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DAA Tutorial-2

(Q1.) What is the time complexity of the below code.

```
void fun(int n)
{
  int j=1, i=0;
  while (i<j)
  {
     i = i+j;
     j++;
  }
}
```

Ans Time complexity = $O(\sqrt{n})$

$\because$ 1st time $i=1$

2nd time $i=3$ $(i=1+2)$

3rd time $i=6$ $(i=1+2+3)$

$\vdots$

x^{th} time $i = \frac{x(x+1)}{2} = x^2 < n$

$\therefore x = O(\sqrt{n})$

(Q2.) Write recurrence relation for the recursive function that prints fibonacci series. Solve the recurrence relation to get complexity of the program. What will be the space complexity of this program & why?

Ans * $fib(n) = fib(n-1) + fib(n-2)$

```
fib(n)
{ if n<=1
     return 1;
```

return (fib(n-1)+fib(n-2));

Time complexity:

Let $T(0)=1$

$T(n) = T(n-1)+T(n-2)+c$

$= 2*T(n-2)+c$ (Let $T(n-1) \simeq T(n-2)$)

$T(n-2) = 2*(2T(n-2-2))$

$= 2*(2T(n-2)+c)+c$

$= 4T(n-2)+3c$

$T(n-4) = 2*(4T(n-2)+3c)+4c$

$= 8T(n-3)+7c$

$= 2^k \times T(n-k)+(2^k-1)c$

$\Rightarrow n-k=0 \Rightarrow n=k$

$T(n) = 2^n * T(0)+(2^n-1)c$

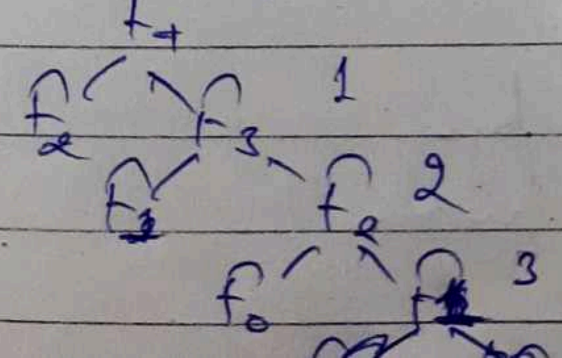
$= 2^n * 1+2^n c-c$

$= 2^n(1+c)-c$

$\simeq 2^n$ // Constants can be ignored

Space complexity: The space is proportional to the maximum depth of the recursion tree.

For e.g.: for f_4



∴ space complexity of fibonacci series = $O(N)$.

(Q3.) Write programs which have complexity - $n(\log n)$, n^3, $\log(\log n)$

Ans Merge sort - $n \log n$

- For time complexity = n^3.

We can use three nested loops - $O(n^3)$

```
for (int i=0; i<n; i++)
 {
  for (int j=0; j<n; j++)
  {
   for (int k=0; k<n; k++)
   {
       // Some O(1) expressions
   }
  }
 }
```

=> for time complexity = $\log(\log n)$

We can use the following function

```
for (int i=2; i<n; i=pow(i,k))
 {
   // some O(1) expression
 }
```

where k is constant

=> for time complexity = $n \log n$

We can use the following function

```
int fun(int n)
{ for (i=1; i<=n; i++)
  {
     for(j=1; j<=n; j+=i
     {// some O(1) expressions } } }
```

Q4.) Solve the following recurrence relation T(n) = T(n/4) + T(n/2) + n^2

$T(n) = 2T(n/2) + cn^2$

Ans. Using master's method $T(n) = aT(n/b) + f(n)$

$a \geq 1$, $b > 1$, $c = \log_b a$ (comparing n^c & $f(n)$)

We get $c = \log_2 2 = 1$

$\Rightarrow f(n) > n^c \quad \because n^2 > 1$

$\therefore T(n) = \Theta(f(n))$

$\Rightarrow T(n) = \Theta(n^2)$

Q5.) What is the time complexity of the following function

```
int fun(int n)
{
  for(int i=1; i<=n; i++)
  {
    for (int j=1; j<n; j+=i)
  } {
      // some O(1) expression
    }
```

Ans. for $i=1 \rightarrow j = 1, 2, 3, 4, \ldots n$ (sum for n terms)

for $i=2 \rightarrow j = 1, 3, 5, 7 \ldots$ (sum for n/2 terms)

for $i=3 \Rightarrow j = 1, 4, 7, \ldots$ (sum for n/3 terms)

$T(n) = n + n/2 + n/3 + n/4 + \ldots$

$= n(1 + 1/2 + 1/3 + 1/4 + \ldots)$

$= n\int_1^n 1/x = n\int_1^n dx/x = \log n \int_1^n dx = n\log n$

$\therefore$ The time complexity is $n\log n$

(Q6.) What should be the time complexity of following function

```
for(int i = 2; i < n; i = pow(i, k))
{
    // Some O(1) expressions of statement
}
```

where k is a constant.

Ans

for first iteration $i = 2$

2^{nd} iteration $i = 2^k$

3^{rd} iteration $i = (2^k)^k = 2^{k^2}$

⋮

n^{th} iteration $i = (2^{k})^{i}$, loop ends at $n = 2^{k^i}$

∴ applying log, $\log n = \log 2^{k^i} \Rightarrow \log n = k^i$

Applying log again, $\log(k^i) = \log(\log n) \Rightarrow i = \log(\log n)$

(Q7.) Write a recurrence relation when quick sort repeatedly divides the array into two parts of 99% & 1%. Derive the time complexity & find the difference in heights of both the extreme parts. What do you understand by analysis?

Ans

n

$1/10\,n \quad 9/10\,n \rightarrow n$

$1/100\,n \quad 9/100\,n \quad 9n/100 \quad 81n/100 \rightarrow n$

$\frac{81n}{1000} \quad \frac{729n}{100} \rightarrow n$

∴ If we split in the maximum

Recurrence relation: $T(n) - T(9n/10) + T(n/10) + O(n)$

where first branch is of $9n/10$ & second is of $n/10$
Solving the above using recursion-tree approach calculating value

At 1^{st} level, value $= n$

At 2^{nd} level, value $= \frac{9n}{10} + \frac{n}{10} = n$

Values remain same at all levels of n.

Time complexity = Summation of values

$= O(n \times \log_{10/9} n)$ (upper bound)

$= \Omega(n \log_{10} n)$ (lower bound)

$= O(n \log_2 n)$

(Q8.) Arrange the following in increasing order of rate of growth:

a.) $n, n!, \log n, \log(\log n), \log(n!), n\log n, \log^2 n, 2^n, 2^{e^n}, 4^n, n^2, 100.$

Ans $100 < \log(\log n) < \log n < \log^2 n < \sqrt{n} < n < n\log n < n^2 < 2^n < 4^n < 2^{e^n} < \log(n!) < n!$

b.) $2(2^n), 4n, 2n, 1, \log(n), \log(\log n), \sqrt{\log(n)}, \log 2n, 2\log(n), n, \log(n!), n!, n^2, n\log n.$

Ans $1 < \log(\log n)) < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < 2n < 4n < n\log n < n^2 < \log(n!) < n! < 2(2^n)$

c.) $8^{2n}, \log_k n, n\log_k n, \log(n!), n!, \log_8(n), 96, 8n^2, 7n^3, 5n.$

Ans $96 < \log_8(n) < \log_2(n) < \log_2 5n < n\log_c n < n\log_e n < 7n\log_2 n < n! < \log n! < n^k < 8^{2n}$