Université libre de Bruxelles

INFO-F-409 - Learning Dynamics

Assignment Two

Evolutionary dynamics in a spatial context

Raymond Lochner - 000443637

raymond.lochner@ulb.ac.be

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Preliminary information

Each game configuration was being simulated 100 times to receive a good picture of the various possible outcomes. Rounds were played until convergence was certain. For the visualizations:

- Red signifies the action *cooperation*
- Blue signifies the action *defection*

The graphic displays one specific game, whereas the cooperation graph shows information of all games combined.

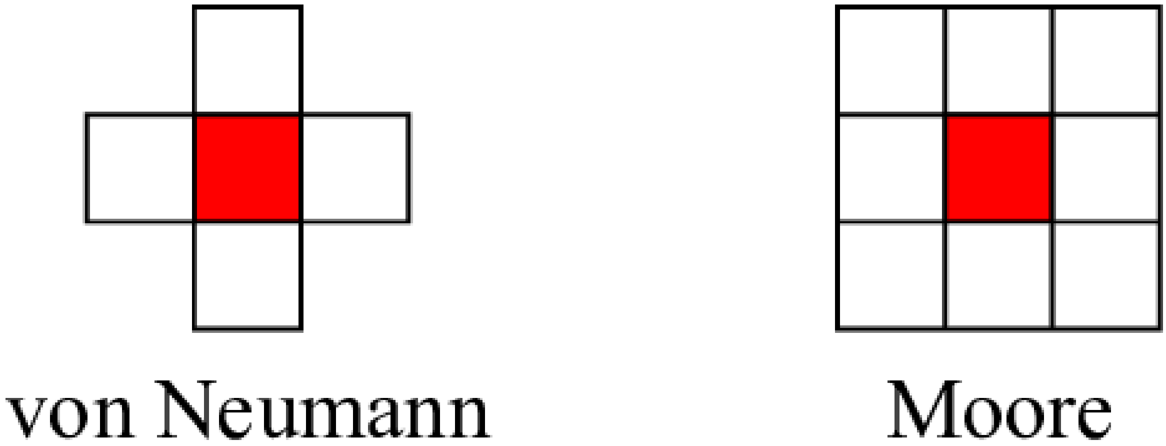


Figure 1: Two Neighborhood types

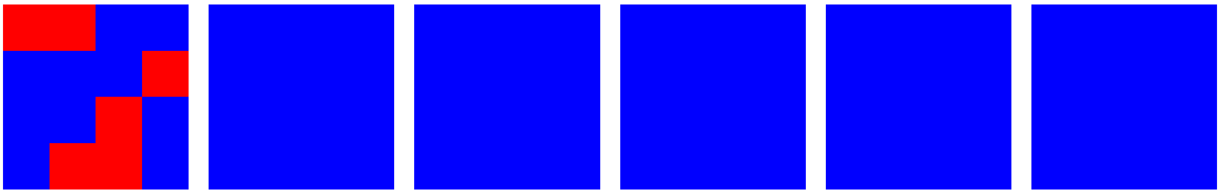
The tested games are:

- Weak Prisonners Dilemma - (T=10, R=7, P=S=0)
- Snowdrift Game - (T=12, R=7, P= 0, S=3)

1 Part One - Spatial Prisoners Dilemma

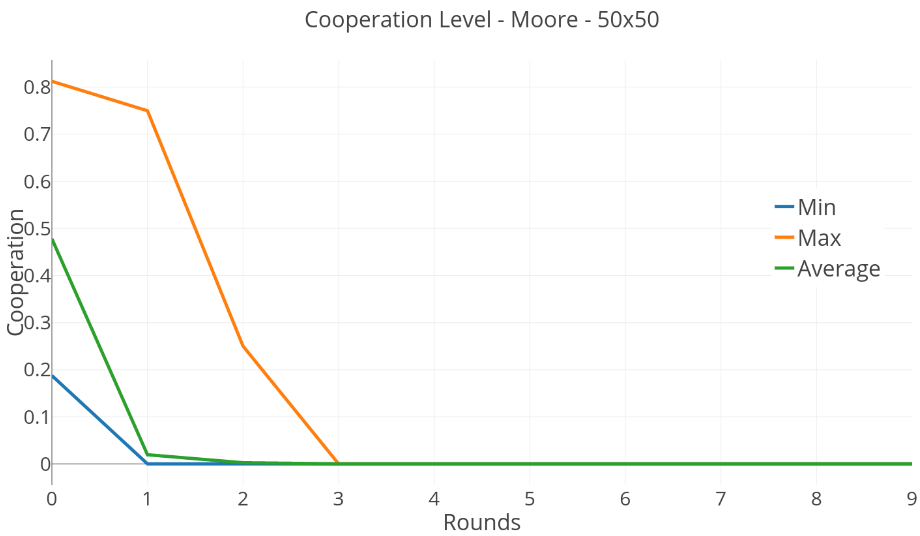
1.1 Moore Neighborhood

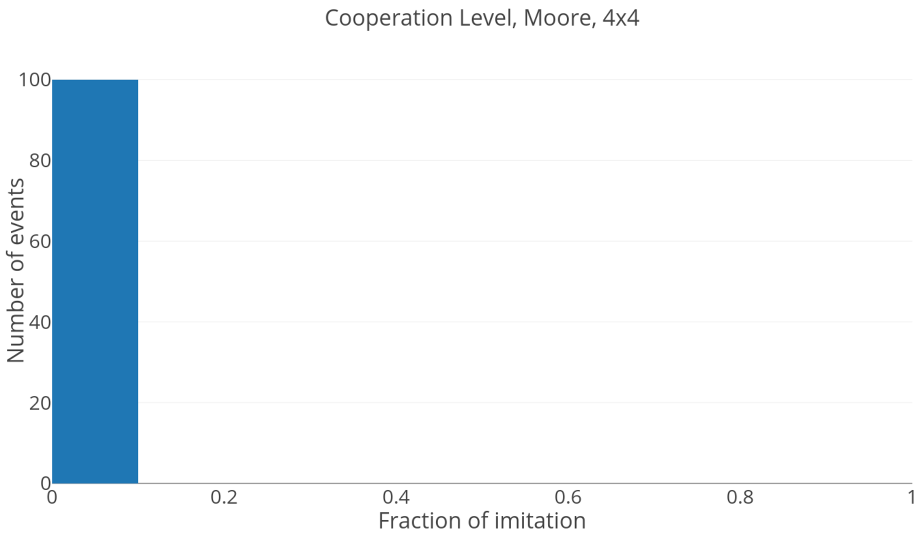
1.1.1 4x4



(a) t=0 (b) t=1 (c) t=5 (d) t=10 (e) t=20 (f) t=50

Figure 2: Prisoners Dilemma, Moore, 4x4

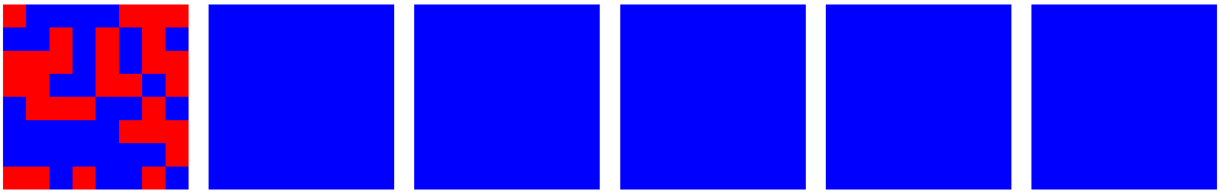




mean = 0
deviation = 0

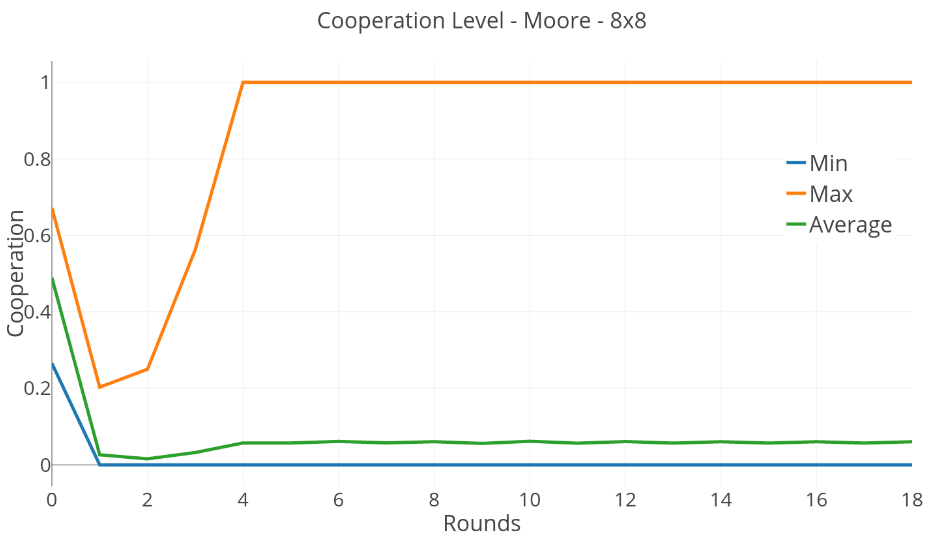
From simulating 100 runs we observe that all converge to the pure strategy of *defecting* after 3 rounds. Nevertheless, it is however possible that a 4x4 configuration converges to a total cooperative field, but it requires that we have a sub-matrix of 2x2 with only cooperators and all other players being defectors. This did obviously not happen during one of the simulations.

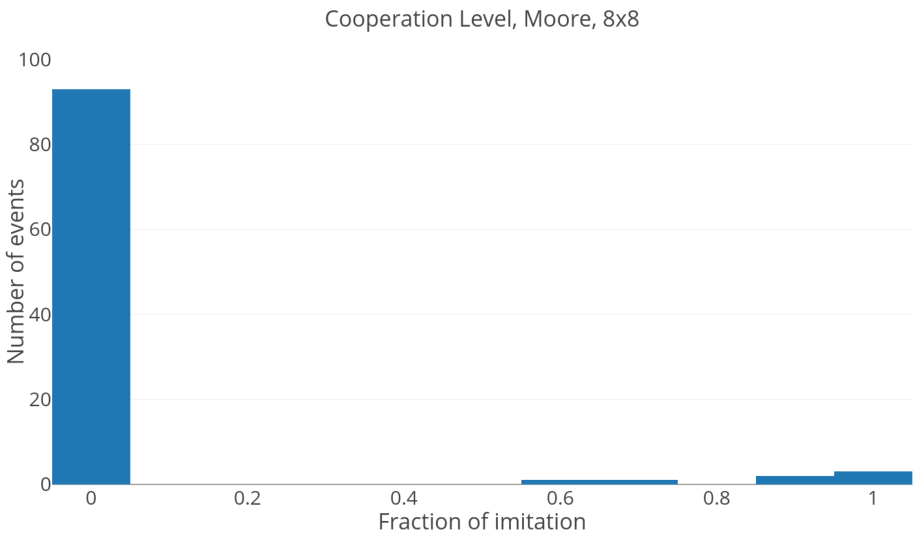
1.1.2 8x8



(a) t=0 (b) t=1 (c) t=5 (d) t=10 (e) t=20 (f) t=50

Figure 4: Prisoners Dilemma, Moore, 8x8

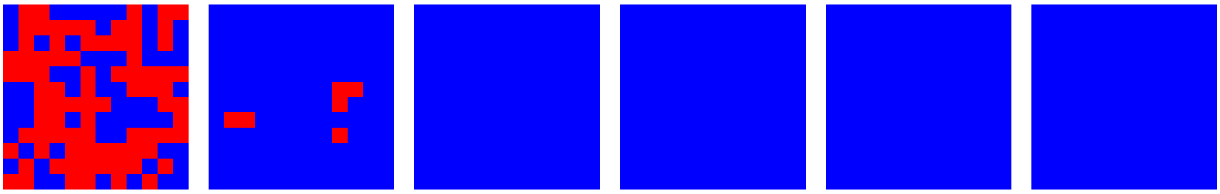




mean = 0.0605
deviation = 0.2251

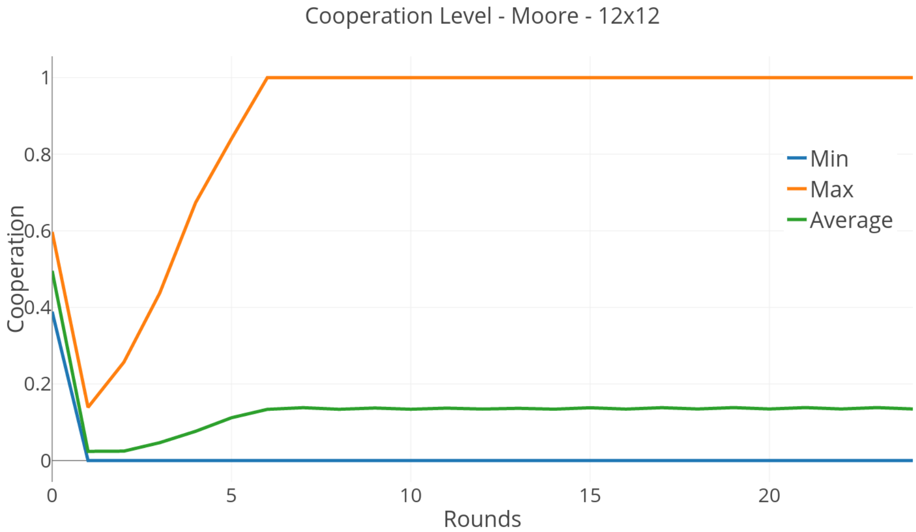
Increasing the lattice to 8x8, we get our first pure cooperation and mixed strategy fields. The configuration converges after 4 rounds, but most fields end up as being pure defector lattices.

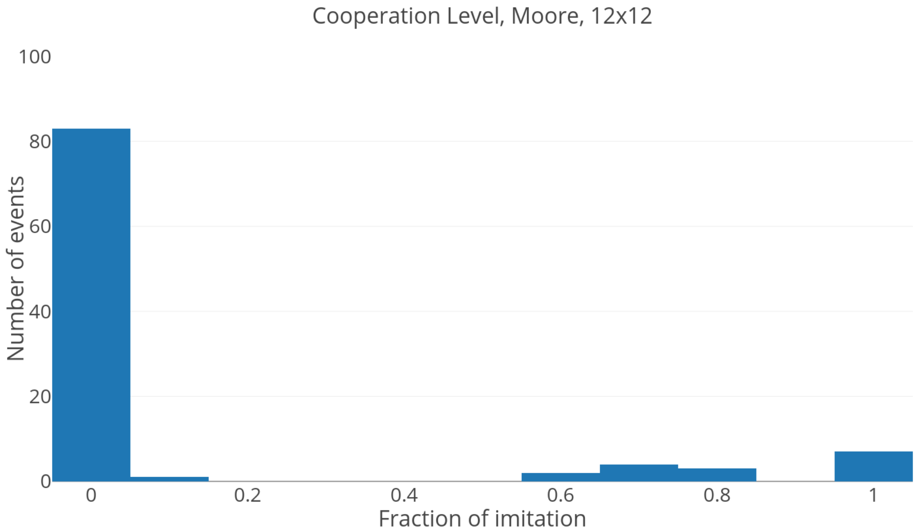
1.1.3 12x12



(a) t=0 (b) t=1 (c) t=5 (d) t=10 (e) t=20 (f) t=50

Figure 6: Prisoners Dilemma, Moore, 12x12

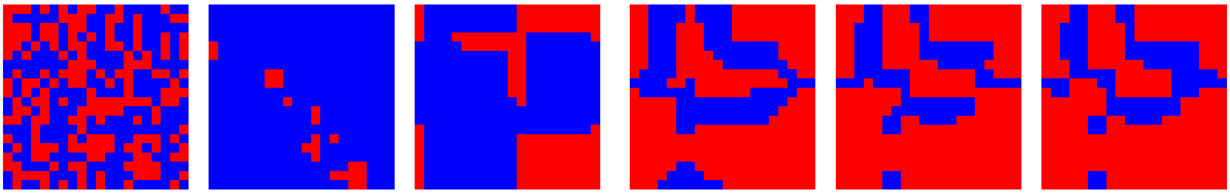




mean = 0.1347
deviation = 0.3115

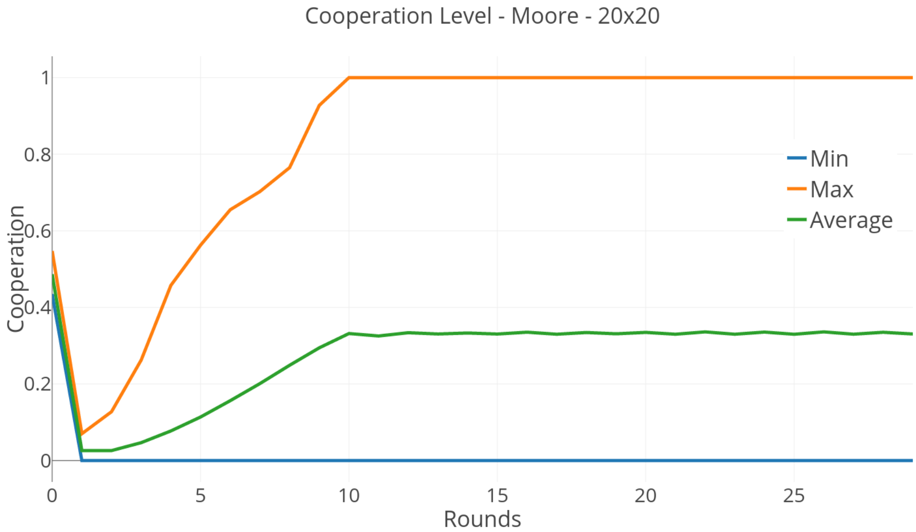
A lattice configuration of 12x12 increases the chance slightly that the whole lattice does not end up being only defectors. More mixed strategy lattices at 0.7 and some more pure strategy cooperation lattices. Convergence after 7 rounds.

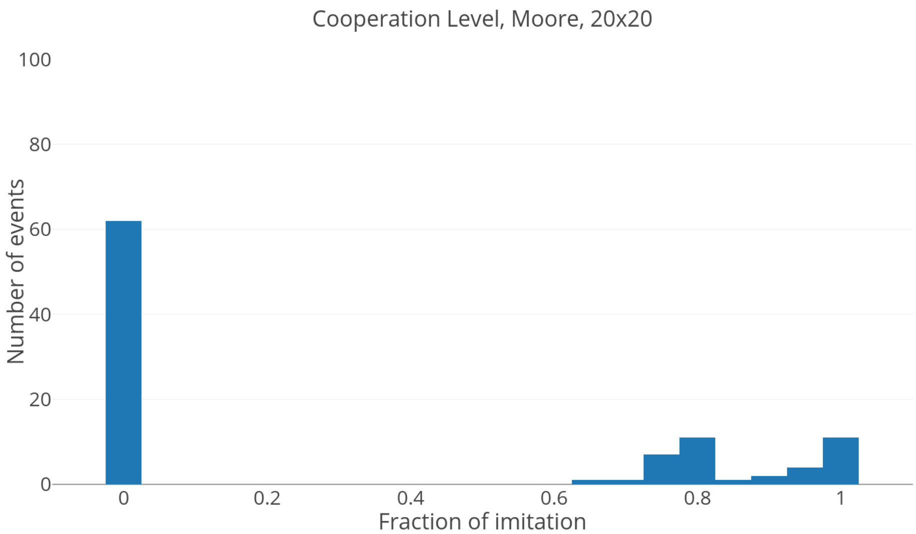
1.1.4 20x20



(a) t=0 (b) t=1 (c) t=5 (d) t=10 (e) t=20 (f) t=50

Figure 8: Prisoners Dilemma, Moore, 20x20

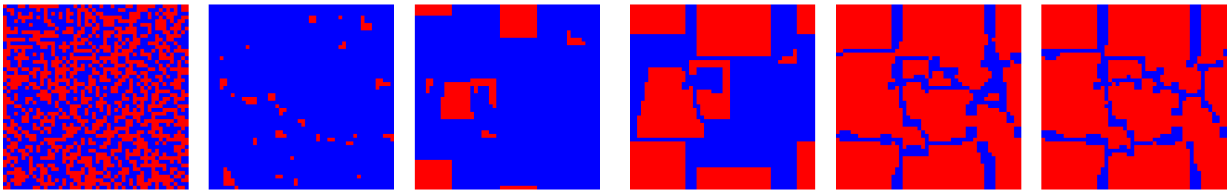




mean = 0.3305
deviation = 0.4262

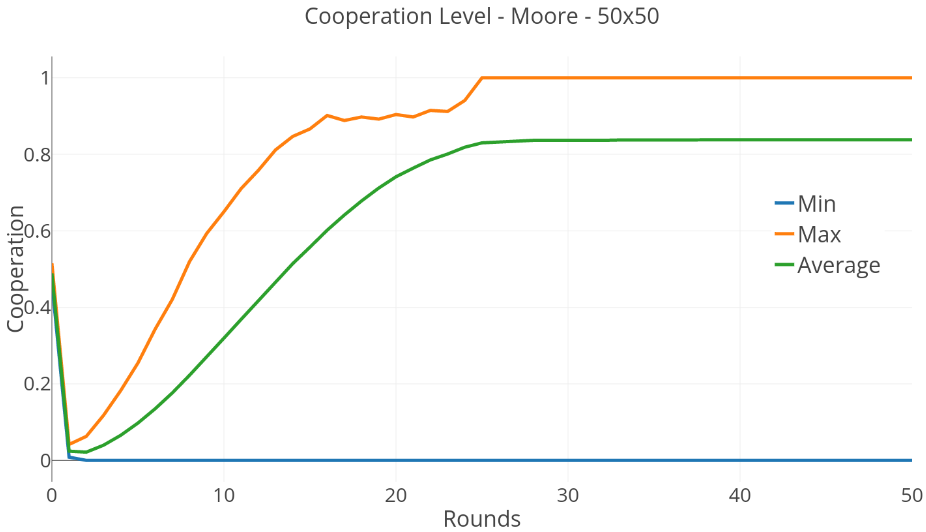
The 20x20 lattice configuration results in 60% of games being purely defectors and the rest being either purely cooperative or mostly cooperative. The graphical representation shows the creation of cooperation blocks after time, with defector *rivers* in between.

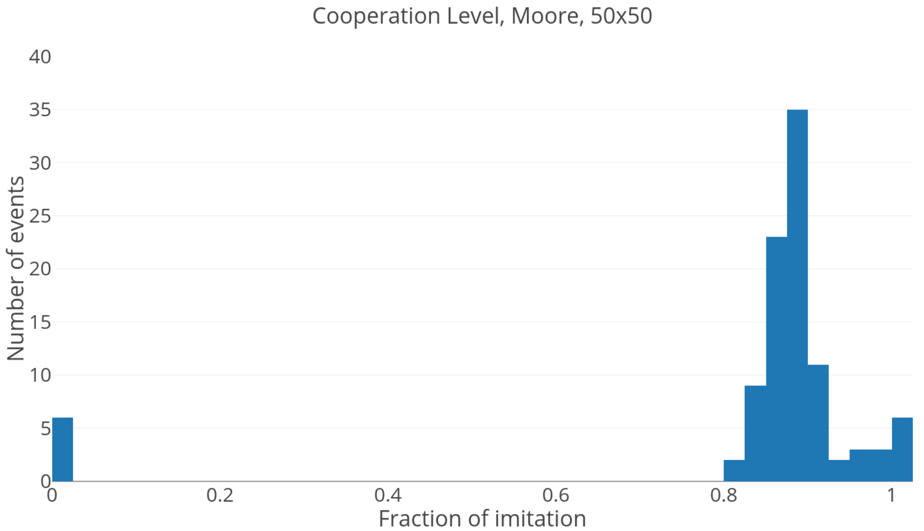
1.1.5 50x50



(a) t=0 (b) t=1 (c) t=5 (d) t=10 (e) t=20 (f) t=50

Figure 10: Prisoners Dilemma, Moore, 50x50



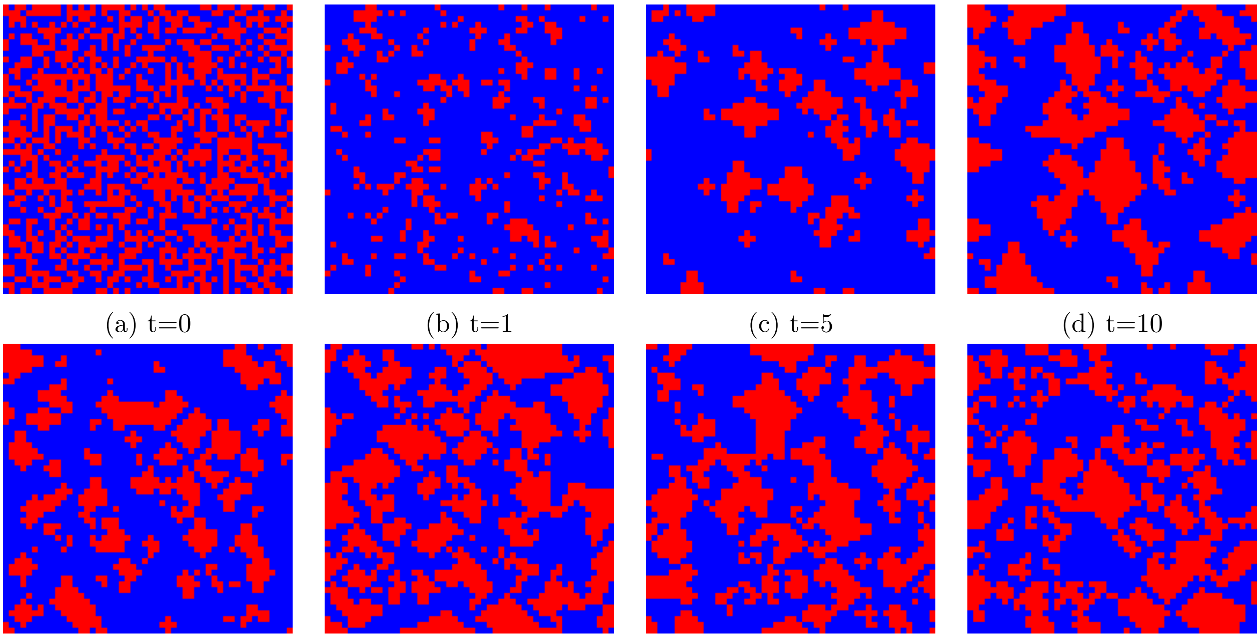


mean = 0.8379
deviation = 0.2165

A 50x50 lattice configuration results in a highly cooperative environment about 94% of the time. Convergence after 25 rounds. Looking at the graphical representation we can see that clusters of cooperation with *rivers* of defection are being formed. The distribution starts to look like a normal distribution.

1.2 Von Neumann Neighborhood

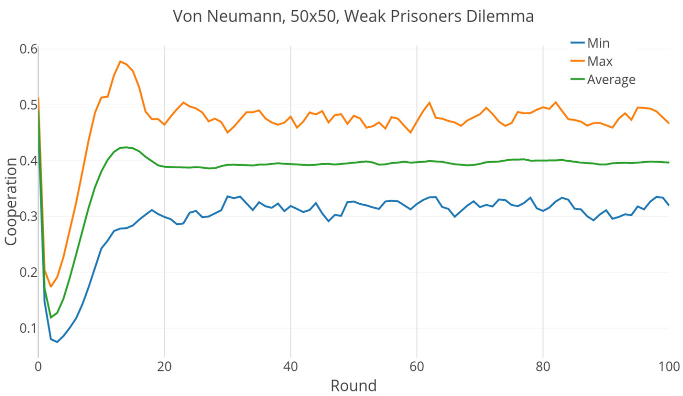
1.2.1 50x50

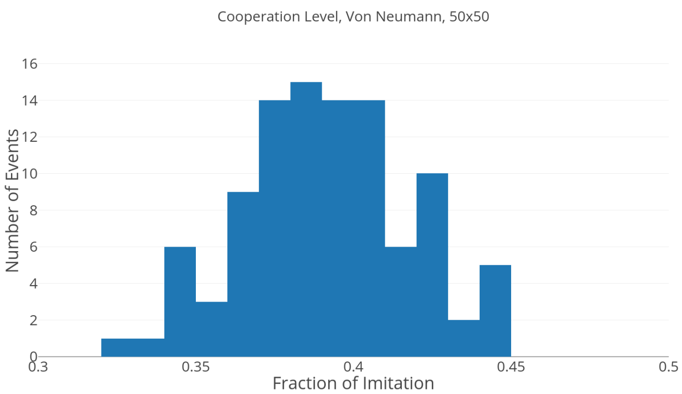


(a) t=0 (b) t=1 (c) t=5 (d) t=10

(e) t=20 (f) t=50 (g) t=75 (h) t=100

Figure 12: Prisoners Dilemma, Von Neumann, 50x50





Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.3914 and deviation = 0.0263. The mean with this neighborhood type is about half as the mean from a Moore neighborhood game with the same lattice size. The deviation is however much smaller.
The configuration converges after 20 rounds. Looking at the differences of the graphical representation, using the Von Neumann neighborhood results in non stationary clusters as we have with a Moore neighborhood. The cooperation level over time also changes, which we can observe in the curve. It does not drop too much at the first few rounds and then quickly converges to $\sim 0.4\%$ with the maximum and minimum level not being too far away which is why the deviation is much smaller compared to the Moore neighborhood. This is because of the influence caused by the defector players in the first few rounds is reduced with fewer neighbors.

1.3 Analysing the results

We can now have a look at the results of the experiments and investigate their differences.

Table 1: Combined Experiment Results

	Moore					Von Neumann
Lattice	4x4	8x8	12x12	20x20	50x50	50x50
Mean	0	0.0605	0.1347	0.3305	0.8379	0.3914
Deviation	0	0.2251	0.3115	0.4262	0.2165	0.0263
Convergence	3	4	7	10	25	20

Looking only at the Moore neighborhood configurations, we can observe that the mean increases and the deviation decreases with an increase in the lattice size. Lattices that converge with cooperation blocks divided by defector *rivers* do need a large lattice size to occur regularly - this shown by the increase of the mean. The time to converge to a constant level does depend on the lattice size according to our results - the bigger the size, the higher the time to converge.

2 Part Two - Spatial Snowdrift Game - Replicator Rule

Replicator rule

$$P_{ij} = \frac{1 + \frac{W_j - W_i}{N \times (max\{P,R,T,S\} - min\{P,R,T,S\})}}{2}$$

With the Snowdrift game, this formula becomes

$$P_{ij} = \frac{1 + \frac{W_j - W_i}{80}}{2}$$

with the Moore neighborhood or

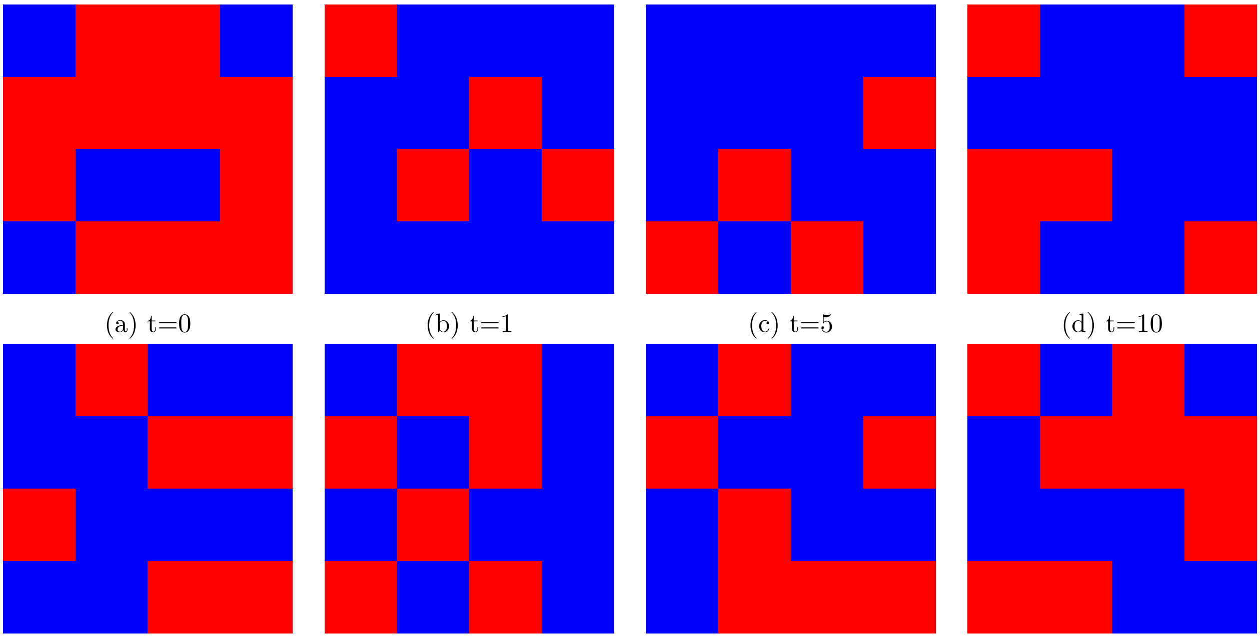
$$P_{ij} = \frac{1 + \frac{W_j - W_i}{40}}{2}$$

with the Von Neumann neighborhood. This probabilistic method assures enables a *mixed* strategy of replicating. Contrary to a *pure* strategy as experimented with in part one, the replicator rule assigns a probability of choosing the strategy of the other player by taking the difference of the payoff between the two players. The greater the difference of $W_j - W_i$, the more likely it is to replicate the action

The N variable reduces this probability - more neighbors lead to a slightly smaller probability of replicating the opponent strategy. It makes sense to have a probabilistic update mechanism as many real *games* have mixed strategies instead of pure ones - it allows for the modelling of phenomena's we can see in nature for example.

2.1 Moore Neighborhood

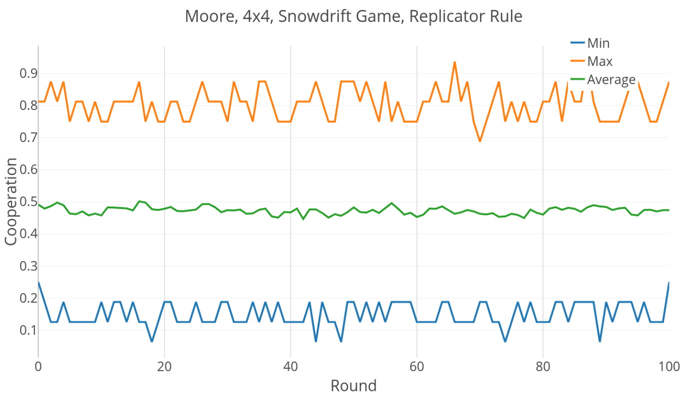
2.1.1 4x4

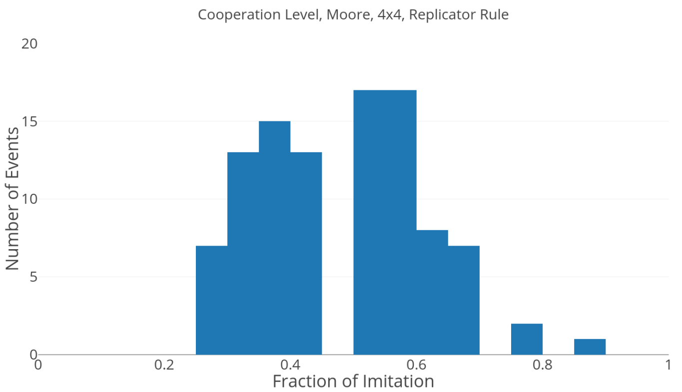


(a) t=0 (b) t=1 (c) t=5 (d) t=10

(e) t=20 (f) t=50 (g) t=75 (h) t=100

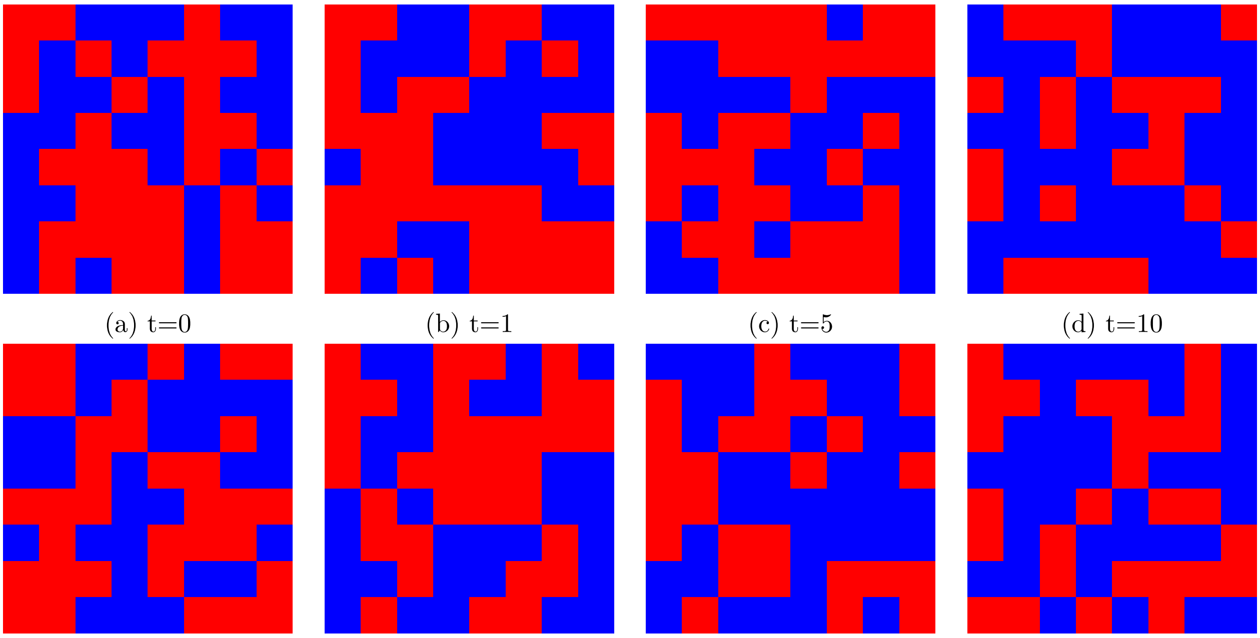
Figure 14: Snowdrift Game, Moore, 4x4





We get a mean = 0.4738 and deviation = 0.1354. The simulation converges after about 10 rounds. Contrary to the old update strategy, we don't have a single pure strategy lattice and the distribution looks already somewhat like a normal distribution.

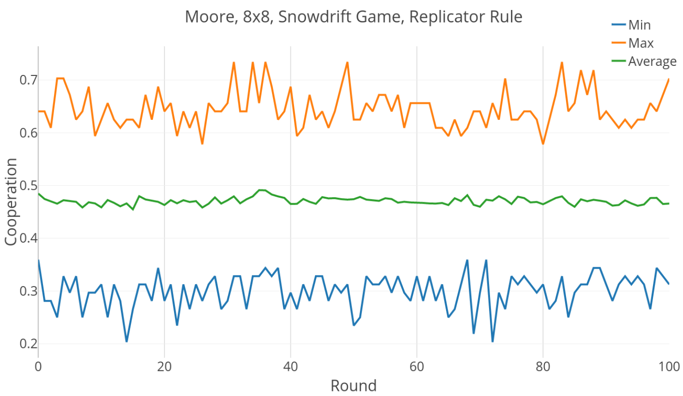
2.1.2 8x8

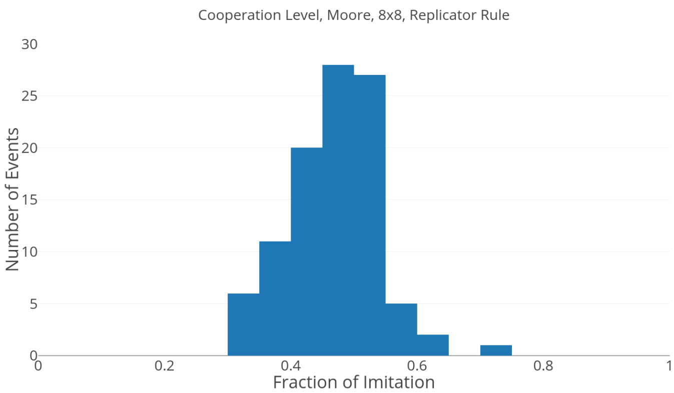


(a) t=0 (b) t=1 (c) t=5 (d) t=10

(e) t=20 (f) t=50 (g) t=75 (h) t=100

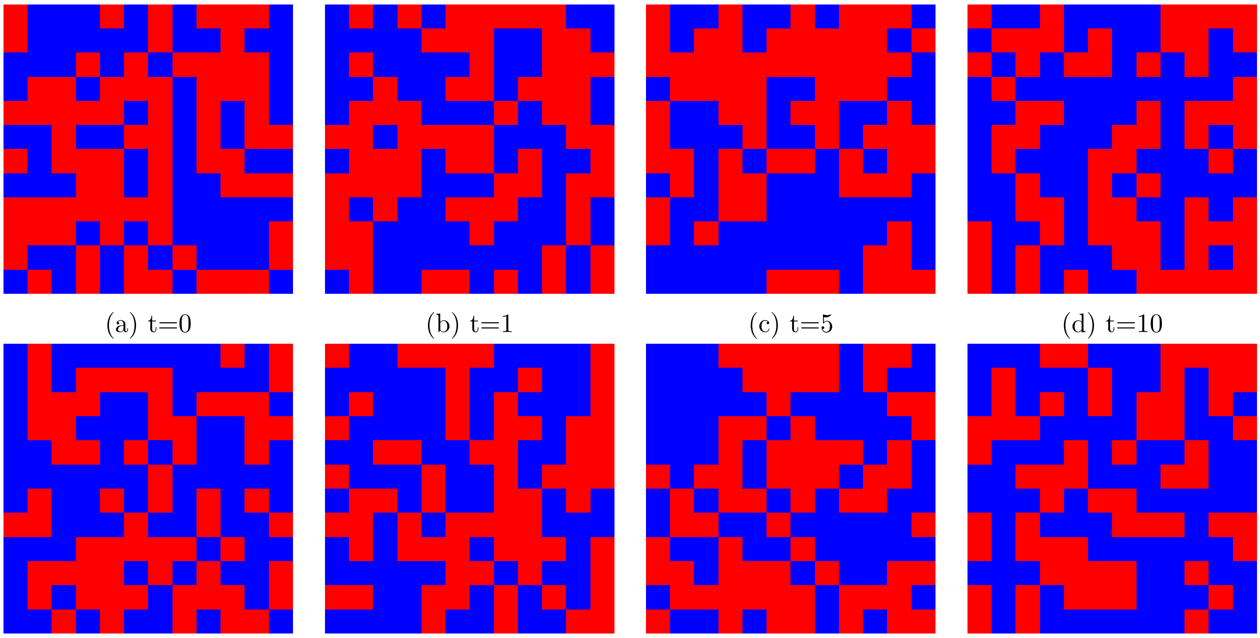
Figure 16: Snowdrift Game, Moore, 8x8





Again no pure strategy lattice and we get a mean = 0.4658 and deviation = 0.0698. A reduction in the deviation from the previous lattice size. The simulation converges at about the same time, after 10 rounds.

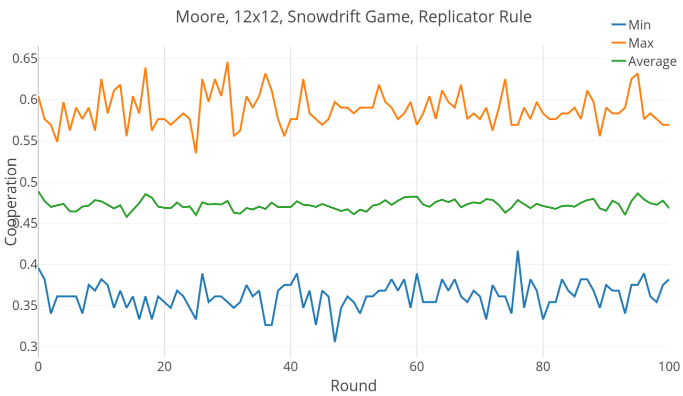
2.1.3 12x12

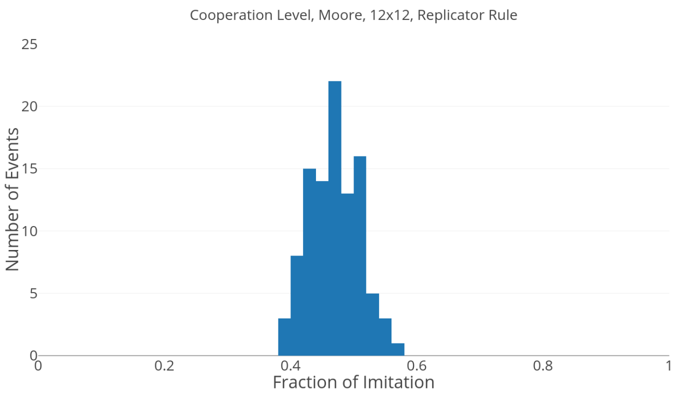


(a) t=0 (b) t=1 (c) t=5 (d) t=10

(e) t=20 (f) t=50 (g) t=75 (h) t=100

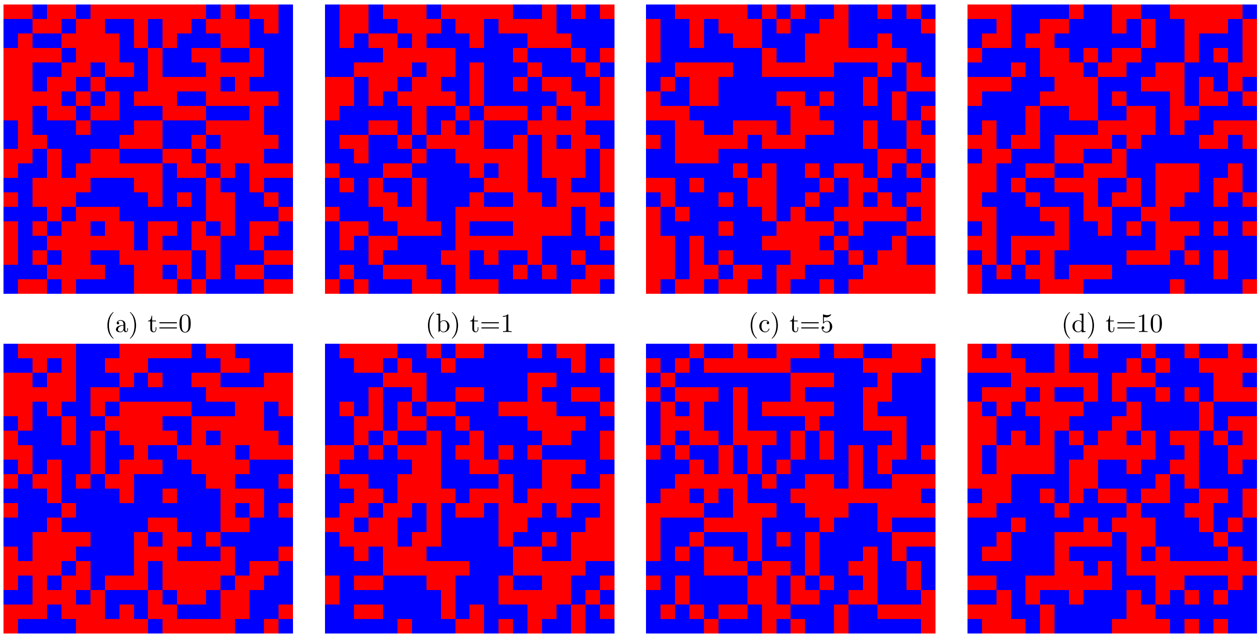
Figure 18: Snowdrift Game, Moore, 12x12





Mean = 0.4682 and deviation = 0.0384. Convergence also after about 10 rounds. Also a reduction in the deviation, but the mean stays at the same level.

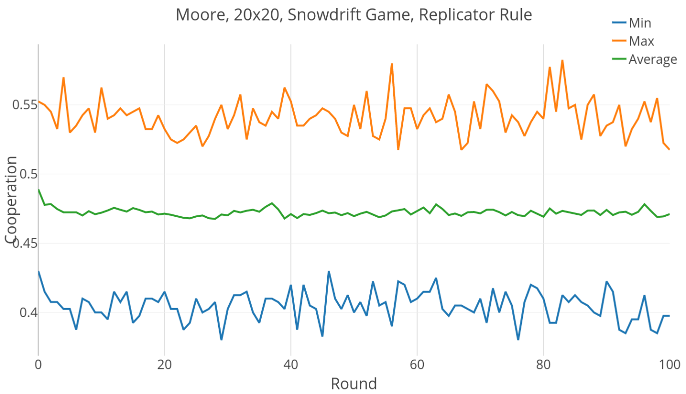
2.1.4 20x20

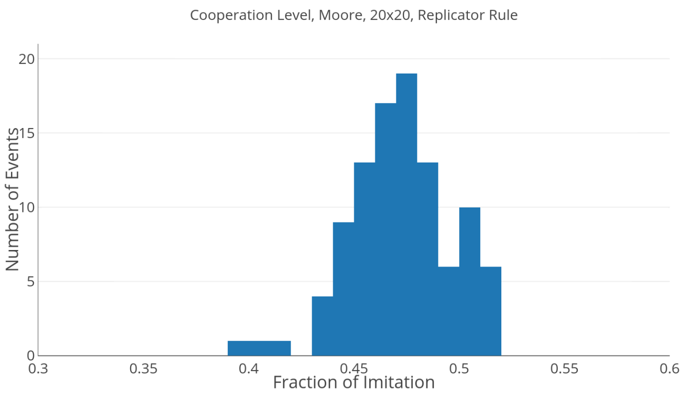


(a) t=0 (b) t=1 (c) t=5 (d) t=10

(e) t=20 (f) t=50 (g) t=75 (h) t=100

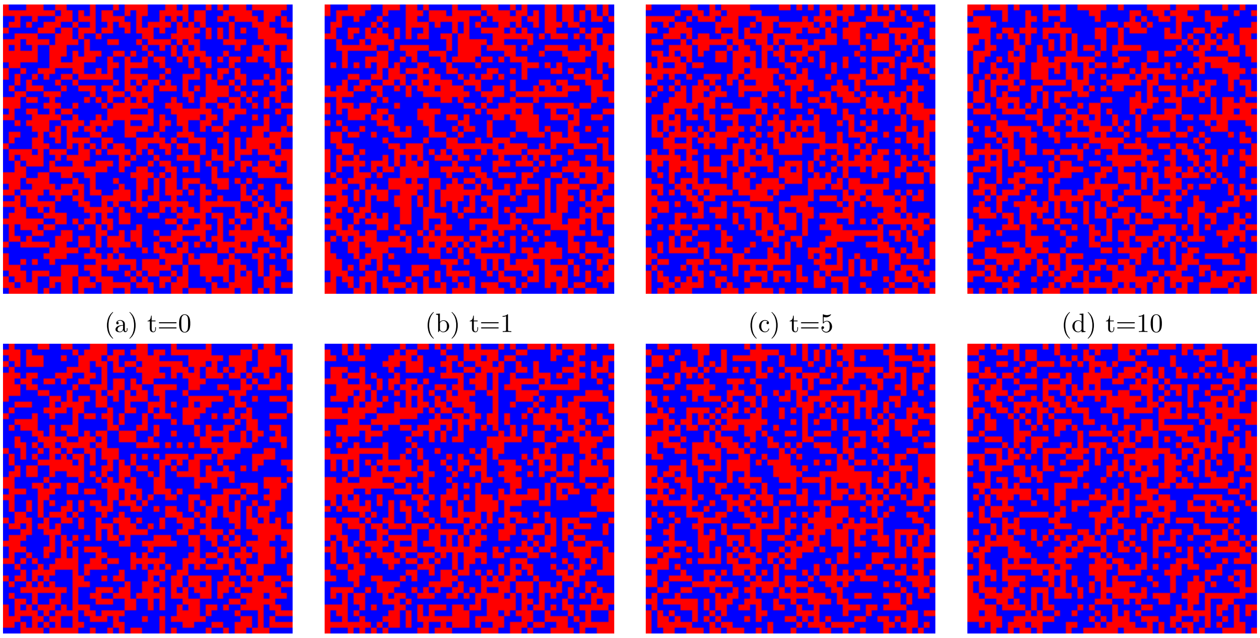
Figure 20: Snowdrift Game, Moore, 20x20





Mean = 0.4711 and deviation = 0.0239. The trend continues: lower deviation, convergence after about 10 rounds. Comparing this lattice size to the old update mechanism, we don't observe the cooperative block creation.

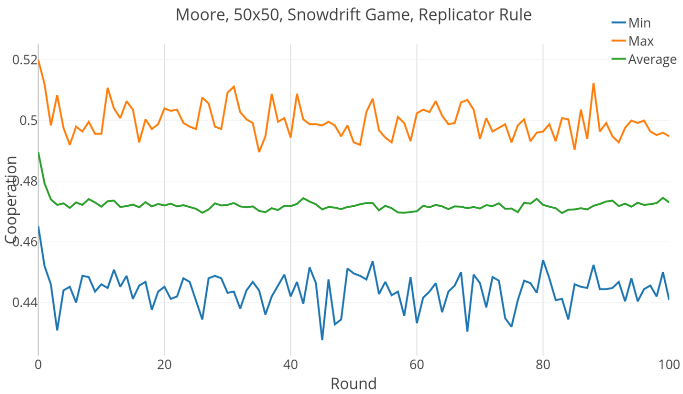
2.1.5 50x50



(a) t=0 (b) t=1 (c) t=5 (d) t=10

(e) t=20 (f) t=50 (g) t=75 (h) t=100

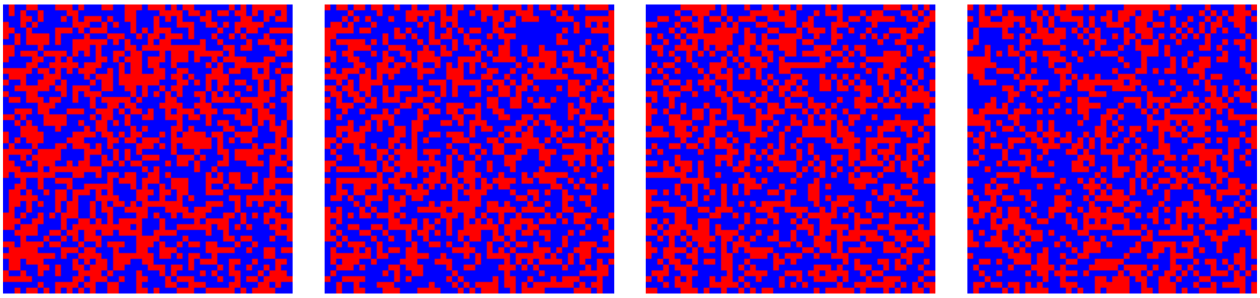
Figure 22: Snowdrift Game, Moore, 50x50



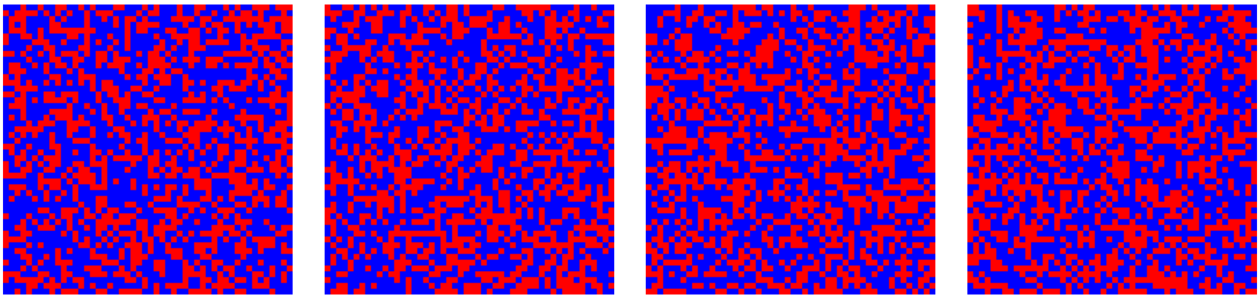
Mean = 0.473 and deviation = 0.0117. Same observation as previous lattice size. In the graphical representation we can observe that no stationary blocks are being formed.

2.2 Von Neumann Neighborhood

2.2.1 50x50

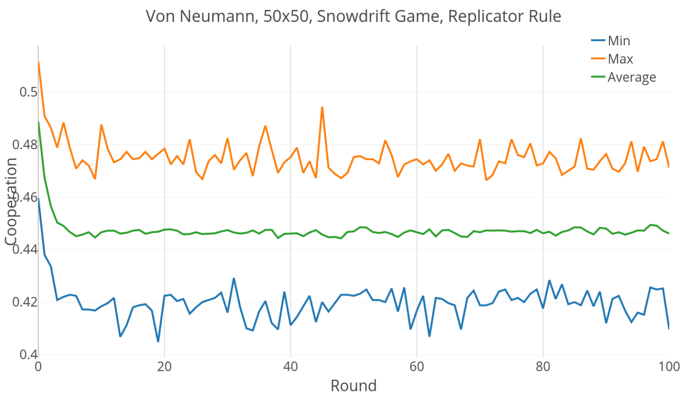


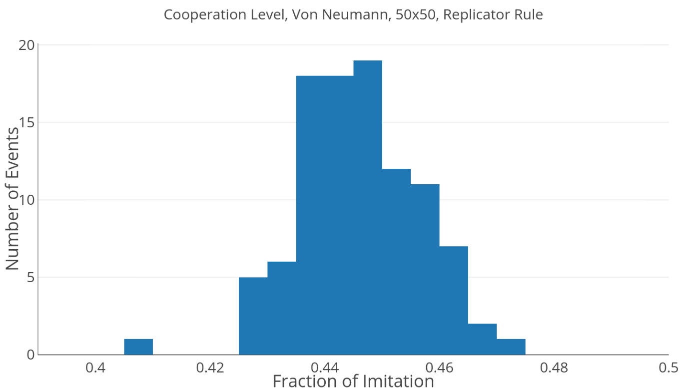
(a) t=0 (b) t=1 (c) t=5 (d) t=10



(e) t=20 (f) t=50 (g) t=75 (h) t=100

Figure 24: Snowdrift Game, Von Neumann, 50x50





Changing the neighborhood to the *Von Neumann* mode, we get a mean = 0.4461 and deviation = 0.0104. Contrary to the observations of part one when changing the neighborhood type, we can not observe a large difference when changing to the Von Neumann method. The only difference is a slight reduction of the deviation

Table 2: Combined Experiment Results

	Moore					Von Neumann
Lattice	4x4	8x8	12x12	20x20	50x50	50x50
Mean	0.4738	0.4658	0.4682	0.4711	0.473	0.4461
Deviation	0.1354	0.0698	0.0384	0.0239	0.0117	0.0104
Convergence	10	10	10	10	10	10

2.3 Analysing the results

We can now have a combined look at the results of the experiments and investigate their differences.

From these results we can conclude that the mean stays relatively the same, no matter the lattice size. The deviation is being reduced with an increasing lattice size and the fields converge all at about the same number of rounds player: 10. Changing the neighborhood method to Von Neumann, the only difference we can observe is a slight reduction of the deviation.

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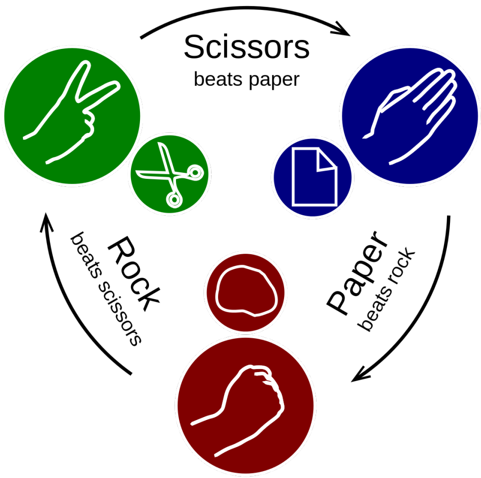
3 Part Three - Rock, Paper, Scissors

For this extra part we will investigate a game with three actions: Rock, Paper, Scissors!



		Player Two		
		Rock	Paper	Scissors
Player One	Rock	0,0	-1,1	1,-1
	Paper	1,-1	0,0	-1,1
	Scissors	-1,1	1,-1	0,0

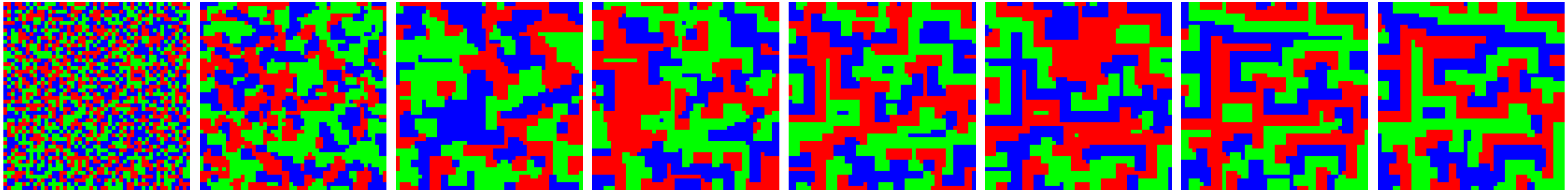
Figure 26: Rock, Paper, Scissors

This game has obvious mixed strategy Nash Equilibria with p(Rock)=p(Paper)=p(Scissors)=$\frac{1}{3}$. We shall investigate the effect of various starting population distributions and the effect this may have on the population of the lattice.

The following graphical lattice representation will use the following colors with the following actions:

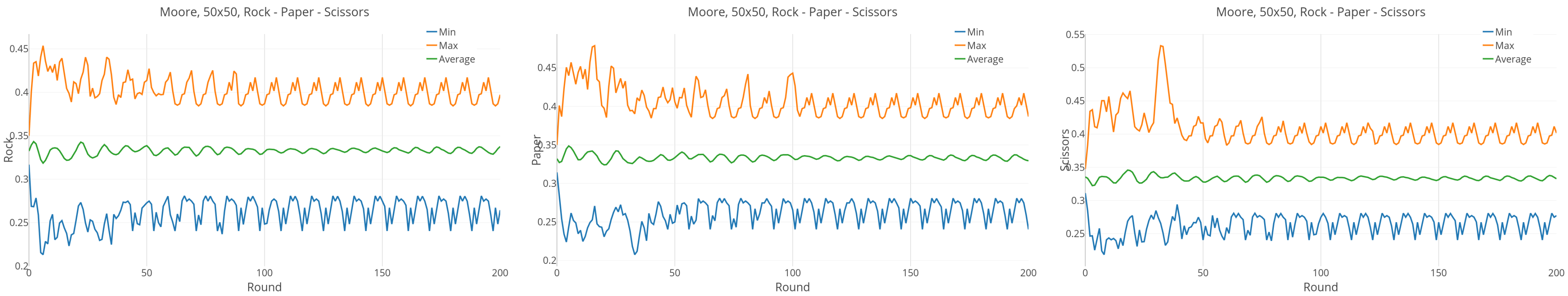
- Rock - Red
- Paper - Blue
- Scissors - Green

3.1 50x50, Equal starting distribution, Highest earner, Moore



(a) t=0 (b) t=1 (c) t=5 (d) t=10 (e) t=20 (f) t=50 (g) t=75 (h) t=100

Figure 27: Rock - Paper - Scissors, Moore, 50x50



(a) Rock population (b) Paper population (c) Scissors population

In the graphical representation we can observe a non stationary population containing all three actions. Contrary to the experiments done in the first two parts, in this experiment we don't observe a clear convergence point. Indeed, population percentage of all three actions seem to be oscillating around $\frac{1}{3}$. Interesting also is the initial population change: Rocks increase their population while paper decreases its population for a very short while before rising. The Scissors population is decreasing in the beginning even more before going into a oscillating state.