Université libre de Bruxelles

INFO-F-409 - Learning Dynamics

Assignment One

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Contents

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1 The Hawk-Dove game

The Hawk-Dove game was first formulated by John Maynard Smith and Georg Prince in 1973 [1]. The aim of the game is to gain a better understanding of conflicts in the animal kingdom. It consits of two players {Player One, Player Two} who have each two actions {Hawk, Dove}.The resulting payoff matrix can bee seen in Table 1 where:

- V = fitness value of winning resources in fight
- D = fitness costs of injury
- T = fitness costs of wasting time

and we assume that V,D,T $\geq$ 0.

Table 1: Hawk-Dove Payoff Matrix

		Player Two	
		Hawk	Dove
Player One	Hawk	(V-D)/2 (V-D)/2	0 V
	Dove	V 0	V/2-T V/2-T

In a mixed strategy game, we consider each player performing his action with a certain probability p or q, which results in the following payoff matrix displayed in Table 2.

Table 2: Hawk-Dove Probability Payoff Matrix

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	(V-D)/2 (V-D)/2	0 V
	P(Dove)=(1-p)	V 0	V/2-T V/2-T

With this table we are able to calculate the payoff of the actions Hawk and Dove for Player One:

Player One's expected payoff for the strategy Hawk(p) is:

$$q \times \frac{V-D}{2} + (1-q) \times V$$

Player One's expected payoff for the strategy Dove(1-p) is:

$$q \times 0 + (1-q) \times (\frac{V}{2} - T) = (1-q) \times (\frac{V}{2} - T)$$

The best response set is {Hawk} if:

$$q \times \frac{V-D}{2} + (1-q) \times V < (1-q) \times (\frac{V}{2} - T)$$

The best response set is {Dove} if:

$$q \times \frac{V-D}{2} + (1-q) \times V > (1-q) \times (\frac{V}{2} - T)$$

All the players mixed strategies are best responses if:

$$q \times \frac{V-D}{2} + (1-q) \times V = (1-q) \times (\frac{V}{2} - T)$$

As we are dealing with unknown variables, let's rewrite the equation for q:

$$q \times \frac{V-D}{2} + (1-q) \times V = (1-q) \times (\frac{V}{2} - T)$$

$$\frac{qV}{2} - \frac{qD}{2} + V - qV = \frac{V}{2} - T - \frac{qV}{2} + qT$$

$$-\frac{qD}{2} + V = \frac{V}{2} - T + qT$$

$$\frac{V}{2} + T = \frac{qD}{2} + qT$$

$$\frac{V}{2} + T = q(\frac{D}{2} + T)$$

$$V + 2T = q(D + 2T)$$

$$q = \frac{V + 2T}{D + 2T}$$

To find any pure or mixed strategy of this scenario, we have to look at different values for the variables V,D and T. In this, we must respect the following constraints:

$$0 \leq q \leq 1 \quad \text{and} \quad 0 \leq \frac{V+2T}{D+2T} \leq 1 \quad \text{and} \quad V, T, D \geq 0$$

1.1 T=0

Let's start by considering the case where $T = 0$. This leaves gives us the following options:

- $V = 0$ with $D = 0$
- $V = 0$ with $D > 0$
- $V > 0$ with $D = 0$
- $V > D > 0$
- $D > V > 0$

1.1.1 T=0, V=0, D=0

Any pure or mixed strategy is a best response. This yields the mixed strategy NE $(p, q) \in \{(0 \leq p \leq 1), (0 \leq q \leq 1)\}$.

Table 3: V=T=0, D>0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	0, 0	0, 0
	P(Dove)=(1-p)	0, 0	0, 0

1.1.2 T=0, V=0, D>0

We have three pure strategy NE: {Dove,Hawk},{Dove,Dove} and {Hawk,Dove}.

Player One's expected payoff for the strategy Hawk(p) is $-q \times \frac{D}{2}$. Player One's expected payoff for the strategy Dove(1-p) is 0. Solving for q gives $q = 0$ which does not yields a mixed strategy NE.

Table 4: D>V=T=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	-D/2 -D/2	0 0
	P(Dove)=(1-p)	0 0	0 0

1.1.3 T=0, V>0, D=0

We find the pure strategy NE {Hawk,Hawk}.

Table 5: V>T=D=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	V/2 V/2	0 V
	P(Dove)=(1-p)	V 0	V/2 V/2

1.1.4 T=0, V>D>0

The action {Hawk, Hawk} is always positive as we have $V > D$. We find the pure strategy NE {Hawk,Hawk}.

Table 6: V>D>T=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	(V-D)/2 (V-D)/2	0 V
	P(Dove)=(1-p)	V 0	V/2 V/2

1.1.5 T=0, D>V>0

The action {Hawk, Hawk} is always negative as we have $D > V$. We find two pure strategy NE's: {Dove,Hawk} and {Hawk,Dove}.

Table 7: D>V>T=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	(V-D)/2, (V-D)/2	V, 0
	P(Dove)=(1-p)	0, V	V/2, V/2

Player One's expected payoff for the strategy Hawk(p) is:

$$q \times \frac{V-D}{2} + (1-q) \times V = \frac{qV}{2} - \frac{qD}{2} + V - qV$$

Player One's expected payoff for the strategy Dove(1-p) is:

$$q \times 0 + (1-q) \times V/2 = (1-q) \times (V/2) = \frac{V}{2} - \frac{qV}{2}$$

All the players mixed strategies are best responses if:

$$\frac{qV}{2} - \frac{qD}{2} + V - qV = \frac{V}{2} - \frac{qV}{2}$$

$$V - \frac{qD}{2} = \frac{V}{2}$$

$$2V - qD = V$$

$$-qD = -V$$

$$q = \frac{V}{D}$$

1.2 old

1.3 V+2T = 0

This is only possible if $V = 0$ and $T = 0$ which gives $q = \frac{0}{D}$. We have to consider two cases: $D = 0$ and $D > 0$

1.3.1 D>0

1.4 T=0

With $T = 0$ we get the following payoff matrix:

We have the following scenarios: $V > D$,$V < D$ and $V = D$.

1.4.1 V=D

If we set $V = D$, we eliminate D from the payoff matrix:

Table 8: T=0

		Player Two	
		P(Hawk)=q	P(Dove)=(1-q)
Player One	P(Hawk)=p	0 / 0	V / 0
	P(Dove)=(1-p)	0 / V	V/2 / V/2

With three pure strategy NE's: {Dove,Hawk},{Hawk,Hawk} and {Hawk,Dove}. To find mixed strategies, we need the expected payoff for both strategies:

Player One's expected payoff for the strategy Hawk(p) is:

$$q \times 0 + (1 - q) \times V = (1 - q) \times V$$

Player One's expected payoff for the strategy Dove(1-p) is:

$$q \times 0 + (1 - q) \times V/2 = (1 - q) \times (V/2)$$

All the players mixed strategies are best responses if:

$$(1 - q) \times V = (1 - q) \times (V/2)$$

1.4.2 V>D

Not viable as

1.4.3 V<D

2 Which social dilemma?

Player A is confronted with one of three social dilemma's - the corresponding payoff matrix is shown in tables 9, 10 and 11. The player has to decide whether to cooperate (C) or to defect (D) without knowing which game he is actually facing. Each dilemma has the same probability 1/3 of being played. Opponent B knows the game.

Table 9: Prisonners dilemma

	C	D
C	2, 2	0, 5
D	5, 0	1, 1

Table 10: Stag-Hunt game

	C	D
C	5, 5	0, 2
D	2, 0	1, 1

Table 11: Snowdrift game

	C	D
C	2, 2	1, 5
D	5, 1	0, 0

with this information we can calculate the expected payoff for player A for every possible strategy of Player B - Table 12.

Table 12: Expected payoff for Player A

		Player B							
		(C,C,C)	(C,C,D)	(C,D,C)	(C,D,D)	(D,C,C)	(D,C,D)	(D,D,C	(D,D,D)
Player A	C	9/3	8/3	4/3	3/3	7/3	6/3	2/3	1/3
	D	12/3	7/3	11/3	6/3	8/3	3/3	7/3	2/3

From this table we can select the best response for Player A for each strategy of Player B - cells marked red in Table 13.

Table 13: Best responses for Player A

		Player B							
		(C,C,C)	(C,C,D)	(C,D,C)	(C,D,D)	(D,C,C)	(D,C,D)	(D,D,C	(D,D,D)
Player A	C	9/3	**8/3**	4/3	3/3	7/3	**6/3**	2/3	1/3
	D	**12/3**	7/3	**11/3**	**6/3**	**8/3**	3/3	**7/3**	**2/3**

Now we have to determine the best responses of Player B against Player A of the three different strategies - marked by green cells in Tables 14, 15 and 16.

Table 14: Prisonners dilemma

	C	D
C	2	**5**
D	0	**1**

Table 15: Stag-Hunt game

	C	D
C	**5**	2
D	0	**1**

Table 16: Snowdrift game

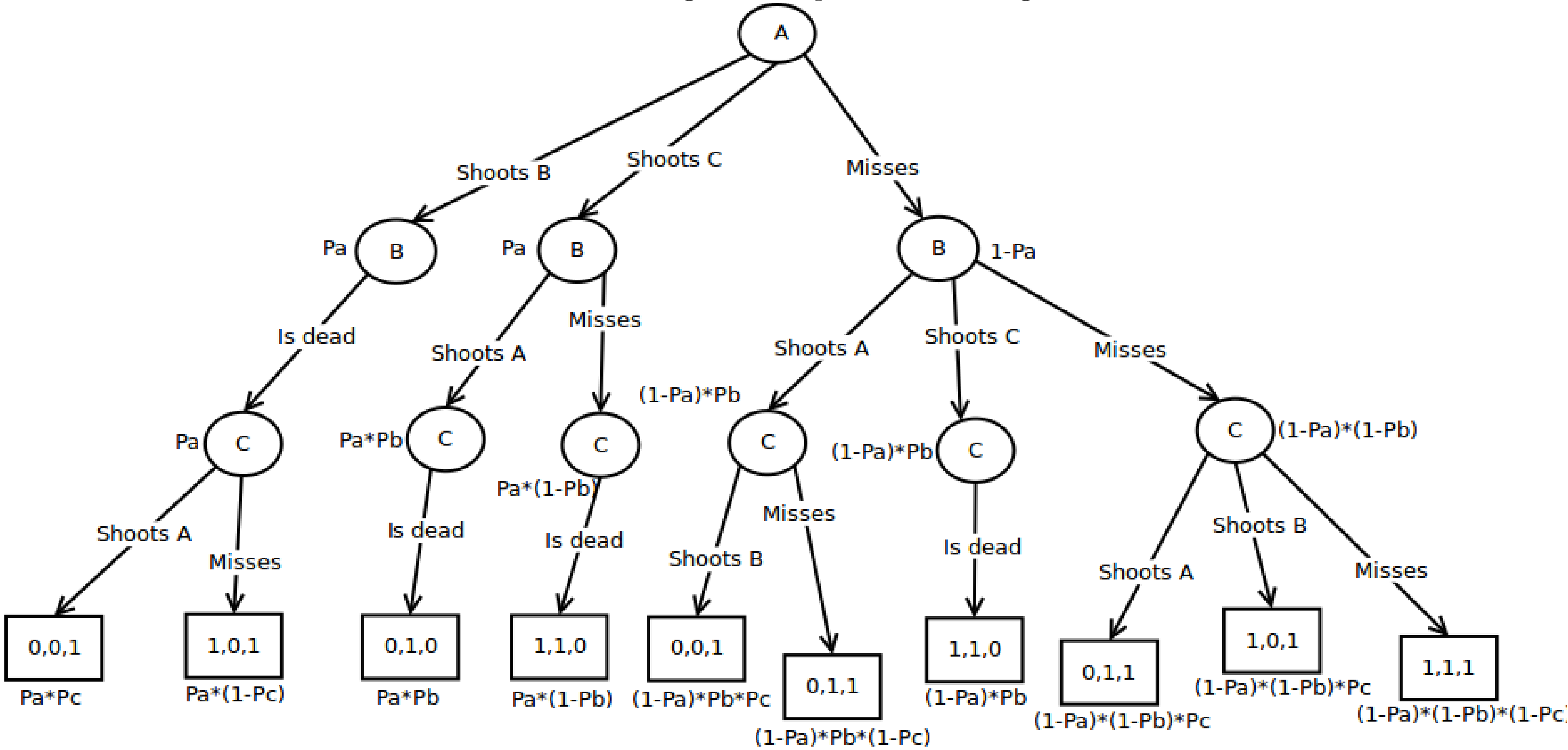
	C	D
C	2	**5**
D	**1**	0

The pure strategy Nash Equilibria can now be determined by matching these results. We find two Nash Equilibria at {C,(D,C,D)} and {D,(D,D,C)}.

3 Sequential truel

This scenario considers three persons A,B and C, each of whom has a gun with a single bullet. If alive, each person may shoot at any surviving person. The order in which the scenario is played out is A, then B and then C. The probability that player i hits their target is denoted by p_i where $0 \leq p_i \leq 1$. Every player wishes to optimize her probability of survival. For this exercise we further assume that a player has to target another living player and is not allowed to miss a shoot consciously. The resulting diagram of this game is shown in Figure 1.

Figure 1: Sequential truel diagram



The Diagram shows all possible actions of all players with all possible outcomes. The lowest level leaf indicates which player is still alive and with which probability - [0,0,1] indicates Player A and B being dead while Player C is alive with a probability of $p_a \times p_c$.

References

[1] J. Maynard Smith and G. R. Price. The logic of animal conflict. *Nature*, 246(5427):15–18, 1973.