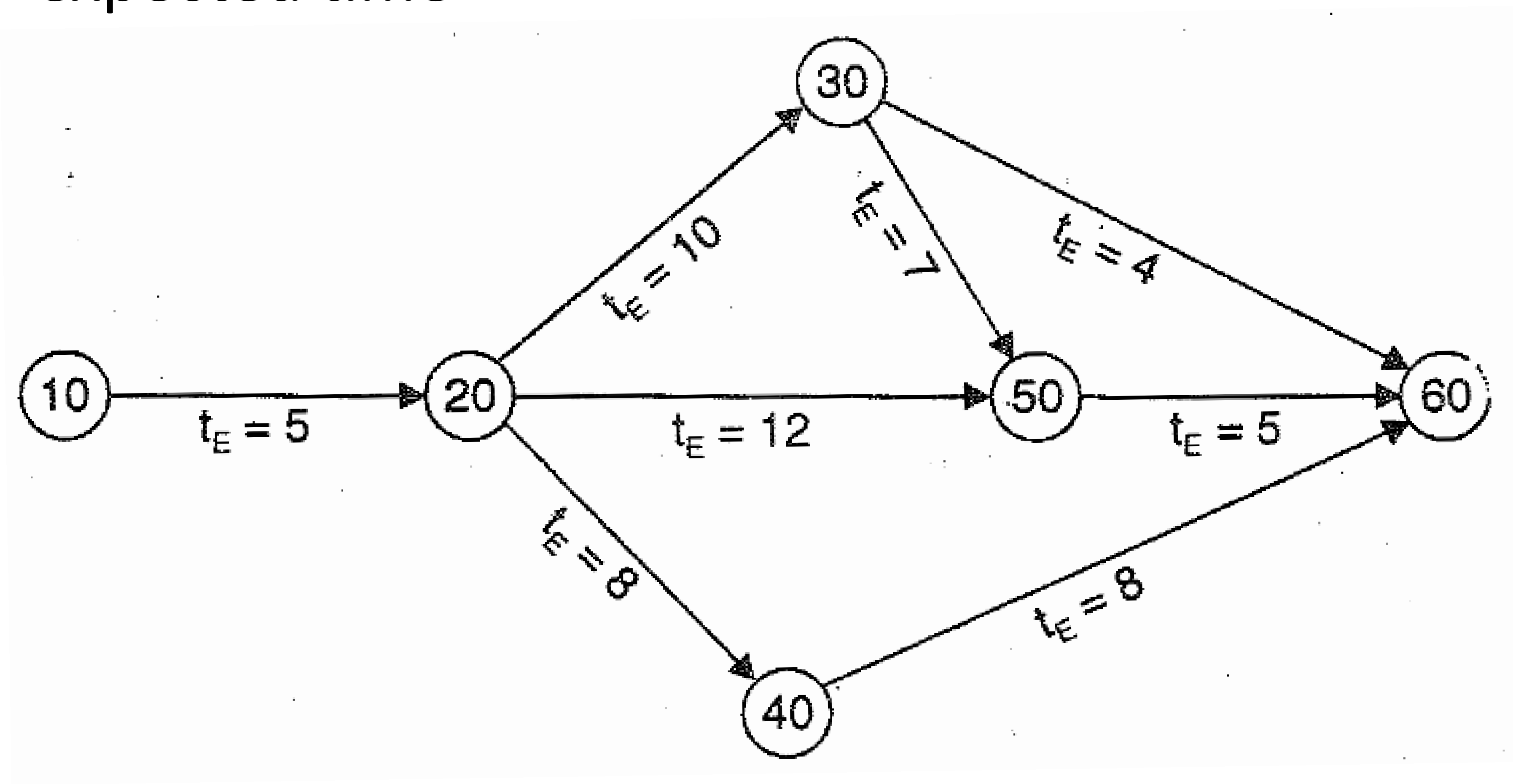
Note: It is represented by symbol T_E and appear above or below the node (event circle) in a network.

For Example: In case of a single path activity as shown in figure, express the earliest expected time





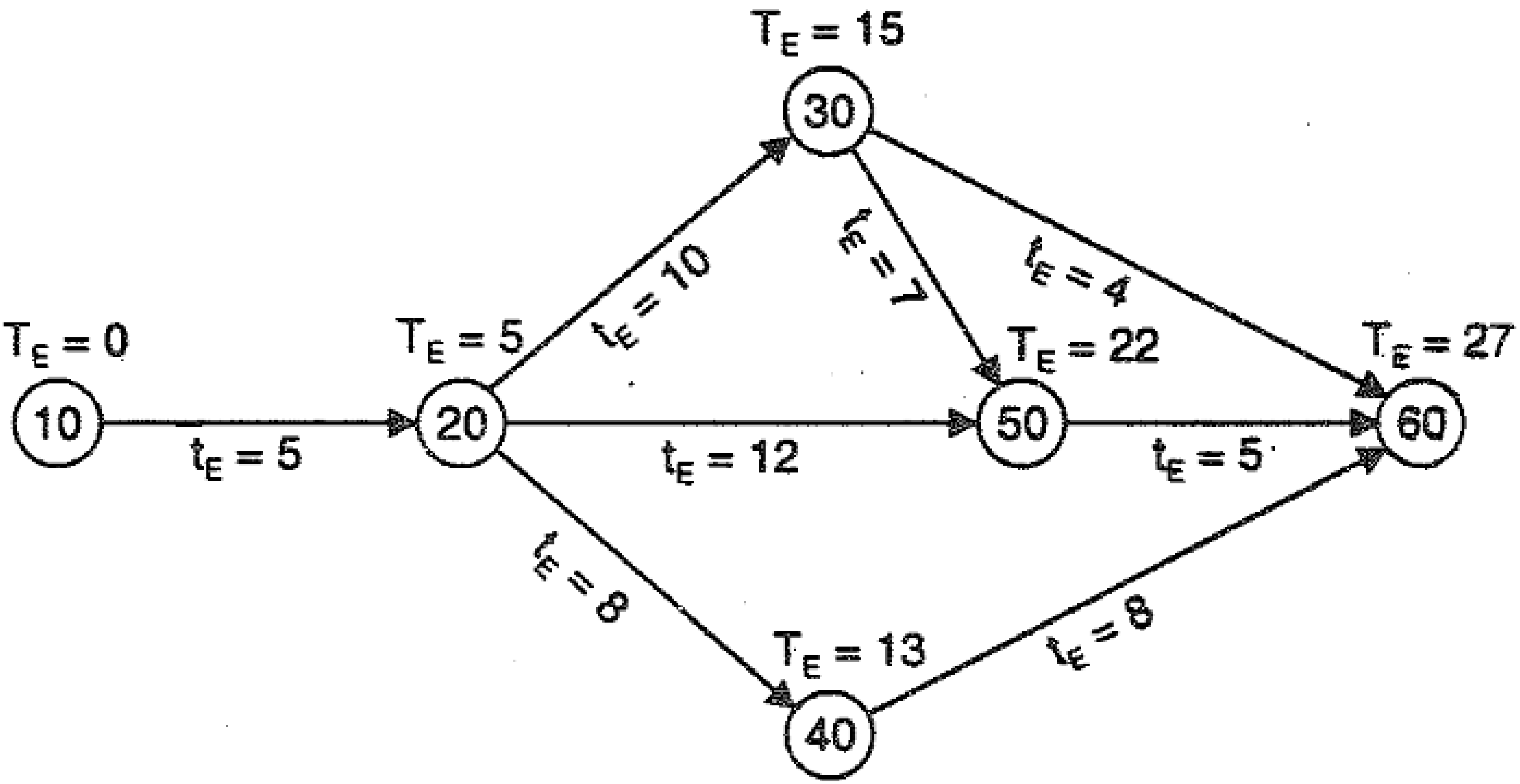
For Example: In case of a multiple-path activity as shown in figure, express the earliest expected time



Note: No event can be considered to have reached until all activities leading to that event are completed.

Hence event 50 cannot be considered to have occurred until all activities along both the paths (10-20-3-50 and 10-20-50) are completed. ***Thus, T_E will be the greater of the two values obtained from the two paths.***

Similar approach will be taken in the case of event 60.



$T_E = 15$
30
$t_E = 10$
$t_E = 7$
$t_E = 4$
$T_E = 0$
$T_E = 5$
$T_E = 22$
$T_E = 27$
10
20
50
60
$t_E = 5$
$t_E = 12$
$t_E = 5$
$t_E = 8$
$t_E = 8$
$T_E = 13$
40

Formulation for T_E

The method described above may be all right for small networks, but for large or complicated networks in which an event under consideration may have many predecessor events, it is better to formulate a rule for computation of T_E so that frequent references to the network may not be necessary.

Let us represent an activity symbolically by ij where i is the predecessor event and j is the successor event, and i—j is the activity connecting the two events. Since T_E for a successor event is equal to T_E for the predecessor event plus the expected activity time (t_E), we have

$$T_E \text{ (successor event)} = T_E \text{ (predecessor event)} + t_E \text{ (activity)}.$$

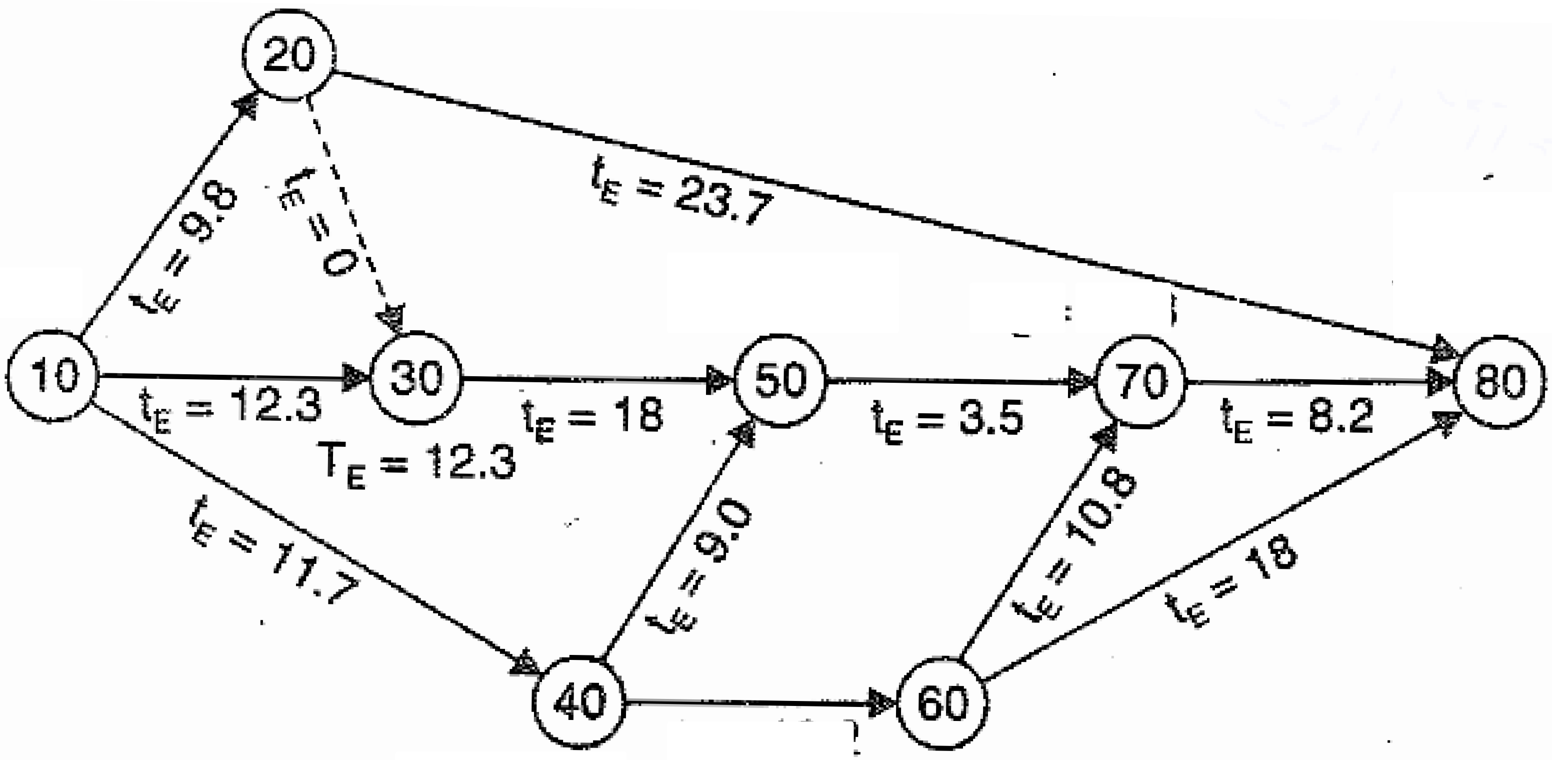
Expressed symbolically,

$$T_E^j = T_E^i + t_E^{ij}$$

The above formulation is true if there is only one predecessor event. If, however, there are more than one predecessor events to the successor event (*i.e.* event under consideration), the above rule needs modification since the event j cannot occur unless all activities leading to it are completed. Hence T_E for the event will be equal to *maximum* of $(T_E^i + t_E^{ij})$ along various activity paths. Hence

$$T_E^j = (T_E^i + t_{ij})_{max}.$$

For Example: Calculate the Earliest Expected Time T_E for the network shown in the following Figure.



Computation of Earliest Expected Time

Successor event j	*Predecessor event* i	*Activity* i—j	t_E^{ij}	t_E^j
80	70	70—80	8.2	<u>42.0</u>
	60	60—80	18.0	39.9
	20	20—80	23.7	33.5
70	60	60—70	10.8	32.7
	50	50—70	3.5	<u>33.8</u>
60	40	40—60	10.2	<u>21.9</u>
50	40	40—50	9.0	20.7
	30	30—50	18.0	<u>30.3</u>
40	10	10—40	11.7	<u>11.7</u>
30	20	20—30	0	9.8
	10	10—30	12.3	<u>12.3</u>
20	10	10—20	9.8	<u>9.8</u>

Latest Allowable Occurrence Time

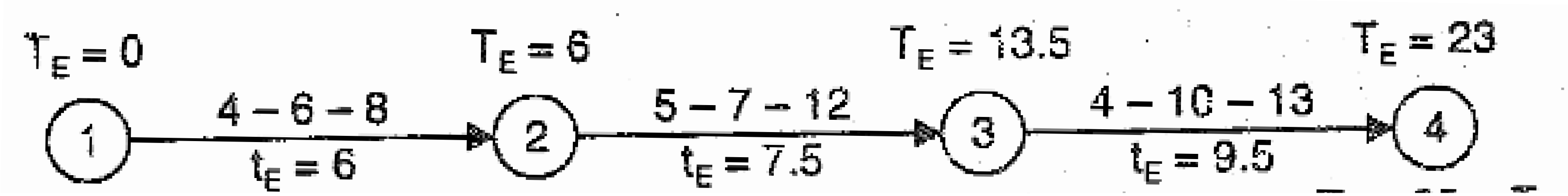
It refers to the latest point in time by which an activity must be completed to avoid delaying the project's overall completion date.

LAOT is usually determined by analyzing the critical path of the project schedule, which is the sequence of activities that determines the shortest possible duration of the project. The LAOT is calculated by subtracting the activity's duration from the project's total duration and adjusting for any lag time or other factors that may affect the project's schedule.

And denoted by T_L.

Whenever a project is taken in hand, decision is taken regarding the time of the completion of the project and the accepted figure is called the scheduled completion time (or the contractual obligation time) and is denoted by symbol T_S.

For Example: To compute the latest occurrence time for the events.



To compute the latest allowable occurrence time for various events, let us again consider the simple network of Fig. 6.2. Let us assume that *scheduled completion time* T_S for the project is 25, meaning thereby that the end event 4 must occur, latest by 25 units of time after the project is initiated.

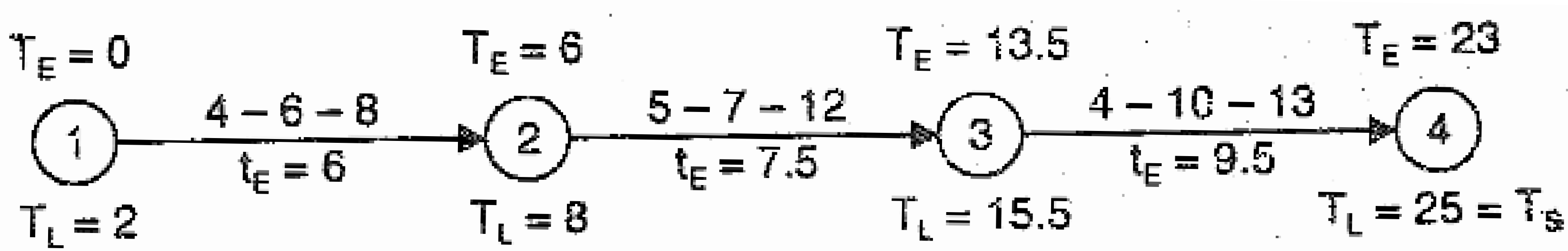
Thus, for the last event, $T_L^4 = T_S = 25$.

The activity 3—4 takes 9.5 units of time for its completion. Hence event 3 cannot occur later than $25 - 9.5 = 15.5$. Thus, $T_L^3 = 15.5$.

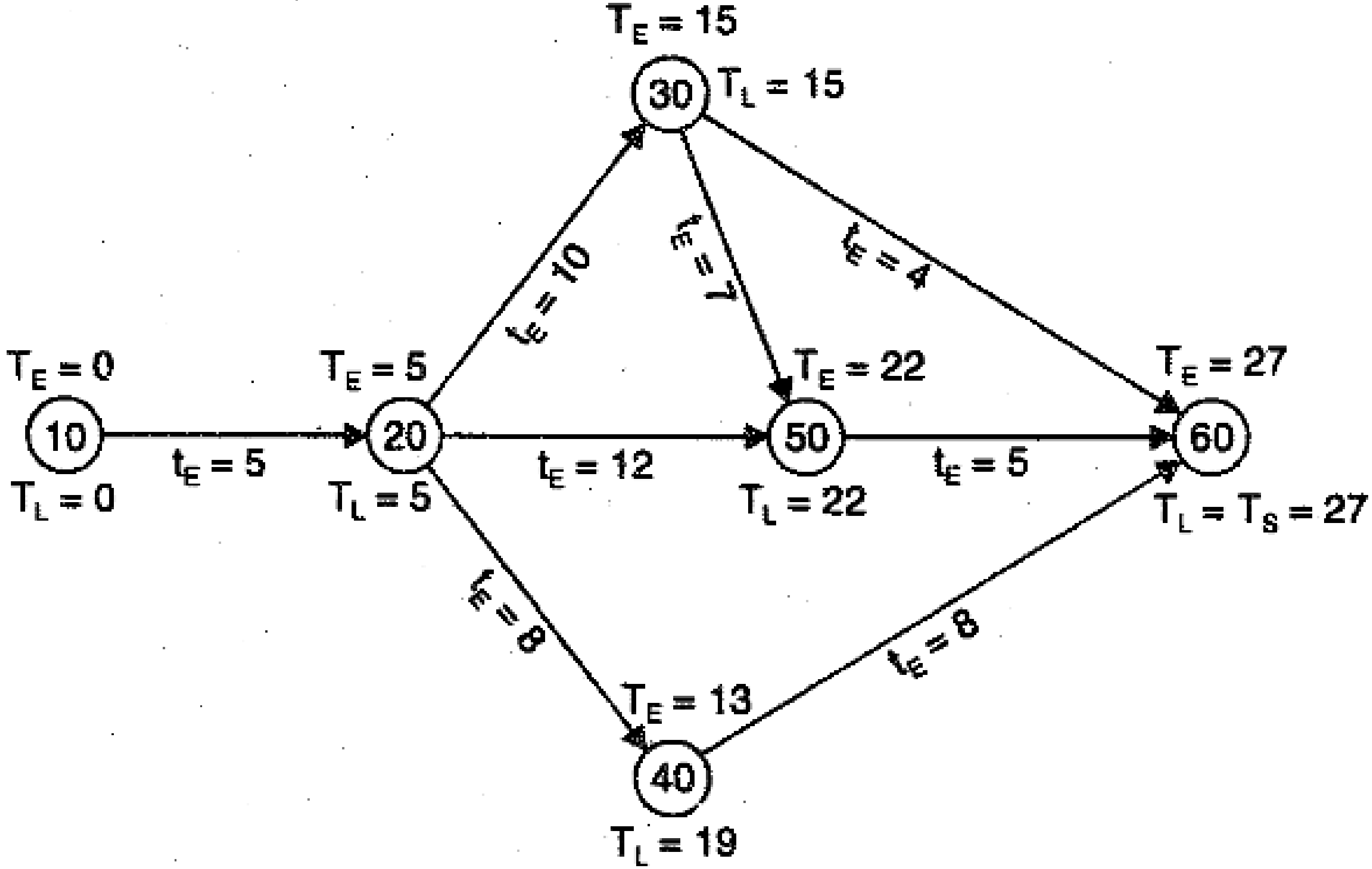
Similarly, for event 2, $T_L^2 = T_L^3 - t_E^{2-3} = 15.5 - 7.5 = 8.0$

and for the initial event.

$$T_E^1 = T_L^2 - t_E^{1-2} = 8.0 - 6 = 2.0.$$



For Example: To compute the latest occurrence time for the events.



Let us assume $T_S = 27$

For event 50, latest occurrence time is given by

$$T_L^{50} = T_L \text{ for } 60 - t_E \text{ for } (50—60)$$
$$= 27 - 5 = 22.$$

For event 40,

$$T_L^{40} = T_L \text{ for } 60 - t_E \text{ for } (40—60)$$
$$= 27 - 8 = 19.$$

Event 30 has two successor events : event 40 and event 50. Hence two values of T_L are obtained as under :

$$T_L^{30} = T_L \text{ for } 60 - t_E \text{ for } (30—60)$$
$$= 27 - 4 = 23$$

and

$$T_L^{30} = T_L \text{ for } 50 - t_E \text{ for } (30—50)$$
$$= 22 - 7 = 15.$$

Out of this, the *minimum* value (*i.e.* 15) will be the appropriate value of T_L^{30}. This is because if event 50 cannot occur later than 22 units of time after the beginning of the project, event 30 cannot occur later than 15 units of time after the initiation of project since activity 30—50 takes 7 units of time for its completion. If a higher value of T_L^{30} (= 23) is permitted, T_L^{50} will be = 23 + 7 = 30 and the event 50 will be late by 8 units of time. Hence a minimum value, out of the various values, is to be selected.

Similarly, event 20 has three successor events : 50, 40 and 30 having T_L's as 22, 19 and 15 respectively. Therefore, we get three values of T_L^{20} as under :

(*i*) $T_L^{20} = T_L$ for $50 - t_E$ for (20—50)

$= 22 - 12 = 10$

(*ii*) $T_L^{20} = T_L$ for $40 - t_E$ for (20—40)

$= 19 - 8 = 11$

(*iii*) $T_L^{20} = T_L$ for $30 - t_E$ for (20—30)

$= 15 - 10 = 5.$

Out of above, the minimum value (*i.e.* 5) is the appropriate value of T_L^{20}.

For the initial event,

$T_L^{10} = T_L$ for $20 - t_E$ for (10—20)

$= 5 - 5 = 0.$

Formulation of T_L

Consider an activity i-j, in which i is the predecessor event and j is the successor event.

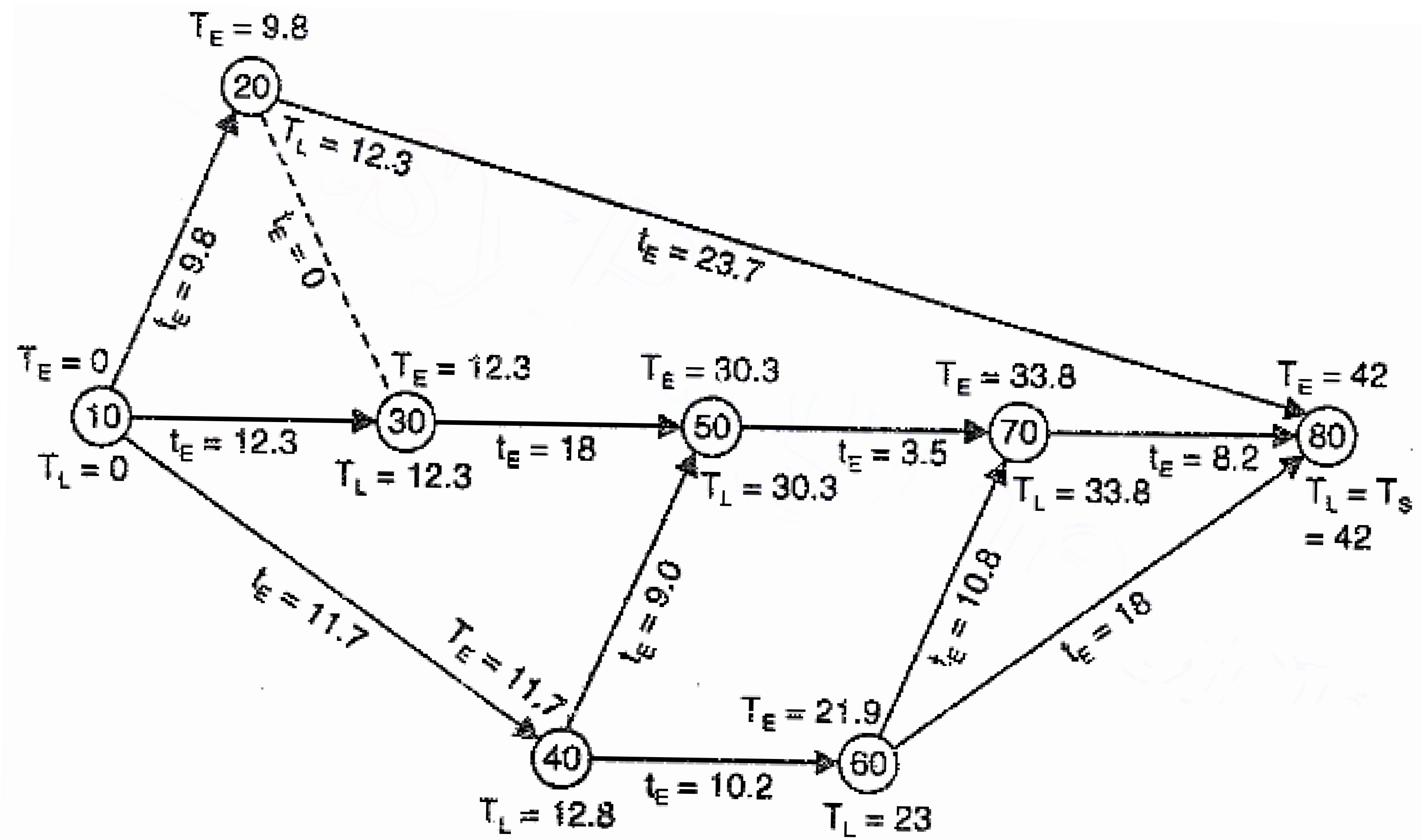
Then the *Latest Occurrence Time* for the predecessor event is given by

$$T_{\mathrm{L}}^{i} = T_{\mathrm{L}}^{j} - t_{\mathrm{E}}^{ij}$$

The above formulation is useful when the event i under consideration has only one successor event (j). If however, there are more than one successor events (j', j'' j''' etc.), the *minimum* of ($T^j_L - t^{ij}_E$) will be the appropriate latest occurrence time T^i_L for event i. This is so because if the higher of the various values is taken, the latest occurrence time for the successor events will be also increased, suggesting a delay in the project completion.

Thus, $$T^i_L = (T^j_L - t^{ij}_E)_{min}$$

For Example: To compute the latest occurrence time for the given network.



Note: If scheduled time is not given, In such a circumstance T_S can be taken equal to T_E of the end event.

Computation of Latest Allowable Occurrence Time

Predecessor event (i)	Successor event (j)	Activity i–j	t_E^{ij}	T_L^i
70	80	70—80	8.2	33.8
60	80	60—80	18.0	24.0
	70	60—70	10.8	23.0
50	70	50—70	3.5	30.3
40	60	40—60	10.2	12.8
	50	40—50	9.0	21.3
30	50	30—50	18.0	12.3
20	80	20—80	23.7	18.3
	30	20—30	0.0	12.3
10	40	10—40	11.7	1.1
	30	10—30	12.3	0.0
	20	10—20	9.8	2.5

- The computation of T_E is done by forward pass, starting from the initial event. For such events, predecessor events form the base.
- Whereas, the computation of T_L is done by backward pass, starting from the end event. For such events, successor events form the base.

However, for most of the networks, computations of both T_E and T_L for each event is required. A combined table is therefore suggested.