Université libre de Bruxelles

INFO-F-409 - Learning Dynamics

Assignment Two

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Preliminary information

Each game configuration was being simulated 100 times to receive a good picture of the various possible outcomes. Rounds were played until convergence was certain. For the visualizations:

- ullet Red signifies the action cooperation
- Blue signifies the action defection

The graphic displays one specific game, whereas the cooperation graph shows information of all games combined.

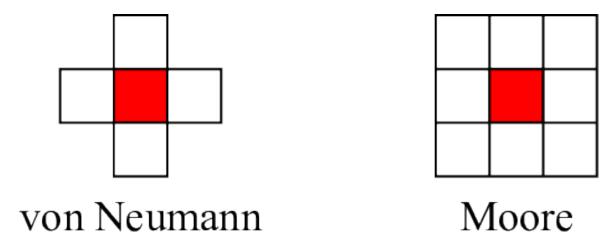


Figure 1: Two Neighborhood types

The tested games are:

- Weak Prisonners Dilemma (T=10, R=7, P=S=0)
- Snowdrift Game (T=12, R=7, P=0, S=3)

1 Part One - Spatial Prisoners Dilemma

1.1 Moore Neighborhood

1.1.1 4x4

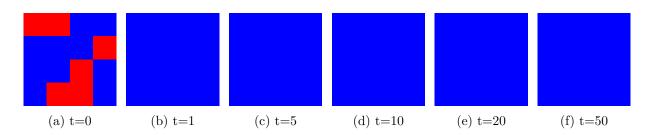
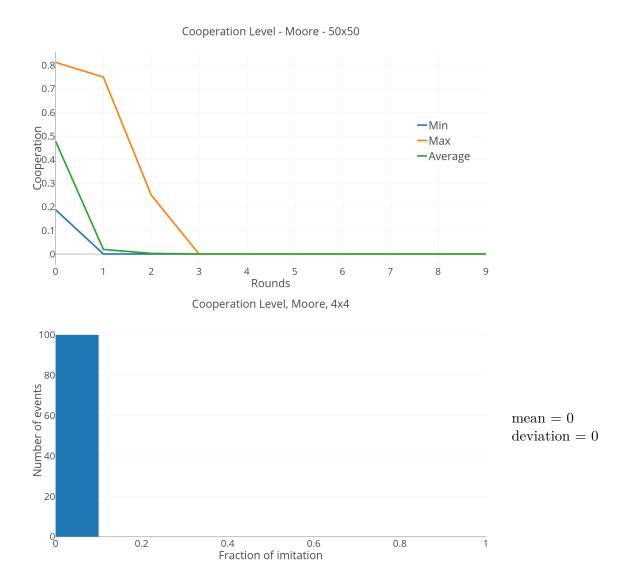


Figure 2: Prisoners Dilemma, Moore, 4x4



From simulating 100 runs we observe that all converge to the pure strategy of defecting after 3 rounds. Nevertheless, it is however possible that a 4x4 configuration converges to a total cooperative field, but it requires that we have a sub-matrix of 2x2 with only cooperators and all other players being defectors. This did obviously not happen during one of the simulations.

1.1.2 8x8

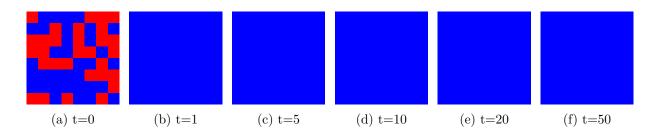
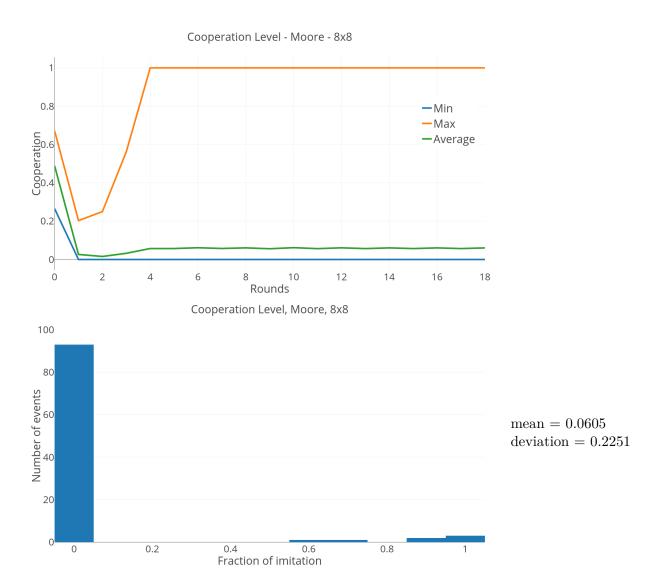


Figure 4: Prisoners Dilemma, Moore, 8x8



Increasing the lattice to 8x8, we get our first pure cooperation and mixed strategy fields. The configuration converges after 4 rounds, but most fields end up as being pure defector lattices.

1.1.3 12x12

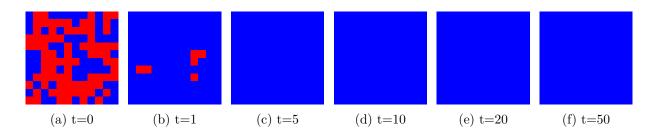
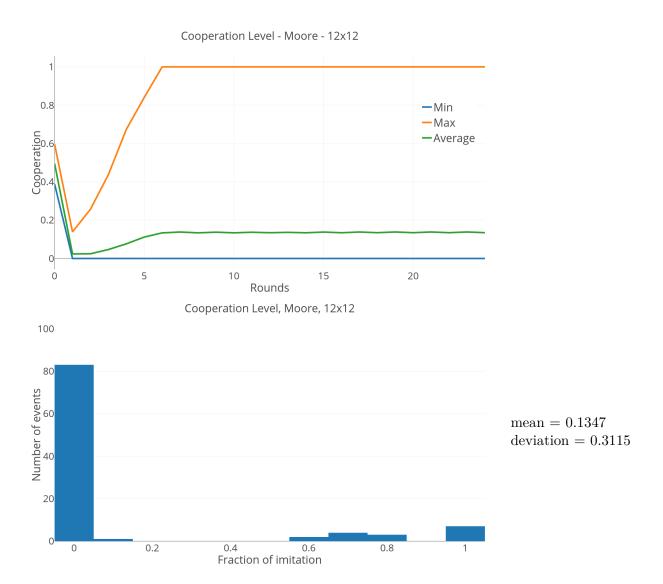


Figure 6: Prisoners Dilemma, Moore, 12x12



A lattice configuration of 12x12 increases the chance slightly that the whole lattice does not end up being only defectors. More mixed strategy lattices at 0.7 and some more pure strategy cooperation lattices. Convergence after 7 rounds.

1.1.4 20x20

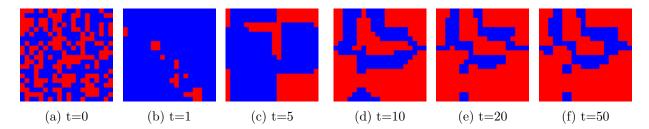
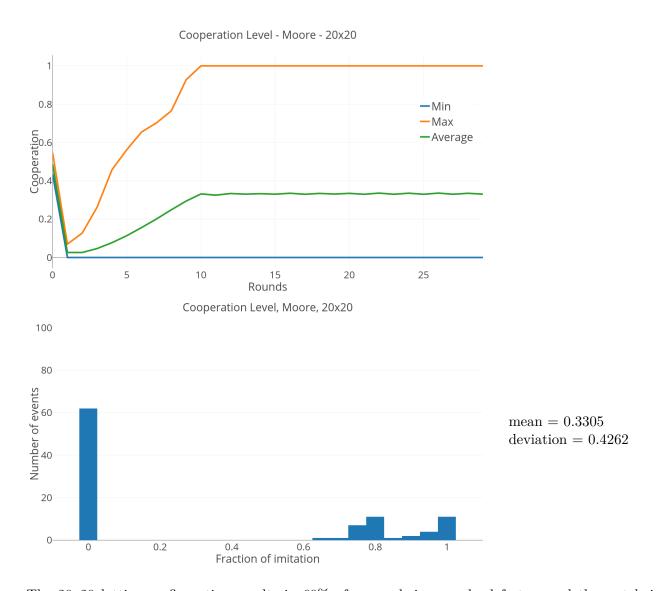


Figure 8: Prisoners Dilemma, Moore, 20x20



The 20x20 lattice configuration results in 60% of games being purely defectors and the rest being either purely cooperative or mostly cooperative. The graphical representation shows the creation of cooperation blocks after time, with defector *rivers* in between.

$1.1.5 \quad 50 \times 50$

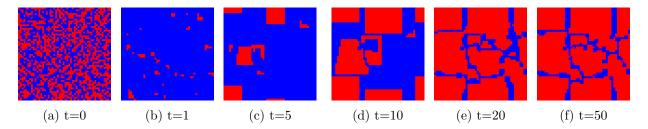
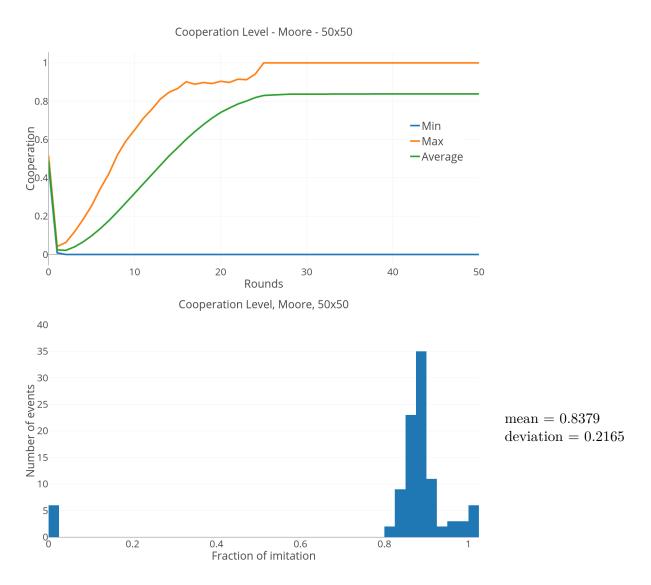


Figure 10: Prisoners Dilemma, Moore, 50x50



A 50x50 lattice configuration results in a highly cooperative environment about 94% of the time. Convergence after 25 rounds. Looking at the graphical representation we can see that clusters of cooperation with rivers of defection are being formed. The distribution starts to look like a normal distribution.

1.2 Von Neumann Neighborhood

$1.2.1 \quad 50 \text{x} 50$

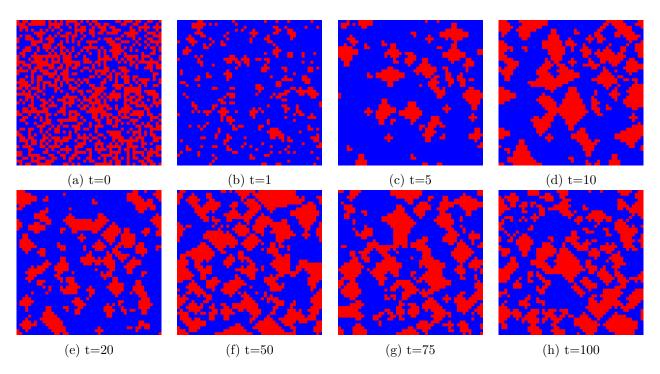
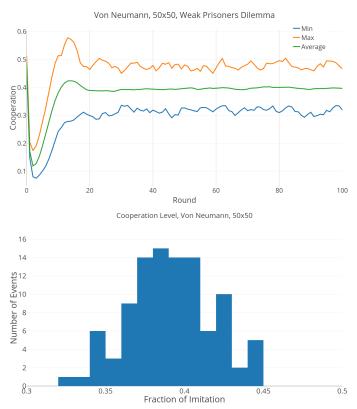


Figure 12: Prisoners Dilemma, Von Neumann, 50x50



Changing the neighborhood to the Von Neumann mode, we get a mean = 0.3914 and deviation = 0.0263. The mean with this neighborhood type is about half as the mean from a Moore neighborhood game with the same lattice size. The deviation is however much smaller.

The configuration converges after 20 rounds. Looking at the differences of the graphical representation, using the Von Neumann neighborhood results in non stationary clusters as we have with a Moore neighborhood. The cooperation level over time also changes, which we can observe in the curve. It does not drop too much at the first few rounds and then quickly converges to $\sim 0.4\%$ with the maximum and minimum level not being too far away which is why the deviation is much smaller compared to the Moore neighborhood.

1.3 Analysing the results

We can now have a look at the results of the experiments and investigate their differences.

Table 1: Combined Experiment Results

			Von Neumann			
Lattice	4x4	8x8	12x12	20x20	50x50	50x50
Mean	0	0.0605	0.1347	0.3305	0.8379	0.3914
Deviation	0	0.2251	0.3115	0.4262	0.2165	0.0263
Convergence	3	4	7	10	25	20

 qsdqsd

2 Part Two - Spatial Snowdrift Game - Replicator Rule

Replicator rule

$$P_{ij} = \frac{1 + \frac{W_j - W_i}{N \times (\max\{P, R, T, S\} - \min\{P, R, T, S\})}}{2}$$

With the Snowdrift game, this formula becomes

$$P_{ij} = \frac{1 + \frac{W_j - W_i}{80}}{2}$$

with the Moore neighborhood or

$$P_{ij} = \frac{1 + \frac{W_j - W_i}{40}}{2}$$

with the Von Neumann neighborhood.

2.1 Moore Neighborhood

2.1.1 4x4

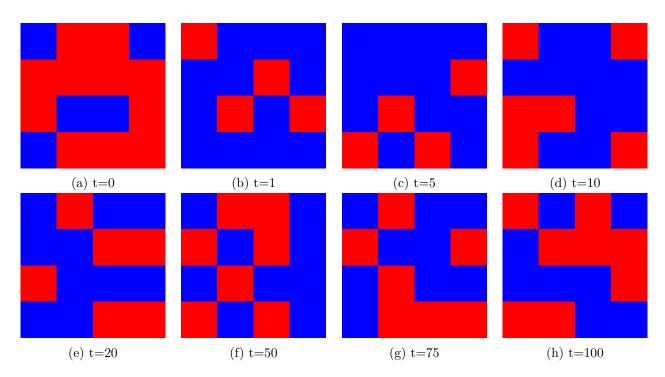
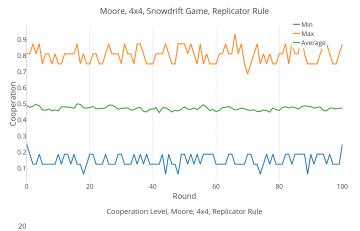
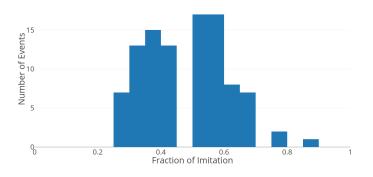


Figure 14: Snowdrift Game, Moore, 4x4



Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.4738 and deviation = 0.1354.



2.1.2 8x8

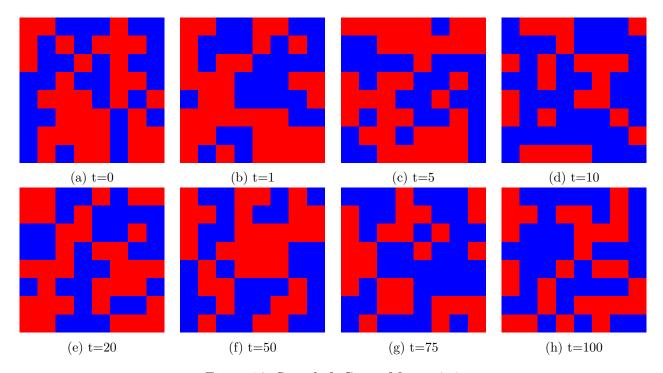
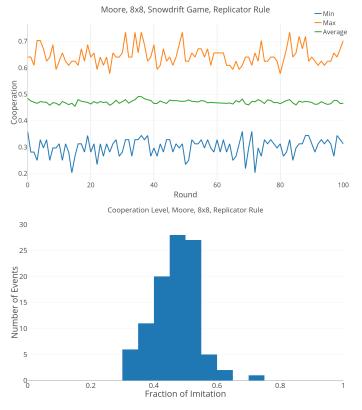


Figure 16: Snowdrift Game, Moore, 8x8



Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.4658 and deviation = 0.0698.

2.1.3 12x12

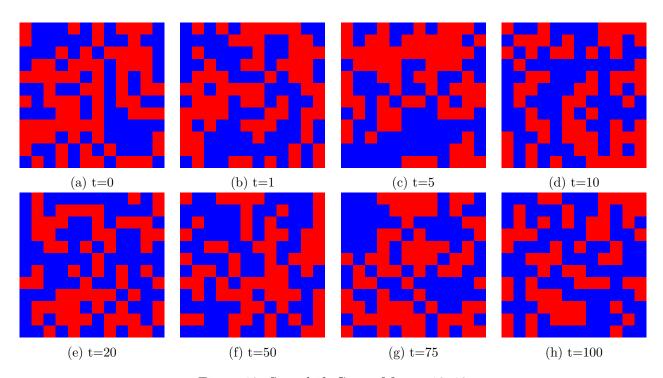
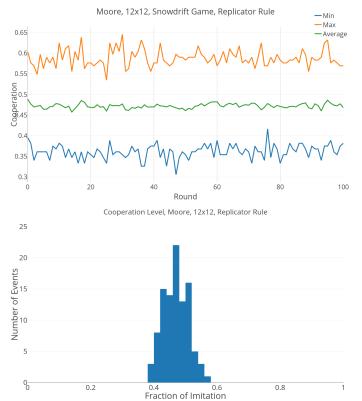


Figure 18: Snowdrift Game, Moore, 12x12



Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.4682 and deviation = 0.0384.

2.1.4 20x20

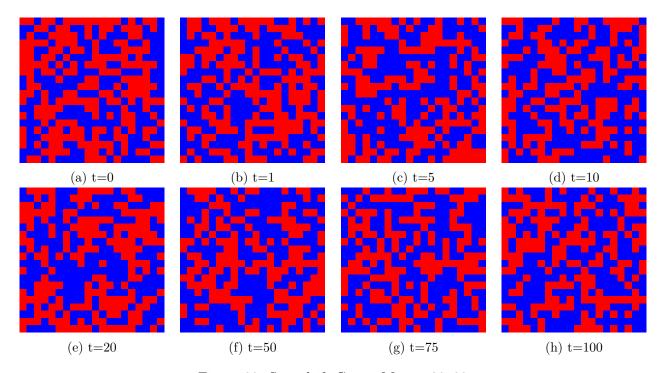
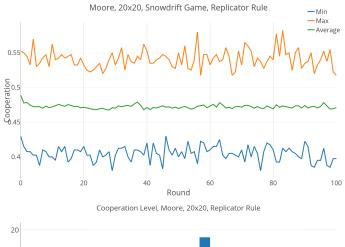


Figure 20: Snowdrift Game, Moore, 20x20



Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.4711 and deviation = 0.0239.

$2.1.5 \quad 50 \times 50$

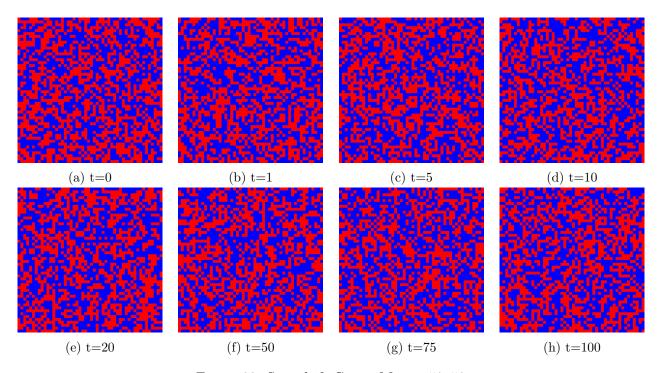
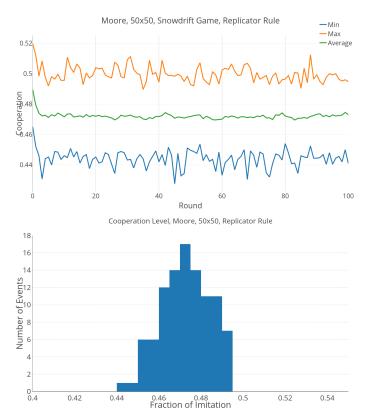


Figure 22: Snowdrift Game, Moore, 50x50



Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.473 and deviation = 0.0117.

2.2 Von Neumann Neighborhood

2.2.1 50x50

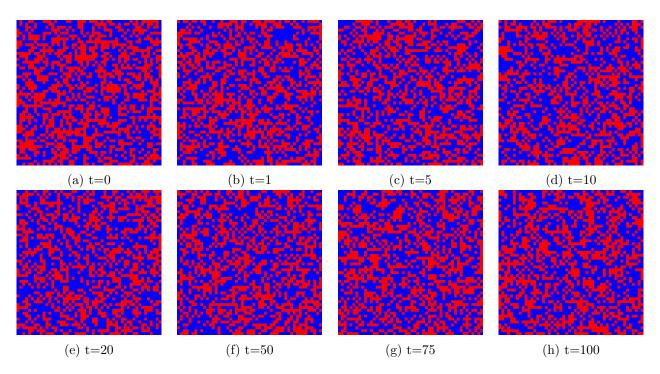
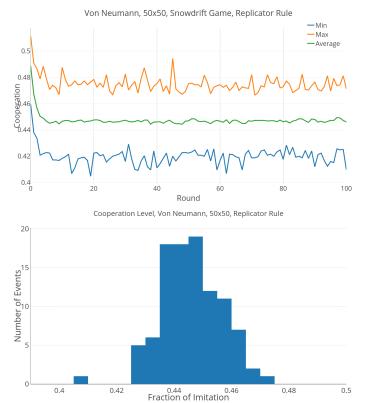


Figure 24: Snowdrift Game, Von Neumann, 50x50



Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.4461 and deviation = 0.0104.

2.3 Analysing the results

We can now have a look at the results of the experiments and investigate their differences.

Table 2: Combined Experiment Results

		Von Neumann				
Lattice	4x4	8x8	12x12	20x20	50x50	50x50
Mean	0.4738	0.4658	0.4682	0.4711	0.473	0.4461
Deviation	0.1354	0.0698	0.0384	0.0239	0.0117	0.0104
Convergence	10	10	20	10	10	10

 qsdqsd

3 Part Three