

1. Write the number of all possible matrices of order  $2 \times 3$  with each entry 1 or 2.

2. If  $(A) = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$  and  $(B) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$  and  $(BA) = (b_{ij})$ , Find  $b_{21} + b_{32}$ .

3. If  $\vec{a} = 4$ ,  $\vec{b} = 3$  and  $\vec{a}, \vec{b} = 6\sqrt{3}$ , the find the value of  $\vec{a} \times \vec{b}$

4. What are the coordinates of the point which is the reflection of the point  $(\alpha, \beta, \gamma)$  in the  $XZ$  - plane?

5. Write the value of  $(A) = \begin{bmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{bmatrix}$

6. Find the position vector of the point which divides the join of points with position  $\vec{a} + 3\vec{b}$  and  $\vec{a} \cdot \vec{b}$  internally in the ratio 1 : 3.

7. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less. Using matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision?

8. Show that the function  $f$  is given by:  $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$  is discontinuous at  $x = 0$ .

9. Evaluate:

$$\int_1^5 |x-1| + |x-2| + |x-3|, dx$$

10. Evaluate:

$$\int_0^\pi \frac{x \sin x}{1 + 3 \cos^2 x}, dx$$

11. Find:

$$\int (3x + 5) \sqrt{5 + 4x - 2x^2} dx$$

12. Solve the differential equation :

$$(x^2 + 3xy + y^2), dx - x^2, dy = 0 \text{ given that } y = 0, \text{ when } x = 1.$$

13. Solve for

$$x : \tan^{-1} \frac{2-x}{2+x} = \frac{1}{2} \tan^{-1} \frac{x}{2}, x > 0$$

14. Prove that  $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$

15. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

16. A random variable  $X$  has the following probability distributio:

X	0	1	2	3	4	5	6
P(X)	c	2c	2c	3c	$c^2$	$2c^2$	$7c^2+c$

Find the value of  $c$  and also calculate mean of the distribution.

17. Show that the lines  $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z-1}{0}$  and  $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$  intersect. Find their point of intersection.

18.  $x = e^{\cos 2t}$  and  $y = e^{\sin 2t}$ , prove that  $\frac{dy}{dx} = \frac{-y \log x}{x \log y}$ .

19. Verify mean value theorem for the function  $f(x) = 2 \sin x + \sin 2x$  on  $[0, \pi]$ .

20. Find the angle between the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  if  $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ , and hence find a vector perpendicular to both  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$ .

21. Find the equation of the tangent line to the curve  $y = \sqrt{5x-3} - 5$ , which is parallel to line  $4x - 2y + 5 = 0$ .

22. Solve the differential equation :  $x \frac{dy}{dx} + y - x + xycotx = 0; x \neq 0$ .

23. Find :

$$\int \frac{2x+1}{(x^2+1)(x^2+4)} dx$$

24. Solve for  $x$  :  $\begin{bmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{bmatrix} = 0$ , using properties of determinants.

25. Using elementary row operations find the inverse of matrix  $(A) = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$  and hence solve the following system of equations  $3x - 3y + 4z = 21, 2x - 3y + 4z = 20, -y + z = 5$ .

26. Using integration, find the area of the angle formed by negative x-axis and tangent and normal to the circle  $x^2 + y^2 = 9$  at  $(-1, 2\sqrt{2})$ .

27. A company manufactures two types of cardigans type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹72,000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B. Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.

28. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning. if A starts first.

29. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height  $h$  and semi-vertical angle  $\alpha$  is one-third that of the cone and the greatest volume of the cylinder is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ .

30. Find the intervals in which the function  $f(x) = \frac{4 \sin x}{2 + \cos x} - x; 0 \leq x \leq 2\pi$  is strictly increasing or strictly decreasing.

31. Find the coordinates of the foot of perpendicular and perpendicular distance from the point  $P(4, 3, 2)$  to the plane  $x + 2y + 3z = 2$ . Also find the image of  $P$  in the plane.
32. Show that the relation  $R$  defined by  $(a, b) R (c, d) \Rightarrow a + d = b + c$  on the  $A \times A$ , where  $a = \{1, 2, 3, \dots, 10\}$  is an equivalence relation. Hence write the equivalence class  $[(3, )]$ ;  $a, b, c, d \in A$ .