

1. Let  $p_n(k)$  be the number of permutations of the set  $\{1, \dots, n\}$ ,  $n \geq 1$ , which have exactly  $k$  fixed points. Prove that

$$\sum_{k=0}^n k \cdot p_n(k) = n$$

(Remark: A permutation  $f$  of a set  $S$  is one-to-one mapping of  $S$  onto itself. An element  $i$  in  $S$  is called a fixed point of the permutation  $f$  if  $f(i)=i$ .)

2. In an acute-angled triangle  $ABC$  the interior bisector of the angle  $A$  intersects  $BC$  at  $L$  and intersects the circumcircle of  $ABC$  again at  $N$ . From point  $L$  perpendiculars are drawn to  $AB$  and  $AC$ , the feet of these perpendiculars being  $K$  and  $M$  respectively. Prove that the quadrilateral  $AKNM$  and the triangle  $ABC$  have equal areas.
3. Let  $x_1, x_2, \dots, x_n$  be real numbers satisfying  $x_1^2 + x_2^2 + \dots + x_n^2 = 1$ . Prove that for every integer  $k \geq 2$  there are integers  $a_1, a_2, \dots, a_n$ , not all 0, such that  $|a_i| \leq k - 1$  for all  $i$  and

$$|a_1x_1 + a_2x_2 + \dots + a_nx_n| \leq \frac{(k-1)\sqrt{n}}{k^n - 1}$$

4. Prove that there is no function  $f$  from the set of non-negative integers into itself such that  $f(f(n)) = n + 1987$  for every  $n$ .
5. Let  $n$  be an integer greater than or equal to 3. Prove that there is a set of  $n$  points in the plane such that the distance between any two points is irrational and each set of three points determines a non-degenerate triangle with rational area.
6. Let  $n$  be an integer greater than or equal to 2. Prove that if  $k^2 + k + n$  is prime for all integers  $k$  such that  $0 \leq k \leq \sqrt{n/3}$ , then  $k^2 + k + n$  is prime for all integers  $k$  such that  $0 \leq k \leq n - 2$