Université libre de Bruxelles

INFO-F-409 - Learning Dynamics

Assignment One

Raymond Lochner - 000443637

raymond.lochner@ulb.ac.be

Contents

October 20, 2017

1 The Hawk-Dove game

The Hawk-Dove game was first formulated by John Maynard Smith and Georg Prince in 1973 [1]. The aim of the game is to gain a better understanding of conflicts in the animal kingdom. It consits of two players {Player One, Player Two} who have each two actions {Hawk, Dove}.The resulting payoff matrix can bee seen in Table 1 where:

- V = fitness value of winning resources in fight
- D = fitness costs of injury
- T = fitness costs of wasting time

and we assume that V,D,T $\geq$ 0.

Table 1: Hawk-Dove Payoff Matrix

		Player Two	
		Hawk	Dove
Player One	Hawk	(V-D)/2 (V-D)/2	0 V
	Dove	V 0	V/2-T V/2-T

In a mixed strategy game, we consider each player performing his action with a certain probability p or q, which results in the following payoff matrix displayed in Table 2.

Table 2: Hawk-Dove Probability Payoff Matrix

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	(V-D)/2 (V-D)/2	0 V
	P(Dove)=(1-p)	V 0	V/2-T V/2-T

With this table we are able to calculate the payoff of the actions Hawk and Dove for Player One:

Player One's expected payoff for the strategy Hawk(p) is:

$$q \times \frac{V-D}{2} + (1-q) \times V$$

Player One's expected payoff for the strategy Dove(1-p) is:

$$q \times 0 + (1-q) \times (\frac{V}{2} - T) = (1-q) \times (\frac{V}{2} - T)$$

The best response set is {Hawk} if:

$$q \times \frac{V-D}{2} + (1-q) \times V < (1-q) \times (\frac{V}{2} - T)$$

The best response set is {Dove} if:

$$q \times \frac{V-D}{2} + (1-q) \times V > (1-q) \times (\frac{V}{2} - T)$$

All the players mixed strategies are best responses if:

$$q \times \frac{V-D}{2} + (1-q) \times V = (1-q) \times (\frac{V}{2} - T)$$

As we are dealing with unknown variables, let's rewrite the equation for q:

$$q \times \frac{V-D}{2} + (1-q) \times V = (1-q) \times (\frac{V}{2} - T)$$

$$\frac{qV}{2} - \frac{qD}{2} + V - qV = \frac{V}{2} - T - \frac{qV}{2} + qT$$

$$-\frac{qD}{2} + V = \frac{V}{2} - T + qT$$

$$\frac{V}{2} + T = \frac{qD}{2} + qT$$

$$\frac{V}{2} + T = q(\frac{D}{2} + T)$$

$$V + 2T = q(D + 2T)$$

$$q = \frac{V+2T}{D+2T}$$

To find any pure or mixed strategy of this scenario, we have to look at different values for the variables V,D and T. In this, we must respect the following constraints:

$$0 \leq q \leq 1 \quad \text{and} \quad 0 \leq \frac{V+2T}{D+2T} \leq 1 \quad \text{and} \quad V,T,D \geq 0$$

1.1 V+2T = 0

This is only possible if $V = 0$ and $T = 0$ which gives $q = \frac{0}{D}$. We have to consider two cases: $D = 0$ and $D > 0$

1.1.1 D=0

Any pure or mixed strategy is a best response. This yields the mixed strategy NE $(p, q) \in \{(0 \leq p \leq 1), (0 \leq q \leq 1)\}$.

Table 3: V=T=0, D>0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	0, 0	0, 0
	P(Dove)=(1-p)	0, 0	0, 0

1.1.2 D>0

We have three pure strategy NE: {Dove,Hawk},{Hawk,Hawk} and {Hawk,Dove}. Solving for q gives $q = 0$ which does not yield a mixed strategy NE: $(p, q) = (0, 0)$.

Table 4: V=T=0, D>0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	-D/2 -D/2	0 0
	P(Dove)=(1-p)	0 0	0 0

1.2 T=0

With $T = 0$ we get the following payoff matrix:

Table 5: T=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	(V-D)/2 (V-D)/2	0 V
	P(Dove)=(1-p)	V 0	V/2 V/2

We have the following scenarios: $V > D$,$V < D$ and $V = D$.

1.2.1 V=D

If we set $V = D$, we eliminate D from the payoff matrix:

Table 6: T=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	0 0	0 V
	P(Dove)=(1-p)	V 0	V/2 V/2

With three pure strategy NE's: {Dove,Hawk},{Hawk,Hawk} and {Hawk,Dove}. To find mixed strategies, we need the expected payoff for both strategies:

Player One's expected payoff for the strategy Hawk(p) is:

$$q \times 0 + (1 - q) \times V = (1 - q) \times V$$

Player One's expected payoff for the strategy Dove(1-p) is:

$$q \times 0 + (1 - q) \times V/2 = (1 - q) \times (V/2)$$

All the players mixed strategies are best responses if:

$$(1 - q) \times V = (1 - q) \times (V/2)$$

1.2.2 V>D

Not viable as

1.2.3 V<D

2 Which social dilemma?

Player A is confronted with one of three social dilemma's - the corresponding payoff matrix is shown in tables 7, 8 and 9. The player has to decide whether to cooperate (C) or to defect (D) without knowing which game he is actually facing. Each dilemma has the same probability 1/3 of being played. Opponent B knows the game.

Table 7: Prisonners dilemma

	C	D
C	2 / 2	0 / 5
D	5 / 0	1 / 1

Table 8: Stag-Hunt game

	C	D
C	5 / 5	0 / 2
D	2 / 0	1 / 1

Table 9: Snowdrift game

	C	D
C	2 / 2	1 / 5
D	5 / 1	0 / 0

with this information we can calculate the expected payoff for player A for every possible strategy of Player B - Table 10.

Table 10: Expected payoff for Player A

		Player B							
		(C,C,C)	(C,C,D)	(C,D,C)	(C,D,D)	(D,C,C)	(D,C,D)	(D,D,C	(D,D,D)
Player A	C	9/3	8/3	4/3	3/3	7/3	6/3	2/3	1/3
	D	12/3	7/3	11/3	6/3	8/3	3/3	7/3	2/3

From this table we can select the best response for Player A for each strategy of Player B - cells marked red in Table 11.

Table 11: Best responses for Player A

		Player B							
		(C,C,C)	(C,C,D)	(C,D,C)	(C,D,D)	(D,C,C)	(D,C,D)	(D,D,C	(D,D,D)
Player A	C	9/3	**8/3**	4/3	3/3	7/3	**6/3**	2/3	1/3
	D	**12/3**	7/3	**11/3**	**6/3**	**8/3**	3/3	**7/3**	**2/3**

Now we have to determine the best responses of Player B against Player A of the three different strategies - marked by green cells in Tables 12, 13 and 14.

Table 12: Prisonners dilemma

	C	D
C	2	**5**
D	0	**1**

Table 13: Stag-Hunt game

	C	D
C	**5**	2
D	0	**1**

Table 14: Snowdrift game

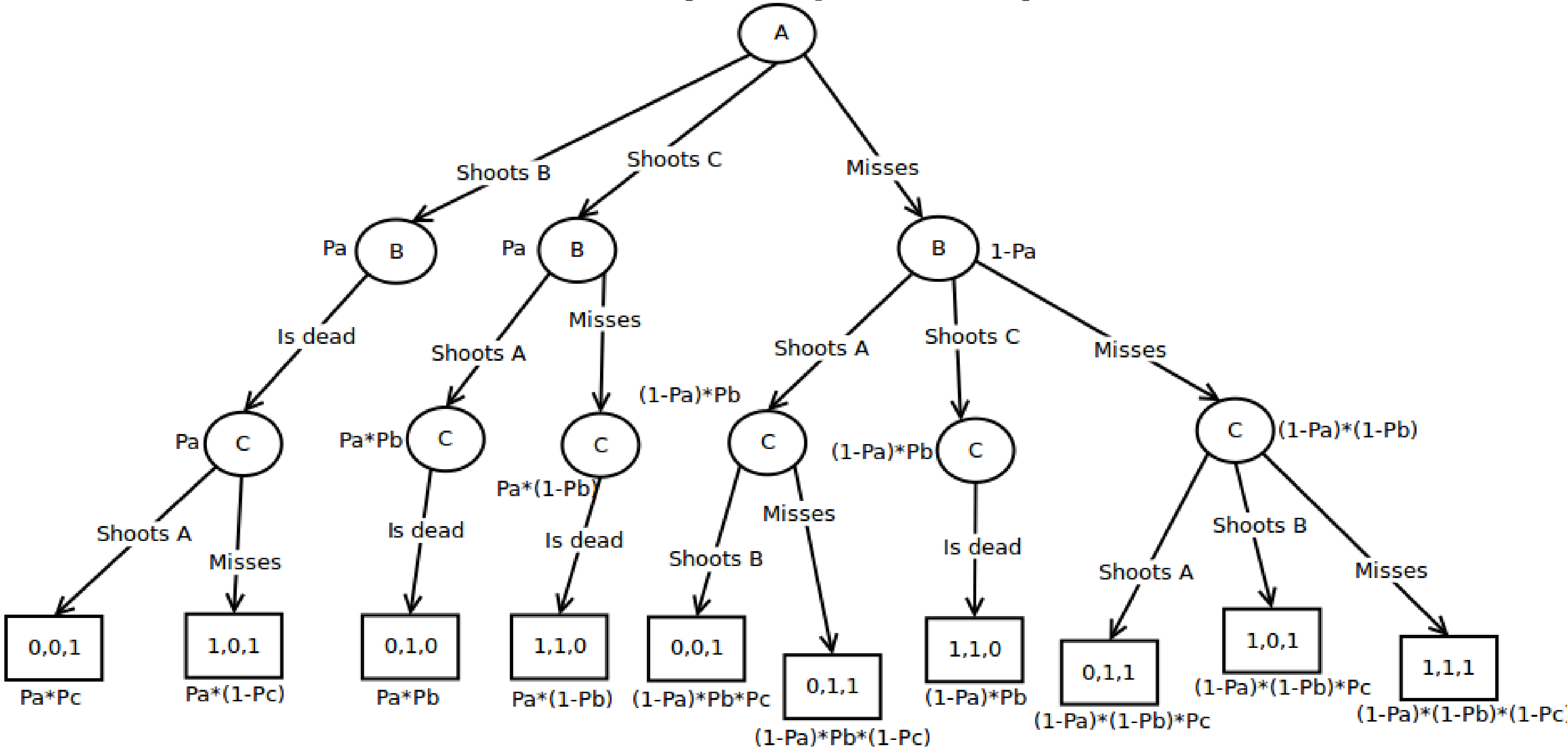
	C	D
C	2	**5**
D	**1**	0

The pure strategy Nash Equilibria can now be determined by matching these results. We find two Nash Equilibria at {C,(D,C,D)} and {D,(D,D,C)}.

3 Sequential truel

This scenario considers three persons A,B and C, each of whom has a gun with a single bullet. If alive, each person may shoot at any surviving person. The order in which the scenario is played out is A, then B and then C. The probability that player i hits their target is denoted by p_i where $0 \leq p_i \leq 1$. Every player wishes to optimize her probability of survival. For this exercise we further assume that a player has to target another living player and is not allowed to miss a shoot consciously. The resulting diagram of this game is shown in Figure 1.

Figure 1: Sequential truel diagram



The Diagram shows all possible actions of all players with all possible outcomes. The lowest level leaf indicates which player is still alive and with which probability - [0,0,1] indicates Player A and B being dead while Player C is alive with a probability of $p_a \times p_c$.

References

[1] J. Maynard Smith and G. R. Price. The logic of animal conflict. *Nature*, 246(5427):15–18, 1973.