

## Practice assignment Solution

### Linear Regression in R

```
#Boston Pricing case study
```

```
setwd("C:\\")
```

```
##Loading Data
```

```
prices<-read.csv("boston_prices.csv",header=TRUE,stringsAsFactors=FALSE)
```

```
##Checking Data Characteristics
```

```
dim(prices)
```

```
## [1] 506 14
```

```
str(prices)
```

```
## 'data.frame': 506 obs. of 14 variables:
```

```
## $ CRIM : num 0.00632 0.02731 0.02729 0.03237 0.06905 ...
```

```
## $ ZN : num 18 0 0 0 0 0 12.5 12.5 12.5 12.5 ...
```

```
## $ INDUS : num 2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 ...
```

```
## $ Charles.River.dummy.variable: num 0 0 0 0 0 0 0 0 0 0 ...
```

```
## $ nitric.oxides.concentration : num 0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 0.524 ...
```

```
## $ X.rooms.dwelling : num 6.58 6.42 7.18 7 7.15 ...
```

```
## $ AGE : num 65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ...
```

```
## $ DIS : num 4.09 4.97 4.97 6.06 6.06 ...
```

```
## $ RAD : int 1 2 2 3 3 3 5 5 5 5 ...
```

```
## $ TAX : num 296 242 242 222 222 222 311 311 311 311 ...
```

```
## $ PTRATIO : num 15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 ...
```

```
## $ B : num 397 397 393 395 397 ...
```

```
## $ LSTAT : num 4.98 9.14 4.03 2.94 5.33 ...
```

```
## $ MEDV : num 24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...
```

```
head(prices)
```

```
##      CRIM  ZN  INDUS Charles.River.dummy.variable
## 1 0.00632 18   2.31                               0
## 2 0.02731  0   7.07                               0
## 3 0.02729  0   7.07                               0
## 4 0.03237  0   2.18                               0
## 5 0.06905  0   2.18                               0
## 6 0.02985  0   2.18                               0

##   nitric.oxides.concentration X.rooms.dwelling  AGE    DIS RAD TAX PTRATIO
## 1                        0.538                6.575 65.2 4.0900    1 296    15.3
## 2                        0.469                6.421 78.9 4.9671    2 242    17.8
## 3                        0.469                7.185 61.1 4.9671    2 242    17.8
## 4                        0.458                6.998 45.8 6.0622    3 222    18.7
## 5                        0.458                7.147 54.2 6.0622    3 222    18.7
## 6                        0.458                6.430 58.7 6.0622    3 222    18.7

##      B LSTAT MEDV
## 1 396.90  4.98 24.0
## 2 396.90  9.14 21.6
## 3 392.83  4.03 34.7
## 4 394.63  2.94 33.4
## 5 396.90  5.33 36.2
## 6 394.12  5.21 28.7
```

```
names(prices)
```

```
## [1] "CRIM"                "ZN"
## [3] "INDUS"               "Charles.River.dummy.variable"
## [5] "nitric.oxides.concentration" "X.rooms.dwelling"
## [7] "AGE"                 "DIS"
## [9] "RAD"                 "TAX"
## [11] "PTRATIO"            "B"
## [13] "LSTAT"              "MEDV"
```

```
#summary statistics
```

```
summary(prices)
```

```
##      CRIM      ZN      INDUS
##  Min.   :0.00000  Min.   : 0.0  Min.   : 0.000
```

```
## 1st Qu.:0.04944 1st Qu.: 0.0 1st Qu.: 3.440
## Median :0.14466 Median : 0.0 Median : 6.960
## Mean :1.26920 Mean : 13.3 Mean : 9.205
## 3rd Qu.:0.81962 3rd Qu.: 18.1 3rd Qu.:18.100
## Max. :9.96654 Max. :100.0 Max. :27.740
##
## Charles.River.dummy.variable nitric.oxides.concentration
## Min. :0.0000 Min. :0.385
## 1st Qu.:0.0000 1st Qu.:0.449
## Median :0.0000 Median :0.538
## Mean :0.1408 Mean :1.101
## 3rd Qu.:0.0000 3rd Qu.:0.647
## Max. :1.0000 Max. :7.313
##
## X.rooms.dwelling AGE DIS RAD
## Min. : 3.561 Min. : 1.137 Min. : 1.130 Min. : 1.00
## 1st Qu.: 5.962 1st Qu.: 32.000 1st Qu.: 2.431 1st Qu.: 4.00
## Median : 6.322 Median : 65.250 Median : 3.926 Median : 5.00
## Mean : 15.680 Mean : 58.745 Mean : 6.173 Mean : 78.06
## 3rd Qu.: 6.949 3rd Qu.: 89.975 3rd Qu.: 6.332 3rd Qu.: 24.00
## Max. :100.000 Max. :100.000 Max. :24.000 Max. :666.00
##
## TAX PTRATIO B LSTAT
## Min. : 20.2 Min. : 2.60 Min. : 0.32 Min. : 1.730
## 1st Qu.:254.0 1st Qu.: 17.00 1st Qu.:365.00 1st Qu.: 6.878
## Median :307.0 Median : 18.90 Median :390.66 Median :10.380
## Mean :339.3 Mean : 42.62 Mean :332.79 Mean :11.538
## 3rd Qu.:403.0 3rd Qu.: 20.20 3rd Qu.:395.62 3rd Qu.:15.015
## Max. :711.0 Max. :396.90 Max. :396.90 Max. :34.410
##
## MEDV
## Min. : 6.30
## 1st Qu.:18.50
## Median :21.95
```

```
## Mean :23.75
## 3rd Qu.:26.60
## Max. :50.00
## NA's :54

#Missing values treatment
colSums(is.na(prices)) #MEDV has a lot of missing values

##          CRIM          ZN
##          0          0
##      INDUS Charles.River.dummy.variable
##          0          0
## nitric.oxides.concentration      X.rooms.dwelling
##          0          0
##          AGE          DIS
##          0          0
##          RAD          TAX
##          0          0
##      PTRATIO          B
##          0          0
##          LSTAT          MEDV
##          0          54

summary((prices$MEDV))

##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.   NA's
##      6.30  18.50   21.95   23.75   26.60   50.00    54

prices$MEDV[is.na(prices$MEDV)]<-mean(prices$MEDV,na.rm=TRUE)

#Outlier plots
par(mfrow=c(2,7)) #This allows you to plot 14 charts on a single page; It is optional.
list<-names(prices) #Store the names of the dataset in a list format
list<-list[-4]

for(i in 1:length(list)) #Plot the boxplots of all variables and shortlist which ones need outlier treatment.
{
  boxplot(prices[,list[i]],main=list[i])
}
```

```
#Restore the par parameters to normal
dev.off()

## null device
##          1

#In this solution, We have replaced the outlier values by the median values
#You can decide to replace by max or mean values based on business objectives

#Outlier treatment
for(i in 1:length(list)) ##For loop to replace all the outlier values with the
mean value ; if you want you can replace with median value as well.
{
  x<-boxplot (prices[,list[i]])
  out<-x$out
  index<-which (prices[,list[i]] %in% x$out)
  prices [index,list[i]]<-mean(prices[,list[i]])
  rm(x)
  rm(out)
}

#Exploratory analysis
library(ggplot2)
## Warning: package 'ggplot2' was built under R version 3.2.2

#Study the histogram of the DV and the transformed histogram
hist (prices$MEDV)

#hist(prices$log_MEDV) #Once you create the transformations;look down

#You can look at the correlation between each IDV and the DV
#An eg :
ggplot (prices,aes (x=MEDV,y=LSTAT)) +geom_point ()
ggplot (prices,aes (x=MEDV,y=DIS)) +geom_point ()
ggplot (prices,aes (x=MEDV,y=AGE)) +geom_point ()

#Inorder to quicken the process, lets write a function :
```

```
#Below is a function that gives you the correlation values between all IDV's  
and the DV
```

```
#Simply taking a look at the output of this function, you can quickly shortli  
st
```

```
#Which all IDV's are correlated to the DV
```

```
#Function to get the list of correlations between : DV and the IDV's
```

```
list1<-list[-13]
```

```
for(i in 1:length(list1))
```

```
{
```

```
  x<-cor (prices$MEDV,prices[list[i]])
```

```
  print(x)
```

```
}
```

```
##          CRIM
```

```
## [1,] -0.4622118
```

```
##          ZN
```

```
## [1,] 0.4172775
```

```
##          INDUS
```

```
## [1,] -0.4981729
```

```
##      nitric.oxides.concentration
```

```
## [1,]                -0.1336743
```

```
##      X.rooms.dwelling
```

```
## [1,]          0.2442593
```

```
##          AGE
```

```
## [1,] -0.4615408
```

```
##          DIS
```

```
## [1,] 0.3322806
```

```
##          RAD
```

```
## [1,] 0.03218742
```

```
##          TAX
```

```
## [1,] -0.2989045
```

```
##          PTRATIO
```

```
## [1,] 0.01971893
```

```
##          B
```

```
## [1,] 0.1439405
```

```
##          LSTAT
## [1,] -0.6535546

#Significant variables are : B LSTAT AGE X.rooms.dwelling nitric.oxides.conce
ntration INDUS

#You can also try to use data transformations

#Log transformations
#Create the log transformation for all variables
prices$log_CRIM<-log(prices$CRIM)
prices$log_ZN<-log(prices$ZN)
prices$log_NOX<-log(prices$nitric.oxides.concentration)
prices$log_RM<-log(prices$X.rooms.dwelling)
prices$log_AGE<-log(prices$AGE)
prices$log_DIS<-log(prices$DIS)
prices$log_RAD<-log(prices$RAD)
prices$log_TAX<-log(prices$TAX)
prices$log_PTRATIO<-log(prices$PTRATIO)
prices$log_B<-log(prices$B)
prices$log_LSTAT<-log(prices$LSTAT)
prices$log_MEDV<-log(prices$MEDV) #DV
prices$log_INDUS<-log(prices$INDUS)

#Refer to the profiling excel sheet to see all the correlations documented

#Function to get the list of correlations between : log_DV and log of IDV's

list_log<-names(prices)[c(15:25,27)]
for(i in 1:length(list_log))
{
  xlog<-cor(prices$log_MEDV,prices[list_log[i]])
  print(xlog)
}

##          log_CRIM
```

```
## [1,]      NaN
##      log_ZN
## [1,]      NaN
##      log_NOX
## [1,] -0.193495
##      log_RM
## [1,] 0.316107
##      log_AGE
## [1,] -0.3442683
##      log_DIS
## [1,] 0.3981884
##      log_RAD
## [1,] -0.08203473
##      log_TAX
## [1,] -0.2861208
##      log_PTRATIO
## [1,] -0.01558666
##      log_B
## [1,] 0.14549
##      log_LSTAT
## [1,] -0.6326763
##      log_INDUS
## [1,]      NaN

#Function to get the list of correlations between : log_DV and IDV's

list_log_DV<-names(prices)[1:13]
list_log_DV<-list_log_DV[-4]
for(i in 1:length(list_log_DV))
{
  xlogdv<-cor(prices$log_MEDV,prices[list_log_DV[i]])
  print(xlogdv)
}

##      CRIM
## [1,] -0.4942302
```



```
##                ZN
## [1,] 0.4082063
##                INDUS
## [1,] -0.5102838
##      nitric.oxides.concentration
## [1,]                -0.1237709
##      X.rooms.dwelling
## [1,]                0.2651325
##                AGE
## [1,] -0.4876028
##                DIS
## [1,] 0.3526956
##                RAD
## [1,] 0.04812929
##                TAX
## [1,] -0.2916202
##                PTRATIO
## [1,] 0.04396186
##                B
## [1,] 0.1444425
##                LSTAT
## [1,] -0.6669138

sampling<-sort (sample(nrow(prices), nrow(prices) *.7) )

#Select training sample
train<-prices[sampling,]
test<-prices[-sampling,]

##Building Simple Linear Regression Model

#Metrics :
#Rsquare
#Coefficients
#P values : Significance levels of the IDV's
```

```
#Residuals distribution

#Factor variables as IDV's
#All good modelssummm
Reg<-lm(log_MEDV~CRIM+INDUS+RAD+TAX+B+
        Charles.River.dummy.variable+
        DIS+ZN+PTRATIO+LSTAT+AGE+X.rooms.dwelling+nitric.oxides.con
        centration,data=train)

summary(Reg)

##
## Call:
## lm(formula = log_MEDV ~ CRIM + INDUS + RAD + TAX + B + Charles.River.dummy
. variable +
##      DIS + ZN + PTRATIO + LSTAT + AGE + X.rooms.dwelling + nitric.oxides.co
ncentration,
##      data = train)
##
## Residuals:
##      Min        1Q    Median        3Q        Max
## -0.36319 -0.09713 -0.01709  0.09100  0.46697
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      3.6817197   0.2363575   15.577 < 2e-16 ***
## CRIM             -0.0909268   0.0280985    -3.236  0.001331 **
## INDUS            -0.0015383   0.0020858    -0.738  0.461319
## RAD               0.0007643   0.0014872     0.514  0.607650
## TAX              -0.0002513   0.0001576    -1.595  0.111712
## B                -0.0001346   0.0004983    -0.270  0.787217
## Charles.River.dummy.variable  0.0688731  0.0313088     2.200  0.028493 *
## DIS              -0.0272362   0.0070547    -3.861  0.000135 ***
## ZN                0.0011734   0.0009458     1.241  0.215591
## PTRATIO          -0.0036298   0.0027451    -1.322  0.186964
## LSTAT            -0.0180644   0.0021377    -8.451  8.6e-16 ***
```

```
## AGE                -0.0010204  0.0004969  -2.053  0.040793  *
## X.rooms.dwelling   0.0242811  0.0085611   2.836  0.004839  **
## nitric.oxides.concentration -0.3263059  0.1210834  -2.695  0.007391  **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1503 on 340 degrees of freedom
## Multiple R-squared:  0.5855, Adjusted R-squared:  0.5696
## F-statistic: 36.94 on 13 and 340 DF,  p-value: < 2.2e-16

#Getting the formula
formula(Reg)

## log_MEDV ~ CRIM + INDUS + RAD + TAX + B + Charles.River.dummy.variable +
##      DIS + ZN + PTRATIO + LSTAT + AGE + X.rooms.dwelling + nitric.oxides.co
ncentration

#Getting the formula
formula(Reg)

## log_MEDV ~ CRIM + INDUS + RAD + TAX + B + Charles.River.dummy.variable +
##      DIS + ZN + PTRATIO + LSTAT + AGE + X.rooms.dwelling + nitric.oxides.co
ncentration

#Remove insignificant variables :

Reg1<-lm(log_MEDV~
          Charles.River.dummy.variable+
          DIS+PTRATIO+LSTAT+AGE+X.rooms.dwelling+nitric.oxides.concentration,
data=train)
summary(Reg1)

##
## Call:
## lm(formula = log_MEDV ~ Charles.River.dummy.variable + DIS +
##      PTRATIO + LSTAT + AGE + X.rooms.dwelling + nitric.oxides.concentration
##      ,
##      data = train)
##
## Residuals:
##      Min        1Q    Median        3Q       Max
## -0.40671 -0.09551 -0.01474  0.09706  0.48335
```

```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      3.500469   0.065967   53.064 < 2e-16 ***
## Charles.River.dummy.variable  0.073992   0.031930    2.317 0.021069 *
## DIS              -0.019626   0.006458   -3.039 0.002555 **
## PTRATIO          -0.002046   0.002387   -0.857 0.392011
## LSTAT            -0.017785   0.002095   -8.489 6.20e-16 ***
## AGE              -0.001658   0.000468   -3.543 0.000451 ***
## X.rooms.dwelling   0.038625   0.007067    5.466 8.83e-08 ***
## nitric.oxides.concentration -0.524614   0.096037   -5.463 8.98e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1538 on 346 degrees of freedom
## Multiple R-squared:  0.558, Adjusted R-squared:  0.549
## F-statistic: 62.39 on 7 and 346 DF, p-value: < 2.2e-16

#Reg2 : remove insignificant values

Reg2 <- lm(log_MEDV ~CRIM+INDUS+RAD+TAX+B+
           Charles.River.dummy.variable+
           DIS+ZN+PTRATIO+LSTAT+X.rooms.dwelling+nitric.oxides.concentr
ation, data=train)
summary(Reg2)

##
## Call:
## lm(formula = log_MEDV ~ CRIM + INDUS + RAD + TAX + B + Charles.River.dummy
.variable +
##     DIS + ZN + PTRATIO + LSTAT + X.rooms.dwelling + nitric.oxides.concentr
ation,
##     data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.36466 -0.10644 -0.01634  0.09679  0.46954
```

```
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      3.6716657   0.2374187  15.465 < 2e-16 ***
## CRIM             -0.1093257   0.0267569  -4.086 5.48e-05 ***
## INDUS            -0.0017660   0.0020926  -0.844  0.39931
## RAD               0.0014879   0.0014517   1.025  0.30610
## TAX              -0.0002315   0.0001580  -1.465  0.14381
## B                -0.0002661   0.0004965  -0.536  0.59229
## Charles.River.dummy.variable  0.0669525   0.0314421   2.129  0.03394 *
## DIS              -0.0217655   0.0065631  -3.316  0.00101 **
## ZN                0.0012962   0.0009483   1.367  0.17259
## PTRATIO           -0.0033593   0.0027548  -1.219  0.22353
## LSTAT             -0.0197241   0.0019883  -9.920 < 2e-16 ***
## X.rooms.dwelling   0.0230146   0.0085791   2.683  0.00766 **
## nitric.oxides.concentration -0.3426291   0.1213907  -2.823  0.00504 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.151 on 341 degrees of freedom
## Multiple R-squared:  0.5803, Adjusted R-squared:  0.5655
## F-statistic: 39.29 on 12 and 341 DF, p-value: < 2.2e-16

#Reg3 _ remove insignificant values
Reg3 <- lm(log_MEDV ~CRIM+RAD+
            Charles.River.dummy.variable+
            DIS+ZN+PTRATIO+LSTAT+nitric.oxides.concentration, data=train)
summary(Reg3)

##
## Call:
## lm(formula = log_MEDV ~ CRIM + RAD + Charles.River.dummy.variable +
##     DIS + ZN + PTRATIO + LSTAT + nitric.oxides.concentration,
##     data = train)
##
## Residuals:
```

```
##      Min      1Q   Median      3Q      Max
## -0.39258 -0.09839 -0.01661  0.09708  0.48898
##
## Coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      3.6261434   0.0807548  44.903 < 2e-16 ***
## CRIM             -0.1454189   0.0234353  -6.205 1.57e-09 ***
## RAD              0.0040390   0.0012692   3.182 0.001594 **
## Charles.River.dummy.variable  0.0836902   0.0313997   2.665 0.008053 **
## DIS             -0.0206337   0.0061179  -3.373 0.000829 ***
## ZN              0.0016759   0.0009496   1.765 0.078492 .
## PTRATIO         -0.0033216   0.0027816  -1.194 0.233255
## LSTAT           -0.0227573   0.0017663 -12.884 < 2e-16 ***
## nitric.oxides.concentration -0.3007190   0.1132555  -2.655 0.008293 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.153 on 345 degrees of freedom
## Multiple R-squared:  0.5638, Adjusted R-squared:  0.5537
## F-statistic: 55.74 on 8 and 345 DF,  p-value: < 2.2e-16
#Some other combination
Reg4<-lm(log_MEDV~INDUS +ZN + X.rooms.dwelling + LSTAT+CRIM + Charles.River.
dummy.variable,data=train)
summary(Reg4)
##
## Call:
## lm(formula = log_MEDV ~ INDUS + ZN + X.rooms.dwelling + LSTAT +
##      CRIM + Charles.River.dummy.variable, data = train)
##
## Residuals:
##      Min      1Q   Median      3Q      Max
## -0.45312 -0.10307 -0.02106  0.09953  0.51897
##
## Coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
```

```
## (Intercept)          3.3044861  0.0361648  91.373 < 2e-16 ***
## INDUS              -0.0010489  0.0018887  -0.555  0.5790
## ZN                 0.0007041  0.0009199   0.765  0.4446
## X.rooms.dwelling    0.0062007  0.0037141   1.669  0.0959 .
## LSTAT              -0.0225876  0.0017457 -12.939 < 2e-16 ***
## CRIM               -0.1044110  0.0230301  -4.534 7.99e-06 ***
## Charles.River.dummy.variable 0.0633570  0.0320178   1.979  0.0486 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1553 on 347 degrees of freedom
## Multiple R-squared:  0.548, Adjusted R-squared:  0.5402
## F-statistic: 70.12 on 6 and 347 DF,  p-value: < 2.2e-16

#The best model happens to be : Reg3

##Getting predicted values
predicted<-predict(Reg3)
plot(predicted)
length(predicted)

## [1] 354

##Finding Residuals
residuals<-resid(Reg3)
plot(residuals)
length(residuals)

## [1] 354

##Plotting Residuals vs Predicted Values
##Checking Heteroskedastcity
##There should be no trend between predicted values and residual values
plot(predicted,residuals,abline(0,0))

#You can notice that there seems to be an inverse pattern for some points

#So this model may not be the preferred model.
```

```

#atttching predicted values to test data
predicted<-predict (Reg3,newdata=test)
length (predicted)

## [1] 152

test$p<-predicted

#Calculating error in the test dataset - (Actual- predicted)/predicted values
test$error<- (test$log_MEDV-test$p) /test$log_MEDV
mean (test$error)*100 #you get to know the average error in the given dataset
## [1] 0.02728412

##Plotting actual vs predicted values
plot (test$p,col="blue",type="l")
lines (test$log_MEDV,col="red",type="l")

#checking for Correlation between variables
library(car)
vif(Reg3)

##                CRIM                RAD
##                2.172322            13.509577
## Charles.River.dummy.variable        DIS
##                1.515615            2.490395
##                ZN                PTRATIO
##                1.667336            7.083645
##                LSTAT  nitric.oxides.concentration
##                1.412882            8.546674

#You can drop variables if they have a vif>10 ; means high correlation betwee
n variables

```