

# Optimal Dynamic Treatment Regimes and Partial Welfare Ordering

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# Dynamic (i.e., Adaptive) Treatment Regimes

Dynamic treatment regimes are seq's of treatment allocations...

- ▶ ...tailored to individual heterogeneity
- ▶ each period  $t$ , assignment rule  $\delta_t(\cdot)$  maps previous outcome (and covariates) onto a current allocation decision

$$\delta_t(y_{t-1}) \in \{0, 1\}$$

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Optimal dynamic treatment regime is a dynamic regime that maximizes counterfactual welfare

$$\delta^*(\cdot) = \arg \max_{\delta(\cdot) \in \mathcal{D}} W_\delta$$

# Identification of Optimal Dynamic Treatment Regime

$$\delta^*(\cdot) = \arg \max_{\delta(\cdot) \in \mathcal{D}} W_\delta$$

This paper investigates the possibility of identification of  $\delta^*(\cdot)$  when data are from...

- ▶ multi-stage experiments with possible non-compliance,  
or
- ▶ more generally, observational studies

## Motivating Example: Returns to Schooling & Training

$Y_2$  employed after program

$D_2$  receiving job training program

$Y_1$  employed before program

$D_1$  receiving high school diploma

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Let  $Y_1(d_1)$  and  $Y_2(d_2)$  be counterfactual employment status

Treatment effects:  $E[Y_1(1)] - E[Y_1(0)]$  and  $E[Y_2(1)] - E[Y_2(0)]$

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May be interested in the effects of sequence of treatments using  
 $Y_2(d_1, d_2)$

Then, e.g.,  $E[Y_2(1, 0)] - E[Y_2(0, 1)]$  or complementarity:

$E[Y_2(\textcolor{blue}{1}, 1)] - E[Y_2(\textcolor{blue}{1}, 0)]$  vs.  $E[Y_2(\textcolor{blue}{0}, 1)] - E[Y_2(\textcolor{blue}{0}, 0)]$

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thus, instead of  $d_2$ , consider  $\delta_2(Y_1(d_1))$  as hypothetical policy

And, instead of  $Y_2(d_1, d_2)$ , consider

$$Y_2(d_1, \delta_2(Y_1(d_1))) \equiv Y_2(\delta)$$

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Optimal **dynamic regime**: schedule  $\delta(\cdot) = (\delta_1, \delta_2(\cdot))$  of allocation rules that maximizes  $W_\delta = E[Y_2(\delta)]$  where

$$\delta_1 = d_1, \quad \delta_2(Y_1(\delta_1)) = d_2$$

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Policy implication of  $\delta^*(\cdot)$  s.t.  $\delta_2^*(1) = 0, \delta_2^*(0) = 1, \dots$

- ▶ more training resources to disadvantaged workers
- ▶ with  $\delta_1^*$  combined, interaction with earlier schooling

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- ▶ random assignment of job training can be  $Z_2$

Single IV can still be helpful esp. with short horizon

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# Sharp Partial Welfare Ordering in Numerical Exercise

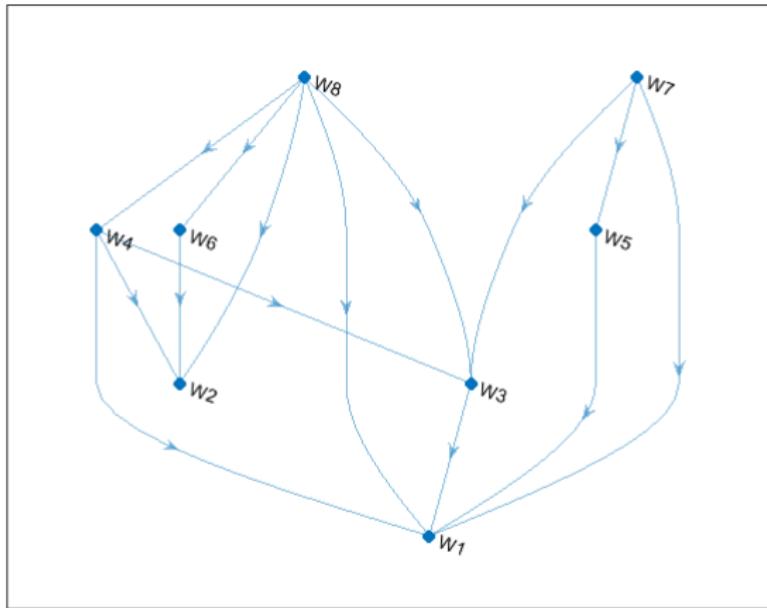


Figure: Partial Ordering as Directed Acyclic Graph

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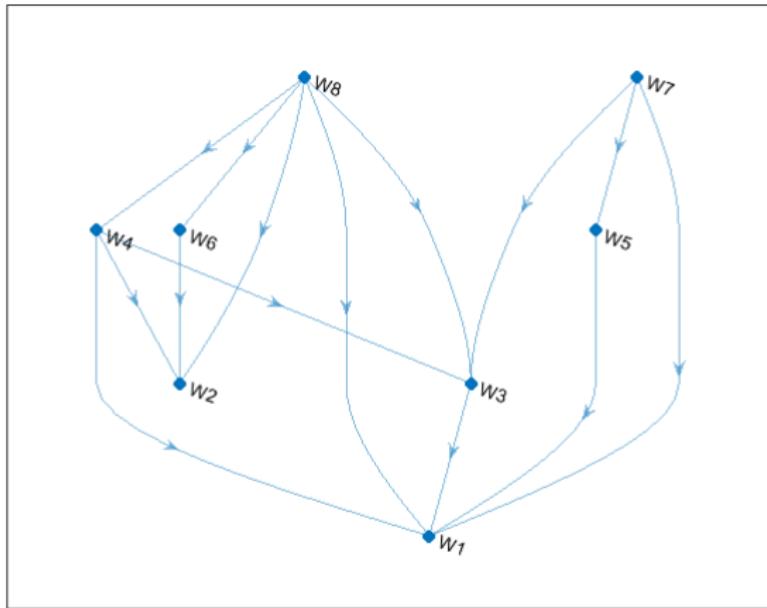


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3. We propose additional assumptions that tighten the ID'ed set
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4. We apply the method in policy analysis using schooling & post-school training as sequence of treatments

# Contribution 1: Treatment Endogeneity

Dynamic treatment regimes:

- ▶ Murphy et al. 01, Murphy 03, Robins 04, ...
- ▶ **sequential randomization**: “randomize treatment in the current period conditional on past treatments and outcomes”

Statistical treatment rules and policy learning:

- ▶ Manski 04, Hirano & Porter 09, Bhattacharya & Dupas 12, Stoye 12, Kitagawa & Tetenov 18, Sakaguchi 19, Athey & Wager 21, Mbakop & Tabord-Meehan 21, ...
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- ▶ versions of unconfoundedness assumption

Not plausible in experiments with partial compliance and many observational studies

This paper: relaxes sequential randomization

## Contribution 2: Partial ID in Multi-Period Settings

ID of optimal regime (as fcn of covariates) using IVs:

- ▶ Cui & Tchetgen Tchetgen 20, Qiu et al. 20, Han 21; Kasy 16, Pu & Zhang 2021
  - ▶ single-period setting
  - ▶ rely on independence of compliance type or rank preservation
  - ▶ or partial ID
- ▶ Han 20
  - ▶ dynamic treatment effects and optimal regime in multi-period setting
  - ▶ rely on existence of extra exogenous variables

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  - ▶ rely on existence of extra exogenous variables

This paper:

- ▶ **partial ID** of optimal adaptive regime and dynamic treatment effects

## Contribution 3: Linear Programming Approach to Partial ID

Calculating bounds using linear programming (LP)

- ▶ Balke & Pearl 97, Manski 07, Mogstad et al. 18, Kitamura & Stoye 19, Torgovitsky 19, Machado et al. 19, Kamat 19, Han & Yang 20,...

This paper:

- ▶ establish partial ordering via a set of LPs...
- ▶ that are governed by the same DGP...
- ▶ and characterize bounds on welfare gaps

Simple estimation and inference procedures for optimal regime

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Simple estimation and inference procedures for optimal regime

Broader applicability:

- ▶ rankings across different counterfactual scenarios

# Roadmap

I. Dynamic treatment regime and counterfactual welfare

II. Partial ID of optimal dynamic regime

- ▶ linear programming
- ▶ partial ordering and ID'ed set

III. Additional identifying assumptions

IV. Numerical illustration

V. Empirical application

VI. Inference

## I. Dynamic Treatment Regime and Counterfactual Welfare

# Dynamic (i.e., Adaptive) Treatment Regimes

Consider two-period case ( $T = 2$ ) only for simplicity

*Dynamic regime* is defined as

$$\delta(\cdot) \equiv (\delta_1, \delta_2(\cdot)) \in \mathcal{D}$$

where

$$\delta_1 = d_1 \in \{0, 1\}$$

$$\delta_2(y_1) = d_2 \in \{0, 1\}$$

- ▶ e.g.,  $y_t$  symptom,  $d_t$  medical treatment
- ▶ (stochastic rules in the paper)

## Dynamic (i.e., Adaptive) Treatment Regimes

Regime #	$\delta_1$	$\delta_2(1)$	$\delta_2(0)$
1	0	0	0
2	1	0	0
3	0	1	0
4	1	1	0
5	0	0	1
6	1	0	1
7	0	1	1
8	1	1	1

Table: Dynamic Regimes  $\delta(\cdot) \equiv (\delta_1, \delta_2(\cdot))$  when  $T = 2$

# Non-Adaptive Treatment Regimes

Regime #	$d_1$	$d_2$
1	0	0
2	1	0
3	0	1
4	1	1

Table: Non-Adaptive Regimes  $\mathbf{d} \equiv (d_1, d_2)$  when  $T = 2$

# Counterfactual Outcomes

Define potential outcome as a function of dynamic regime

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Potential outcomes with non-adaptive regime  $\mathbf{d} = (d_1, d_2)$ :

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Potential outcomes with non-adaptive regime  $\mathbf{d} = (d_1, d_2)$ :

$$\begin{aligned} Y_1(d_1) \\ Y_2(d_1, d_2) \end{aligned}$$

Potential outcomes with dynamic regime  $\delta(\cdot) = (\delta_1, \delta_2(\cdot))$ :

$$\begin{aligned} Y_1(\delta_1) &= Y_1(d_1) \\ Y_2(\delta) &= Y_2(\delta_1, \delta_2(Y_1(\delta_1))) \end{aligned}$$

# Welfare and Optimal Dynamic Regime

Let  $\mathbf{Y}(\delta) \equiv (Y_1(\delta_1), Y_2(\delta))$

Counterfactual welfare as linear funct'l of  $q_\delta(\mathbf{y}) \equiv \Pr[\mathbf{Y}(\delta(\cdot)) = \mathbf{y}]$

$$W_\delta \equiv f(q_\delta)$$

- ▶ e.g.,  $E[Y_T(\delta(\cdot))] = \Pr[Y_T(\delta(\cdot)) = 1]$  ► Details
- ▶ e.g.,  $\sum_{t=1}^T \{\omega_t E[Y_t(\delta^t(\cdot))]\}$  (less the cost of treatments)

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Optimal dynamic regime as

$$\delta^*(\cdot) = \arg \max_{\delta(\cdot) \in \mathcal{D}} W_\delta$$

## II. Partial ID of Optimal Dynamic Regime

## Observed Data

For  $t = 1, \dots, T$  on a finite horizon,

- ▶  $Y_t \in \{0, 1\}$  **outcome** at  $t$  (e.g., symptom indicator)
  - ▶ extension: continuous  $Y_t$  with discretized rule (later)
- ▶  $D_t \in \{0, 1\}$  **treatment** at  $t$  (e.g., medical treatment received)
- ▶  $Z_t \in \{0, 1\}$  **instrument** at  $t$  (e.g., medical treatment assigned)

Large  $N$  small  $T$  panel of  $(Y, D, Z)$

- ▶ (cross-sectional index  $i$  suppressed; covariates suppressed)
- ▶ more generally, e.g., single IV is allowed

# Partial ID of Optimal Dynamic Regime

Let  $\mathbf{Y}(\mathbf{d})$  be vector of  $Y_t(\mathbf{d}^t)$ 's and  $\mathbf{D}(z)$  be vector of  $D_t(z^t)$ 's.

## Assumption SX

$$Z_t \perp (\mathbf{Y}(\mathbf{d}), \mathbf{D}(z)) | Z^{t-1}.$$

- ▶ e.g., sequential randomized experiments, sequential fuzzy RDs

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ID'ed set as a subset of the discrete set  $\mathcal{D}$ :

$$\mathcal{D}^* \subset \mathcal{D}$$

# Partial ID of Optimal Dynamic Regime

As first step, establish *sharp partial ordering* of welfare  $W_\delta$  w.r.t.  $\delta(\cdot)$  based on  $(Y, D, Z)$

- ▶ cf. total ordering is needed for point ID of  $\delta^*(\cdot)$
- ▶ can only recover obs'ly equivalent total orderings

## Partial ID of Optimal Dynamic Regime

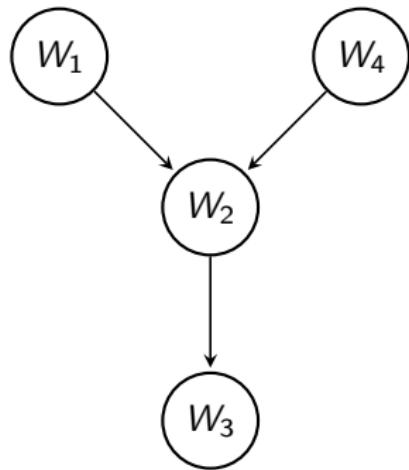
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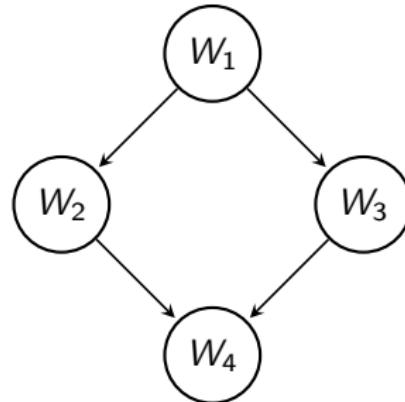
Partial ordering = a *directed acyclic graph* (DAG)

- ▶ parameter of independent interest
- ▶ *topological sorts* of DAG = obs'ly equivalent total orderings

## Partial Ordering of Welfare $W_k \equiv W_{\delta_k}$



(a)



(b)

Figure: Partially Ordered Sets as DAGs

# Sharp Partial Ordering of Welfare $W_\delta$

We want this partial ordering to be *sharp*

Definition (Sharp Partial Ordering, i.e., Sharp DAG)

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**Definition (Sharp Partial Ordering, i.e., Sharp DAG)**

In the DAG, no more edges can be created without additional assumptions.

To guarantee this, characterize sharp lower and upper bounds on

$$W_\delta - W_{\delta'}$$

as optima of linear programming

## Linear Programming for Bounds on Welfare Gap

For each  $\delta, \delta' \in \mathcal{D}$ , welfare gap (i.e., dynamic treatment effect) is

$$W_\delta - W_{\delta'} = (A_\delta - A_{\delta'})q$$

where  $q \in \mathcal{Q}$  is vector of latent distribution

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where  $q \in \mathcal{Q}$  is vector of latent distribution

Sharp lower and upper bounds via linear programming:

$$\begin{aligned} L_{\delta, \delta'} &= \min_{q \in \mathcal{Q}} (A_\delta - A_{\delta'})q \\ U_{\delta, \delta'} &= \max_{q \in \mathcal{Q}} (A_\delta - A_{\delta'})q \end{aligned} \quad \text{s.t. } Bq = p$$

- ▶  $A_\delta$ ,  $A_{\delta'}$ , and  $B$  are known to researcher
- ▶  $p$  is vector of data distribution for  $(Y, D, Z)$
- ▶  $q$  is unknown decision variable in standard simplex  $\mathcal{Q}$

# Sharp Partial Ordering and Identified Set

## Theorem

Suppose SX holds. (i) DAG is sharp with set of edges

$$\{(W_\delta, W_{\delta'}) : L_{\delta, \delta'} > 0 \text{ for } \delta \neq \delta'\}$$

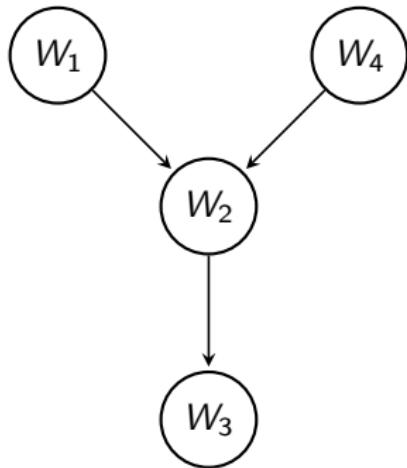
(ii)  $\mathcal{D}_p^*$  satisfies

$$\begin{aligned}\mathcal{D}_p^* &= \{\delta' : \nexists \delta \text{ such that } L_{\delta, \delta'} > 0 \text{ for } \delta \neq \delta'\} \\ &= \{\delta' : L_{\delta, \delta'} \leq 0 \text{ for all } \delta \text{ and } \delta \neq \delta'\}\end{aligned}$$

i.e., the rhs set is sharp

- ▶  $\mathcal{D}_p^*$  is the set of *maximal elements* associated with the DAG
- ▶ key insight: despite separate optimizations, DAG is governed by *common* latent dist  $q$ 's in  $\{q : Bq = p\}$  (i.e., that are obs'ly equivalent)

## Partial Ordering of Welfare $W_k \equiv W_{\delta_k}$



(a)  $\delta^*(\cdot)$  is partially ID'ed

$$\mathcal{D}_p^* = \{\delta_{\#1}, \delta_{\#4}\}$$

(b)  $\delta^*(\cdot)$  is point ID'ed

$$\mathcal{D}_p^* = \{\delta_{\#1}\}$$

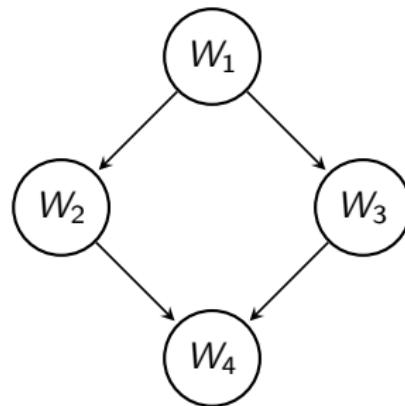


Figure: Partially Ordered Sets as DAGs

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Such  $\mathcal{D}_p^*$  still has implications for policy:

- (i) it recommends the planner to eliminate sub-optimal regimes from her options
- (ii) it warns about the lack of informativeness of data (e.g., even with experimental data)

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- (i) it recommends the planner to eliminate sub-optimal regimes from her options
- (ii) it warns about the lack of informativeness of data (e.g., even with experimental data)

The size of  $\mathcal{D}_p^*$  is related to...

- ▶ the strength of  $Z_t$  (i.e., the size of the complier group at  $t$ ),
- ▶ the strength of the dynamic treatment effects

### III. Additional Identifying Assumptions

## Additional Identifying Assumptions

Researchers are willing to impose more assumptions based on priors about agent's behavior or dynamics

- ▶ monotonicity/uniformity
  - ▶ Imbens & Angrist 94, Manski & Pepper 00
  - ▶ for each  $t$ , either  $Y_t(1) \geq Y_t(0)$  w.p.1 or  $Y_t(1) \leq Y_t(0)$  w.p.1.  
conditional on  $(\mathbf{Y}^{t-1}, \mathbf{D}^{t-1})$
- ▶ agent's learning
  - ▶ Assumption L
- ▶ Markovian structure
  - ▶ Assumption K
- ▶ positive state dependence, stationarity, etc.
  - ▶ Torgovitsky 19

Easy to incorporate within the linear programming

These assumptions tighten the ID'ed set  $\mathcal{D}_p^*$  by...

- ▶ reducing the dimension of the simplex  $\mathcal{Q}$

## IV. Numerical Illustration

## Numerical Illustration

For  $T = 2$ , DGP is

$$D_{i1} = 1\{\pi_1 Z_{i1} + \alpha_i + \nu_{i1} \geq 0\}$$

$$Y_{i1} = 1\{\mu_1 D_{i1} + \alpha_i + e_{i1} \geq 0\}$$

$$D_{i2} = 1\{\pi_{21} Y_{i1} + \pi_{22} D_{i1} + \pi_{23} Z_{i2} + \alpha_i + \nu_{i2} \geq 0\}$$

$$Y_{i2} = 1\{\mu_{21} Y_{i1} + \mu_{22} D_{i2} + \alpha_i + e_{i2} \geq 0\}$$

and

$$W_\delta = E[Y_2(\delta)]$$

Calculate  $[L_{\delta_k, \delta_l}, U_{\delta_k, \delta_l}]$  for  $W_{\delta_k} - W_{\delta_l}$  for all pairs  $k, l \in \{1, \dots, 8\}$

We make  $\binom{8}{2} = 28$  comparisons, i.e.,  $28 \times 2$  linear programs

# Bounds on Welfare Gaps $W_{\delta_k} - W_{\delta_l}$

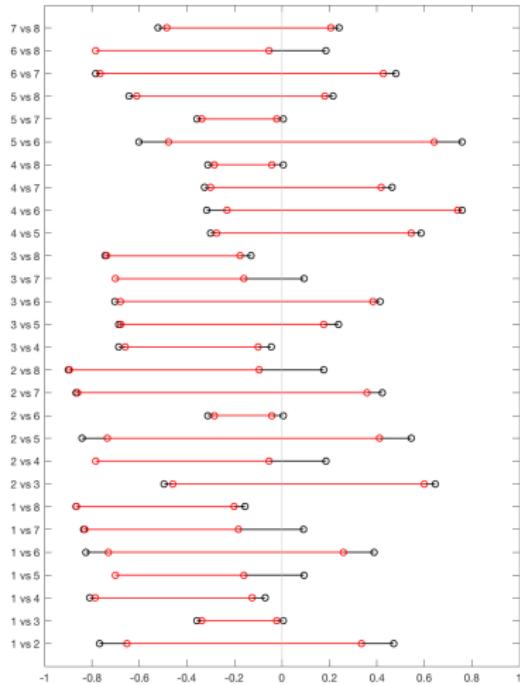


Figure: Sharp Bounds on Welfare Gaps (red: under M2)

# Sharp Partial Welfare Ordering

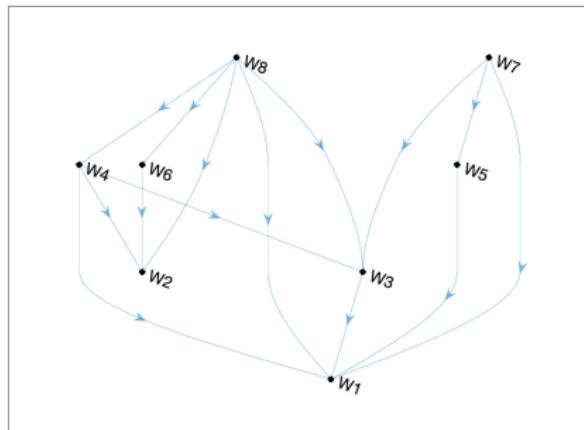


Figure: Partial Ordering as DAG and ID'ed Set for  $\delta^*$  (under M2)

# Sharp Partial Welfare Ordering

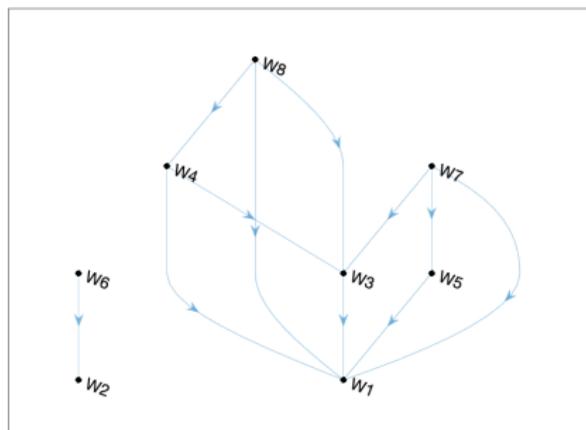


Figure: Partial Ordering as DAG with Only  $Z_1$  (under M2)

## V. Empirical Application: Returns to Schooling and Training

## Empirical Application: Returns to Schooling and Training

Individuals who face “barriers to employment”

$Y_2$  above median 30-mo earnings

$D_2$  receiving job training program

$Z_2$  random assignment of the program

$Y_1$  above 80th pctile pre-program earnings

$D_1$  receiving high school diploma (or GED)

$Z_1$  number of schools per sq mile (e.g., Neal 97)

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Data: JTPA (e.g., [Abadie, Angrist & Imbens 02](#), [Kitagawa & Tetenov 18](#))  
+ NCES + US Census

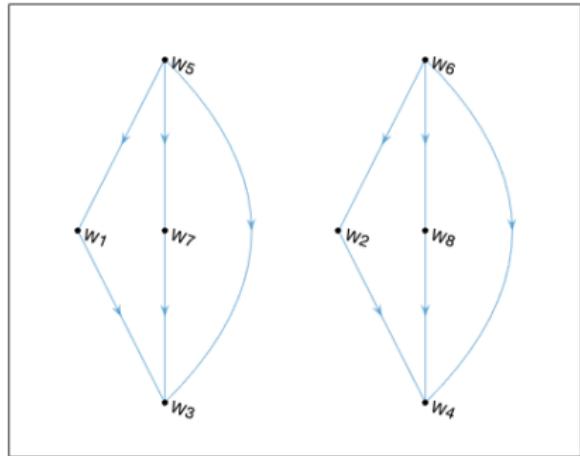
## Estimation

Estimation of DAG and  $\mathcal{D}_p^*$  is straightforward

- ▶ replace data distribution  $p$  in LP with sample frequencies  $\hat{p}$ , a vector of

$$\hat{p}_{\mathbf{y}, \mathbf{d} | \mathbf{z}} = \sum_{i=1}^N \mathbb{1}\{\mathbf{Y}_i = \mathbf{y}, \mathbf{D}_i = \mathbf{d}, \mathbf{Z}_i = \mathbf{z}\} / \sum_{i=1}^N \mathbb{1}\{\mathbf{Z}_i = \mathbf{z}\}$$

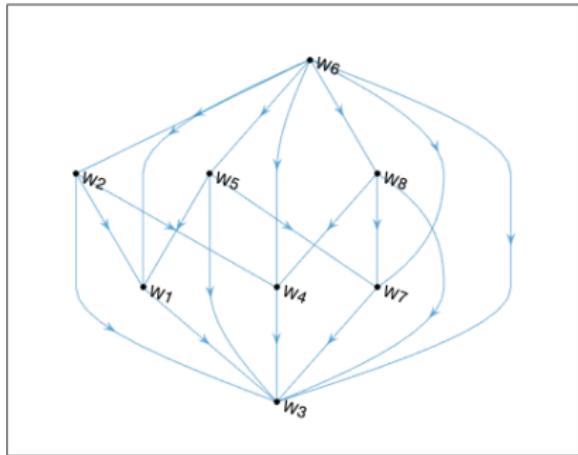
# Policy Analysis with Schooling and Training



Regime #	$\delta_1$	$\delta_2(1)$	$\delta_2(0)$
1	0	0	0
2	1	0	0
3	0	1	0
4	1	1	0
5	0	0	1
6	1	0	1
7	0	1	1
8	1	1	1

Figure: DAG of  $W_\delta = E[Y_2(\delta)]$  and Est'ed Set for  $\delta^*$  (under M2)

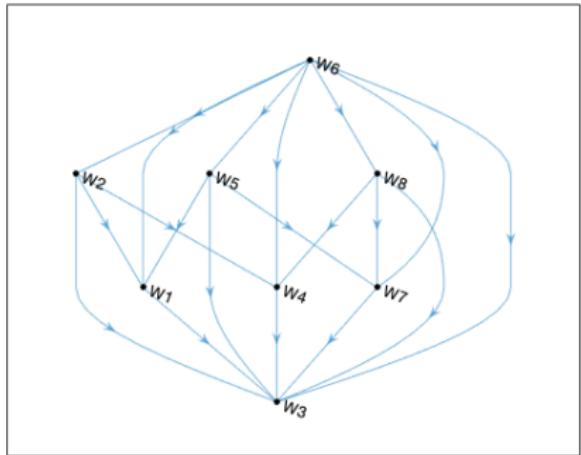
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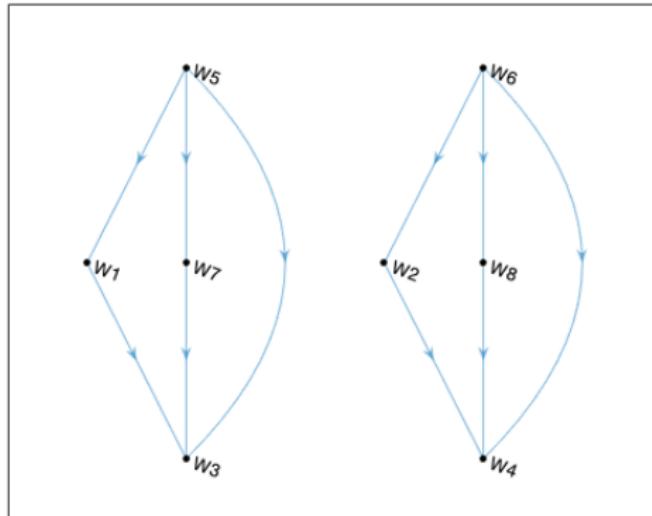


Figure: Partial Ordering with only  $Z_2$  (under M2)

## VI. Inference

## Inference

For the inference on  $\delta^*(\cdot)$ , we construct confidence set for  $\mathcal{D}_p^*$

- ▶ by seq of hypothesis tests (Hansen, Lunde & Nason 11)
  - ▶ to eliminate regimes that are significantly inferior to others
  - ▶ null hypotheses in terms of multiple ineq's as functions of  $p$ 
    - ▶ e.g., Hansen 05, Andrews & Soares 10,...
  - ▶ no need to solve LPs for every bootstrap repetition
    - ▶ by using strong duality and vertex enumeration
- ▶ also useful for specification tests of (less palatable) identifying assumptions

## Inference

Recall  $W_\delta - W_{\delta'} = (A_\delta - A_{\delta'})q$  and

$$\begin{aligned} L_{\delta, \delta'} &= \min_{q \in \mathcal{Q}} (A_\delta - A_{\delta'})q \\ U_{\delta, \delta'} &= \max_{q \in \mathcal{Q}} (A_\delta - A_{\delta'})q \end{aligned} \quad \text{s.t. } Bq = p$$

Dual programs with vertex enumeration (e.g., Avis & Fukuda 92):

$$L_{\delta, \delta'} = \max_{\lambda \in \Lambda_{\delta, \delta'}} -\tilde{p}'\lambda$$

$$U_{\delta, \delta'} = \min_{\lambda \in \tilde{\Lambda}_{\delta, \delta'}} \tilde{p}'\lambda$$

Null hypothesis for sequence of tests:

$$H_{0, \tilde{\mathcal{D}}} : L_{\delta, \delta'} \leq 0 \leq U_{\delta, \delta'} \quad \forall \delta, \delta' \in \tilde{\mathcal{D}}$$

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Null hypothesis for sequence of tests:

$$H_{0, \tilde{\mathcal{D}}} : \tilde{p}'\lambda \geq 0 \quad \forall \lambda \in \bigcup_{\delta, \delta' \in \tilde{\mathcal{D}}} (\Lambda_{\delta, \delta'} \cup \tilde{\Lambda}_{\delta, \delta'})$$

# Inference

Let  $\hat{\mathcal{D}}_{CS}$  be the confidence set for  $\mathcal{D}_p^*$

## Algorithm (Constructing $\hat{\mathcal{D}}_{CS}$ )

*Step 0.* Initially set  $\tilde{\mathcal{D}} = \mathcal{D}$ .

*Step 1.* Test  $H_{0,\tilde{\mathcal{D}}}$  at level  $\alpha$ .

*Step 2.* If  $H_{0,\tilde{\mathcal{D}}}$  is not rejected, define  $\hat{\mathcal{D}}_{CS} = \tilde{\mathcal{D}}$ ;  
otherwise eliminate a regime  $\delta^-$  from  $\tilde{\mathcal{D}}$  and repeat from Step 1.

- ▶ in Step 1,  $T_{\tilde{\mathcal{D}}} \equiv \min_{\delta, \delta' \in \tilde{\mathcal{D}}} t_{\delta, \delta'}$  where  
 $t_{\delta, \delta'} \equiv \min_{\lambda \in \Lambda_{\delta, \delta'} \cup \tilde{\Lambda}_{\delta, \delta'}} t_{\lambda}$  with standard  $t$ -statistic  $t_{\lambda}$ 
  - ▶ distribution of  $T_{\tilde{\mathcal{D}}}$  can be estimated using bootstrap on  $p$
- ▶ in Step 2,  $\delta^- \equiv \arg \min_{\delta \in \tilde{\mathcal{D}}} \min_{\delta' \in \tilde{\mathcal{D}}} t_{\delta, \delta'}$ .

# Inference

## Assumption CS

For any  $\tilde{\mathcal{D}}$ , (i)  $\limsup_{n \rightarrow \infty} \Pr[\phi_{\tilde{\mathcal{D}}} = 1 | H_{0,\tilde{\mathcal{D}}}] \leq \alpha$ ,  
(ii)  $\lim_{n \rightarrow \infty} \Pr[\phi_{\tilde{\mathcal{D}}} = 1 | H_{A,\tilde{\mathcal{D}}}] = 1$ , and  
(iii)  $\lim_{n \rightarrow \infty} \Pr[\delta_{\tilde{\mathcal{D}}}^-(\cdot) \in \mathcal{D}_p^* | H_{A,\tilde{\mathcal{D}}}] = 0$ .

## Proposition

Under Assumption CS, it satisfies that

$$\liminf_{n \rightarrow \infty} \Pr[\mathcal{D}_p^* \subset \hat{\mathcal{D}}_{CS}] \geq 1 - \alpha$$

and  $\lim_{n \rightarrow \infty} \Pr[\delta(\cdot) \in \hat{\mathcal{D}}_{CS}] = 0$  for all  $\delta(\cdot) \notin \mathcal{D}_p^*$

## Extension: Continuous Outcomes

This paper's analysis can be extended to the case of continuous  $Y_t$

But the cost of incremental customization with  $Y_{t-1}$  can be high

- ▶ thus planner may want to employ a threshold-crossing rule:

$$\delta_t(1\{y_{t-1} \geq \gamma_{t-1}\}) \in \{0, 1\}$$

Then a similar analysis can be done for optimal regime  $(\delta^*(\cdot), \gamma^*)$

With continuous  $Y_t$ , two challenges in LP:

- ▶  $q$  is infinite dimensional  $\implies$  approximate using Bernstein polynomials
- ▶ continuum of constraints  $\implies$  use mean absolute deviation of constraints
- ▶ Han & Yang 22

## VI. Conclusions

## Concluding Remarks

Propose a partial ID framework for optimal dynamic treatment regimes and welfares

- ▶ allowing for observational data

Sharp partial welfare ordering and ID'ed set for optimal regime

- ▶ via a set of linear programs

Applicability:

- ▶ e.g., when establishing rankings across multiple treatments or counterfactual policies

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Follow-ups:

1. inference on welfare with selected (set-ID'ed) regime
2. treatment allocation with distributional welfare

*Thank You*

# Contributions to Diversity, Equity and Inclusion

## At University of Texas–Austin

### 1. research collaborations

- ▶ ranks of all levels (from students to full professors)
- ▶ females and males
- ▶ diverse ethnic backgrounds (North and South Americans, Europeans, East Asians, Indian, Russian)
- ▶ various disciplines (econometricians, economists, computer scientists, and designers)

### 2. student advising

- ▶ similarly diverse group

### 3. mentoring

- ▶ e.g., Asian Desi Pacific Islander American Collective (ADPAC)  
Tea Time Discussions

# Contributions to Diversity, Equity and Inclusion

## At University of Bristol

—School of Economics received Bronze Award 2022 in recognition of commitment to gender equality

4. promotions and hiring
  - ▶ supporting ethnic minorities and a researcher with disabled kid
5. seminar and conference organization
  - ▶ balance of gender, opportunity to PhD students

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## On “Mars”

—Hawaii Space Exploration Analog & Simulation, Mission VI, 2018

- ▶ commander of most diverse crew in the history of the simulation
- ▶ managing the crew to minimize conflicts and maximize productivity in extreme environments

# Simulated Mars Mission



# Simulated Mars Mission



# Distribution of Counterfactual Outcome

With  $T = 2$ ,

$$\begin{aligned} & \Pr[Y_2(\delta) = 1] \\ &= \sum_{y_1 \in \{0,1\}} \Pr[Y_2(\delta_1, \delta_2(Y_1(\delta_1))) = 1 | Y_1(\delta_1) = y_1] \Pr[Y_1(\delta_1) = y_1] \end{aligned}$$

- ▶ for example, Regime #4 yields

$$\begin{aligned} \Pr[Y_2(\delta_{\#4}) = 1] &= P[Y_1(1) = 1, Y_2(1, 1) = 1] \\ &\quad + P[Y_1(1) = 0, Y_2(1, 0) = 1] \end{aligned}$$

◀ Return

# Monotonicity/Uniformity in $D_t$

## Assumption M1

Conditional on  $(\mathbf{Y}^{t-1}, \mathbf{D}^{t-1}, \mathbf{Z}^{t-1})$ , either

$$D_t(\mathbf{Z}^{t-1}, 1) \geq D_t(\mathbf{Z}^{t-1}, 0) \text{ w.p.1 or}$$

$$D_t(\mathbf{Z}^{t-1}, 1) \leq D_t(\mathbf{Z}^{t-1}, 0) \text{ w.p.1.}$$

Assumption M1 imposes that there is no defying (complying) behavior in the decision  $D_t$  conditional on  $(\mathbf{Y}^{t-1}, \mathbf{D}^{t-1}, \mathbf{Z}^{t-1})$

- ▶ without conditional on  $(\mathbf{Y}^{t-1}, \mathbf{D}^{t-1}, \mathbf{Z}^{t-1})$ , general non-uniform pattern of  $\mathbf{Z}^t$  influencing  $\mathbf{D}^t$

By extending Vytlacil 02, M1 is implied by

$$D_t = 1\{\pi_t(\mathbf{Y}^{t-1}, \mathbf{D}^{t-1}, \mathbf{Z}^t) \geq \nu_t\}$$

◀ Return

## Monotonicity/Uniformity in $Y_t$

### Assumption M2

M1 holds, and conditional on  $(Y^{t-1}, D^{t-1}, Z^{t-1})$ , either

$$Y_t(D^{t-1}, 1) \geq Y_t(D^{t-1}, 0) \text{ w.p.1 or}$$

$$Y_t(D^{t-1}, 1) \leq Y_t(D^{t-1}, 0) \text{ w.p.1.}$$

Assumption M2 implicitly imposes rank similarity

- ▶ without conditional on  $(Y^{t-1}, D^{t-1}, Z^{t-1})$ , general non-uniform pattern of  $D^t$  influencing  $Y^t$

Assumption M2 (and M1) does not assume the direction of monotonicity

M2 is implied by

$$Y_t = 1\{\mu_t(Y^{t-1}, D^t) \geq \varepsilon_t\}$$

$$D_t = 1\{\pi_t(Y^{t-1}, D^{t-1}, Z^t) \geq \nu_t\}$$

# Agent's Learning

## Assumption L

$D_t(\mathbf{y}^{t-1}, \mathbf{d}^{t-1}, \mathbf{z}^t) \geq D_t(\tilde{\mathbf{y}}^{t-1}, \tilde{\mathbf{d}}^{t-1}, \mathbf{z}^t)$  w.p.1 for  $(\mathbf{y}^{t-1}, \mathbf{d}^{t-1})$  and  $(\tilde{\mathbf{y}}^{t-1}, \tilde{\mathbf{d}}^{t-1})$  s.t.  $\|\mathbf{y}^{t-1} - \mathbf{d}^{t-1}\| < \|\tilde{\mathbf{y}}^{t-1} - \tilde{\mathbf{d}}^{t-1}\|$  (long memory) or  $y_{t-1} - d_{t-1} < \tilde{y}_{t-1} - \tilde{d}_{t-1}$  (short memory).

Assumption L assumes agents have the ability to revise his next period's decision based on his memory

- ▶ e.g., consider  $D_2(y_1, d_1)$
- ▶ agent who would switch his decision had he experienced  $y_1 = 0$  after  $d_1 = 1$ , i.e.,  $D_2(0, 1) = 0$ , would remain to take treatment had he experienced  $y_1 = 1$ , i.e.,  $D_2(1, 1) = 1$
- ▶ more importantly, if  $D_2(0, 1) = 1$ , it should only because of unobserved preference, *not* because he cannot learn from the past, i.e.,  $D_2(1, 1) = 0$  cannot happen

◀ Return

# Markovian Structure

## Assumption K

$$Y_t | (\mathbf{Y}^{t-1}, \mathbf{D}^t) \stackrel{d}{=} Y_t | (Y_{t-1}, D_t) \text{ and}$$
$$D_t | (\mathbf{Y}^{t-1}, \mathbf{D}^{t-1}, \mathbf{Z}^t) \stackrel{d}{=} D_t | (Y_{t-1}, D_{t-1}, Z_t).$$

In terms of the triangular model under M2, Assumption K implies

$$Y_t = 1\{\mu_t(Y_{t-1}, D_t) \geq \varepsilon_t\}$$

$$D_t = 1\{\pi_t(Y_{t-1}, D_{t-1}, Z_t) \geq \nu_t\}$$

- ▶ a familiar structure of dynamic discrete choice models in the literature

[◀ Return](#)