Quantificational Logic

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2.1. Quantifiers

1. Analyse the logical forms of these statements.
(a)Anyone who has forgiven at least one person is a saint.
$\forall x(\exists y Forgives(x, y) \Rightarrow Saint(x)$
(b)Nobody in the calculus class is smarter than everybody in the discrete maths class.
$\neg\exists x \in calculusclass(\forall y \in discretemathsclass, smarter(x, y)$
(c)Everyone likes Mary, except Mary herself.
$\forall x, x \neq Mary \Rightarrow likes(x, Mary)$
(d) Jane saw a police officer, and Roger saw one too.
$\exists a, b \in \{Police\} \wedge saw(Jane, a) \wedge saw(Roger, b)$
(e) Jane saw a police officer, and Roger saw him too.
$\exists p \in \{Police\} \wedge saw(Jane, p) \wedge saw(Roger, p)$

4. Translate the following statements into idiomatic English.
(a)$\forall x[(H(x) \wedge \neg\exists M(x, y)) \Rightarrow U(x)$
All unmarried men are unhappy.
(b)$\exists z(P(z, x) \wedge S(z, y) \wedge W(y))$
There is a parent who has a sister.

7. Are these statements true or false? The universe of discourse is $\mathbb{N}$.
(a)$\forall x\exists y(2x - y = 0)$ True
(b)$\exists y\forall x(2x - y = 0)$ False, because there is no one number which can be subtracted from 2 times all natural numbers to equal zero.
(c)$\forall x\exists y(x - 2y = 0)$ False, because the statement fails for $x = 1$
(d)$\forall x(x < 10 \Rightarrow \forall y(y < x \Rightarrow y < 9))$ True
(e)$\exists y\exists z(y + z = 100)$ True (e.g. $y = 1$ and $z = 99$)
(f)$\forall x\exists y(y > x \wedge \exists z(y + z = 100))$ False, because it is untrue for all cases where $x >= 99$.

Equivalences Involving Quantifiers

1.Negate these statements and then reexpress the results as equivalent positive statements.

(a) Everyone who is majoring in maths has a friend who needs help with his homework.

$\neg\forall x(M(x) \wedge F(x,y) \Rightarrow H(y)$

$\exists x(M(x) \wedge F(x,y) \Rightarrow \neg H(y)$

(b)Everyone has a roommate who dislikes everyone.

$\neg\forall x(R(x,y) \wedge \forall(x)D(y,x))$

$\exists x(R(x,y) \wedge \exists x \neg D(y,x)$

(c) $A \cup B \subseteq C \setminus D$

$\exists x(x \in A \vee x \in B) \wedge (x \notin C \vee x \in D)$

(d) $\exists x \forall y[y > x \Rightarrow \exists z(z^2 + 5z = y)]$

$\neg\exists x \forall y[y > x \Rightarrow \exists z(z^2 + 5z = y)]$

$\forall x \forall y[y > x \Rightarrow \neg\exists z(z^2 + 5z = y)]$