# PERT: Network Analysis

Chapter 7

# Slack

Slack may be simply defined as the difference in the *latest allowable time* and the *earliest expected time* of an event.

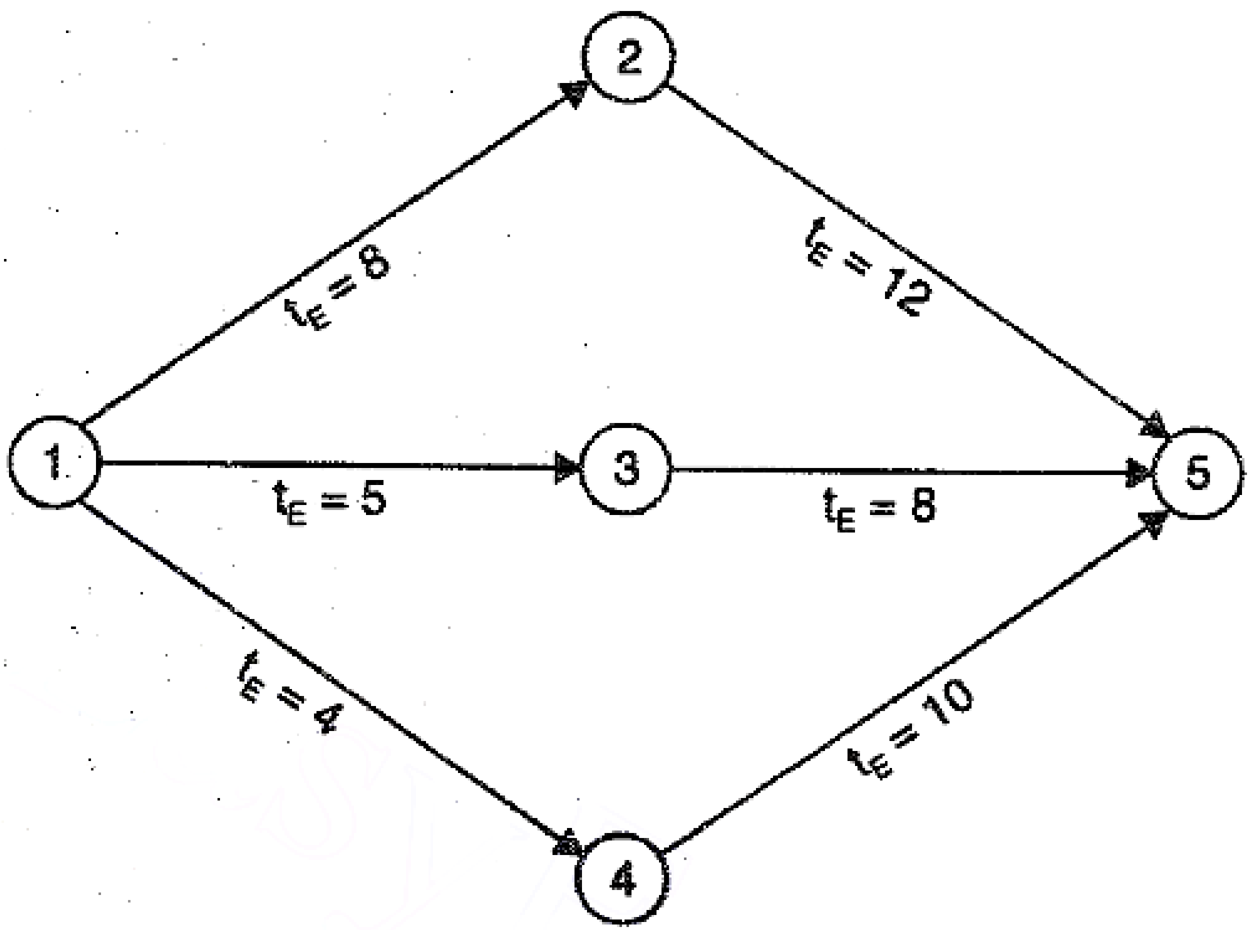
The difference between the two times of an activity indicate the range between which the occurrence time of an event can vary.

Therefore, $S = T_L - T_E$

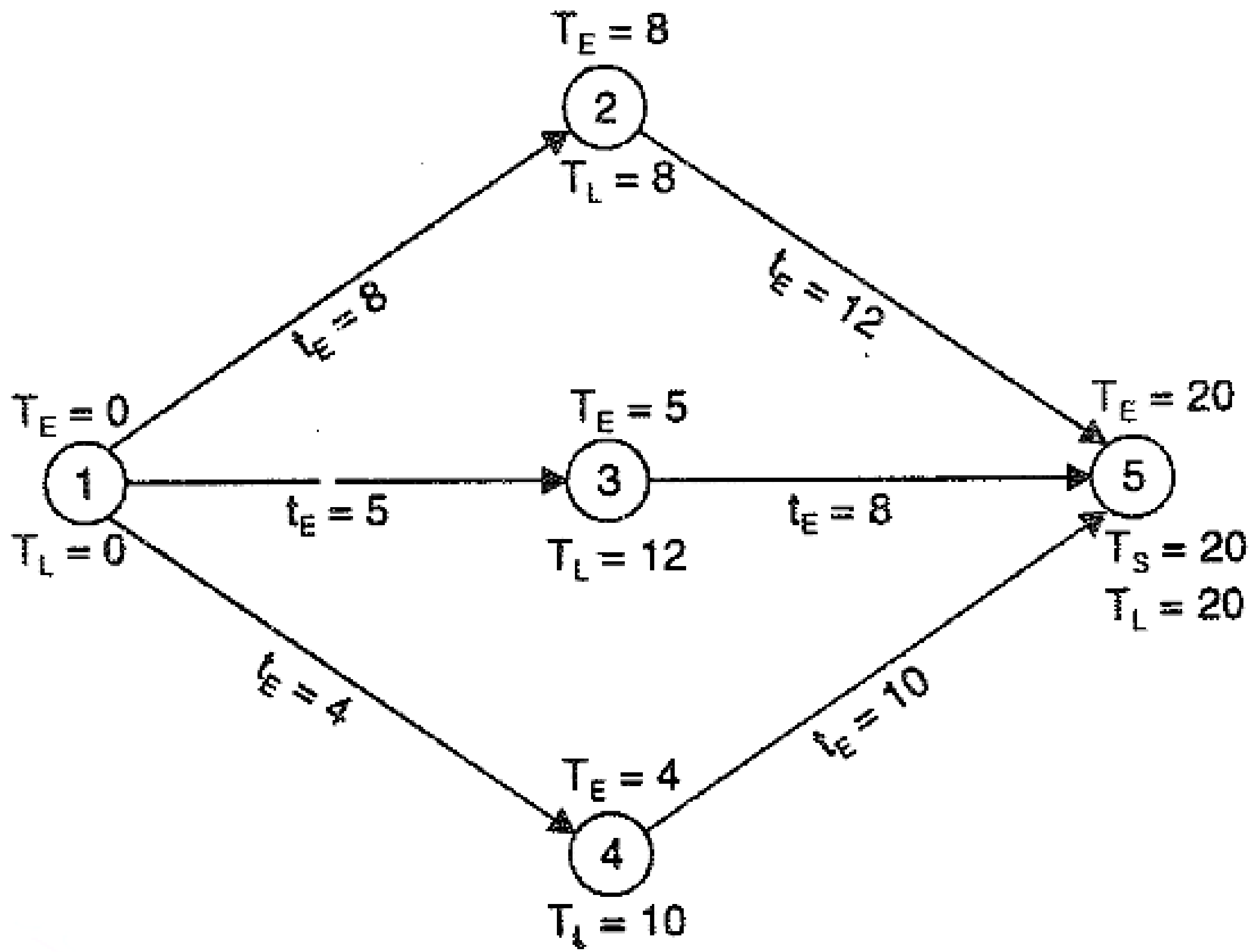
Thus *slack* gives the idea of ‘time to spare’. Slack means more time to work, less to worry about. It reveals about those areas of work which have an excess of resources from which trades-offs can be rearranged. It also spots those areas which are potential trouble areas, *i.e.* Those area of zero or minimum slack.

- **Positive Slack:** It is obtained when *TL* is more than *TE* for an event. It is an indication of an ahead of schedule condition (excess resources).
- **Zero Slack:** It is an indication of a on schedule condition (adequate resources).
- **Negative Slack:** It is an indication of behind of schedule condition (lack of resources).

**Example:** Analyse with respect to resources the network shown in Figure.



| Event No. | Earliest expected time ↓ | | | | Latest occurrence time ↑ | | | | Slack $S = T_L - T_E$ |
|---|---|---|---|---|---|---|---|---|---|
| | Predecessor event $(i)$ | $t_E^{ij}$ | $T_E^i$ | $T_E$ | Successor event $(j)$ | $t_E^{ij}$ | $T_L^i$ | $T_L$ | |
| 1 | — | — | $\underline{0}$ | 0 | 2 | 8 | $\underline{0}$ | 0 | 0 |
| | | | | | 3 | 5 | 7 | | |
| | | | | | 4 | 4 | 6 | | |
| 2 | 1 | 8 | $\underline{8}$ | 8 | 5 | 12 | $\underline{8}$ | 8 | 0 |
| 3 | 1 | 5 | $\underline{5}$ | 5 | 5 | 8 | $\underline{12}$ | 12 | 7 |
| 4 | 1 | 4 | $\underline{4}$ | 4 | 5 | 10 | $\underline{10}$ | 10 | 6 |
| 5 | 2 | 12 | $\underline{20}$ | 20 | — | — | 20 | 20 | 0 |
| | 3 | 8 | 13 | | | | | | |
| | 4 | 10 | 14 | | | | | | |



$T_E = 8$
2
$T_L = 8$
$t_E = 8$
$t_E = 12$
$T_E = 0$
1
$T_L = 0$
$T_E = 5$
3
$T_L = 12$
$t_E = 5$
$t_E = 8$
$T_E = 20$
5
$T_S = 20$
$T_L = 20$
$t_E = 4$
$t_E = 10$
$T_E = 4$
4
$T_L = 10$

The following conclusions are drawn :

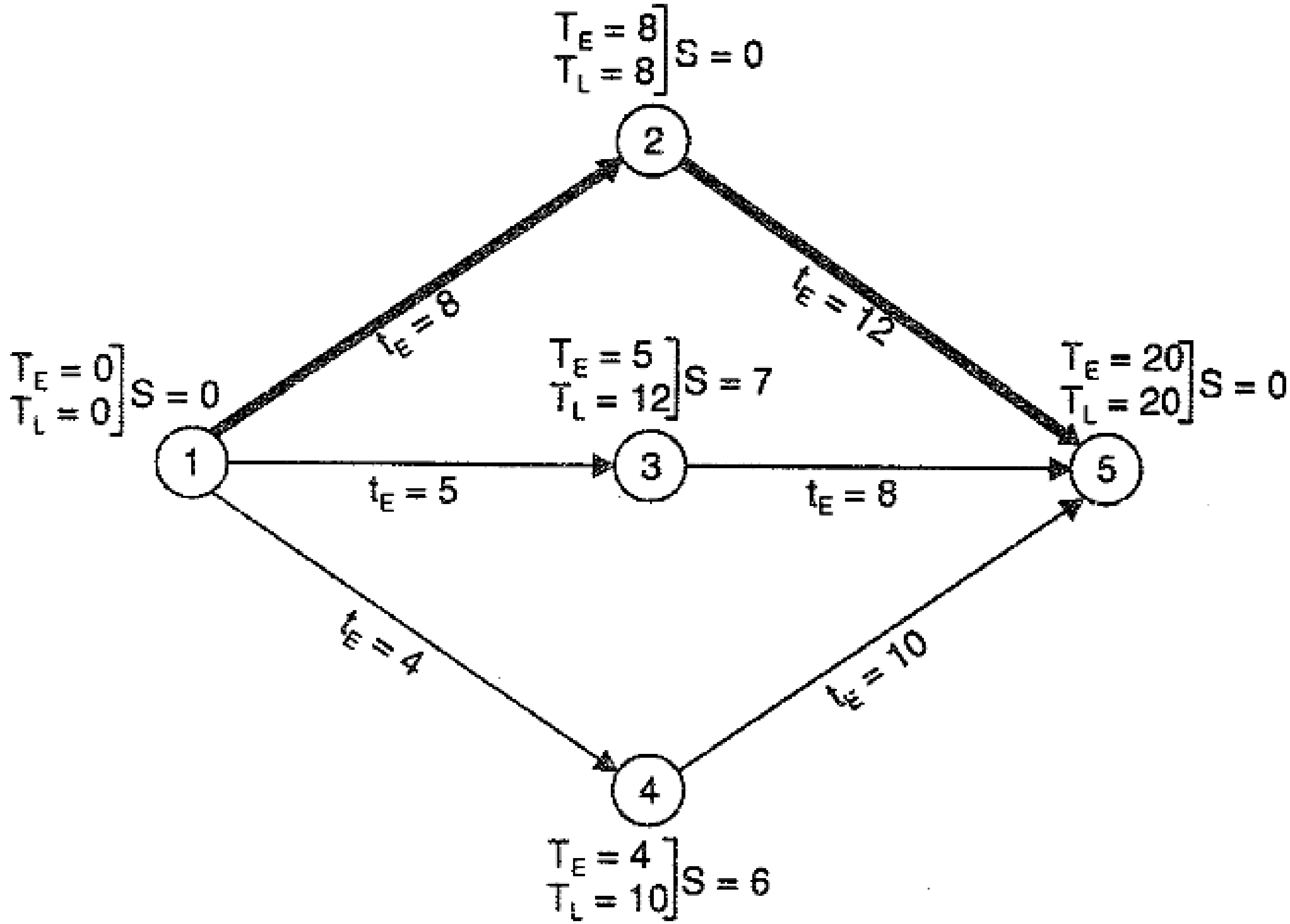
1. Events 1, 2 and 5 have zero slack. Any delay in the activities connecting them would cause corresponding delay in the completion of the project. The occurrence of these events is *critical*.

2. Events 3 and 4 have slack of 7 and 6 days respectively. The activities connecting either of these events can be *delayed* by the slack value without affecting the scheduled completion time of the project.

3. Event 1 must start exactly on schedule. However, the above analysis suggests that the resources of activities 1—3 and 1—4 can be partly shifted to aid activities 1—2 and 2—5.

# Critical Path

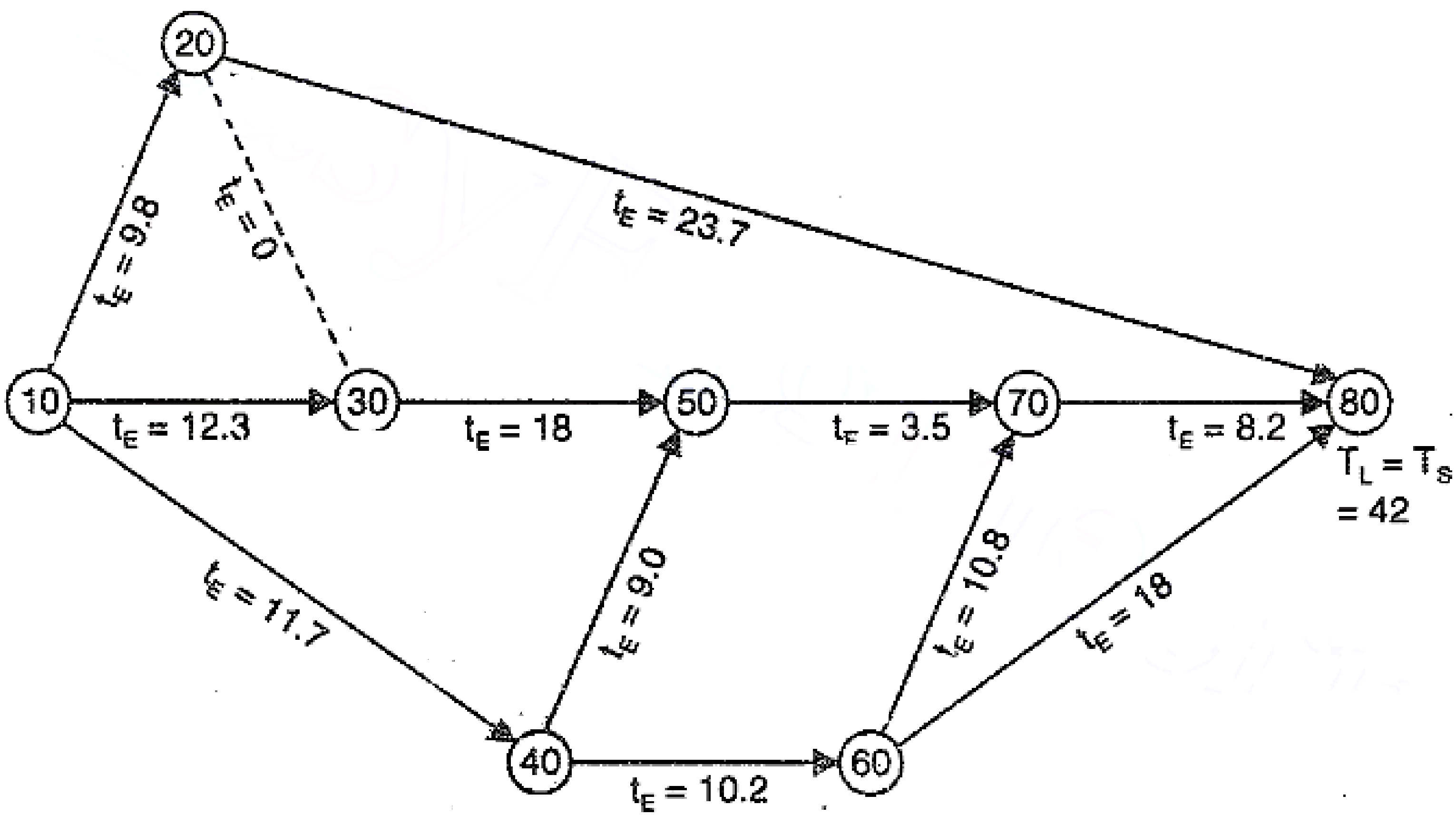
It is important to note that the value of slack, associated with an event, determine how critical that event is. The less the slack (more negative), the more critical an event is. A *critical path* is the one which connects the events having zero or minimum slack times. All the events along the *critical path* are considered to be *critical* in the sense that any delay in their occurrence will result in the delay in the scheduled completion of the project. *Eventually, a critical path is the longest path (time wise) connecting the initial and end event.*



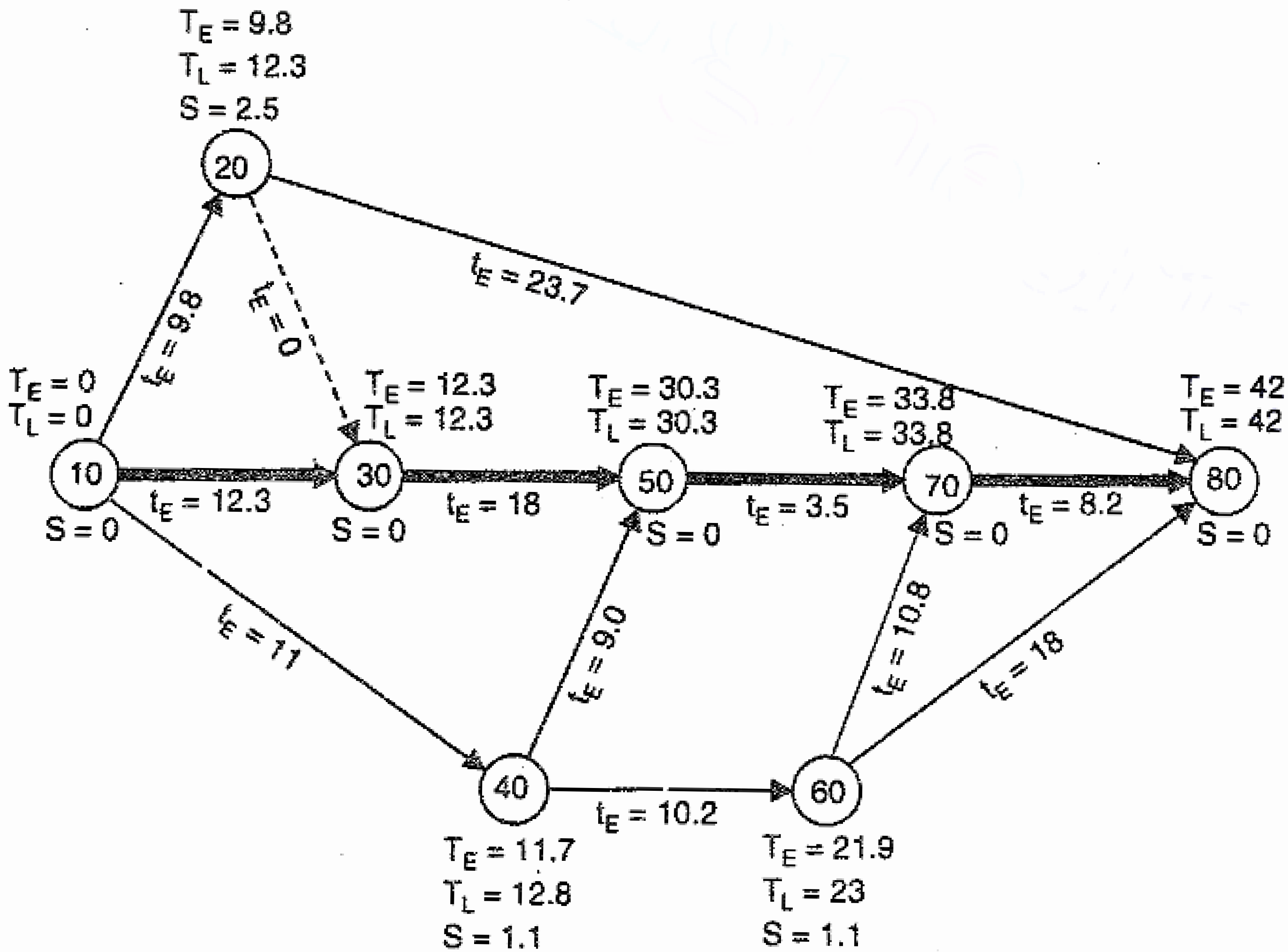

THE CRITICAL PATH.

Note that $\Sigma t_E$ of all the activities along the critical path is equal to $T_E$ of that last event. Along any other path, $\Sigma t_E$ is less than $T_E$ of last event. *Thus, critical path is the longest path.*

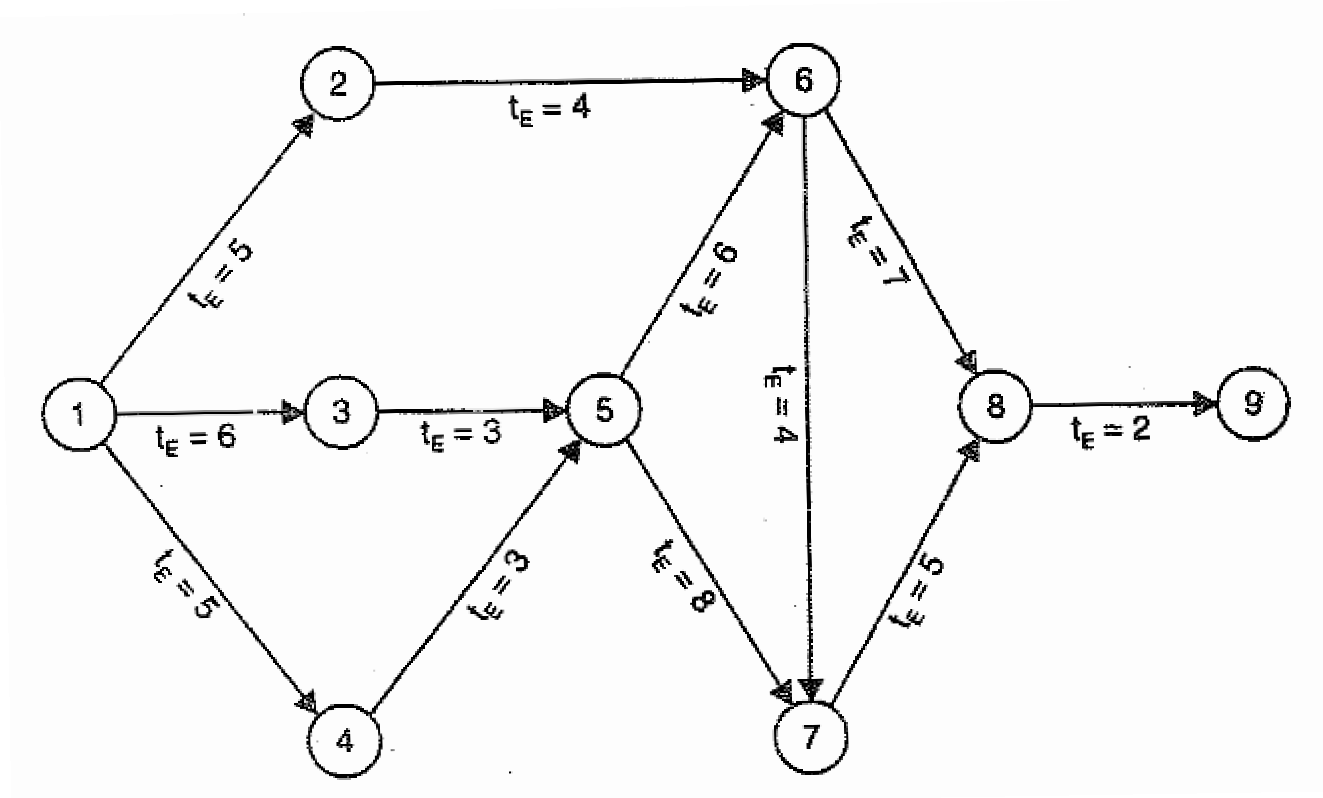
**Example:** Determine the *critical path* for the network shown in figure. Numbers indicate *time in weeks*..



| *Event* | $T_E$ | $T_L$ | *Slack* $S = T_L - T_E$ |
|---|---|---|---|
| 10 | 0 | 0 | 0 |
| 20 | 9.8 | 12.3 | 2.5 |
| 30 | 12.3 | 12.3 | 0 |
| 40 | 11.7 | 12.8 | 1.1 |
| 50 | 30.3 | 30.3 | 0 |
| 60 | 21.9 | 23.0 | 1.1 |
| 70 | 33.8 | 33.8 | 0 |
| 80 | 42.0 | 42.0 | 0 |

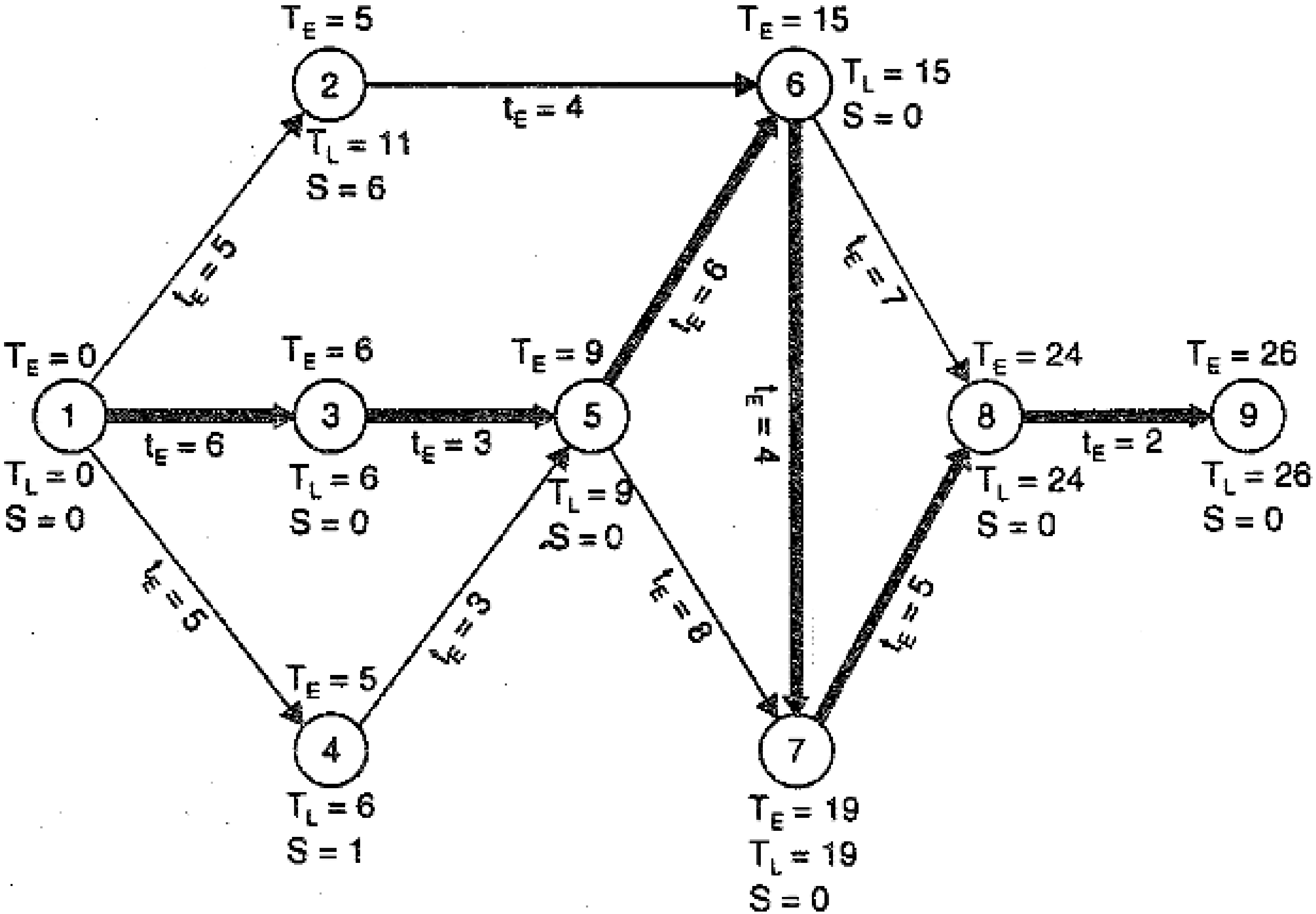

$T_E = 9.8$
$T_L = 12.3$
$S = 2.5$
20
$t_E = 23.7$
$t_E = 9.8$
$t_E = 0$
$T_E = 0$
$T_L = 0$
10
$S = 0$
$t_E = 12.3$
$T_E = 12.3$
$T_L = 12.3$
30
$S = 0$
$t_E = 18$
$T_E = 30.3$
$T_L = 30.3$
50
$S = 0$
$t_E = 3.5$
$T_E = 33.8$
$T_L = 33.8$
70
$S = 0$
$t_E = 8.2$
$T_E = 42$
$T_L = 42$
80
$S = 0$
$t_E = 11$
$t_E = 9.0$
$t_E = 10.8$
$t_E = 18$
40
$t_E = 10.2$
60
$T_E = 11.7$
$T_L = 12.8$
$S = 1.1$
$T_E = 21.9$
$T_L = 23$
$S = 1.1$

**Example:** Determine the *critical path* for the network shown in figure. Numbers indicate *time in weeks*..



| Event No. | Earliest expected time ↓ | | | | Latest occurrence time ↑ | | | | Slack $S = T_L - T_E$ |
|---|---|---|---|---|---|---|---|---|---|
| | Predecessor event (*i*) | $t_E^{ij}$ | $T_L^i$ | $T_E$ | Successor event (*j*) | $t_E^{ij}$ | $T_L^i$ | $T_L$ | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 1 | — | — | 0 | 0 | 2<br>3<br>4 | 5<br>6<br>5 | 6<br>$\underline{0}$<br>1 | 0 | 0 |
| 2 | 1 | 5 | $\underline{5}$ | 5 | 6 | 4 | $\underline{11}$ | 11 | 6 |
| 3 | 1 | 6 | $\underline{6}$ | 6 | 5 | 3 | $\underline{6}$ | 6 | 0 |
| 4 | 1 | 5 | $\underline{5}$ | 5 | 5 | 3 | $\underline{6}$ | 6 | 1 |

| Event No. | Earliest expected time ↓ | | | | Latest occurrence time ↑ | | | | Slack $S = T_L - T_E$ |
|---|---|---|---|---|---|---|---|---|---|
| | Predecessor event $(i)$ | $t_E^{ij}$ | $T_L^i$ | $T_E$ | Successor event $(j)$ | $t_E^{ij}$ | $T_L^i$ | $T_L$ | |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| 5 | 3 | 3 | $\underline{9}$ | 9 | 6 | 6 | $\underline{9}$ | 9 | 0 |
| | 4 | 3 | 8 | | 7 | 8 | 11 | | |
| 6 | 2 | 4 | 9 | 15 | 7 | 4 | $\underline{15}$ | 15 | 0 |
| | 5 | 6 | $\underline{15}$ | | 8 | 7 | 17 | | |
| 7 | 5 | 8 | 17 | 19 | 8 | 5 | $\underline{19}$ | 19 | 0 |
| | 6 | 4 | $\underline{19}$ | | | | | | |
| 8 | 6 | 7 | 22 | 24 | 9 | 2 | $\underline{24}$ | 24 | 0 |
| | 7 | 5 | $\underline{24}$ | | | | | | |
| 9 | 8 | 2 | $\underline{26}$ | 26 | — | — | 26 | 26 | 0 |



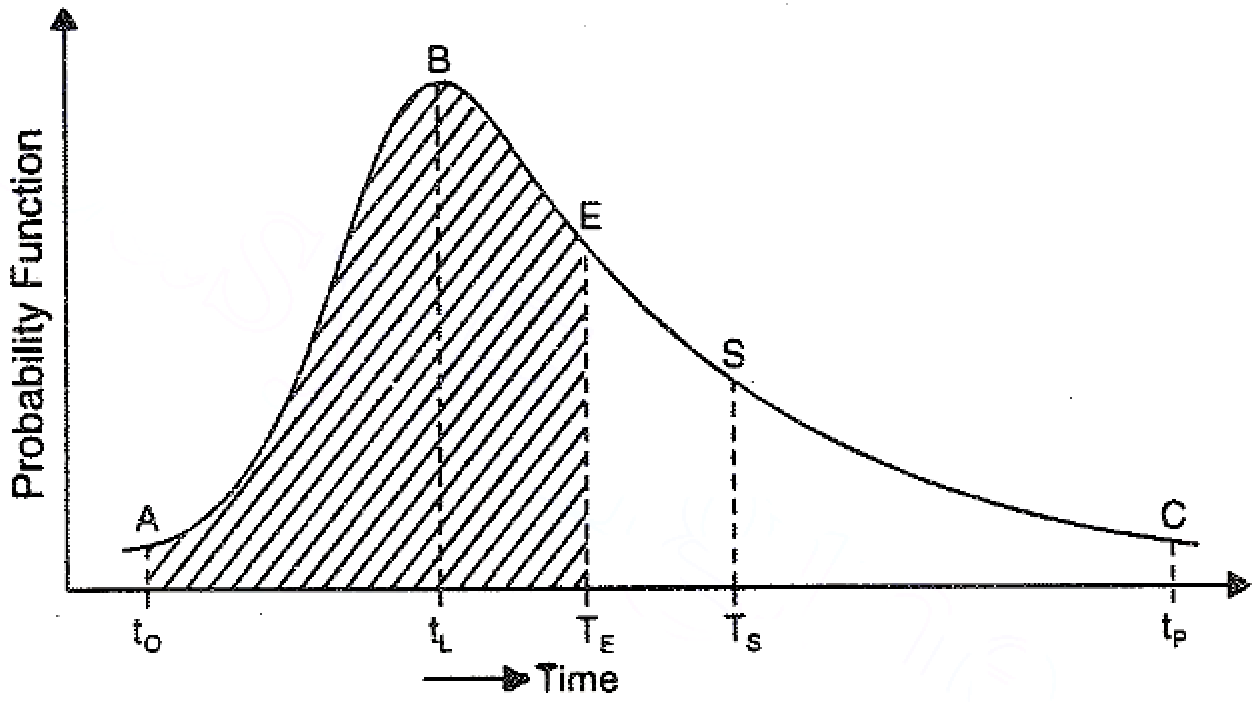
$T_E = 0$
1
$T_L = 0$
$S = 0$
$t_E = 5$
$T_E = 5$
2
$T_L = 11$
$S = 6$
$t_E = 4$
$t_E = 6$
$T_E = 6$
3
$T_L = 6$
$S = 0$
$t_E = 3$
$t_E = 5$
$T_E = 5$
4
$T_L = 6$
$S = 1$
$t_E = 3$
$T_E = 9$
5
$T_L = 9$
$S = 0$
$t_E = 6$
$T_E = 15$
6
$T_L = 15$
$S = 0$
$t_E = 4$
$t_E = 8$
7
$T_E = 19$
$T_L = 19$
$S = 0$
$t_E = 7$
$t_E = 5$
$T_E = 24$
8
$T_L = 24$
$S = 0$
$t_E = 2$
$T_E = 26$
9
$T_L = 26$
$S = 0$

# What is the probability of meeting scheduled time?

**Answer:** *by applying the probability theory to the network analysis*

The expected time ($t_E$) is such that there is fifty-fifty chance of completion of the activity in this time.

$$t_E = \frac{t_O + 4t_L + t_P}{6}$$



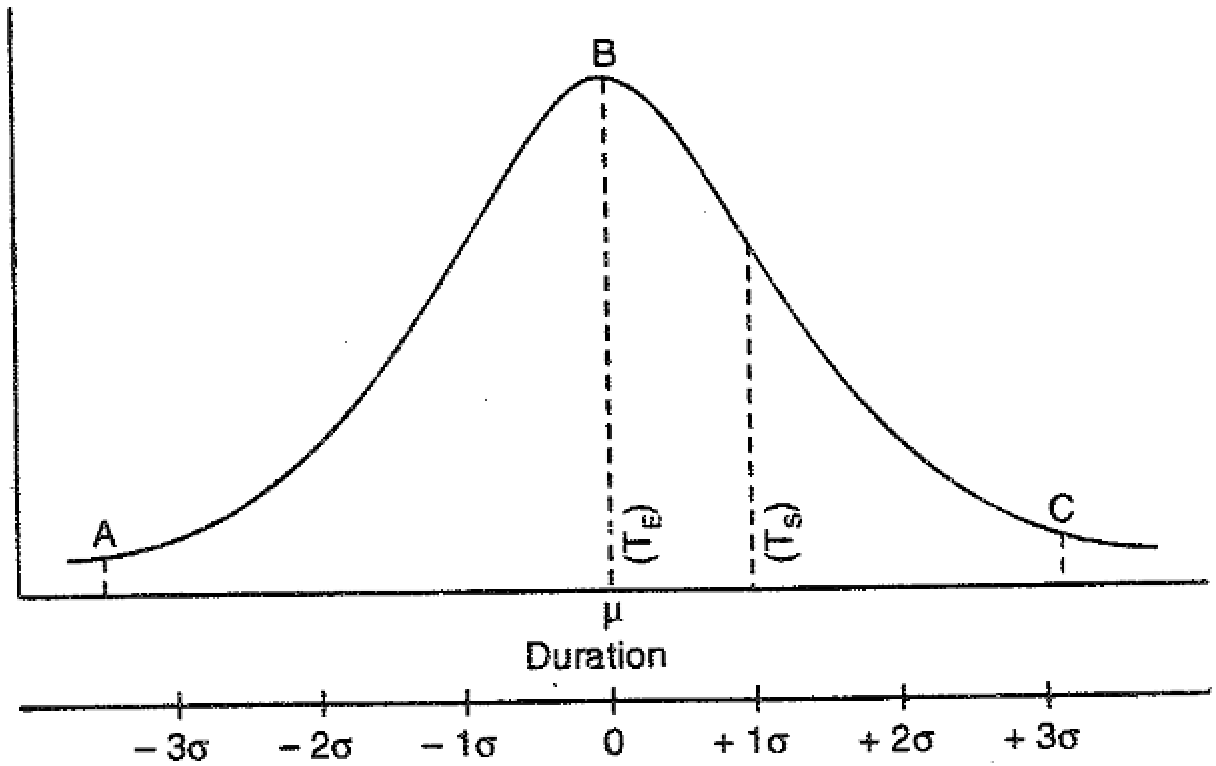

BETA DISTRIBUTION.

Hence the activity *ij*, for which the above probability distribution curve refers, has 50% probability of its completion within time $t_E$.

*Similarly,* the probability of completion of the activity within some other time $t_s$ will be equal to the area under the curve up to vertical line through $t_s$ divided by the total area of the curve:

$$\text{Probability} = \frac{\text{area under } ABS}{\text{area under } ABC}$$

So, in order to find the shaded area, we take the help of the important property of the normal distribution curve...




If mean of the normal distribution is denoted by $\mu$, it can be proved that

($a$) Approximately 68 per cent of the value of normal distribution lie within $\pm\ \sigma$ from the average, where $\sigma$ is the standard deviation. This means that the area of the curve between $x = \mu - \sigma$ to $x = \mu + \sigma$ is 68% of the total area (*b*) Approximately 95 per cent of all the values lie within $\pm 2\sigma$ from the average. This means that area of the curve between $x = \mu - 2\sigma$ to $x = \mu + 2\sigma$ is 95% of the total area.

(*c*) Approximately 99.7 per cent of all the values lie within $\pm 3\sigma$ from the average. This means that the area of the curve between $x = \mu - 3\sigma$ to $x = \mu + 3\sigma$ is 99.7%.

# **Steps** to find out the probability of meeting the schedule time of completion

**Step 1.** Determine the standard deviation ($\sigma$) appropriate to the critical path, for the network, using the relation.

$$\sigma = \sqrt{\text{sum of variances along critical path}}$$

or

$$\sigma = \sqrt{\Sigma \sigma_{ij}^2}$$

where $\sigma_{ij}^2$ = variance for the activity $i$—$j$ along the critical path

$$= \left( \frac{t_P^{ij} - t_O^{ij}}{6} \right)^2$$

**Step 2.** Knowing the scheduled time of completion ($t_S$) and earliest expected time of completion ($T_E$), find the time distance $T_S - T_E$ and express it in terms of *probability factor Z by the relation :*

$$Z = \frac{T_S - T_E}{\sigma}$$

or

$$Z = \frac{T_S - T_E}{\sqrt{\Sigma\sigma_{ij}^2}}$$

The probability factor ($Z$) is the same as *normal deviate* of Table

The probability factor ($Z$) can be positive, zero or negative.

When $Z$ is *positive* (*i.e.* $T_S$ to the right of $T_E$), the chances of completing the project in time are *more* than 50%.

When $Z$ is *zero* (*i.e.* $T_S$ coinciding with $T_E$), the chances of completing the project in time is fifty-fifty.

When $Z$ is *negative* (*i.e.* $T_S$ to the left of $T_E$), the chances of completing the project in time is *less* than 50%.

**Step 3.** Find % probability with respect to the normal deviate $Z$ from Table

**Standard Normal Distribution Function**

| $Z(+)$ | *Probability* $(P_r)$ (%) | $Z(-)$ | *Probability* $(P_r)$ (%) |
|---|---|---|---|
| 0 | 50.0 | 0 | 50.0 |
| + 0.1 | 53.98 | – 0.1 | 46.02 |
| + 0.2 | 57.93 | – 0.2 | 42.07 |
| + 0.3 | 61.79 | – 0.3 | 38.21 |
| + 0.4 | 65.54 | – 0.4 | 34.46 |
| + 0.5 | 69.15 | – 0.5 | 30.85 |
| + 0.6 | 72.57 | – 0.6 | 27.43 |
| + 0.7 | 75.80 | – 0.7 | 24.20 |
| + 0.8 | 78.81 | – 0.8 | 21.19 |
| + 0.9 | 81.59 | – 0.9 | 18.41 |
| + 1.0 | 84.13 | – 1.0 | 15.87 |

| $Z(+)$ | Probability ($P_r$) (%) | $Z(-)$ | Probability ($P_r$) (%) |
|---|---|---|---|
| + 1.0 | 84.13 | – 1.0 | 15.87 |
| + 1.1 | 86.43 | – 1.1 | 13.57 |
| + 1.2 | 88.49 | – 1.2 | 11.51 |
| + 1.3 | 90.32 | – 1.3 | 9.68 |
| + 1.4 | 91.92 | – 1.4 | 8.08 |
| + 1.5 | 93.32 | – 1.5 | 6.68 |
| + 1.6 | 94.52 | – 1.6 | 5.48 |
| + 1.7 | 95.54 | – 1.7 | 4.46 |
| + 1.8 | 96.41 | – 1.8 | 3.59 |
| + 1.9 | 97.13 | – 1.9 | 2.87 |
| + 2.0 | 97.72 | – 2.0 | 2.28 |
| + 2.1 | 98.21 | – 2.1 | 1.79 |
| + 2.2 | 98.61 | – 2.2 | 1.39 |
| + 2.3 | 98.93 | – 2.3 | 1.07 |
| + 2.4 | 99.18 | – 2.4 | 0.82 |
| + 2.5 | 99.38 | – 2.5 | 0.62 |
| + 2.6 | 99.53 | – 2.6 | 0.47 |
| + 2.7 | 99.65 | – 2.7 | 0.35 |
| + 2.8 | 99.74 | – 2.8 | 0.26 |
| + 2.9 | 99.81 | – 2.9 | 0.19 |
| + 3.0 | 99.87 | – 3.0 | 0.13 |

**Example:** For the network shown in figure, determine the probability of completing the project in 35 days. The time estimates for each activity are in term of days.