Chapter 3 - Proofs

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3.1. Proof Strategies

1. **Theorem.** *Suppose n is an integer larger than 1 and n is not prime. Then* $2^n - 1$ *is not prime.*
(a) Hypotheses:
n is an integer larger than 1
n is not prime
Conclusion:
$2^n - 1$ is not prime
When $n = 6$ both the hypotheses are true, because $6 > 1$ and 6 is not prime $(2 * 3 = 6)$.
The theorem says that $2^6 - 1$ is not prime, and this is correct since $2^6 - 1 = 63$ and 63 is not prime $(9 * 7 = 63)$.

(b) For the case $n = 15$, $n > 1$ and n is not prime $(15 = 3 * 5)$, so the hypotheses are true. The conclusion is also true because $2^{15} - 1 = 32767$ and 32767 is not prime $(32767 = 151 * 31 * 7)$.

(c) For the case $n = 1$, $11 > 1$ and 11 is prime. Because one of the hypotheses is false, nothing can be concluded from the theorem in this case.

2. **Theorem.** *Suppose that* $b^2 > 4ac$. *Then the quadratic equation* $ax^2+bx+c = 0$ *has exactly two real solutions.*
(a) Hypothesis: $b^2 > 4ac$ Conclusion: The quadratic equation $ax^2 + bx + c = 0$ has exactly two real solutions.

(b) To give an instance of the theorem, values of a,b and c must be specified, but not x. This is because the values of a, b and c must be known to decide if the hypothesis is true. The value of x is unspecified because the theorem applies to all possible values of x.

(c)$a = 2, b = 5, c = 3$
$5^2 > 4 * 2 * 3 \equiv 25 > 24$ which is true, so the hypothesis holds.
$2x^2 + 5x + 3 = 0$
$2 * 3 = 6$

$2 + 3 = 5$
$2x^2 + 2x + 3x + 3$
$2x(x + 1) + 3(x + 1)$
$(2x + 3)(x + 1) = 0$
$x = -1$ or $x = -3/2$ so there are exactly two real solutions.
The conclusion is correct.

(d)$a = 2, b = 4, c = 3$
$4^2 > 4 * 2 * 3 \equiv 16 > 24$ which is false. Since the hypothesis is false, nothing can be concluded from the theorem in this case.

3. **Incorrect Theorem.** *Suppose n is a natural number larger than 2, and n is not a prime number. Then* $2n + 13$ *is not a prime number.*
Hypotheses: n is a natural number larger than 2
n is not a prime number
Conclusion: $2n + 13$ is not a prime number
Counterexample: $n = 8$
$8 > 2$ and 8 is not prime, so the hypotheses are true.
$2 * 8 + 13 = 29$ which is prime, so the conclusion is false.
The hypotheses are true, but the conclusion is false, so the theorem is incorrect.