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Bakalářská práce

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Aspekty přechodu od kvantové mechaniky ke kvantové teorii pole

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Abstrakt

V této bakalářské/diplomové/rigorózní práci se věnujeme ...

Abstract

In this thesis we study ...

Místo tohoto listu vložte kopii oficiálního (podepsaného) zadání práce.

Poděkování

Na tomto místě bych chtěl(-a) poděkovat ...

Prohlášení

Prohlašuji, že jsem svoji bakalářskou/diplomovou/rigorózní práci vypracoval(-a) samostatně s využitím informačních zdrojů, které jsou v práci citovány.

Brno 1. května 2018 . . . .

Milan Suk

Contents

Nomenclature . . . . **xv**

Introduction **1**

Kapitola 1. Relativistic quantum mechanics **3**
- 1.1 Canonical quantization 3
- 1.2 Klein-Gordon equation 4
- 1.3 Dirac equation 4

Kapitola 2. Failures of the Relativistic Quantum Mechanics **5**
- 2.1 5
- 2.2 Klein-Gordon equation 5
- 2.3 Dirac equation 5

Conclusion **7**

Bibliography **9**

Nomenclature

$\mathbb{C}$ množina všech komplexních čísel
$\mathbb{R}$ množina všech reálných čísel
$\mathbb{Z}$ množina všech celých čísel

Introduction

The Quantum Mechanics (QM) is a fundamental theory in physics which aims to describe the world at the lowest scales of measured units. It had a great success of explaining a lot of new observations in the very beginning of 20th century, e.g. spectra of atoms, the black-body radiation or photoelectric effect. In 1905, Albert Einstein published his famous article *On the Electrodynamics of Moving Bodies* and proposed the Special theory of relativity (STR). STR corrects the classical mechanics in a way that it can predict a state of a physical system having *relativistic velocities* and setups a new restrictions for mathematical formulations of physical theories.

A logical step is to develop a physical theory which preserves STR and is capable of describing particles in quantum mechanics. This theory is known as Relativistic Quantum Mechanics (RQM) and it has been successful in prediction of interesting phenomena in physics like antimatter or spin. Nevertheless, RQM can't be introduced without formal inconsistencies and it cant deal with varying number of particles. The modern framework which consistently combines QM and STR is the Quantum Field Theory (QFT).

The thesis consists of three main chapters. In the first one we will discuss the formulation of RQM, i.e. we will construct two relativistic quantum mechanical eqautions - the Klein-Gordon equation and the Dirac equation. In the second chapter we will focus on the failures of these formulations and we show the need for a new theory. In the last chapter we will briefly introduce the new framework of QFT. In the terms of Quantum scalar fields we will try to check whether the new formulation solves the problems described in the second chapter.

Chapter 1

Relativistic quantum mechanics

In this chapter (TODO)

1.1 Canonical quantization

Canonical transformation is a process of transition from a classical theory to the quantum one, i.e. procedure for quantizing the theory. The term *canonical* comes from (TODO).

Definition 1. *Given two functions $f(\vec{q}, \vec{p}, t)$ and $g(\vec{q}, \vec{p}, t)$, the binary operation $\{f, g\}$ is called the* ***Poisson bracket*** *of these functions and takes the form*

$$\{f, g\} = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right) \tag{1.1}$$

Theorem 1. *The Poisson brackets of canonical coordinates q_i and p_i are*

$$\begin{aligned} \{q_i, q_j\} &= 0 \\ \{p_i, p_j\} &= 0 \\ \{q_i, p_j\} &= \delta_{ij} \end{aligned} \tag{1.2}$$

Proof. The proof of 1 can be easily done from the definition 1. The first two equations are manifestly zero because $\frac{\partial q_i}{\partial p_j} = \frac{\partial p_i}{\partial q_j} = 0$ and the last can one be proved as follows.

$$\{q_i, p_j\} = \sum_{k=1}^{N} \left(\frac{\partial q_i}{\partial q_k} \frac{\partial p_j}{\partial p_k} - \frac{\partial q_i}{\partial p_k} \frac{\partial p_j}{\partial q_k} \right) = \sum_{k=1}^{N} \frac{\partial q_i}{\partial q_k} \frac{\partial p_j}{\partial p_k} = \begin{cases} 1 & \text{if } i = j = k \\ 0 & \text{otherwise} \end{cases} \tag{1.3}$$
$$\implies \{q_i, p_j\} = \delta_{ij}$$

□

Using the definition 1

1.2 Klein-Gordon equation

1.3 Dirac equation

Chapter 2

Failures of the Relativistic Quantum Mechanics

In this chapter

2.1

2.2 Klein-Gordon equation

2.3 Dirac equation

Conclusion

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