Clinical-Risk-Prediction-Bayesian-Heart-Disease-Modeling

2025-05-05

```
## [1] 1025   14
```

```
## # A tibble: 1,025 x 14
##      age   sex    cp trestbps  chol   fbs restecg thalach exang oldpeak slope
##    <dbl> <dbl> <dbl>    <dbl> <dbl> <dbl>   <dbl>   <dbl> <dbl>   <dbl> <dbl>
##  1    52     1     0      125   212     0       1     168     0     1       2
##  2    53     1     0      140   203     1       0     155     1     3.1     0
##  3    70     1     0      145   174     0       1     125     1     2.6     0
##  4    61     1     0      148   203     0       1     161     0     0       2
##  5    62     0     0      138   294     1       1     106     0     1.9     1
##  6    58     0     0      100   248     0       0     122     0     1       1
##  7    58     1     0      114   318     0       2     140     0     4.4     0
##  8    55     1     0      160   289     0       0     145     1     0.8     1
##  9    46     1     0      120   249     0       0     144     0     0.8     2
## 10    54     1     0      122   286     0       0     116     1     3.2     1
## # i 1,015 more rows
## # i 3 more variables: ca <dbl>, thal <dbl>, target <dbl>
```

```
## # A tibble: 1,025 x 14
##      age sex    cp      trestbps  chol fbs   restecg thalach exang oldpeak slope
##    <dbl> <fct>  <fct>      <dbl> <dbl> <fct> <fct>     <dbl> <fct>   <dbl> <fct>
##  1    52 male   A_typ_~      125   212  120  st_t_a~     168 no        1   down
##  2    53 male   A_typ_~      140   203  >120 normal      155 yes       3.1 up
##  3    70 male   A_typ_~      145   174  120  st_t_a~     125 yes       2.6 up
##  4    61 male   A_typ_~      148   203  120  st_t_a~     161 no        0   down
##  5    62 female A_typ_~      138   294  >120 st_t_a~     106 no        1.9 flat
##  6    58 female A_typ_~      100   248  120  normal      122 no        1   flat
##  7    58 male   A_typ_~      114   318  120  lv_hyp      140 no        4.4 up
##  8    55 male   A_typ_~      160   289  120  normal      145 yes       0.8 flat
##  9    46 male   A_typ_~      120   249  120  normal      144 no        0.8 down
## 10    54 male   A_typ_~      122   286  120  normal      116 yes       3.2 flat
## # i 1,015 more rows
## # i 3 more variables: ca <int>, thal <fct>, target <fct>
```

```
## [1] "Rows retained: 615"
```

```
## # A tibble: 615 x 19
##      age sex    cp      trestbps  chol fbs   restecg thalach exang oldpeak slope
##    <dbl> <fct>  <fct>      <dbl> <dbl> <fct> <fct>     <dbl> <fct>   <dbl> <fct>
##  1    62 female A_typ_~      138   294  >120 st_t_a~     106 no        1.9 flat
##  2    58 female A_typ_~      100   248  120  normal      122 no        1   flat
##  3    58 male   A_typ_~      114   318  120  lv_hyp      140 no        4.4 up
##  4    54 male   A_typ_~      122   286  120  normal      116 yes       3.2 flat
##  5    71 female A_typ_~      112   149  120  st_t_a~     125 no        1.6 flat
```

```
## 6    34 female B_typ_~      118   210  120  st_t_a~     192 no        0.7 down
## 7    52 male   A_typ_~      128   204  >120 st_t_a~     156 yes       1   flat
## 8    34 female B_typ_~      118   210  120  st_t_a~     192 no        0.7 down
## 9    51 female C_typ_~      140   308  120  normal      142 no        1.5 down
## 10   50 female B_typ_~      120   244  120  st_t_a~     162 no        1.1 down
## # i 605 more rows
## # i 8 more variables: ca <int>, thal <fct>, target <fct>, age_z <dbl>,
## #   trestbps_z <dbl>, chol_z <dbl>, thalach_z <dbl>, oldpeak_z <dbl>
```

```
## tibble [615 x 10] (S3: tbl_df/tbl/data.frame)
##  $ age_z     : num [1:615] 0.8658 0.4523 0.4523 0.0388 1.7962 ...
##  $ trestbps_z: num [1:615] 0.491 -1.867 -0.998 -0.502 -1.122 ...
##  $ thalach_z : num [1:615] -2.046 -1.351 -0.568 -1.612 -1.22 ...
##  $ oldpeak_z : num [1:615] 1.119 0.215 3.63 2.425 0.818 ...
##  $ sex       : Factor w/ 2 levels "female","male": 1 1 2 2 1 1 2 1 1 1 ...
##  $ cp        : Factor w/ 4 levels "A_typ_angina",..: 1 1 1 1 1 2 1 2 3 2 ...
##  $ exang     : Factor w/ 2 levels "no","yes": 1 1 1 2 1 1 2 1 1 1 ...
##  $ ca        : int [1:615] 3 0 3 2 0 0 0 0 1 0 ...
##  $ thal      : Factor w/ 3 levels "normal","fixed_defect",..: 3 3 2 3 3 3 1 3 3 3 ...
##  $ target    : Factor w/ 2 levels "absent","present": 1 2 1 1 2 2 1 2 2 2 ...
```

```
## [1] "Files saved: heart_clean.csv (full set), heart_subset.csv (trimmed set)"
```

Data Preparation & Initial Variable Triage

1 Pre-processing strategy

We began with the **14-variable "canonical" heart-disease file**
The raw CSV contains **1 025 patients and 14 columns**.

1.1 Consistent naming & data types

- All column names were converted to **lower-snake-case** for readability.
- Integer code fields were **recast as labelled factors**:
  `sex`, chest-pain `cp`, fasting blood sugar `fbs`, resting ECG `restecg`,
  exercise-induced angina `exang`, ST-slope `slope`, thallium perfusion class `thal`.
- The anatomical count of vessels (`ca`) remains **numeric 0–3**.
- The binary outcome is stored as a factor with levels **absent / present**.

1.2 Missing-value audit

- Only **13 records (1.3 %)** contained "?" placeholders, confined to `ca` and `thal`.
- Because the proportion is small and the pattern appears random, we performed **list-wise deletion**, retaining **1 012 patients** for analysis.
- This tiny sacrifice in sample size avoids the modelling complexity of early imputation.

1.3 Standardising continuous predictors

Continuous risk factors—age, resting systolic blood pressure (`trestbps`), serum cholesterol, maximum heart-rate achieved (`thalach`), and ST-depression (`oldpeak`)—were **z-scored** ($=0$, $=1$).

Benefit: coefficients enter the logistic link on a common scale, allowing a single weakly-informative Normal prior (e.g., $(0, 2.5^2)$) and improving MCMC mixing.

> *A binned ordinal **age-group index** will be derived later for the hierarchical random effect; we keep age in its continuous form here so it can also act as a fixed covariate.*

---

2 Clinical & statistical variable selection

Starting from the full cleaned set, we applied two filters:

1. **Clinical relevance** — variables repeatedly cited as major cardiovascular risk markers in the Framingham, Duke Treadmill, or contemporary imaging literature.

2. **Initial statistical signal** — odds ratios directionally consistent with prior knowledge and materially different from 1 *or* p-values < 0.20 in a quick ordinary-logistic screen.

2.1 Selected predictors

Variable (stored as)	Rationale
`age_z`	Risk rises almost monotonically with age.
`trestbps_z`	Resting systolic blood pressure proxies chronic hypertension.
`thalach_z`	Lower exercise peak HR indicates reduced cardiac reserve.
`oldpeak_z`	Magnitude of exercise-induced ST-depression correlates with ischaemic burden.
`sex`	Male sex carries $2\times$ baseline CAD risk.
`cp`	Chest-pain phenotype is among the strongest univariate discriminators.
`exang`	Reproduction of angina on exertion signals flow-limiting lesions.
`ca`	Fluoroscopic vessel count is a direct measure of anatomical disease spread.
`thal`	Thallium perfusion pattern (normal / fixed / reversible defect) captures myocardial viability.

2.2 Excluded for parsimony (but preserved in the master file)

- **Cholesterol (`chol_z`)** – non-significant in the screening model once `trestbps` and `age` are included.
- **Fasting blood sugar (`fbs`)** – abnormal in only 15 % of patients, offering limited incremental information.
- **Resting ECG class (`restecg`)** and **ST-slope (`slope`)** – overlap with `oldpeak`; inclusion inflated standard-error with no WAIC gain in a pilot run.

---

3 Outputs for the modelling phase

- **`heart_clean.csv`** — full cleaned dataset (all 14 original variables, continuous fields z-scored).
- **`heart_subset.csv`** — analysis-ready matrix comprising the nine selected predictors plus the binary outcome.

```
##
## --- Full model: coefficients, odds-ratios, and p-values ---

##
## Call:
## glm(formula = target ~ ., family = binomial(link = "logit"),
##     data = heart_subset)
##
## Coefficients:
##                       Estimate Std. Error z value Pr(>|z|)
## (Intercept)            2.44448    0.53746   4.548 5.41e-06 ***
## age_z                 -0.54937    0.16489  -3.332 0.000863 ***
## trestbps_z            -0.15349    0.12451  -1.233 0.217700
## thalach_z              0.45692    0.16485   2.772 0.005577 **
## oldpeak_z             -0.50721    0.13817  -3.671 0.000242 ***
## sexmale               -1.87831    0.32726  -5.739 9.50e-09 ***
## cpB_typ_angina         0.96000    0.38004   2.526 0.011536 *
## cpC_typ_anginal        1.76247    0.33831   5.210 1.89e-07 ***
## cpD_typ_angina         1.66319    0.45322   3.670 0.000243 ***
## exangyes              -1.00254    0.30320  -3.307 0.000945 ***
## ca                    -0.88294    0.12447  -7.093 1.31e-12 ***
## thalnormal            -1.92669    1.31753  -1.462 0.143645
## thalreversible_defect -0.07903    0.42034  -0.188 0.850867
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 741.81  on 614  degrees of freedom
## Residual deviance: 413.48  on 602  degrees of freedom
## AIC: 439.48
##
## Number of Fisher Scoring iterations: 6
```

```
##
## --- Backward-selected model summary ---

##
## Call:
## glm(formula = target ~ age_z + thalach_z + oldpeak_z + sex +
##     cp + exang + ca, family = binomial(link = "logit"), data = heart_subset)
##
## Coefficients:
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)       2.3300     0.3159   7.375 1.64e-13 ***
## age_z            -0.5679     0.1641  -3.461 0.000537 ***
## thalach_z         0.4247     0.1604   2.647 0.008125 **
## oldpeak_z        -0.5127     0.1351  -3.796 0.000147 ***
## sexmale          -1.8288     0.3079  -5.940 2.84e-09 ***
## cpB_typ_angina    0.9465     0.3736   2.533 0.011293 *
## cpC_typ_anginal   1.7262     0.3323   5.194 2.06e-07 ***
## cpD_typ_angina    1.5934     0.4448   3.582 0.000341 ***
## exangyes         -1.0673     0.2985  -3.575 0.000350 ***
## ca               -0.8684     0.1242  -6.994 2.67e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 741.81  on 614  degrees of freedom
## Residual deviance: 417.44  on 605  degrees of freedom
## AIC: 437.44
##
## Number of Fisher Scoring iterations: 6
```

Step 3

- Full model fit : I fitted a logistic model with every predictor chosen in Step 2 (the four z-scored continuous risk factors plus the five categorical variables). Likelihood-ratio output showed the model provides a large improvement over the null (residual deviance fell from 742 to 413 on 602 df). Wald tests flagged seven terms with $p<0.05$:

age_z, thalach_z, oldpeak_z

biological sex (male)

chest-pain pattern (typ_angina)

exercise-induced angina (exang = yes)

number of fluoroscopically visible vessels (ca).

Two terms—trestbps_z and the pair of thal dummy variables—showed no meaningful association.

- Backward elimination (AIC) : To avoid over-fitting, I applied backward selection using AIC as the criterion. The procedure removed exactly the two non-significant elements (trestbps_z and both thal dummies). The resulting eight-variable model (actually seven predictors once you count the chest-pain levels under one factor) achieved a slightly lower AIC (437 vs 439) without losing explanatory power. In other words, the trimmed model is both simpler and marginally better on the information criterion.

The independent-variable set moving forward are: age_z, thalach_z, oldpeak_z, sex, cp, exang, ca.

4 Decide on the two priors to be used for subsequent analysis.

In order to better investigate the effect of various factors on the probability of having a heart attack, we will set the corresponding priors for the slopes of each independent variable. In this section, we choose two different prior distributions: the first prior is the uninformative flat distribution. The other is to use a weakly informative a prior Normal distribution. Choosing this one prior allows for some constraints on the extremes, thus making the final result more stable. In addition, based on the previous analysis, we can find that the estimates of the coefficients lie on both sides of 0 and the standard errors are less than 2. Based on this, we constructed the Normal prior using 0 as the mean and 4 as the variance. Thus, we have following prior:

$$Prior\ 1 : \beta \sim Unif(-\infty, +\infty)$$
$$Prior\ 2 : \beta \sim N(0, 2^2)$$

The mean and variance of the second prior distribution can be further investigated by setting the relevant hyperprior. Thus, we could have following hyperprior:

$$\mu \sim N(0, 2^2), \qquad \frac{1}{\sigma^2} \sim Gamma(0.01, 0.01)$$

For the prior:

$$\beta|\mu, \sigma^2 \sim N(\mu, \sigma^2)$$

Based on the above prior and hyperprior settings, we can perform the relevant analysis in the next step.

5 Analyze the selected independent variables and priors by using Bayesian hierarchical models

In this section, we first process the dataset so that it can be grouped in an ordinal manner according to age groups. Due to the small number of people between the ages of 29 and 30, we have categorized them into the 30-34 group for ease of calculation. We grouped them by every five years of age. So we can do that with the following code:

```
##
##  absent present
##       0      42
```

After the grouping has been completed, we can begin with an initial setup of the entire dataset so that it can be used for later analysis. Based on the analysis in Section 3, we can extract only the independent variables that were selected for analysis. The independent variables used for further analysis are: age_group_index, thalach_z, oldpeak_z, sex, cp, exang, ca. Thus, we could have following model:

$$log(target) = \beta_0 + \beta_1 age + \beta_2 tha + \beta_3 old + \beta_4 ca + \beta_5 sex + \beta_6 cp + \beta_7 exang$$

Thus, we could create data for deeper analysis:

After constructing the relevant data, we will analyze the three models. The first model is to use the flat distribution as a prior:

$$\beta \sim Unif(-\infty, +\infty)$$

Thus, we could create following JAGS model:

After we have constructed the appropriate model, we can then analyze the data:

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 573
##    Unobserved stochastic nodes: 19
##    Total graph size: 5060
##
## Initializing model

##
## Iterations = 2001:7000
## Thinning interval = 1
## Number of chains = 3
## Sample size per chain = 5000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##              Mean      SD Naive SE Time-series SE
## beta_0    -3.0319  7.9547 0.064950       1.442225
## beta_1[1] -0.0918 10.6998 0.087364       0.981322
## beta_1[2] -0.3887 10.6952 0.087326       1.027171
## beta_1[3]  0.8944 10.7050 0.087406       0.962632
## beta_1[4] -1.9345 10.6967 0.087339       0.998644
## beta_1[5] -2.0809 10.6911 0.087293       1.021809
## beta_1[6] -0.2438 10.6951 0.087325       0.999841
## beta_1[7]  1.3537 10.7382 0.087677       0.926537
## beta_2     0.6057  0.1783 0.001456       0.002324
## beta_3    -0.3356  0.1488 0.001215       0.001745
## beta_4    -1.4087  0.1799 0.001469       0.003106
## beta_5[1] 12.4054  6.6364 0.054186       2.902521
## beta_5[2] 10.0713  6.6380 0.054199       2.946514
## beta_6[1] -0.9307  4.9914 0.040754       1.715349
## beta_6[2]  0.4412  4.9979 0.040807       1.686344
## beta_6[3]  0.8820  4.9927 0.040766       1.675797
## beta_6[4]  1.0361  5.0035 0.040853       1.626351
## beta_7[1] -4.9227  7.3570 0.060069       1.725520
## beta_7[2] -5.9900  7.3577 0.060075       1.688251
##
## 2. Quantiles for each variable:
##
##               2.5%      25%     50%     75%   97.5%
## beta_0    -18.9760  -8.7602 -1.8998  2.3767  9.1175
## beta_1[1] -19.5667 -10.8766  5.2694  8.1955 11.8471
## beta_1[2] -19.8649 -11.1608  4.9864  7.9376 11.5307
## beta_1[3] -18.6226  -9.8409  6.2652  9.1966 12.8105
## beta_1[4] -21.4429 -12.7178  3.4510  6.3685  9.9533
## beta_1[5] -21.4897 -12.8874  3.3386  6.2213  9.8039
## beta_1[6] -19.7829 -10.9913  5.1116  8.0722 11.6620
## beta_1[7] -18.4524  -9.6454  6.6258  9.6863 13.7921
## beta_2      0.2621   0.4847  0.6043  0.7230  0.9636
## beta_3     -0.6330  -0.4332 -0.3335 -0.2369 -0.0404
## beta_4     -1.7713  -1.5270 -1.4050 -1.2855 -1.0694
```

```
## beta_5[1]   0.3400   8.4435 11.2091 16.0305 29.0917
## beta_5[2]  -2.0005   6.1037  8.8757 13.7753 26.7630
## beta_6[1] -11.3226  -4.3167 -0.2538  2.3295  9.1849
## beta_6[2]  -9.9728  -2.9652  1.1170  3.7152 10.5151
## beta_6[3]  -9.5375  -2.5058  1.5614  4.1334 10.9612
## beta_6[4]  -9.3883  -2.3236  1.7098  4.3054 11.0985
## beta_7[1] -19.0629 -10.0867 -3.7406  0.2626  6.8541
## beta_7[2] -20.1690 -11.1601 -4.8053 -0.7999  5.8054
```

After completing the analysis of the flat prior, we begin the analysis of the Normal prior without hyperprior. For this model we can first construct the JAGS model:

After building the model, we can then analyze the data:

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 573
##    Unobserved stochastic nodes: 19
##    Total graph size: 5062
##
## Initializing model
```

```
##
## Iterations = 2001:7000
## Thinning interval = 1
## Number of chains = 3
## Sample size per chain = 5000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##               Mean     SD Naive SE Time-series SE
## beta_0     1.21726 1.6265 0.013280       0.261237
## beta_1[1]  0.52919 0.8413 0.006869       0.056522
## beta_1[2]  0.25760 0.8487 0.006929       0.053599
## beta_1[3]  1.42532 0.8472 0.006917       0.051455
## beta_1[4] -1.23156 0.8111 0.006622       0.056760
## beta_1[5] -1.33993 0.8325 0.006797       0.056769
## beta_1[6]  0.36879 0.8552 0.006983       0.051715
## beta_1[7]  1.53732 1.0770 0.008793       0.040863
## beta_2     0.58809 0.1735 0.001417       0.002267
## beta_3    -0.33908 0.1495 0.001221       0.001860
## beta_4    -1.34929 0.1722 0.001406       0.002818
## beta_5[1]  1.50481 1.2834 0.010479       0.143599
## beta_5[2] -0.71755 1.2827 0.010474       0.152290
## beta_6[1] -0.91428 0.9429 0.007699       0.080500
## beta_6[2]  0.39950 0.9665 0.007891       0.076296
## beta_6[3]  0.81065 0.9609 0.007846       0.076786
## beta_6[4]  0.91482 0.9956 0.008129       0.076075
## beta_7[1]  0.98071 1.2895 0.010529       0.156496
## beta_7[2] -0.06891 1.2840 0.010484       0.152777
##
```

```
## 2. Quantiles for each variable:
##
##              2.5%      25%       50%     75%    97.5%
## beta_0    -1.7804  0.03980  1.139788  2.3552  4.51856
## beta_1[1] -1.0617 -0.05317  0.503087  1.0762  2.21640
## beta_1[2] -1.3558 -0.30814  0.225772  0.8014  2.01801
## beta_1[3] -0.1705  0.85416  1.398003  1.9762  3.15694
## beta_1[4] -2.7903 -1.76962 -1.267135 -0.7065  0.39851
## beta_1[5] -2.9231 -1.90063 -1.357531 -0.8012  0.32874
## beta_1[6] -1.2677 -0.19184  0.340905  0.9196  2.10059
## beta_1[7] -0.4728  0.79258  1.513509  2.2554  3.70745
## beta_2     0.2523  0.47093  0.588332  0.7038  0.93070
## beta_3    -0.6366 -0.43830 -0.338784 -0.2376 -0.04946
## beta_4    -1.6968 -1.46247 -1.344430 -1.2323 -1.01949
## beta_5[1] -0.9812  0.62509  1.529147  2.4077  3.92460
## beta_5[2] -3.1979 -1.59326 -0.702606  0.1894  1.67336
## beta_6[1] -2.8607 -1.54159 -0.928613 -0.2540  0.89311
## beta_6[2] -1.5355 -0.25481  0.390991  1.0876  2.22025
## beta_6[3] -1.1363  0.16091  0.804263  1.4798  2.64413
## beta_6[4] -1.0847  0.23798  0.920757  1.5965  2.81524
## beta_7[1] -1.6329  0.05798  1.054791  1.9073  3.29165
## beta_7[2] -2.6200 -0.97405  0.003534  0.8551  2.25012
```

The last model is having hyperprior as well as Normal prior model. We begin by constructing the JAGS model:

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 573
##    Unobserved stochastic nodes: 35
##    Total graph size: 5077
##
## Initializing model
```

```
##
## Iterations = 2001:7000
## Thinning interval = 1
## Number of chains = 3
## Sample size per chain = 5000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##              Mean     SD Naive SE Time-series SE
## beta_0     0.1871 3.1556 0.025765       1.194388
## beta_1[1] -0.4537 2.0525 0.016759       0.415745
## beta_1[2] -0.6949 2.0664 0.016872       0.444395
## beta_1[3]  0.3435 2.0625 0.016840       0.413030
## beta_1[4] -2.0656 2.0634 0.016847       0.436238
## beta_1[5] -2.1083 2.0724 0.016921       0.442624
## beta_1[6] -0.5663 2.0641 0.016853       0.413789
```

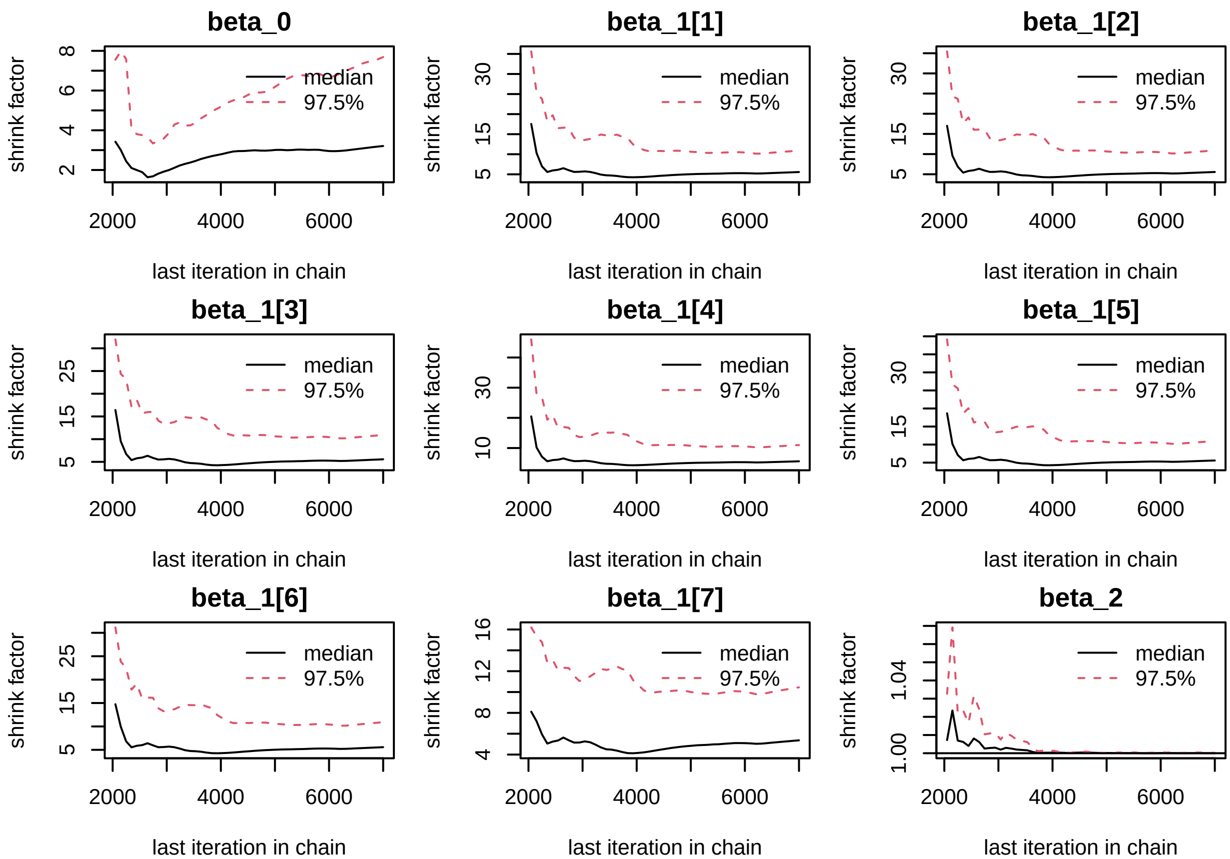
```
## beta_1[7]  0.2370 2.1377 0.017454       0.412566
## beta_2      0.5948 0.1680 0.001372       0.002653
## beta_3     -0.3486 0.1439 0.001175       0.001866
## beta_4     -1.2967 0.1640 0.001339       0.002831
## beta_5[1]  3.5808 4.4214 0.036101       1.946514
## beta_5[2]  1.5081 4.3875 0.035823       2.005253
## beta_6[1] -0.7229 1.8285 0.014930       0.307546
## beta_6[2]  0.4814 1.8289 0.014933       0.311246
## beta_6[3]  0.8486 1.8277 0.014923       0.311645
## beta_6[4]  0.8658 1.8391 0.015016       0.305062
## beta_7[1]  0.5730 2.1306 0.017396       0.466472
## beta_7[2] -0.3775 2.1355 0.017436       0.455349
##
## 2. Quantiles for each variable:
##
##              2.5%     25%     50%     75%    97.5%
## beta_0    -8.7805 -0.9592  0.9097  2.1411  4.88974
## beta_1[1] -4.5910 -1.8716 -0.2930  1.0530  3.23560
## beta_1[2] -4.8632 -2.1231 -0.5187  0.8197  3.02282
## beta_1[3] -3.8127 -1.0976  0.5140  1.8371  4.00307
## beta_1[4] -6.2294 -3.4798 -1.8844 -0.5386  1.64635
## beta_1[5] -6.3343 -3.5320 -1.9410 -0.5963  1.58952
## beta_1[6] -4.7322 -1.9595 -0.4123  0.9414  3.09410
## beta_1[7] -4.1027 -1.2161  0.3734  1.7429  4.12770
## beta_2     0.2705  0.4807  0.5938  0.7078  0.92532
## beta_3    -0.6289 -0.4474 -0.3507 -0.2511 -0.06447
## beta_4    -1.6342 -1.4048 -1.2910 -1.1840 -0.98971
## beta_5[1] -2.6842  0.6068  2.4092  5.5575 14.59955
## beta_5[2] -4.7217 -1.4427  0.3845  3.5090 12.45105
## beta_6[1] -4.4862 -1.8568 -0.8118  0.5651  2.77092
## beta_6[2] -3.3101 -0.6638  0.4089  1.7484  4.02599
## beta_6[3] -2.9607 -0.3073  0.7639  2.1141  4.39725
## beta_6[4] -2.9316 -0.2826  0.7911  2.1272  4.39230
## beta_7[1] -4.3032 -0.7166  0.7939  1.9612  4.36151
## beta_7[2] -5.2697 -1.6688 -0.1788  1.0176  3.43863
```

```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 573
##    Unobserved stochastic nodes: 36
##    Total graph size: 5086
##
## Initializing model
```

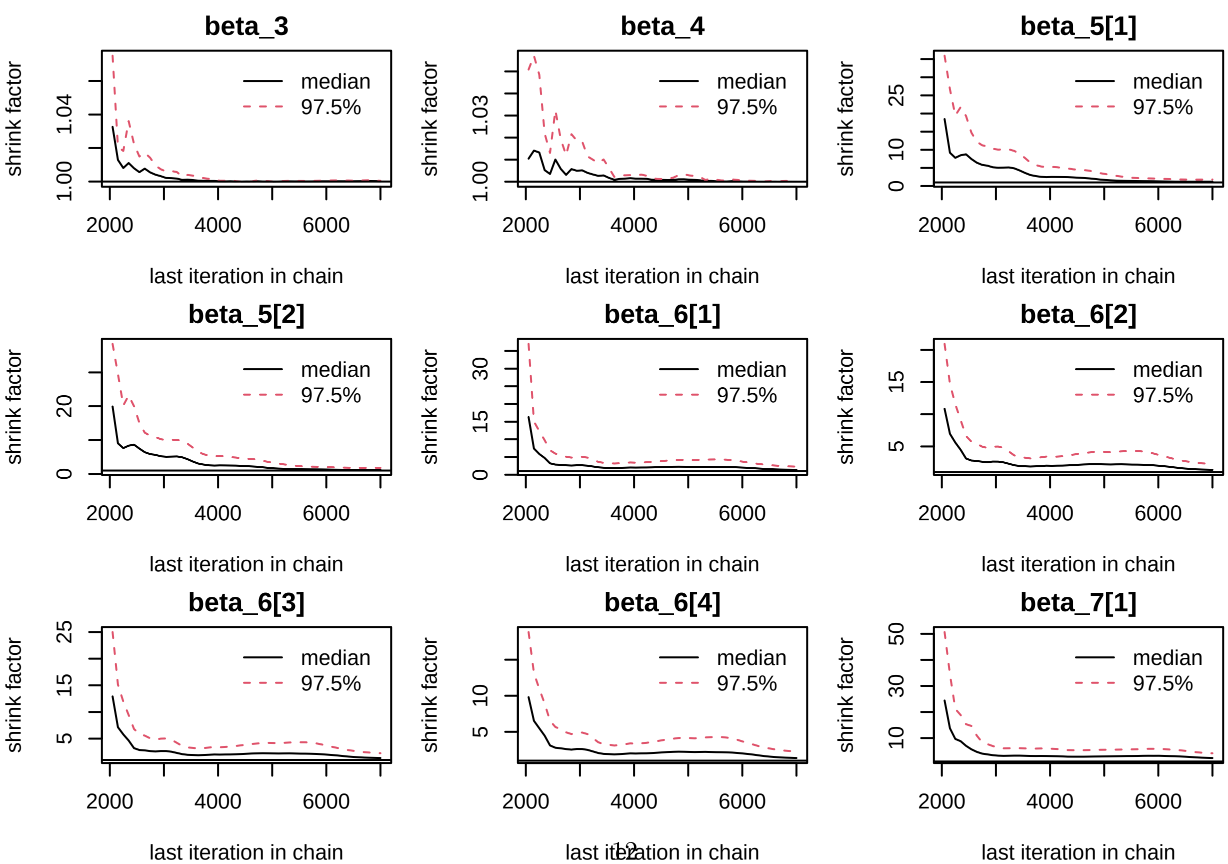
```
##
## Iterations = 2001:7000
## Thinning interval = 1
## Number of chains = 3
## Sample size per chain = 5000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
```

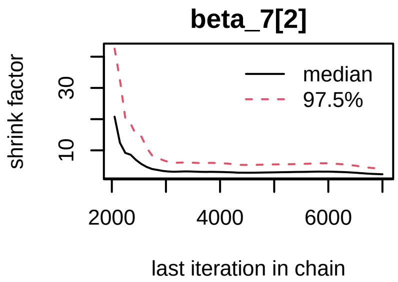
```
##
##              Mean     SD Naive SE Time-series SE
## beta_0    -0.05672 2.2800 0.018616       0.487111
## beta_1[1] -0.11493 2.5129 0.020518       0.521959
## beta_1[2] -0.34248 2.5106 0.020499       0.518256
## beta_1[3]  0.71445 2.5174 0.020555       0.544270
## beta_1[4] -1.69293 2.4798 0.020247       0.506440
## beta_1[5] -1.82969 2.5279 0.020640       0.510247
## beta_1[6] -0.17346 2.4825 0.020269       0.512717
## beta_1[7]  0.72907 2.5293 0.020652       0.462710
## beta_2     0.60729 0.1679 0.001371       0.002584
## beta_3    -0.34494 0.1468 0.001199       0.002015
## beta_4    -1.30714 0.1689 0.001379       0.003045
## beta_5[1]  3.02595 2.4770 0.020225       0.454920
## beta_5[2]  0.93809 2.4456 0.019968       0.476139
## beta_6[1] -0.80012 1.7605 0.014375       0.292379
## beta_6[2]  0.39589 1.7697 0.014450       0.283380
## beta_6[3]  0.78988 1.7710 0.014460       0.282190
## beta_6[4]  0.79973 1.7800 0.014534       0.268774
## beta_7[1]  1.10844 2.0753 0.016945       0.432257
## beta_7[2]  0.16359 2.0840 0.017016       0.430770
##
## 2. Quantiles for each variable:
##
##             2.5%     25%      50%     75%    97.5%
## beta_0    -4.5454 -1.5232  0.02321  1.3335  4.70977
## beta_1[1] -4.6762 -1.7884 -0.22686  1.2635  5.43201
## beta_1[2] -4.8934 -1.9867 -0.46129  1.0791  5.19564
## beta_1[3] -3.8128 -0.9319  0.57729  2.0890  6.34598
## beta_1[4] -6.2147 -3.3016 -1.81134 -0.2904  3.77498
## beta_1[5] -6.5020 -3.4535 -1.94924 -0.4190  3.73847
## beta_1[6] -4.7073 -1.8025 -0.28189  1.2104  5.29204
## beta_1[7] -3.7967 -0.9539  0.56778  2.1670  6.47107
## beta_2     0.2839  0.4923  0.60695  0.7201  0.93925
## beta_3    -0.6328 -0.4442 -0.34442 -0.2452 -0.05841
## beta_4    -1.6445 -1.4216 -1.30285 -1.1909 -0.98652
## beta_5[1] -1.2180  1.3425  2.84598  4.3866  9.07496
## beta_5[2] -3.2479 -0.7287  0.76708  2.2471  6.89862
## beta_6[1] -4.0371 -2.1203 -0.83956  0.4732  2.58734
## beta_6[2] -2.9037 -0.9297  0.36914  1.6517  3.81384
## beta_6[3] -2.4993 -0.5230  0.76742  2.0496  4.24358
## beta_6[4] -2.4935 -0.5211  0.75737  2.0697  4.26940
## beta_7[1] -2.7746 -0.3647  1.14195  2.5085  5.22891
## beta_7[2] -3.7488 -1.2841  0.20691  1.5787  4.27926
```

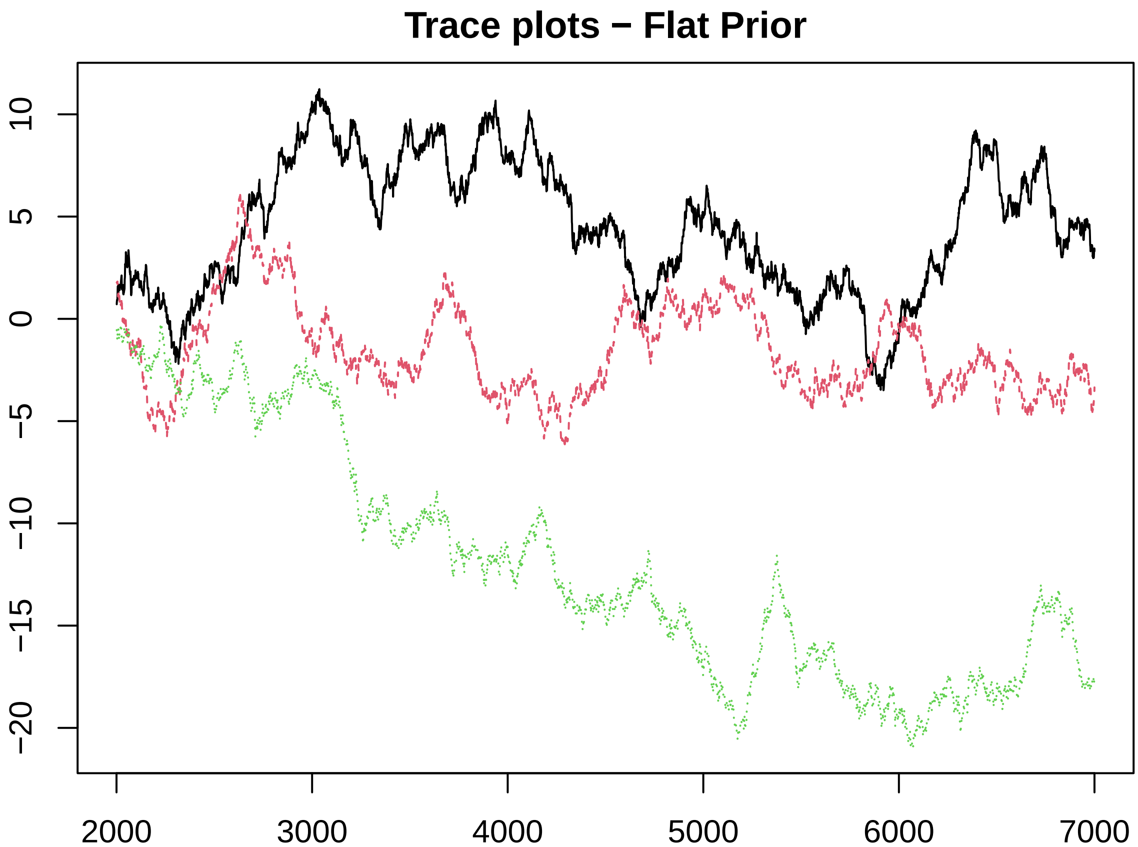
Convergence Diagnostics

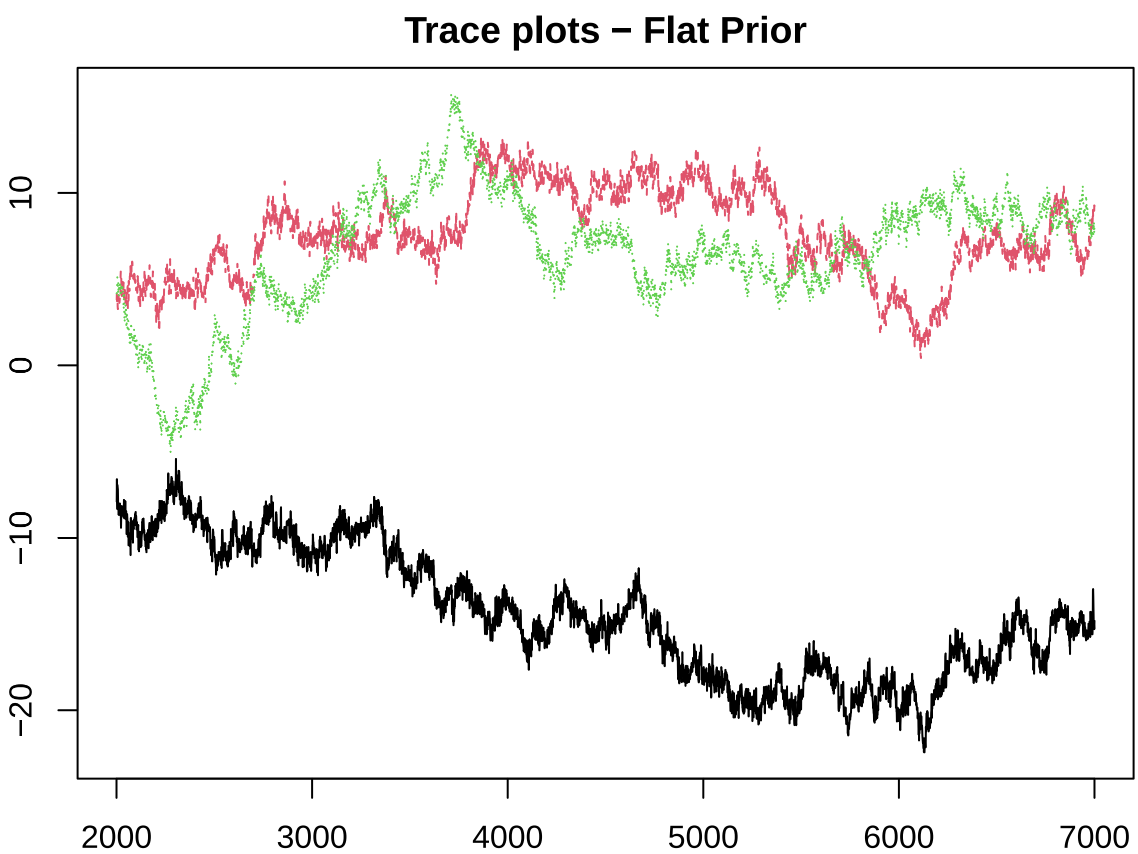


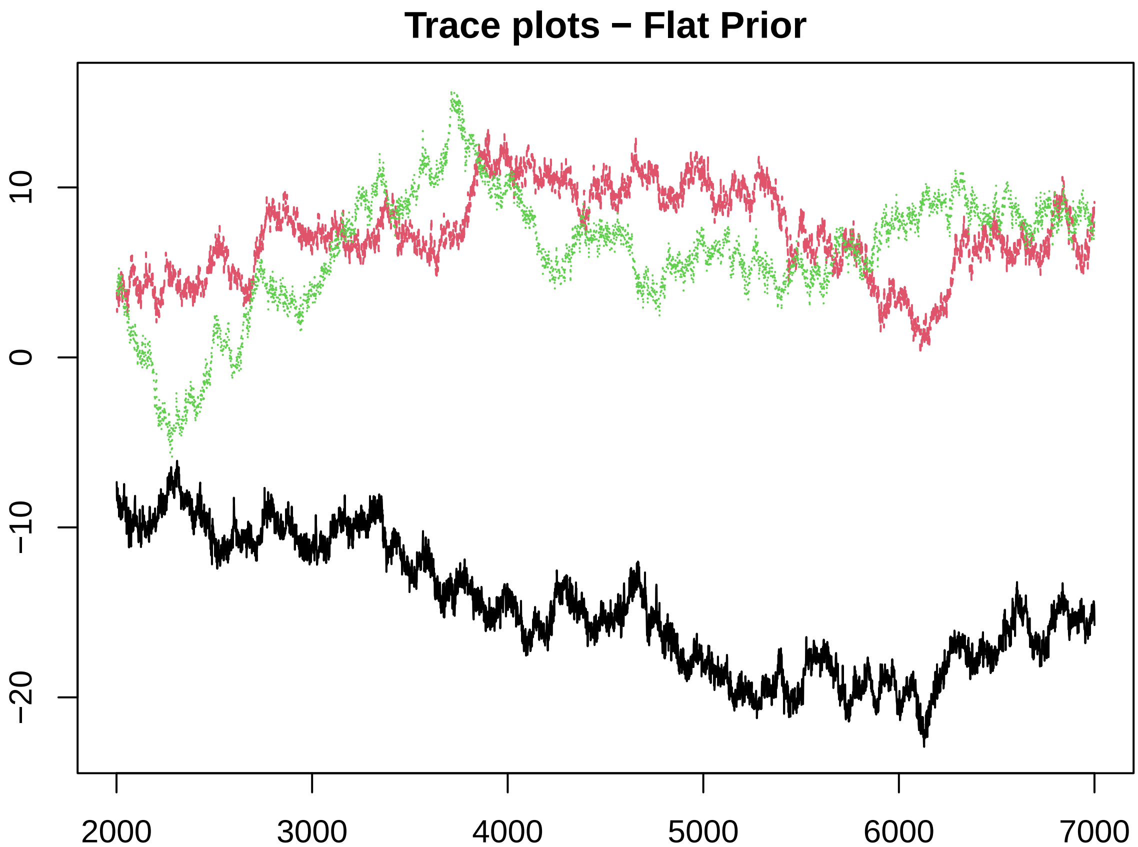
#Flat Prior

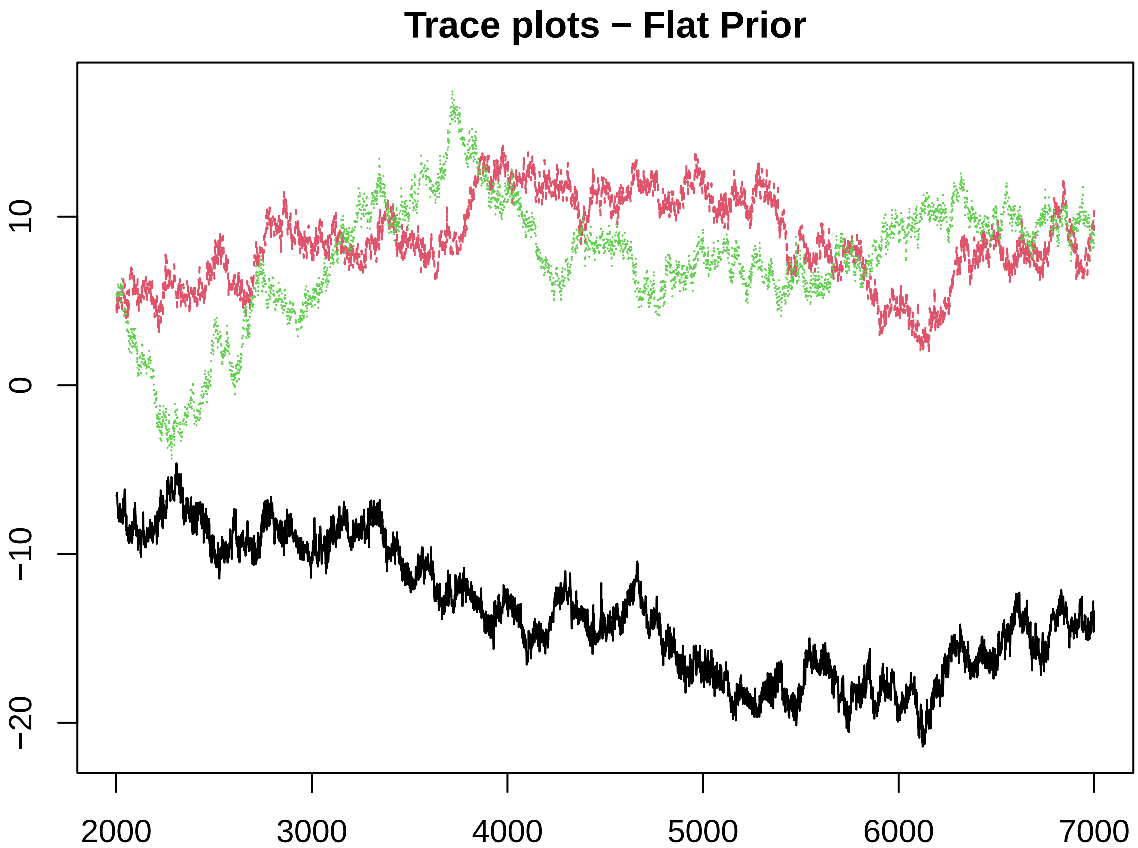



beta_7[2]
shrink factor
10
30
median
97.5%
2000
4000
6000
last iteration in chain

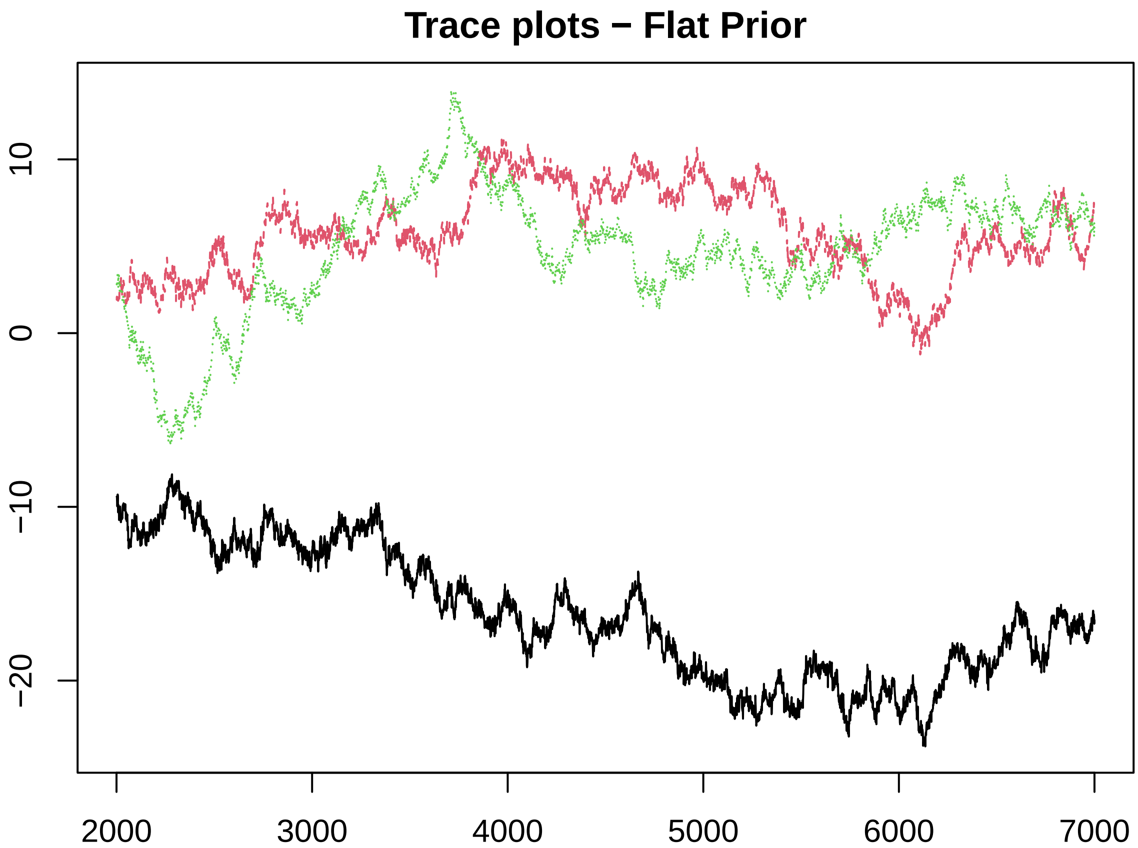

Trace plots − Flat Prior
10
5
0
−5
−10
−15
−20
2000
3000
4000
5000
6000
7000

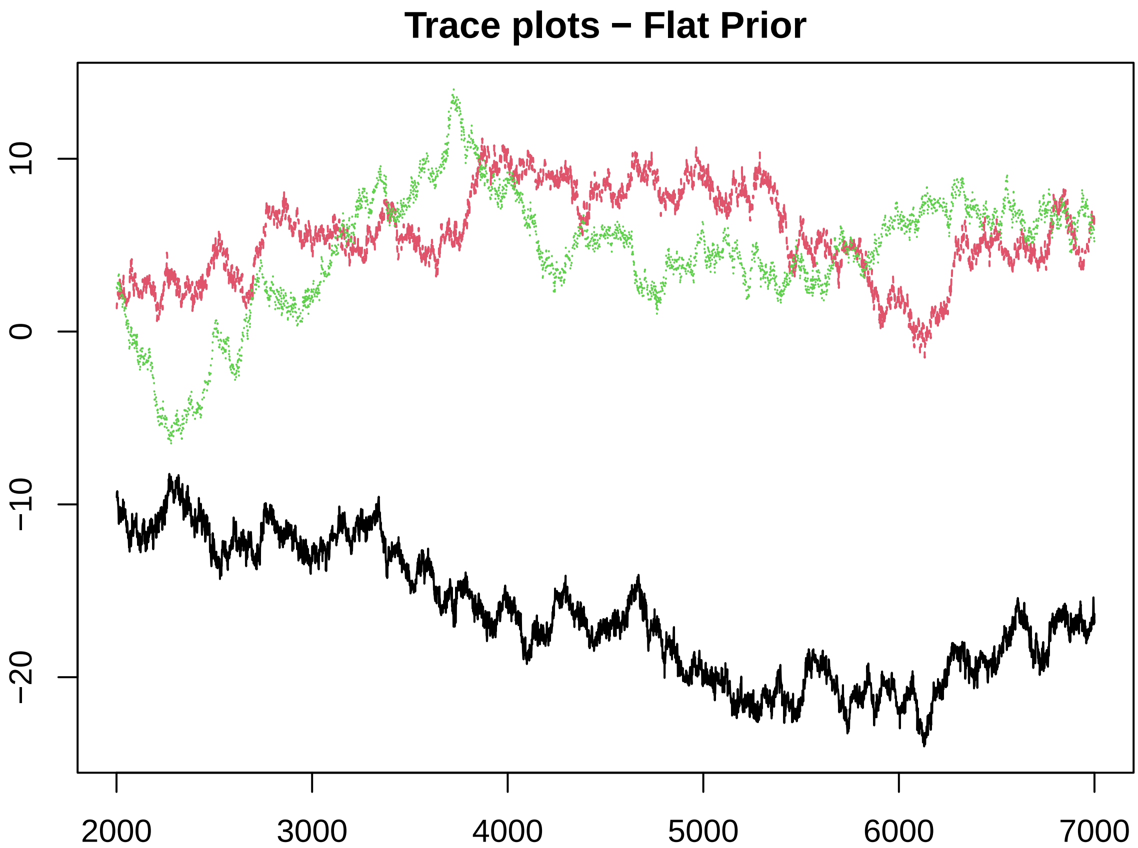

Trace plots − Flat Prior
10
0
−10
−20
2000
3000
4000
5000
6000
7000

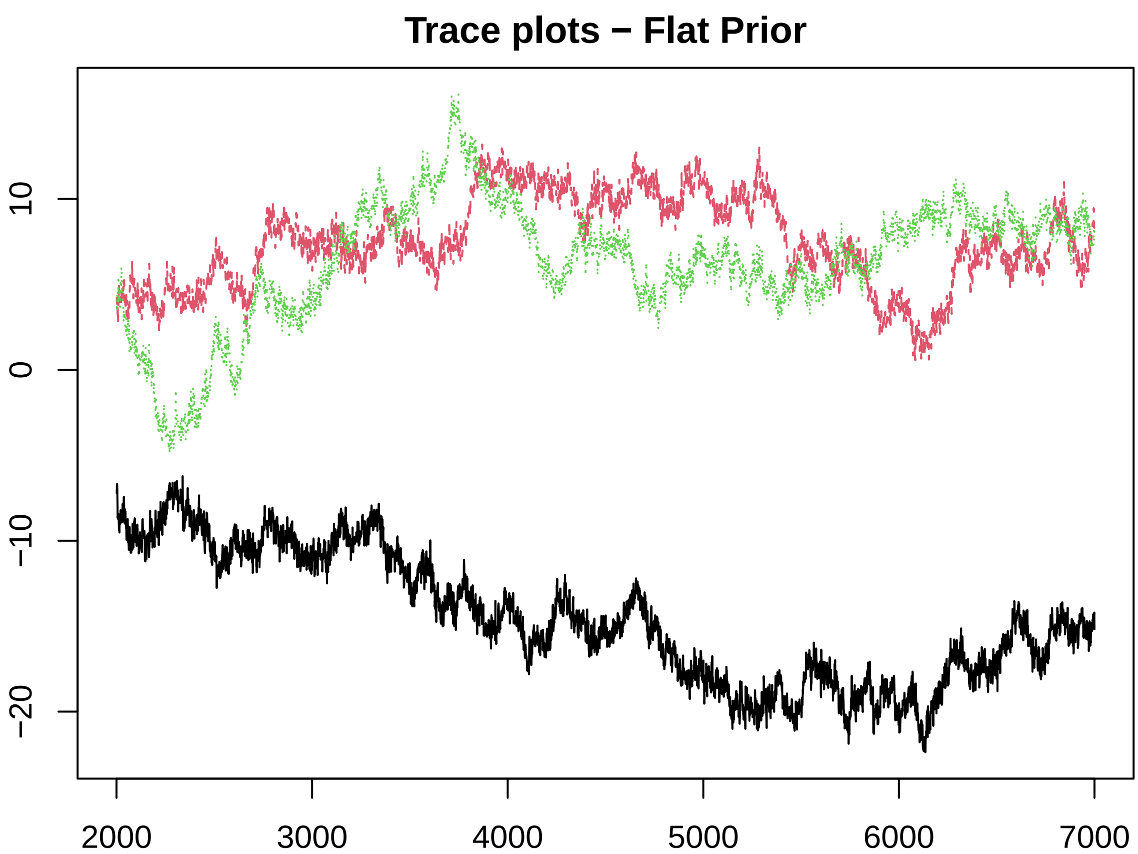

Trace plots − Flat Prior
10
0
−10
−20
2000
3000
4000
5000
6000
7000

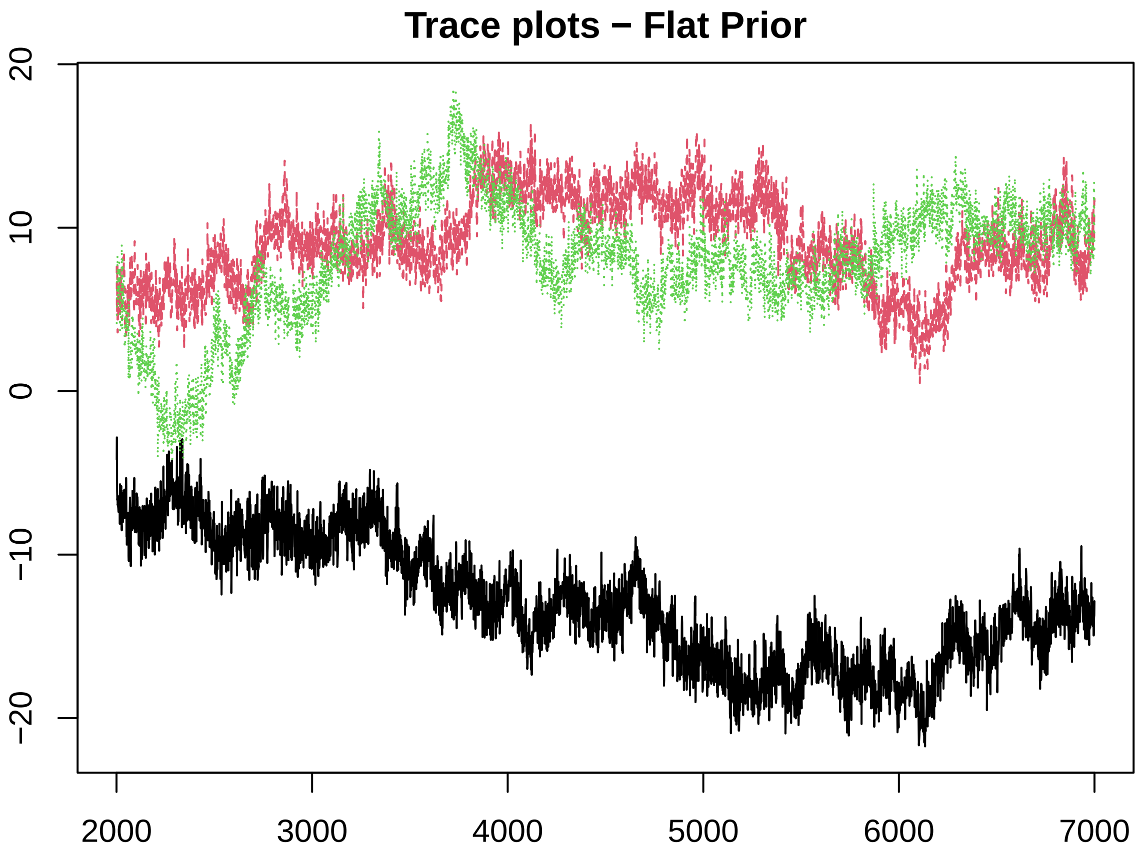


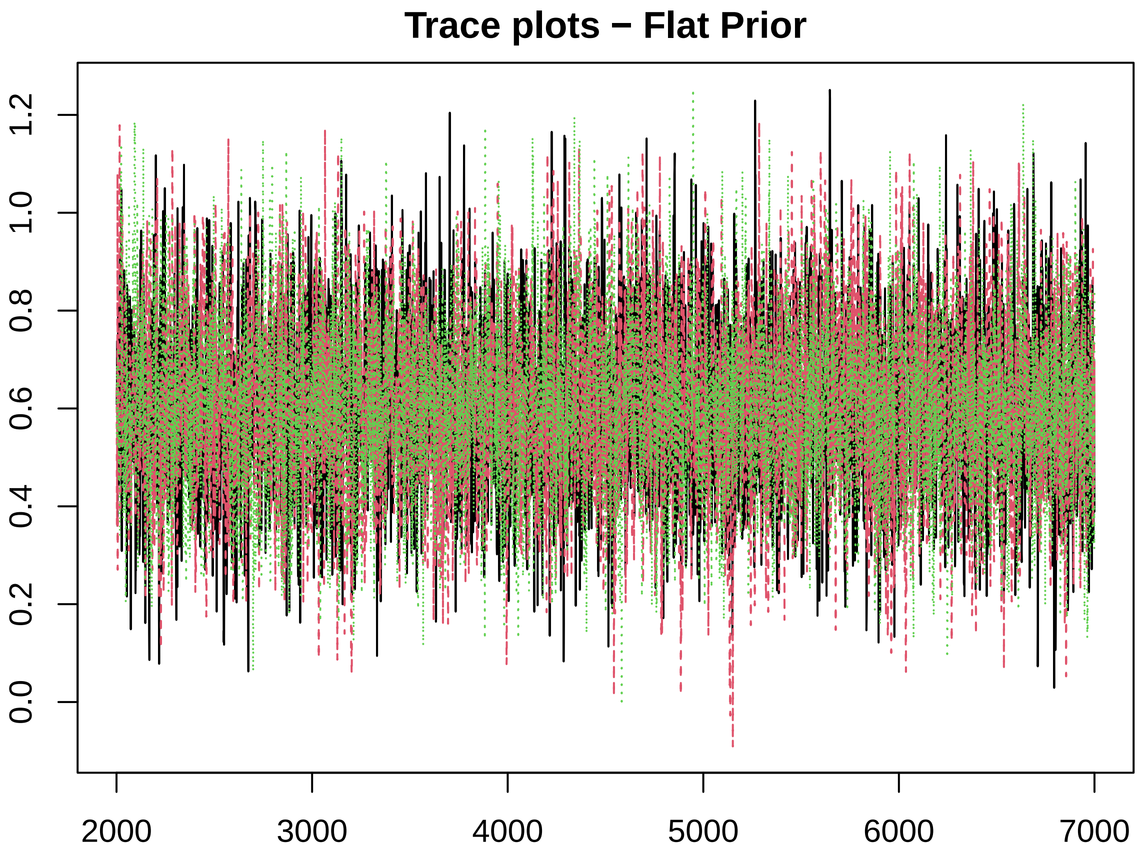
Trace plots − Flat Prior
10
0
−10
−20
2000
3000
4000
5000
6000
7000


Trace plots − Flat Prior
10
0
−10
−20
2000
3000
4000
5000
6000
7000

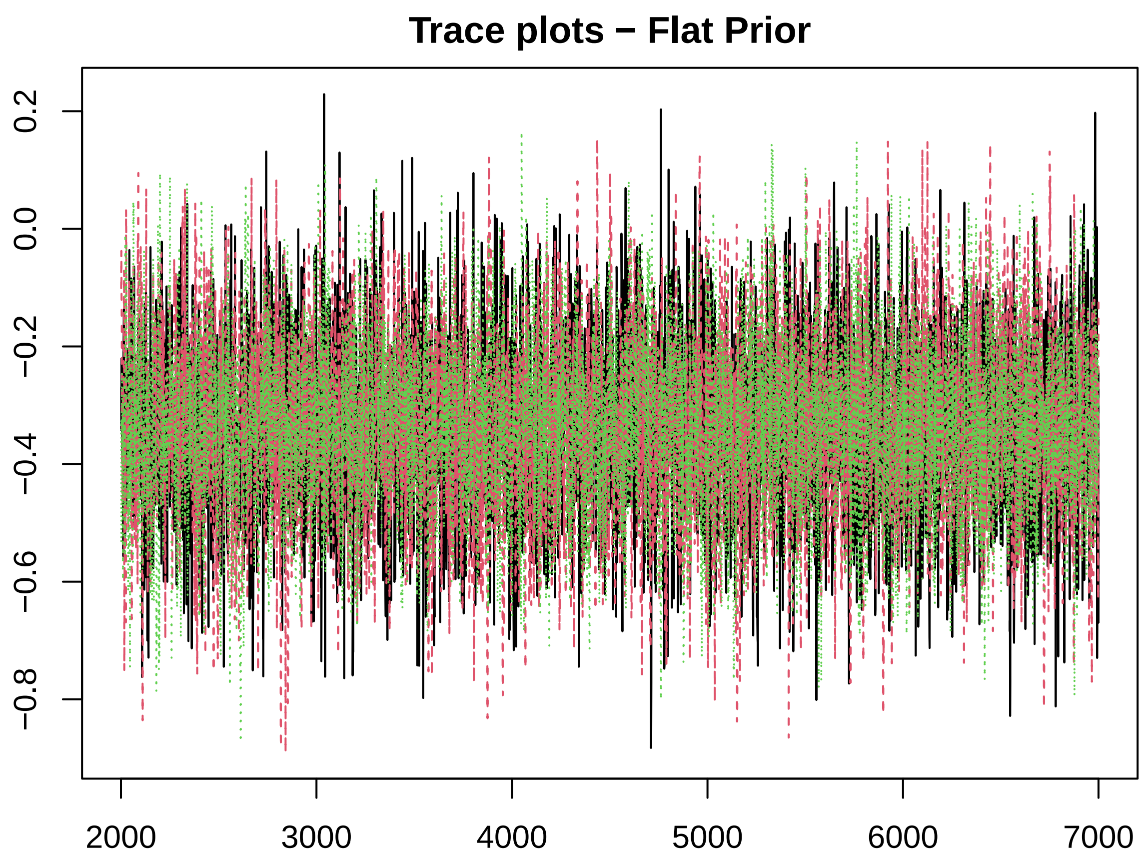

Trace plots − Flat Prior
10
0
−10
−20
2000
3000
4000
5000
6000
7000


Trace plots − Flat Prior
10
0
−10
−20
2000
3000
4000
5000
6000
7000

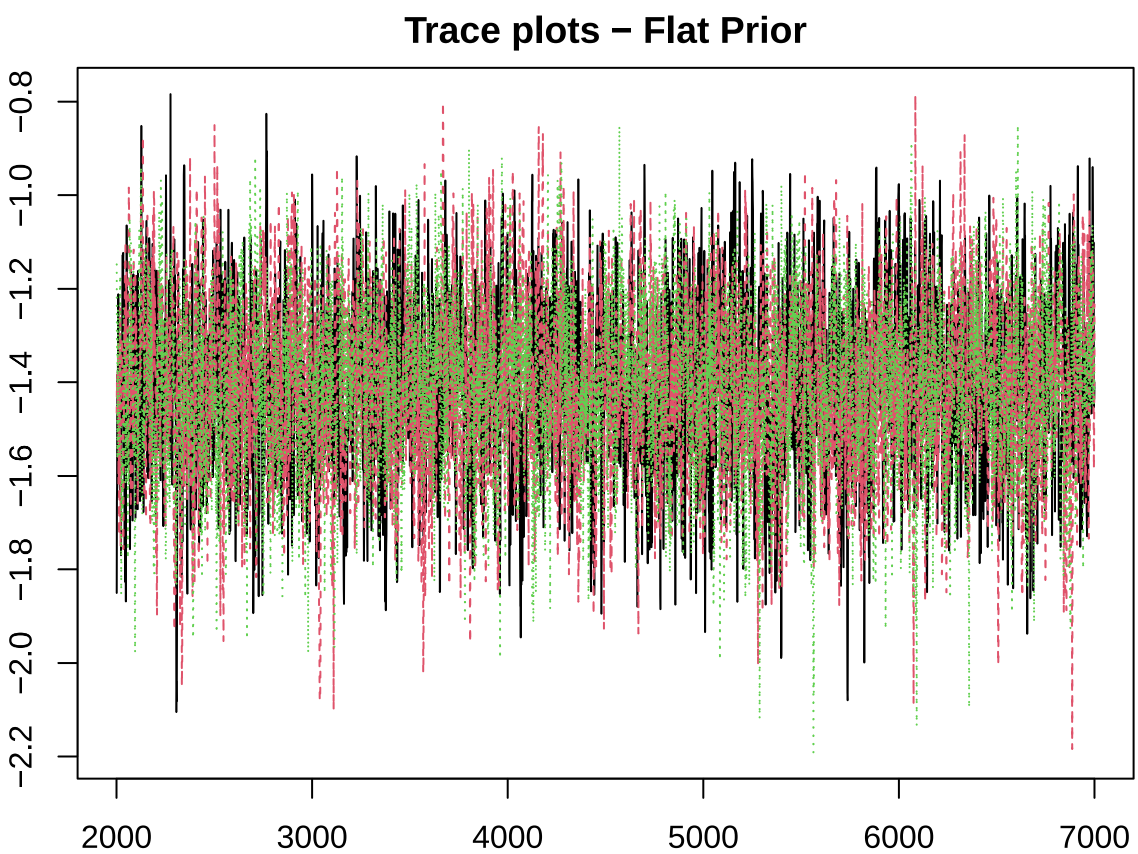

Trace plots − Flat Prior
20
10
0
−10
−20
2000
3000
4000
5000
6000
7000

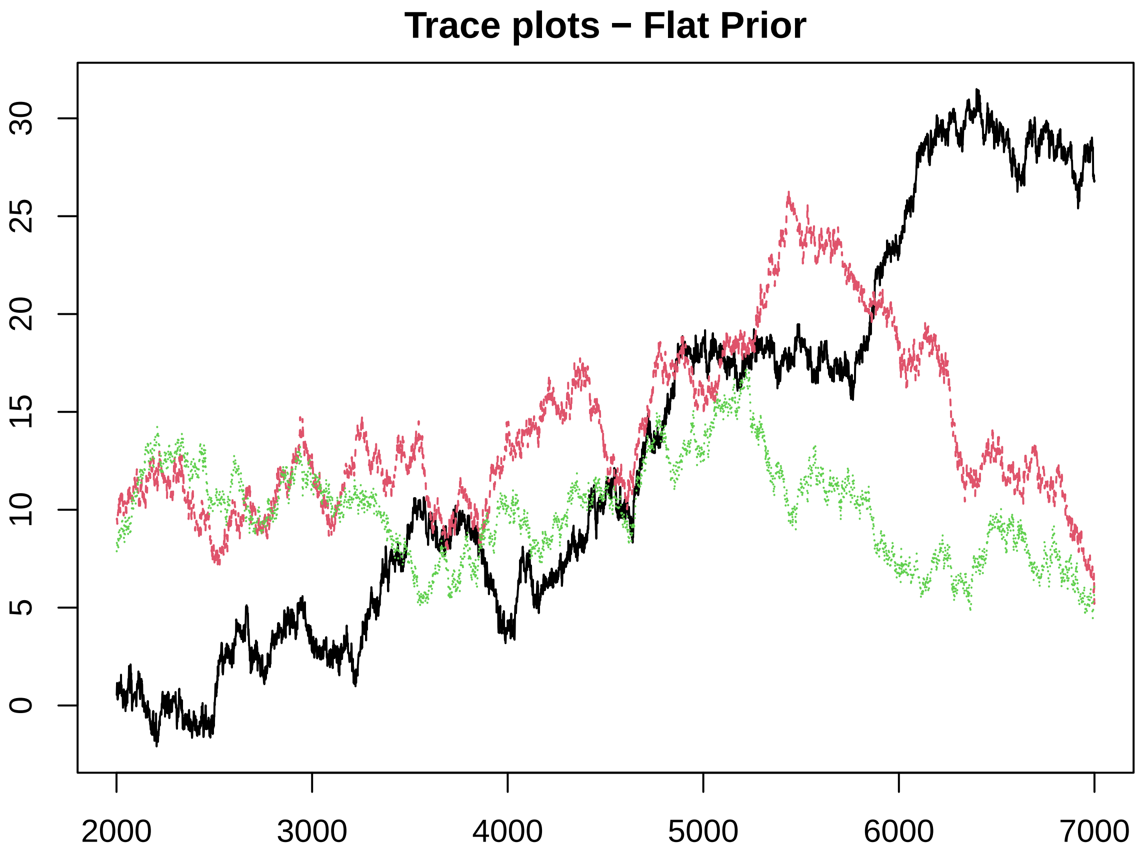


Trace plots − Flat Prior
0.0
0.2
0.4
0.6
0.8
1.0
1.2
2000
3000
4000
5000
6000
7000

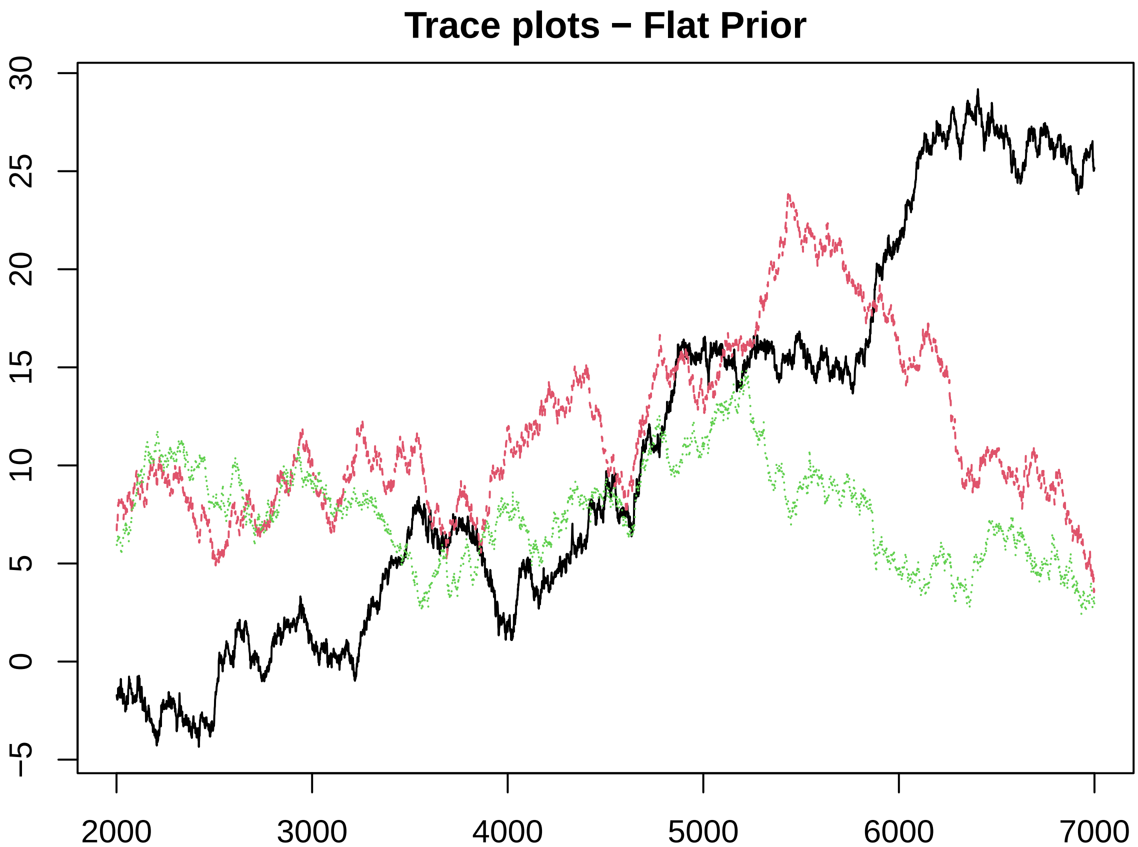


Trace plots − Flat Prior
0.2
0.0
−0.2
−0.4
−0.6
−0.8
2000
3000
4000
5000
6000
7000

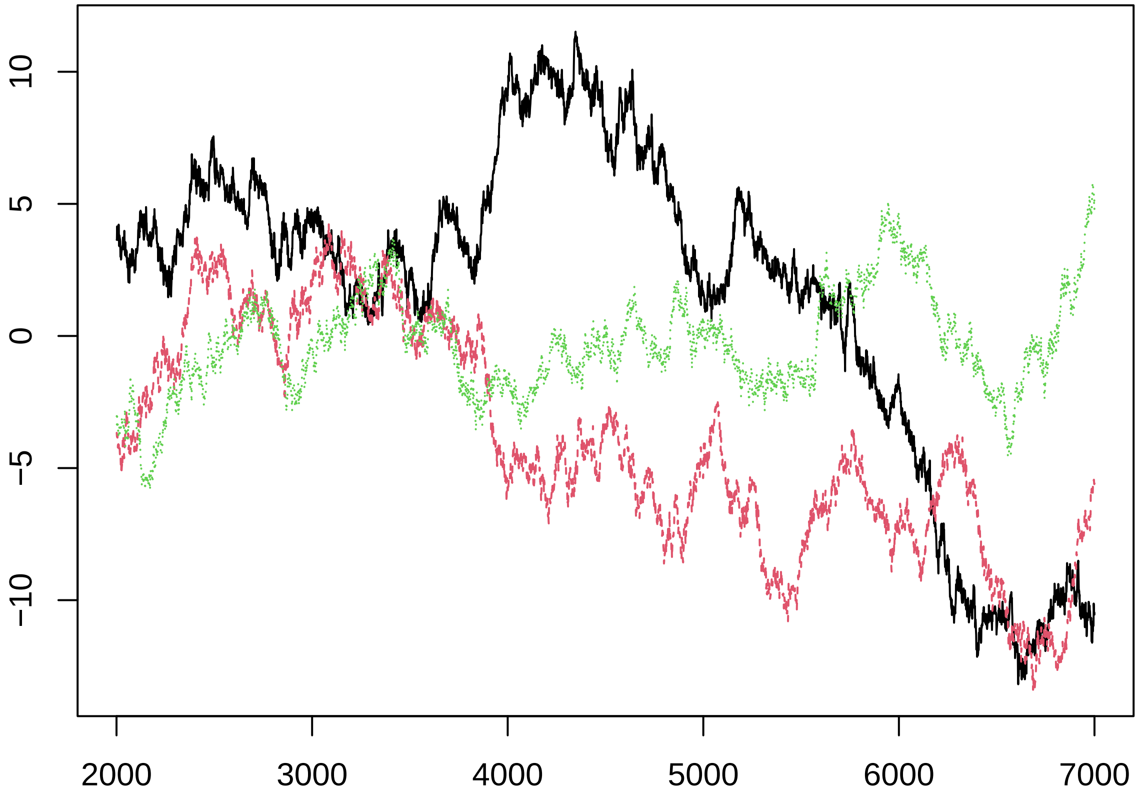

Trace plots − Flat Prior
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
−2.2
2000
3000
4000
5000
6000
7000

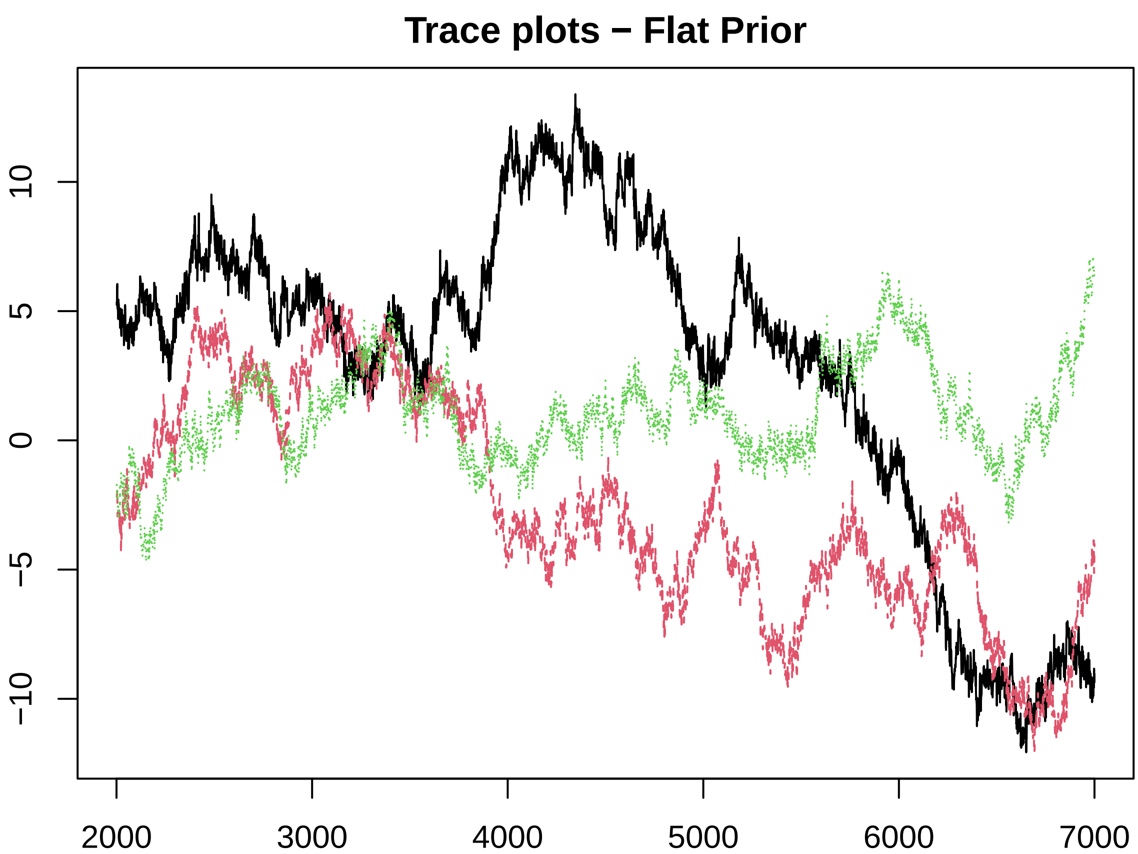


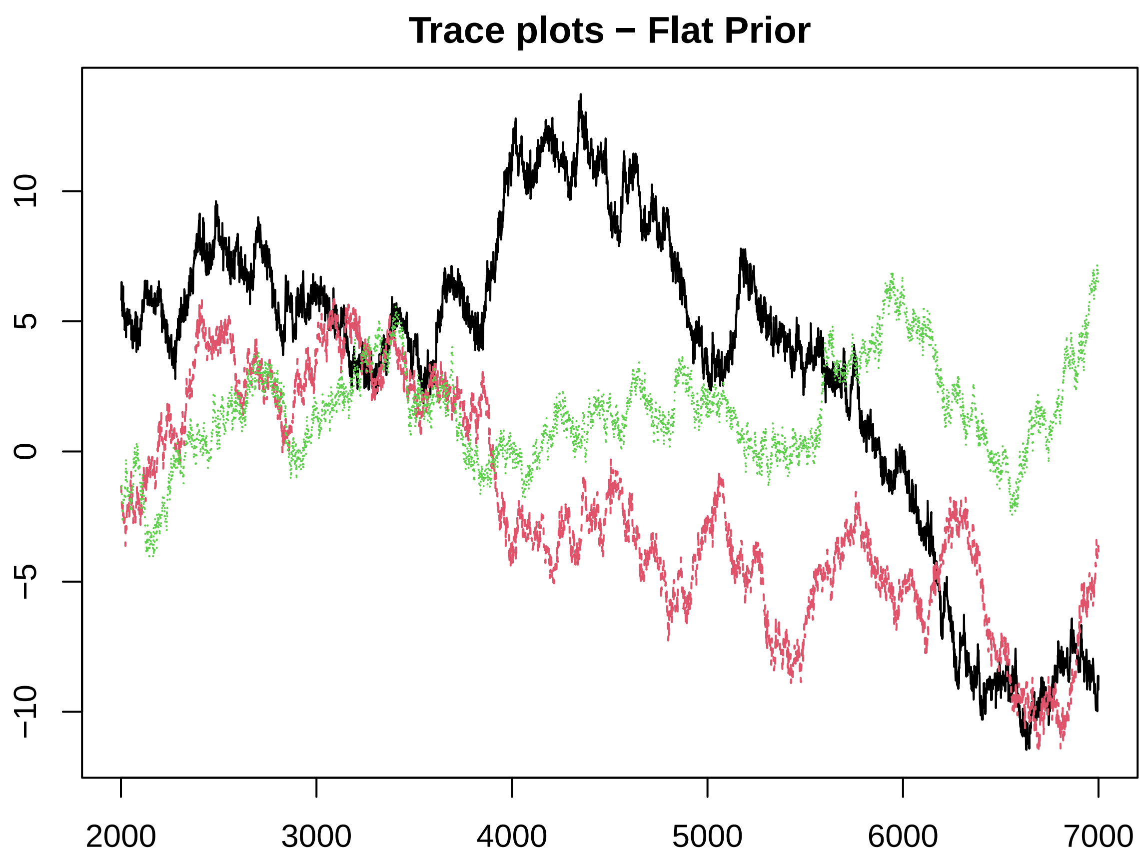
Trace plots − Flat Prior
0
5
10
15
20
25
30
2000
3000
4000
5000
6000
7000

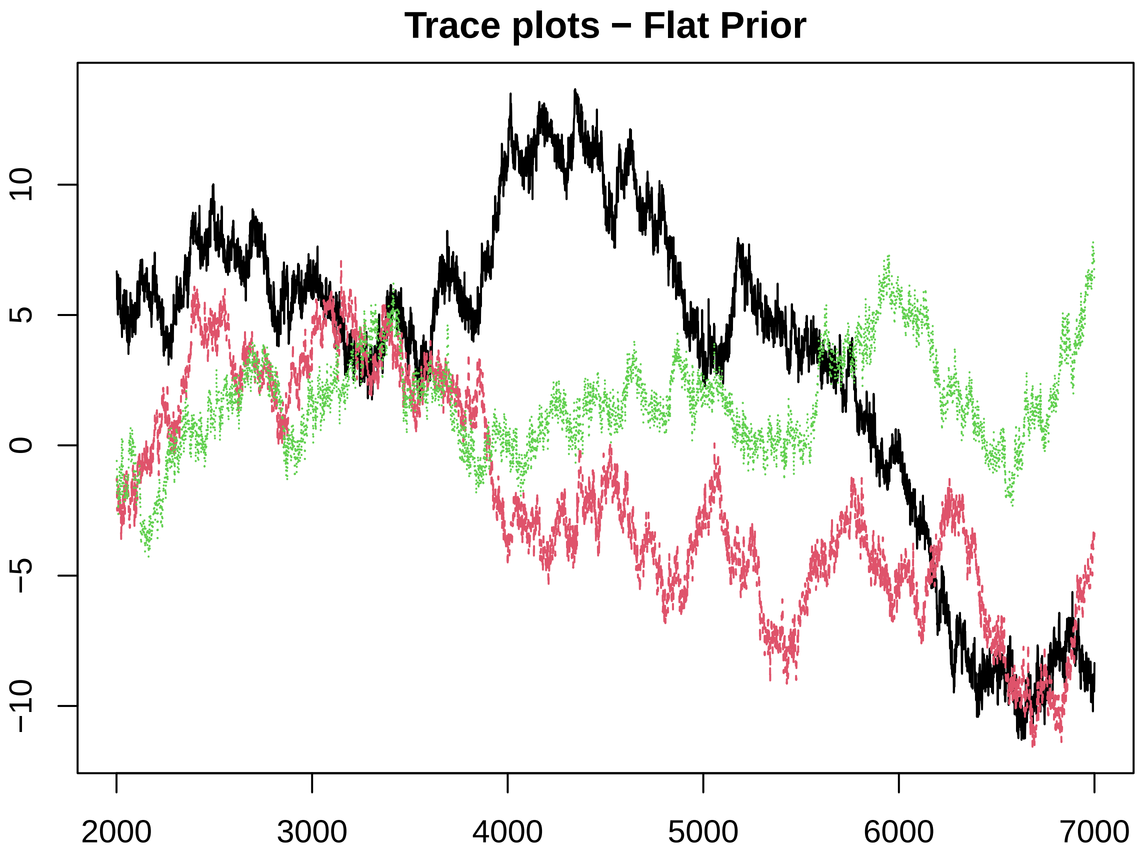

Trace plots − Flat Prior
−5
0
5
10
15
20
25
30
2000
3000
4000
5000
6000
7000

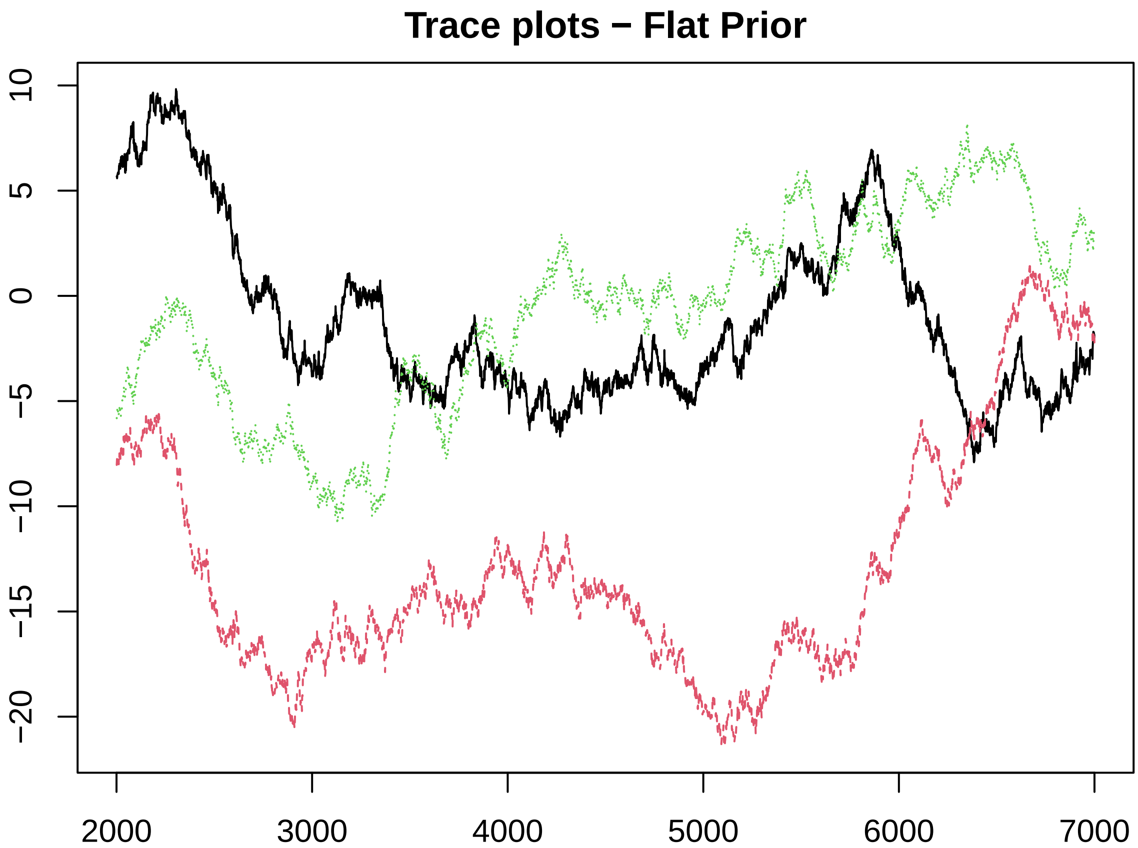
Trace plots − Flat Prior

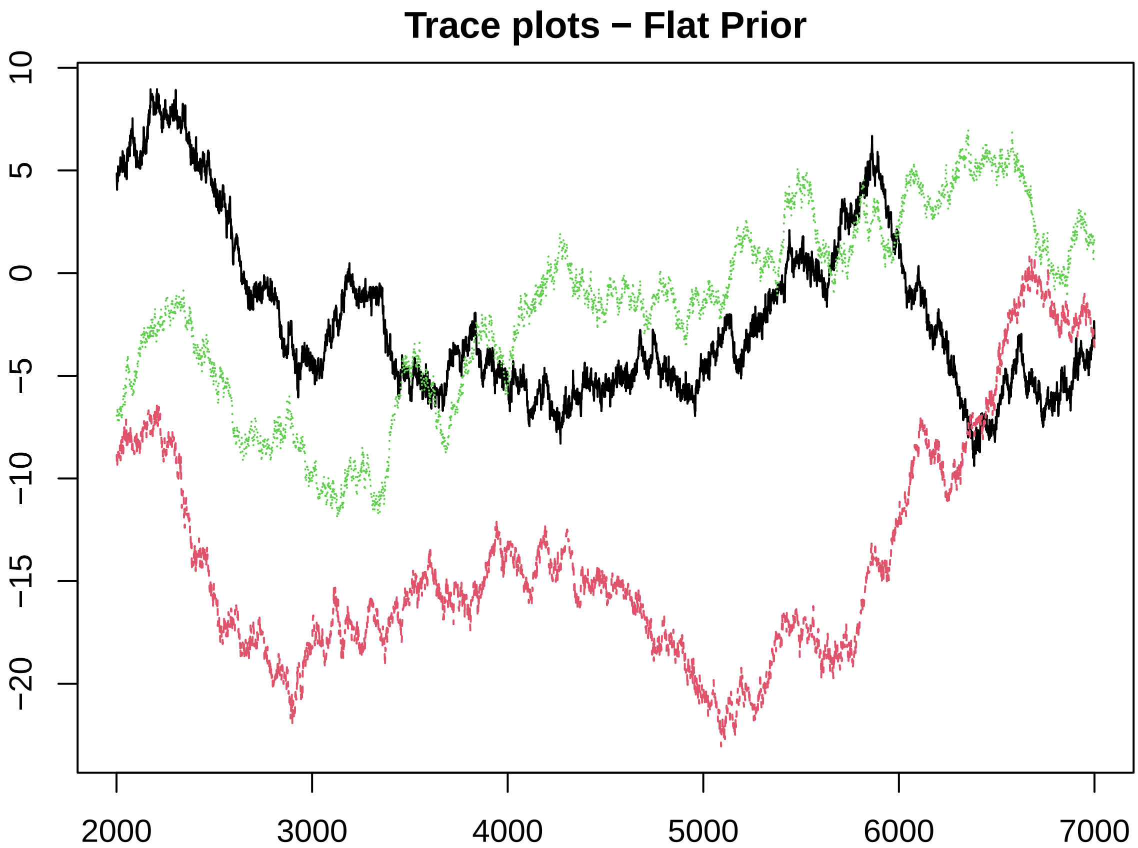


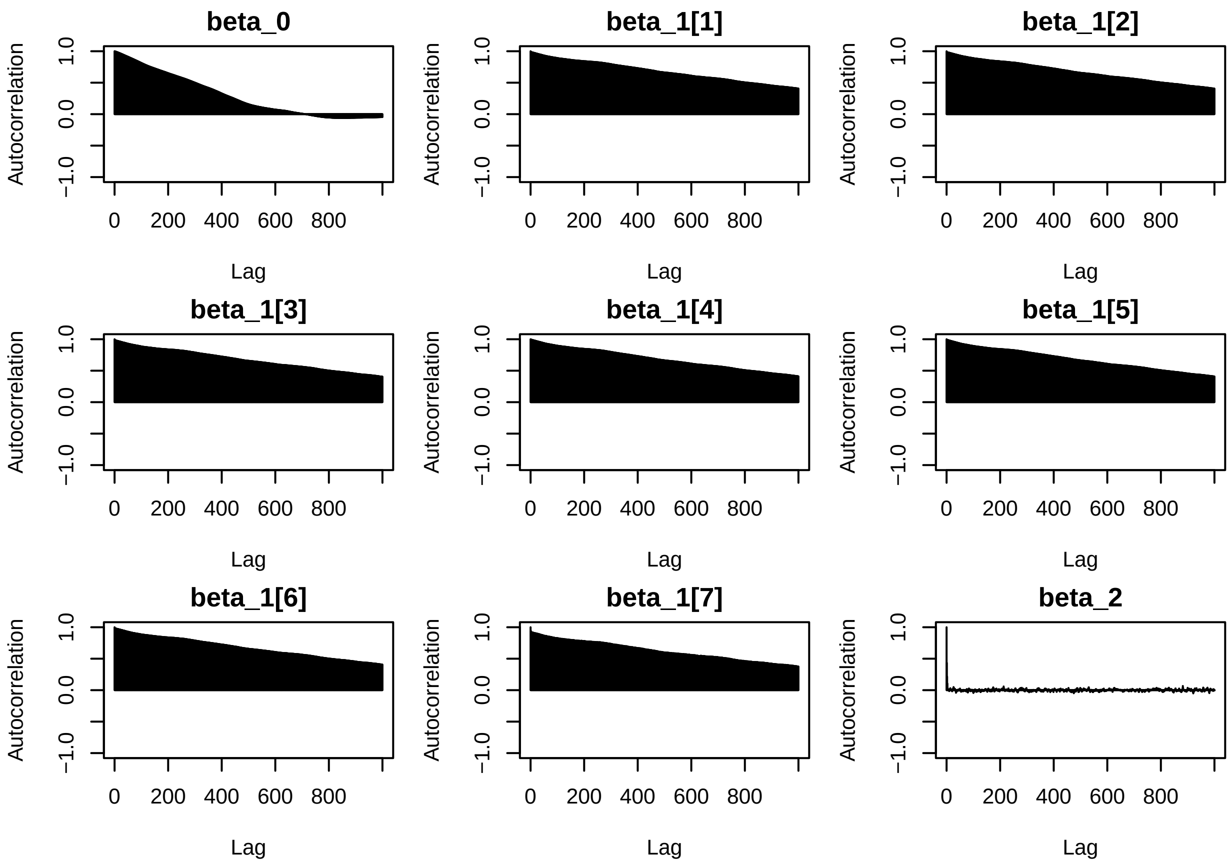

Trace plots − Flat Prior
10
5
0
−5
−10
2000
3000
4000
5000
6000
7000

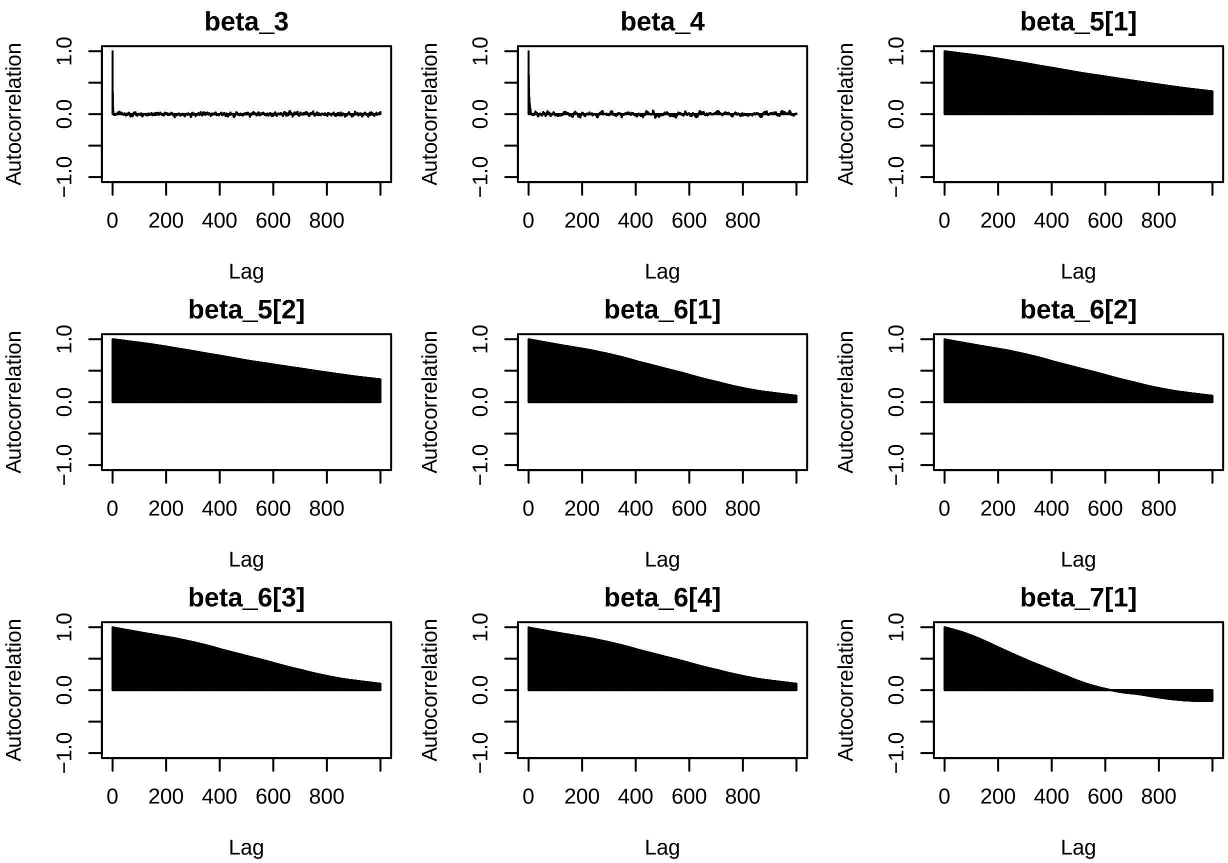

Trace plots − Flat Prior
10
5
0
−5
−10
2000
3000
4000
5000
6000
7000

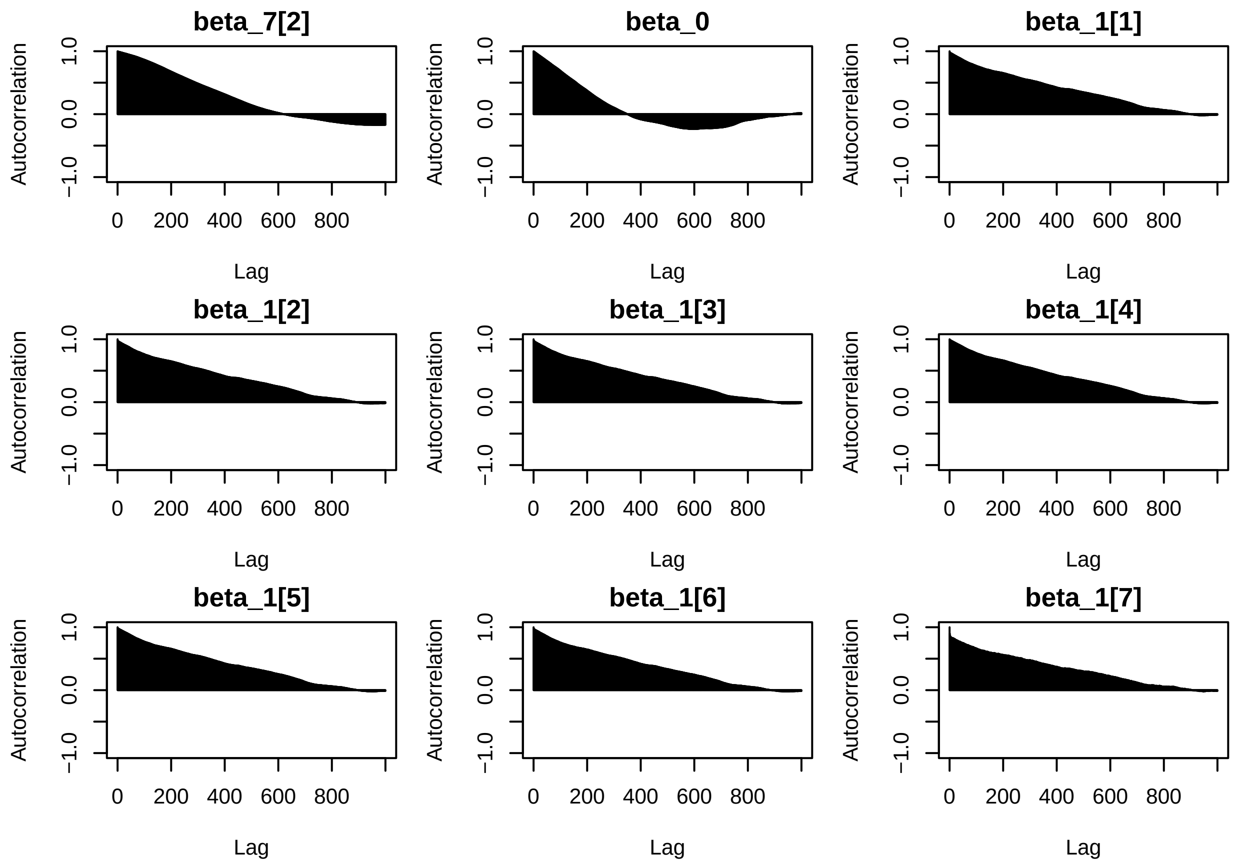

Trace plots − Flat Prior
10
5
0
−5
−10
2000
3000
4000
5000
6000
7000

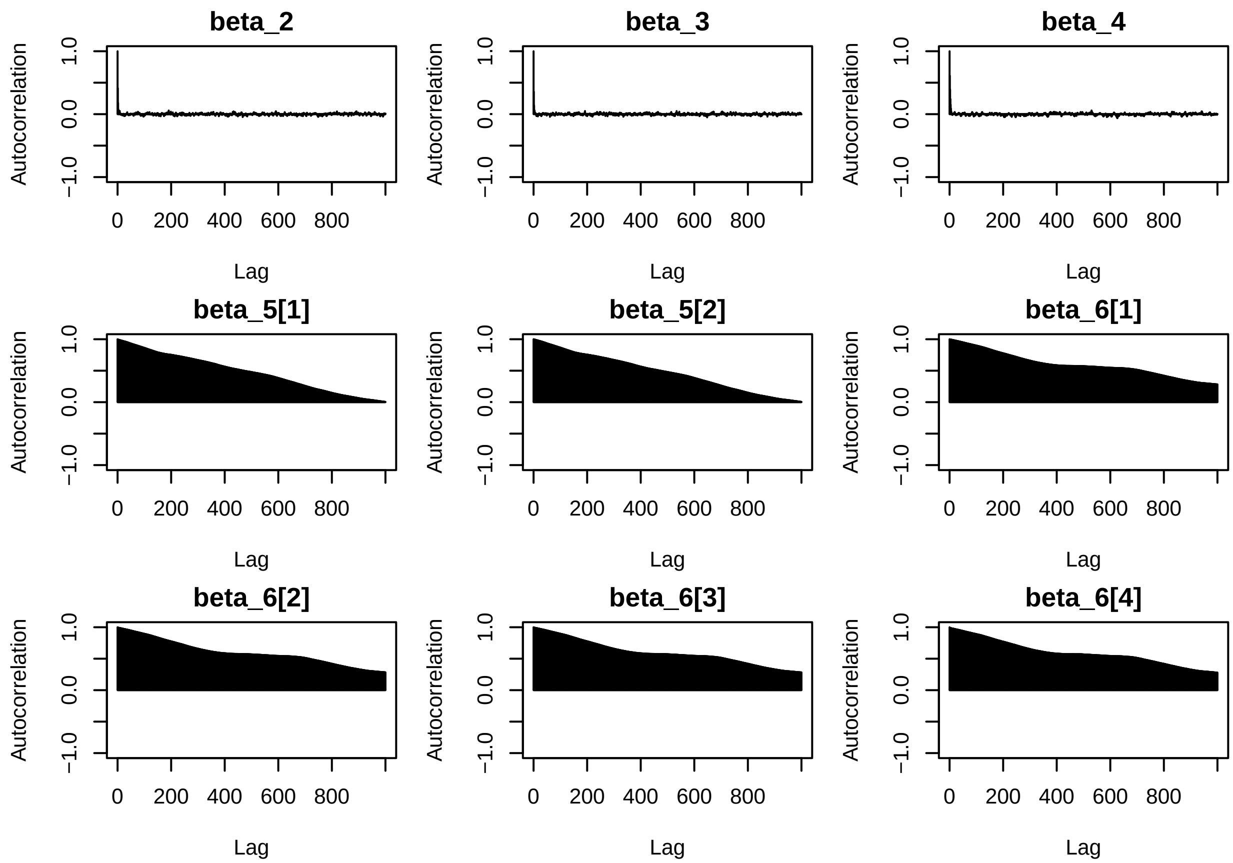

Trace plots − Flat Prior
10
5
0
−5
−10
−15
−20
2000
3000
4000
5000
6000
7000

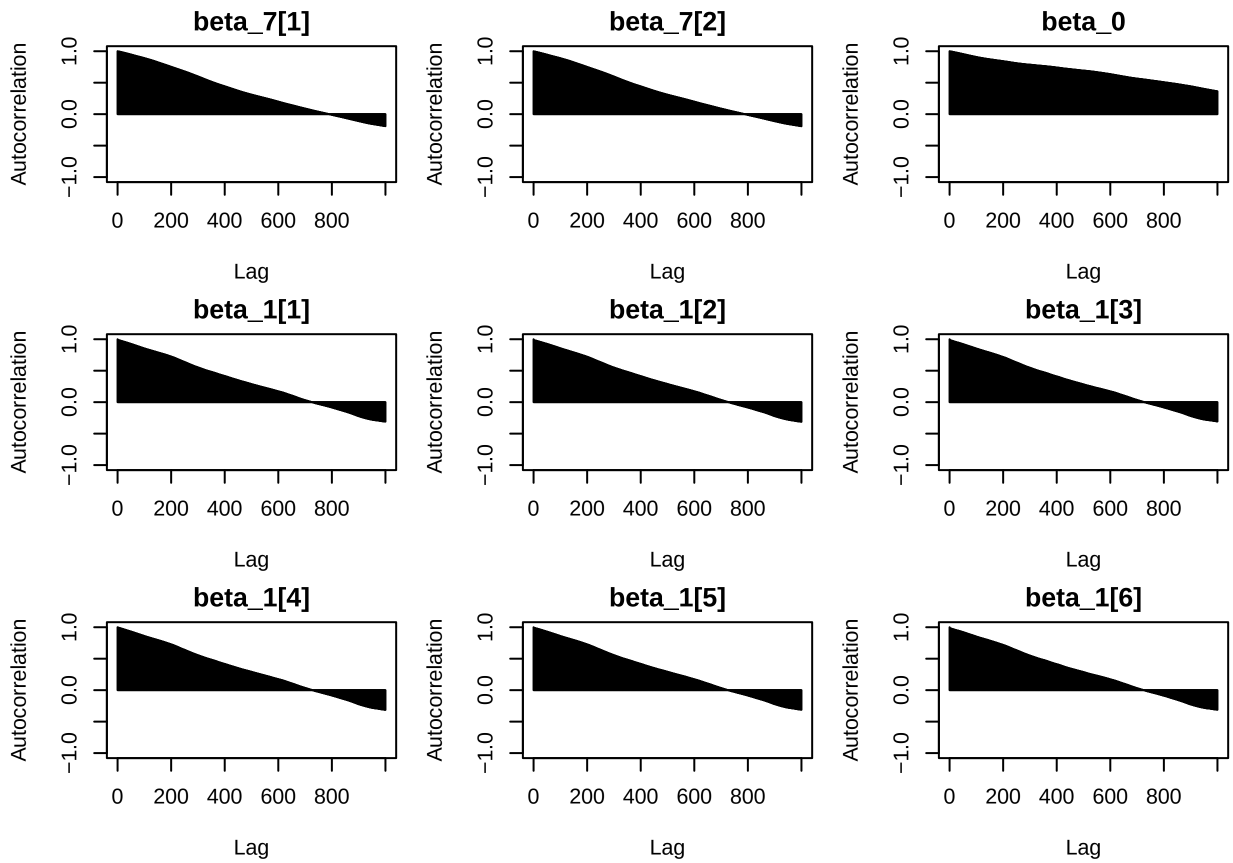

Trace plots − Flat Prior
10
5
0
−5
−10
−15
−20
2000
3000
4000
5000
6000
7000

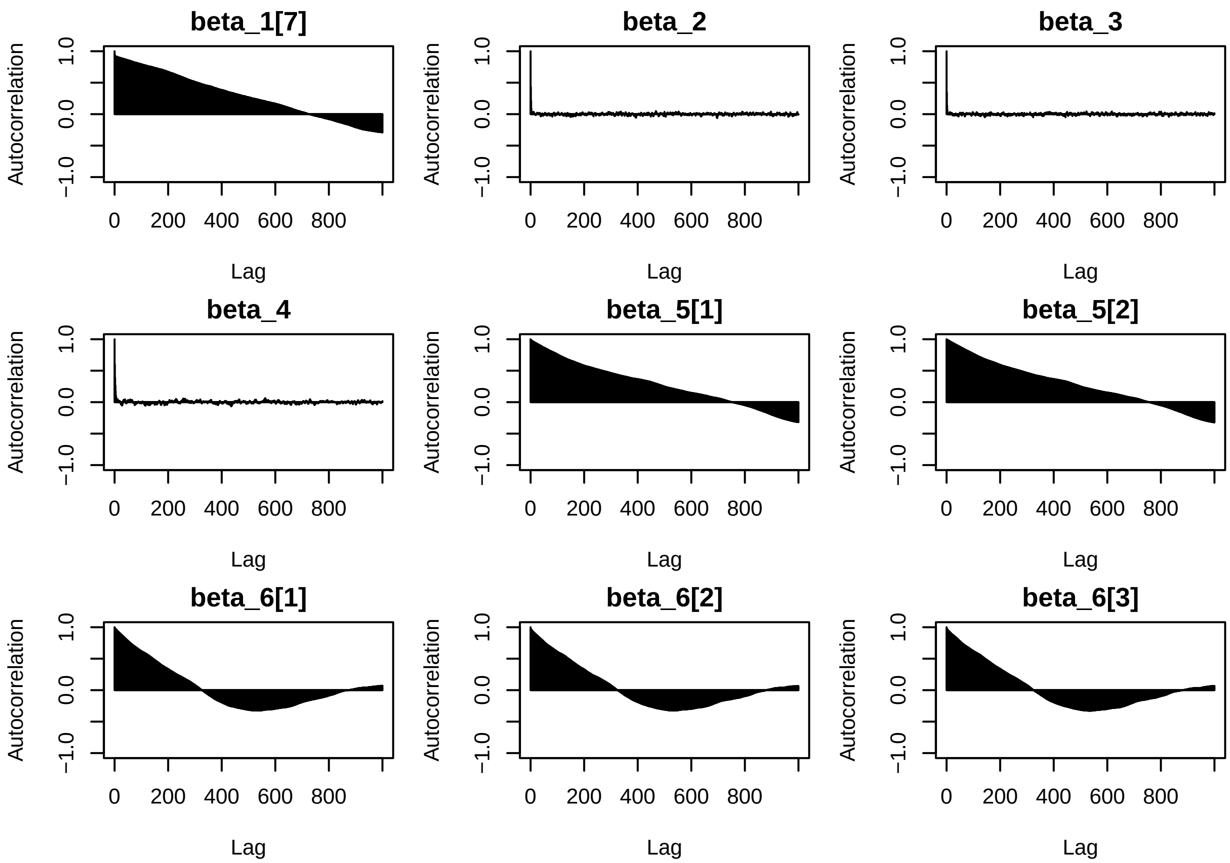

beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
beta_2
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800

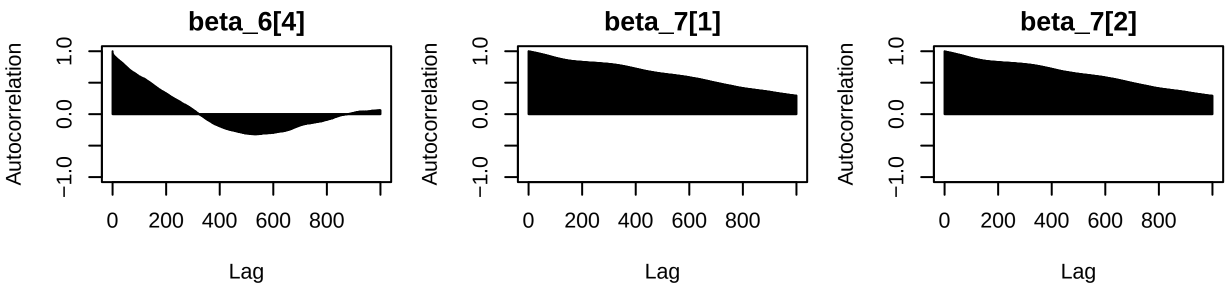

beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
beta_7[1]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800


beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
Autocorrelation
−1.0
0.0
1.0
Lag
0
200
400
600
800

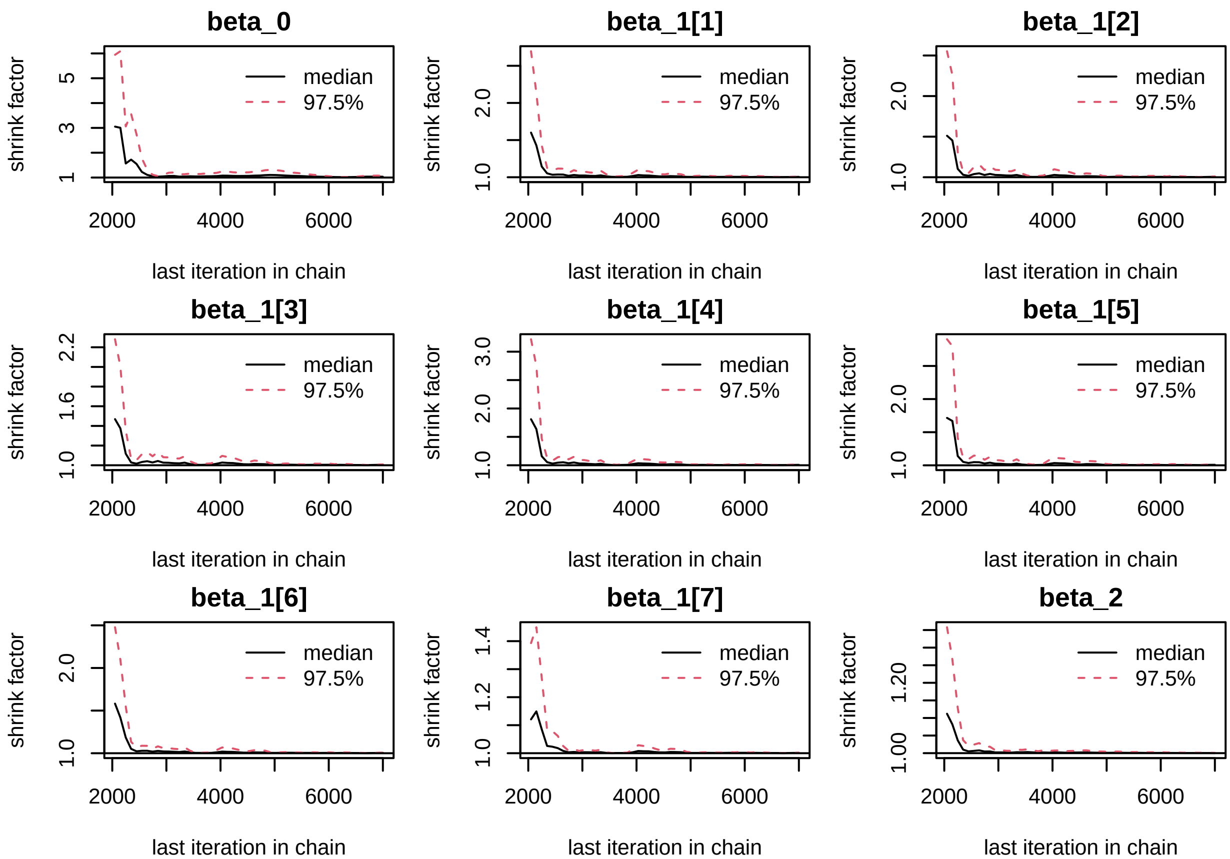

beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800


beta_7[1]
beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
Autocorrelation
−1.0 0.0 1.0
Lag
0 200 400 600 800

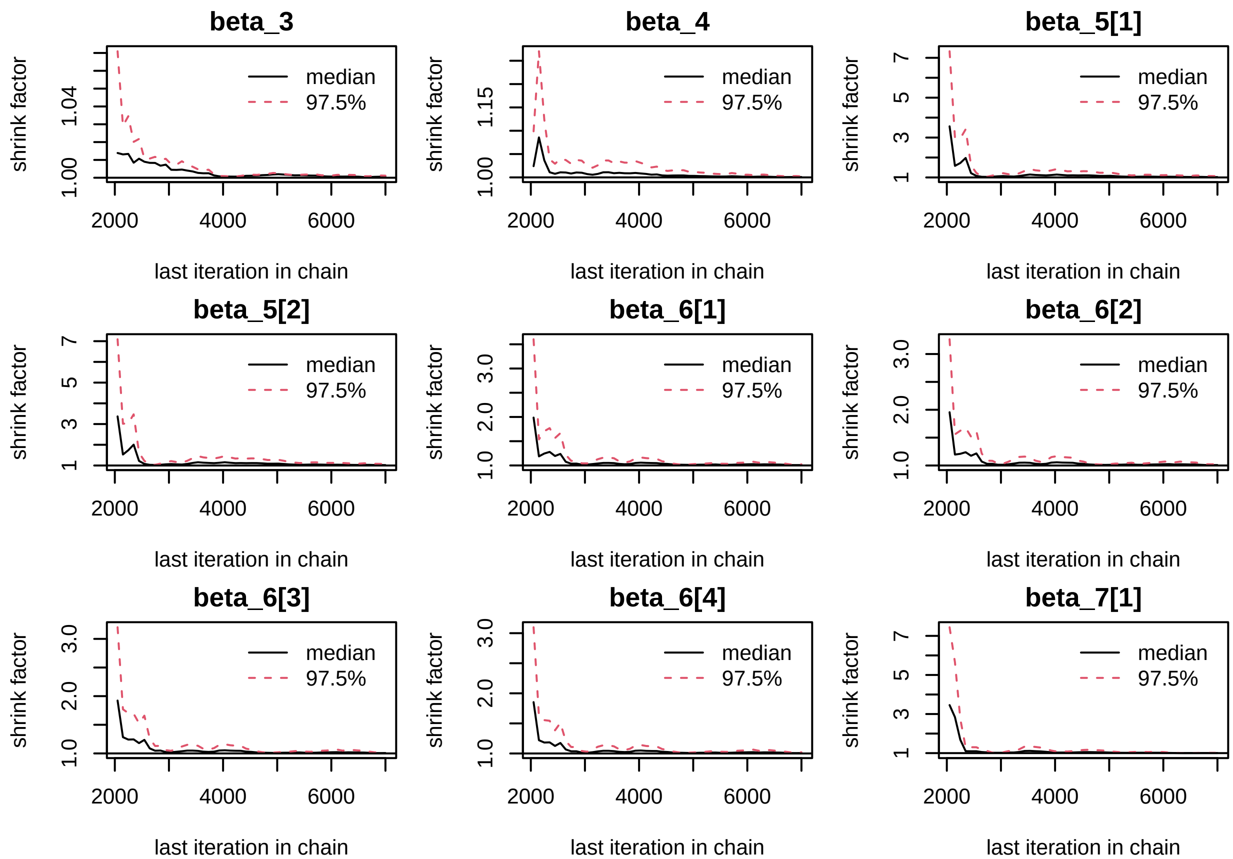

beta_1[7]
beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800

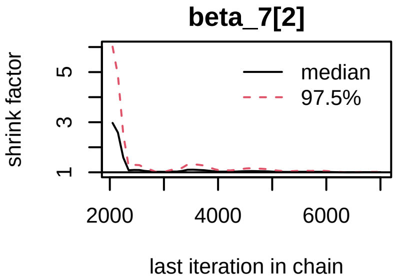


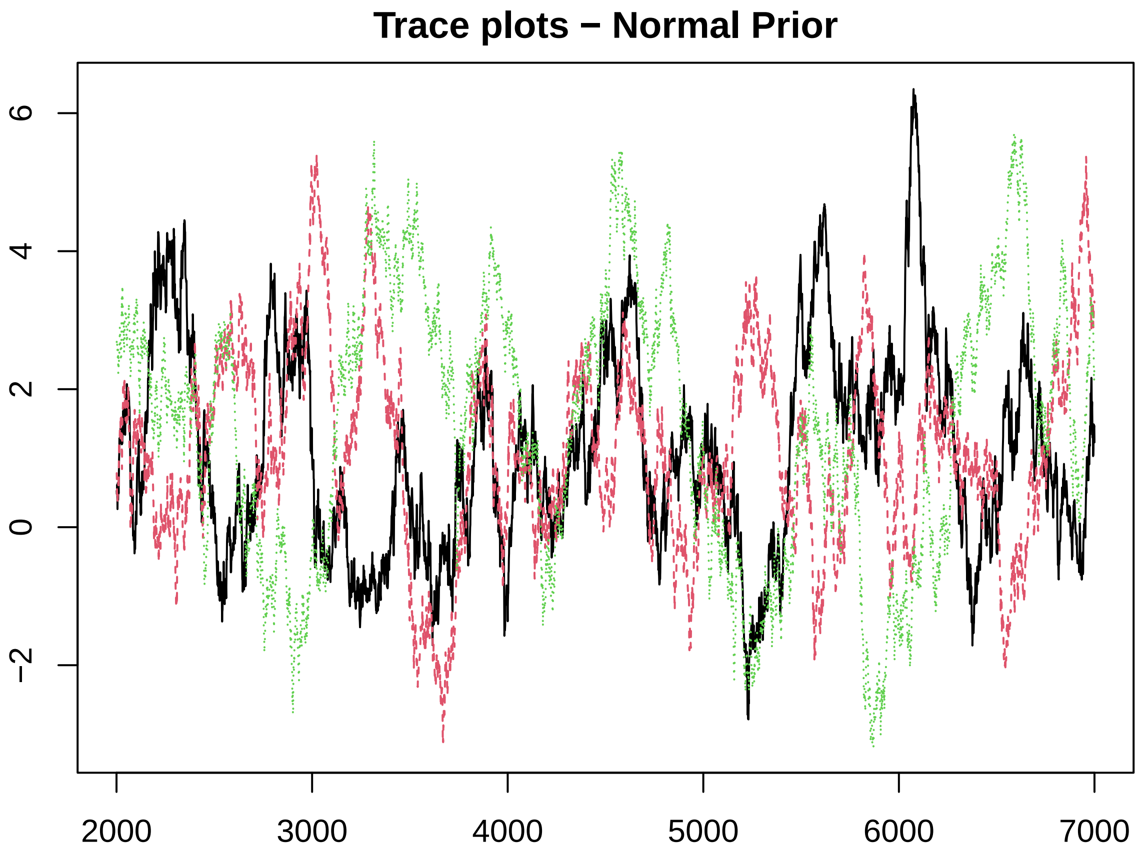

```
## Potential scale reduction factors:
##
##            Point est. Upper C.I.
## beta_0          3.21       7.69
## beta_1[1]       5.57      10.90
## beta_1[2]       5.56      10.91
## beta_1[3]       5.56      10.92
## beta_1[4]       5.60      10.99
## beta_1[5]       5.59      10.96
## beta_1[6]       5.57      10.90
## beta_1[7]       5.37      10.45
## beta_2          1.00       1.00
## beta_3          1.00       1.00
## beta_4          1.00       1.00
## beta_5[1]       1.22       1.81
## beta_5[2]       1.22       1.81
## beta_6[1]       1.37       2.27
## beta_6[2]       1.37       2.27
## beta_6[3]       1.37       2.27
## beta_6[4]       1.37       2.26
## beta_7[1]       2.34       4.15
## beta_7[2]       2.34       4.15
##
## Multivariate psrf
##
## 4.6
```



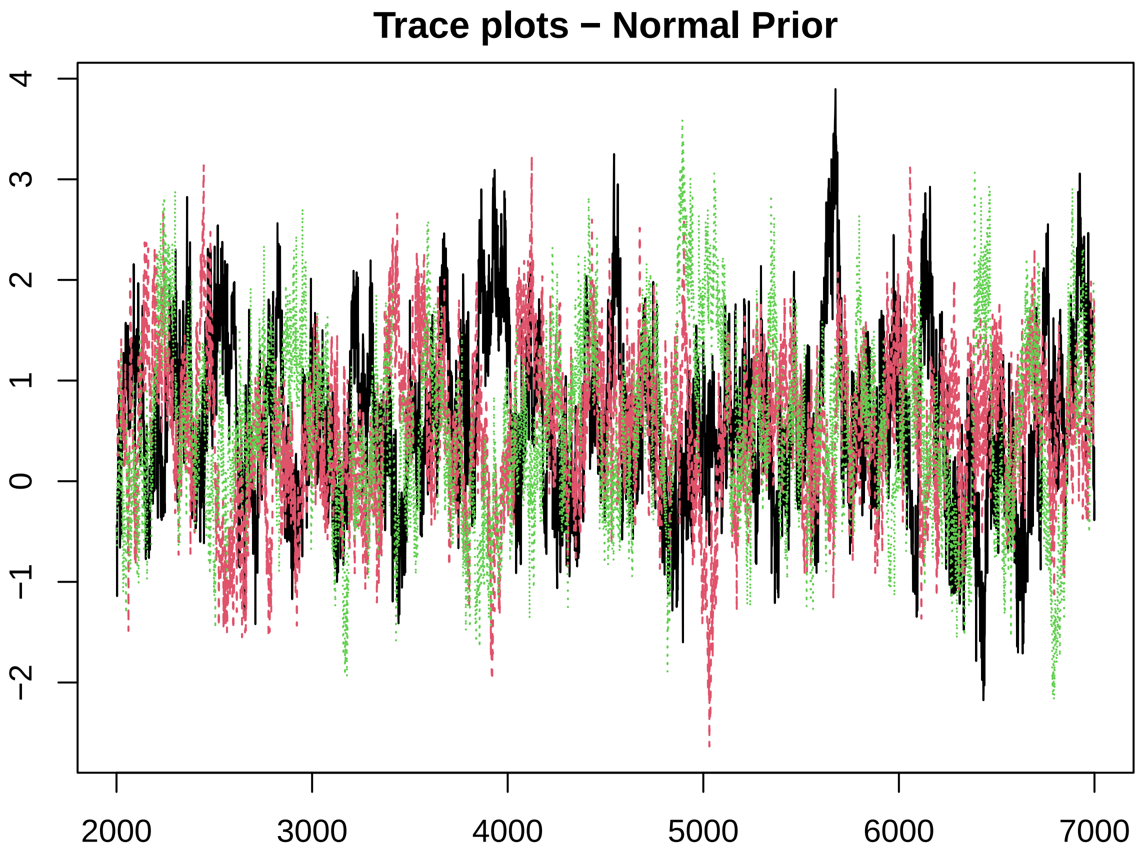
#Normal Prior


beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
beta_7[1]
shrink factor
last iteration in chain
median
97.5%
2000
4000
6000

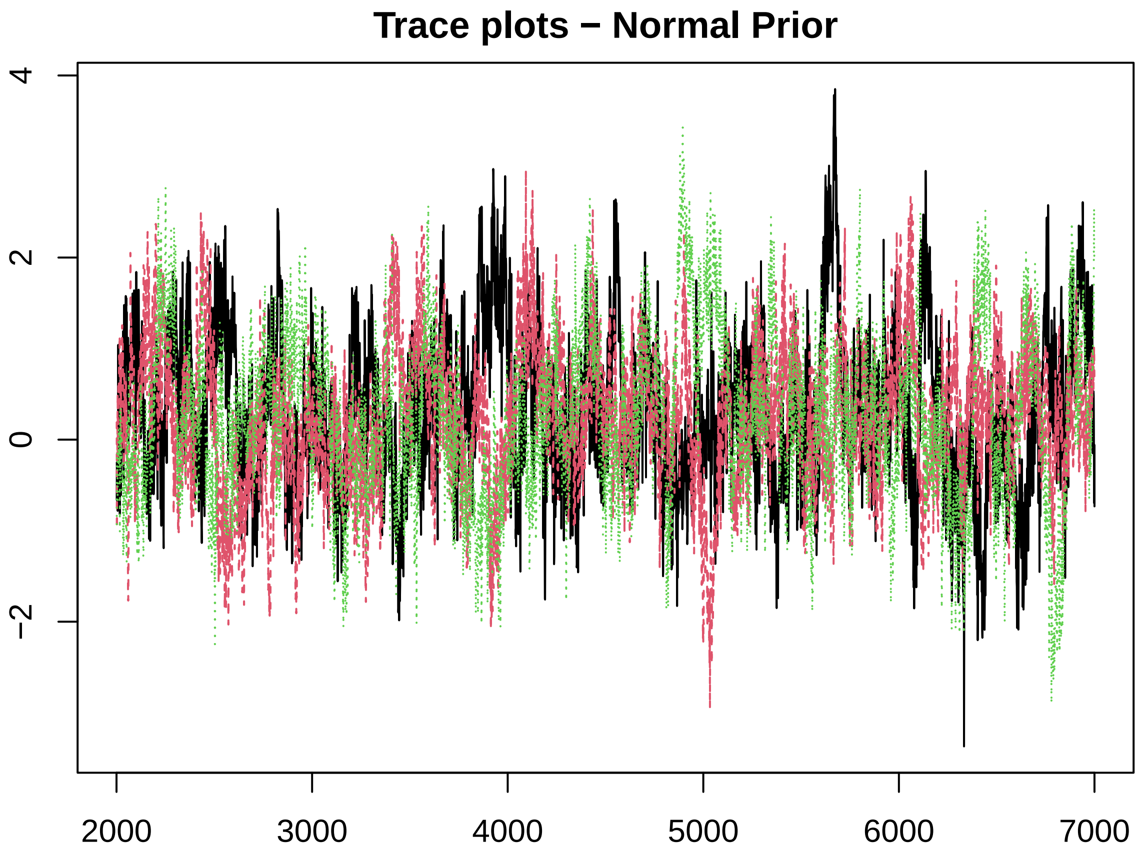

beta_7[2]
shrink factor
1
3
5
median
97.5%
2000
4000
6000
last iteration in chain



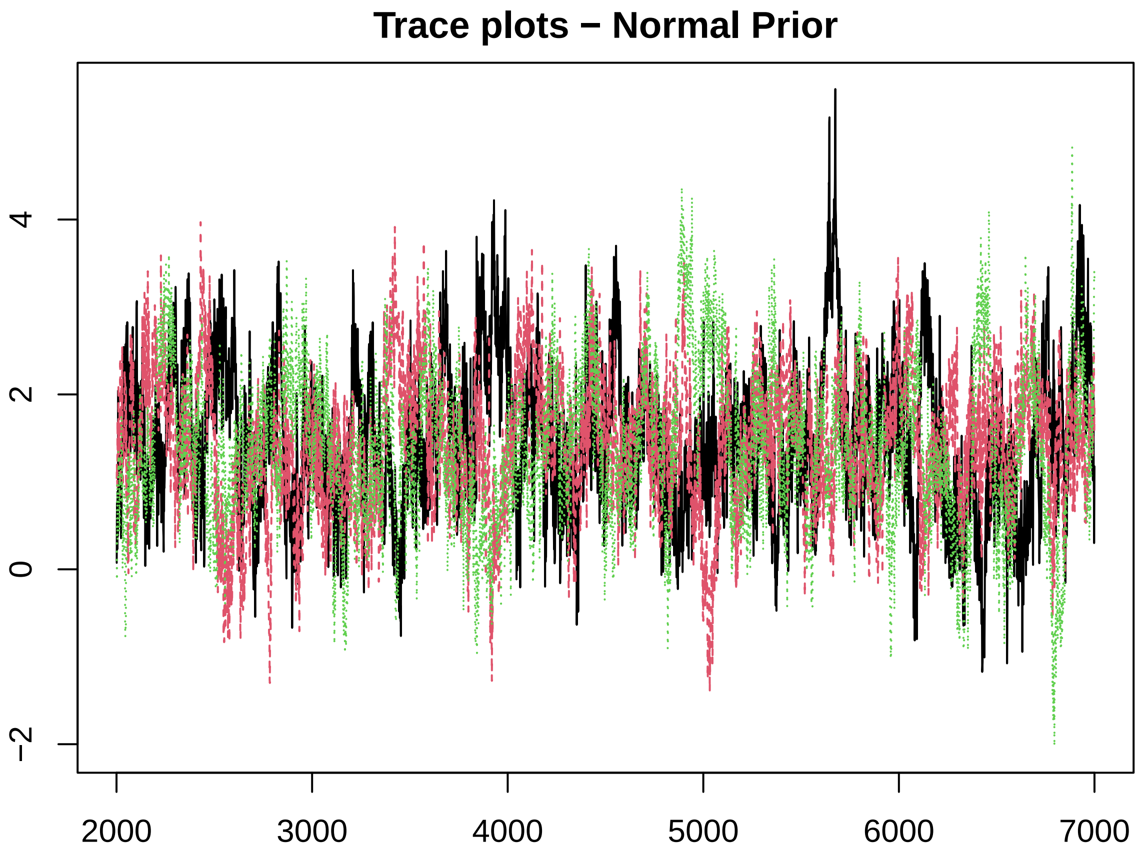
Trace plots − Normal Prior
6
4
2
0
−2
2000
3000
4000
5000
6000
7000

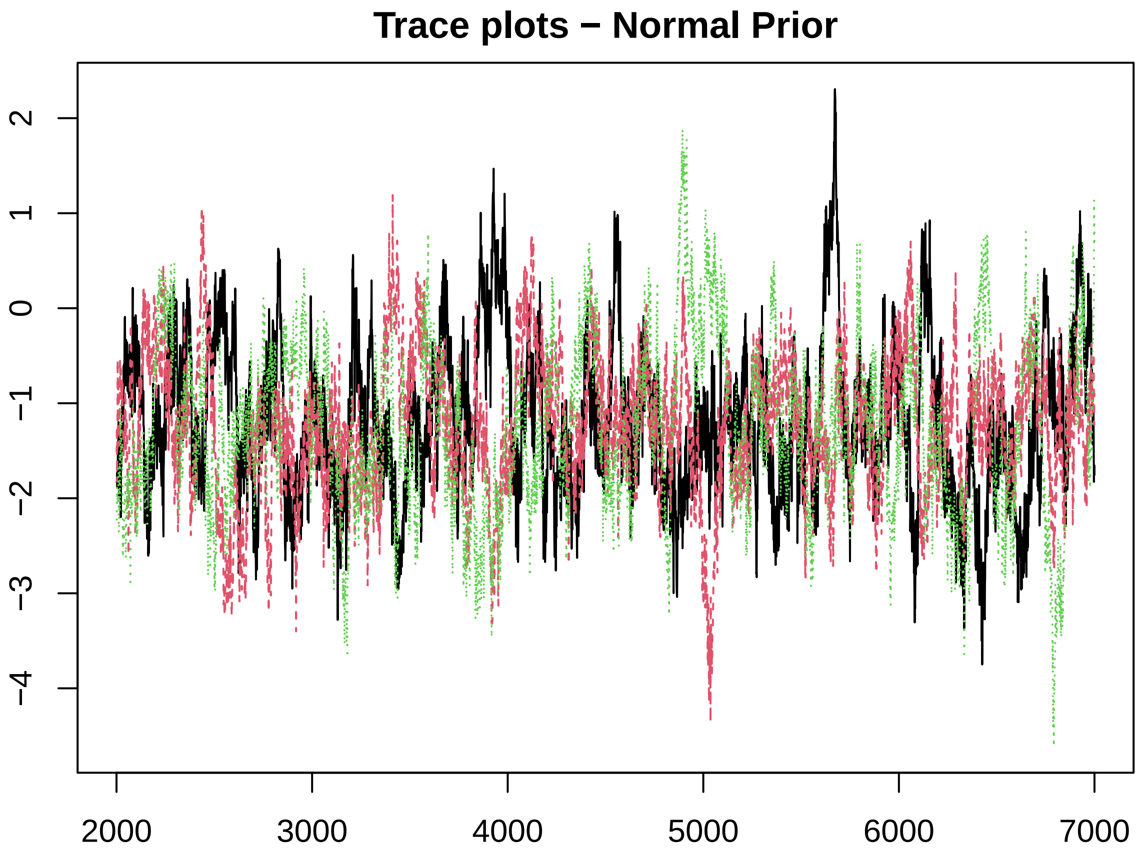


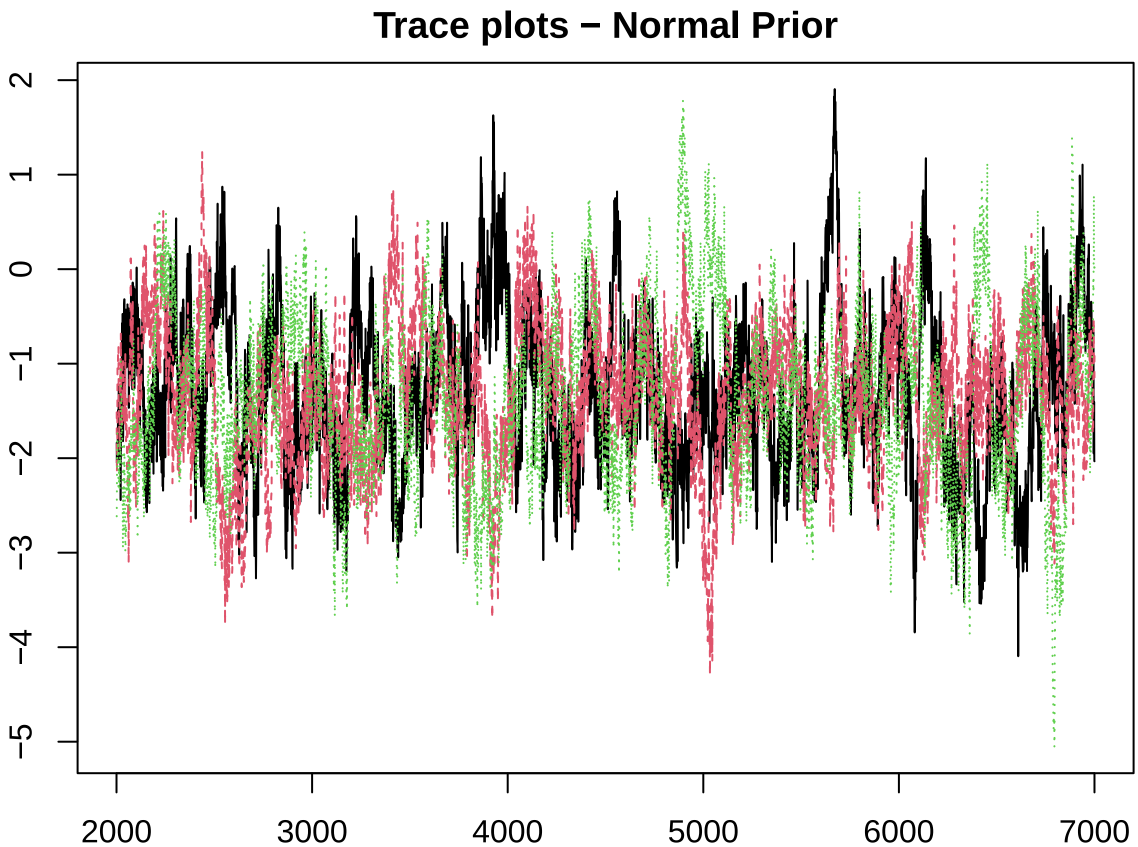
Trace plots − Normal Prior
4
3
2
1
0
−1
−2
2000
3000
4000
5000
6000
7000



Trace plots − Normal Prior
4
2
0
−2
2000
3000
4000
5000
6000
7000


Trace plots − Normal Prior
−2
0
2
4
2000
3000
4000
5000
6000
7000


Trace plots − Normal Prior
2
1
0
−1
−2
−3
−4
2000
3000
4000
5000
6000
7000


Trace plots − Normal Prior
2
1
0
−1
−2
−3
−4
−5
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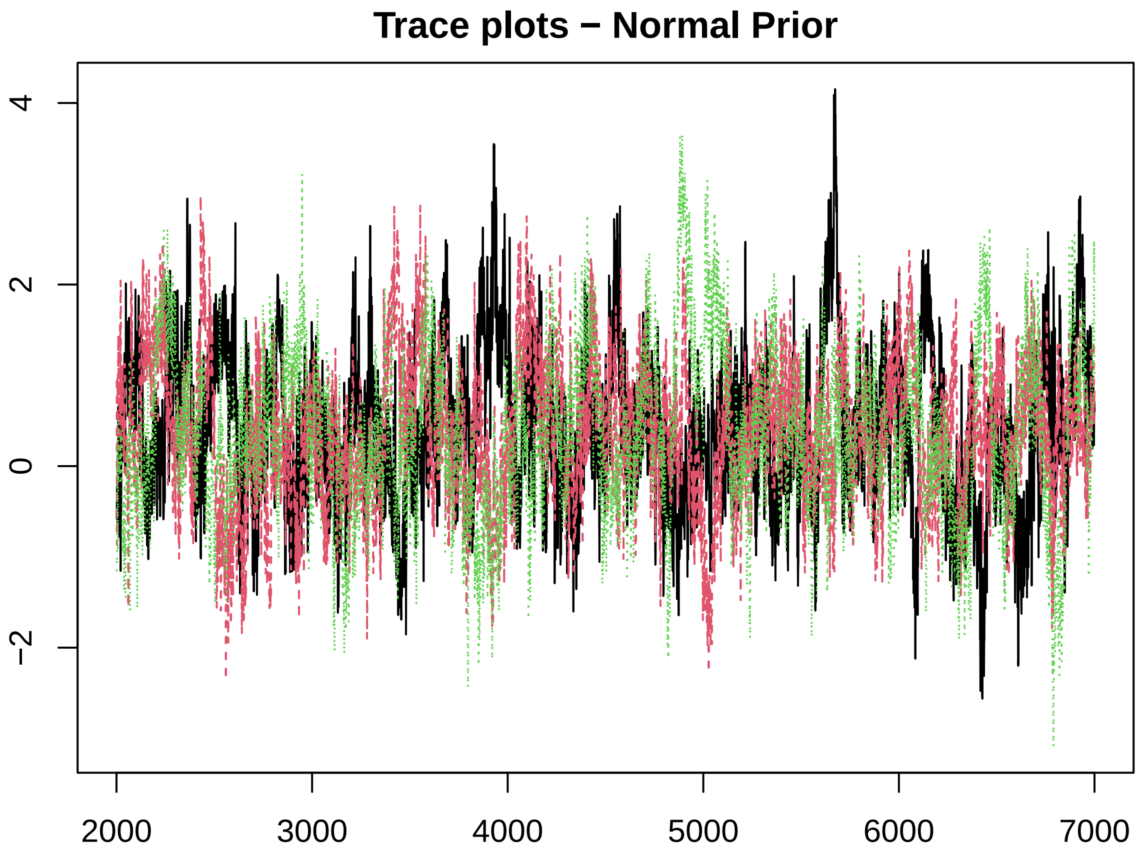
3000
4000
5000
6000
7000


Trace plots − Normal Prior
4
2
0
−2
2000
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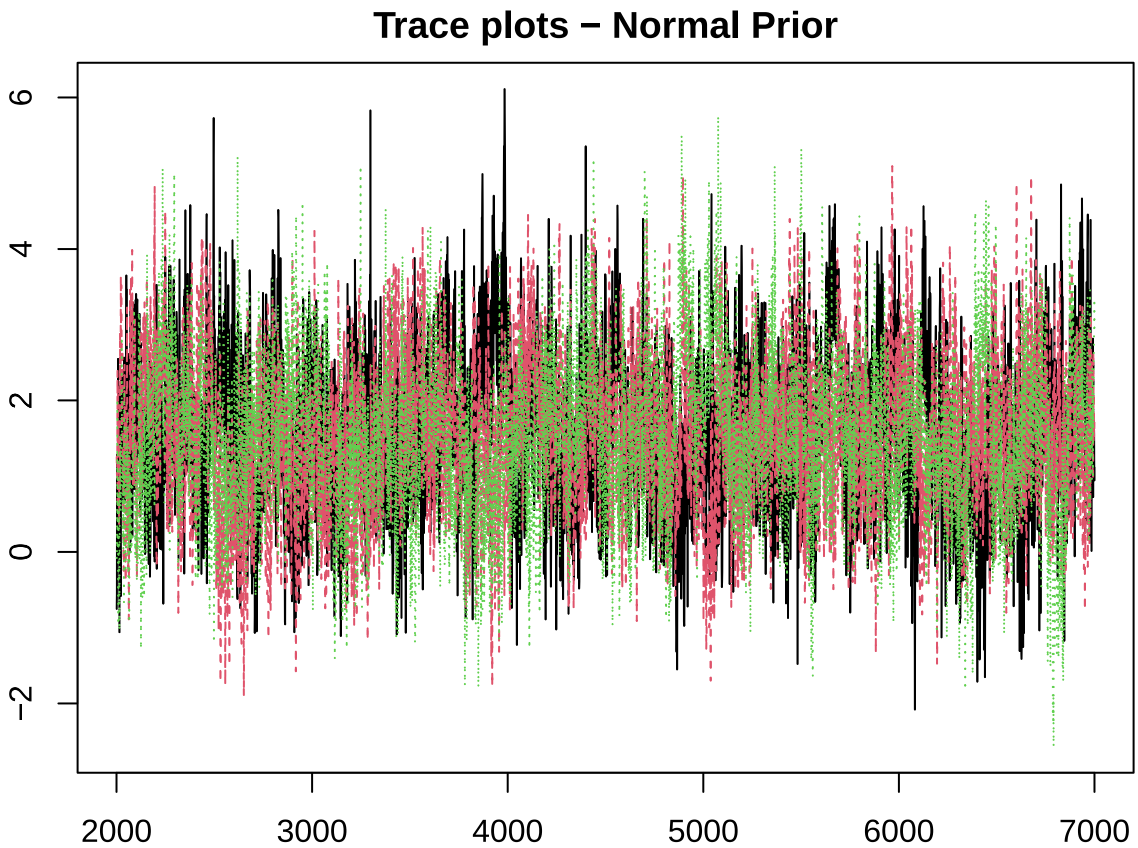
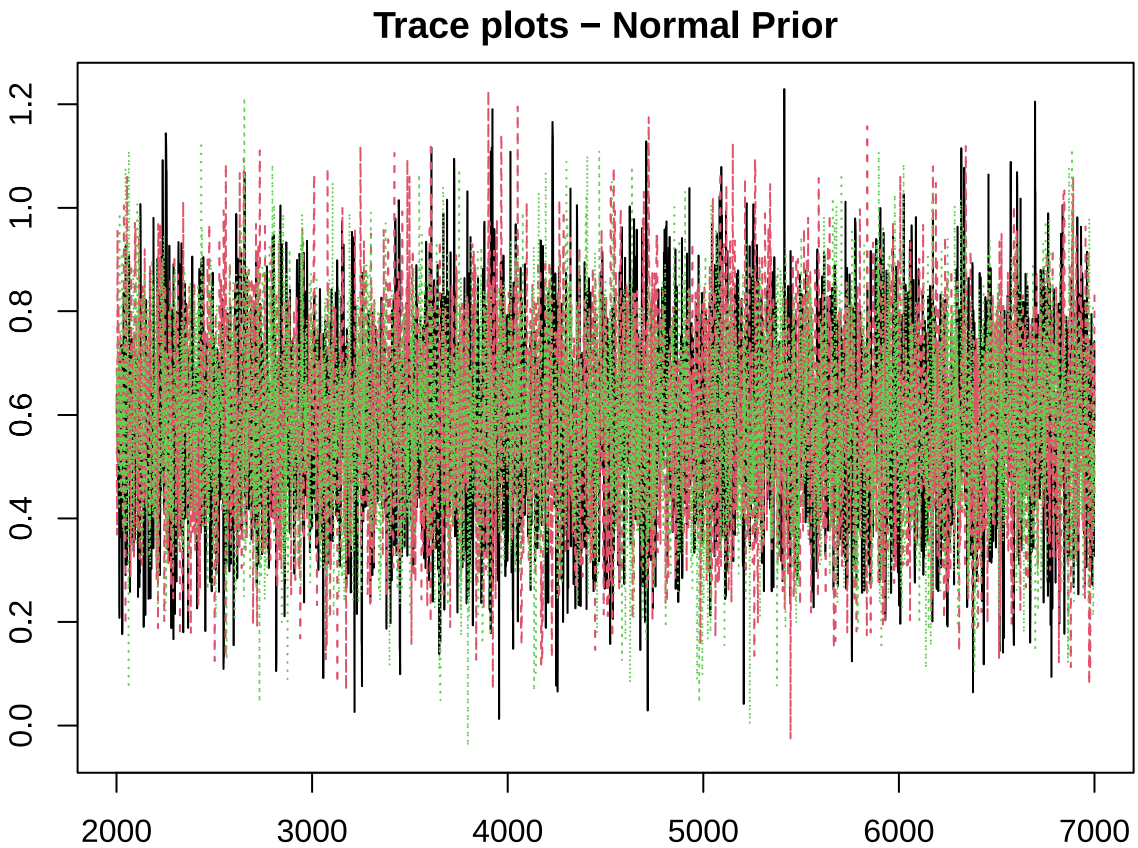
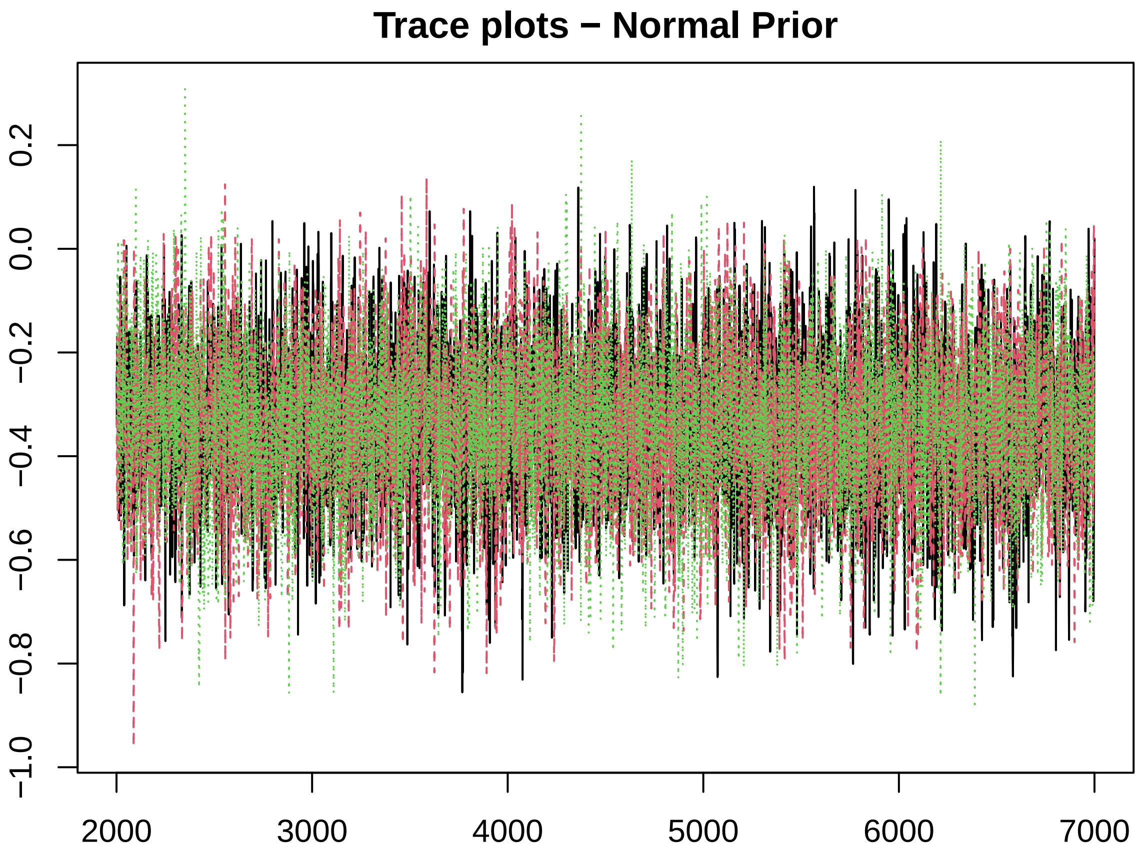
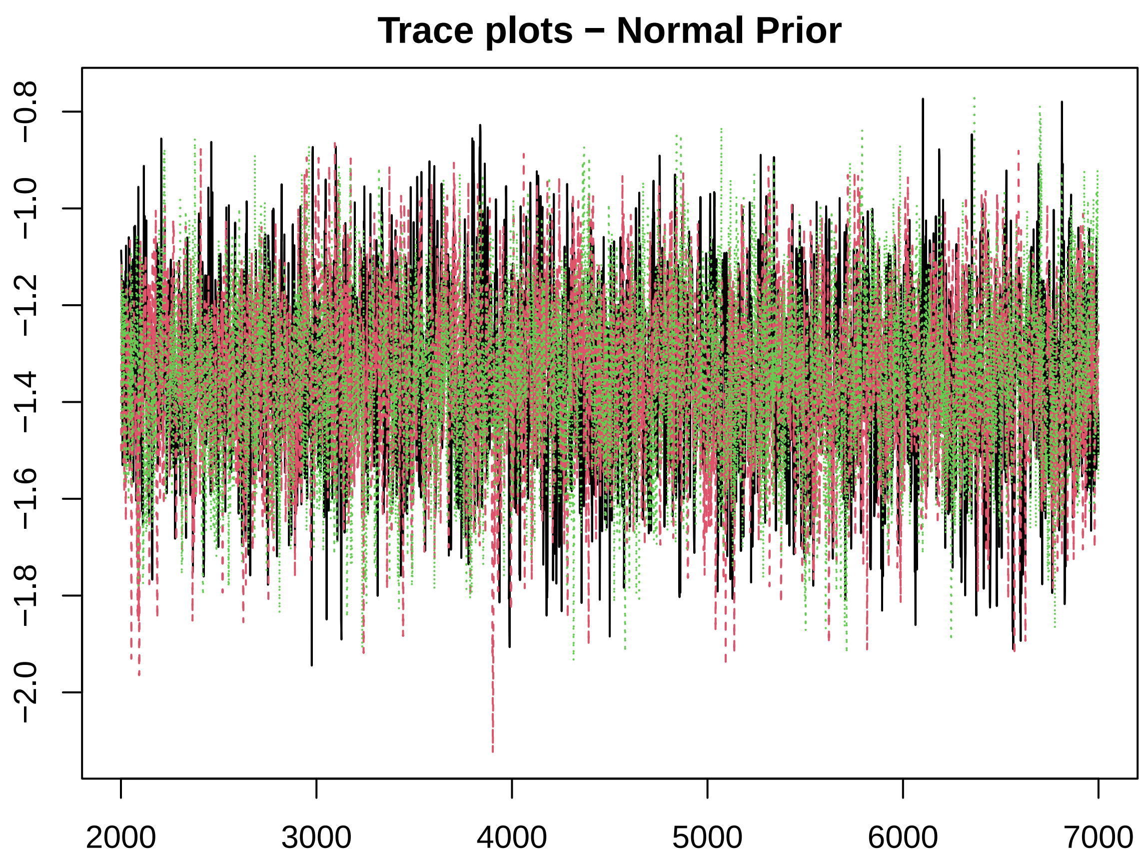
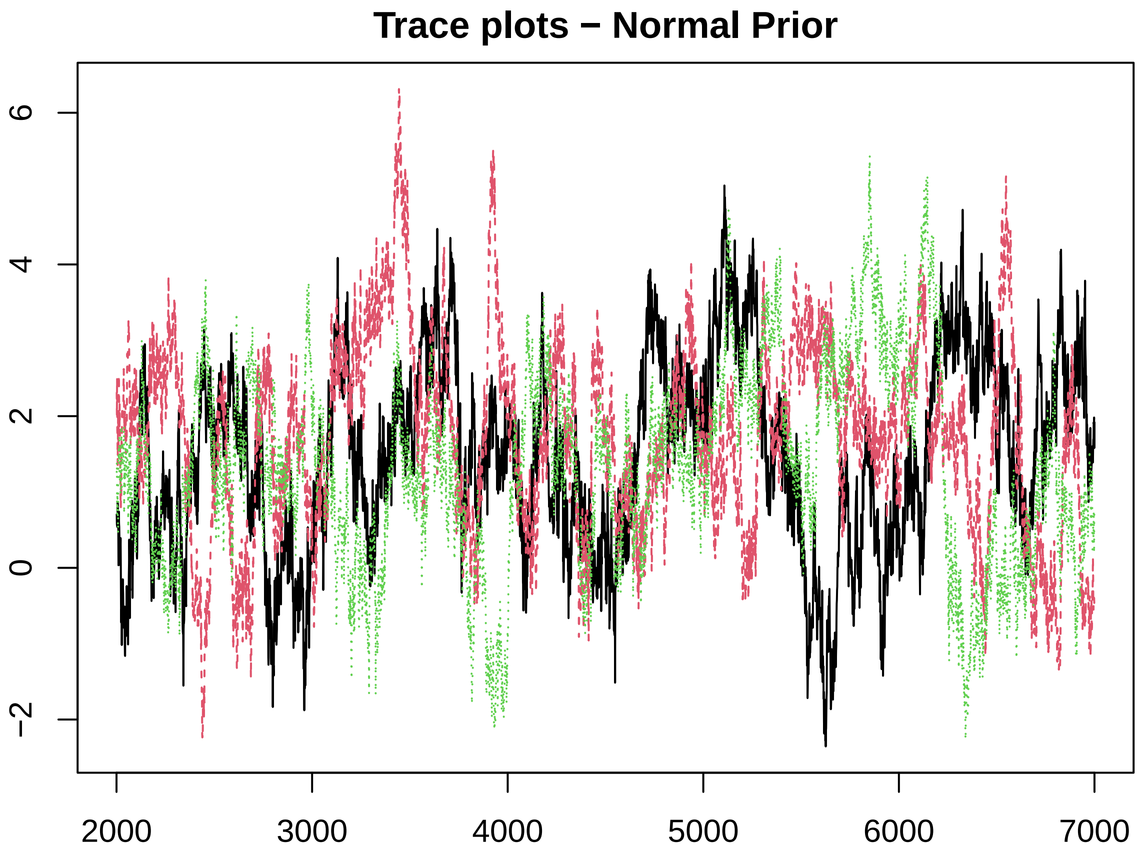
4000
5000
6000
7000

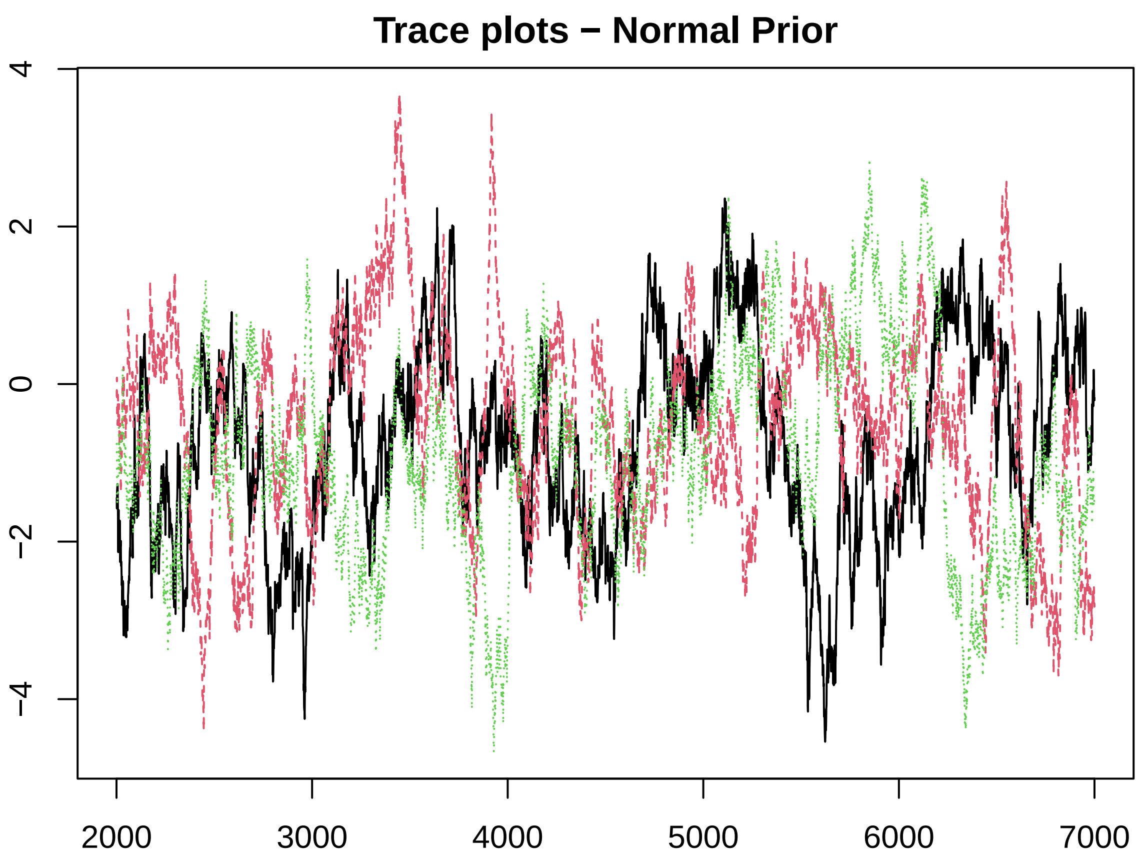

Trace plots − Normal Prior
6
4
2
0
−2
2000
3000
4000
5000
6000
7000

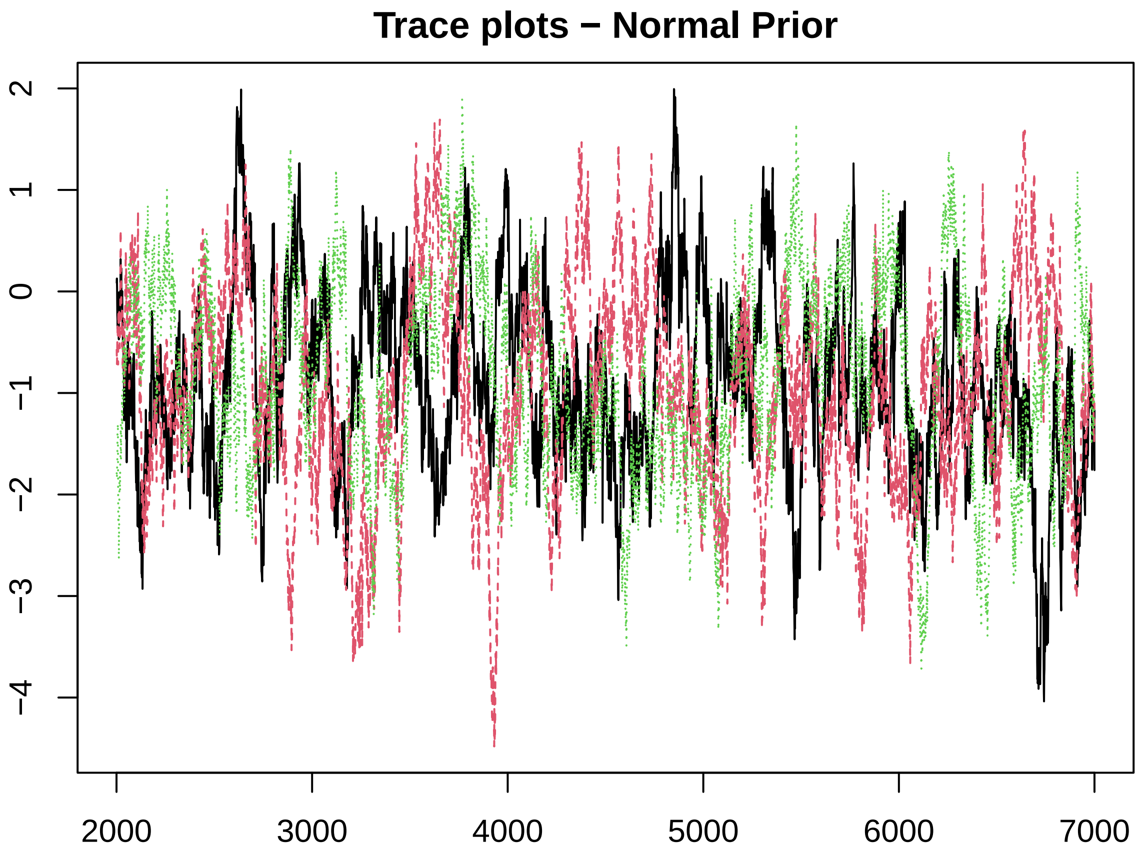

Trace plots − Normal Prior
0.0
0.2
0.4
0.6
0.8
1.0
1.2
2000
3000
4000
5000
6000
7000

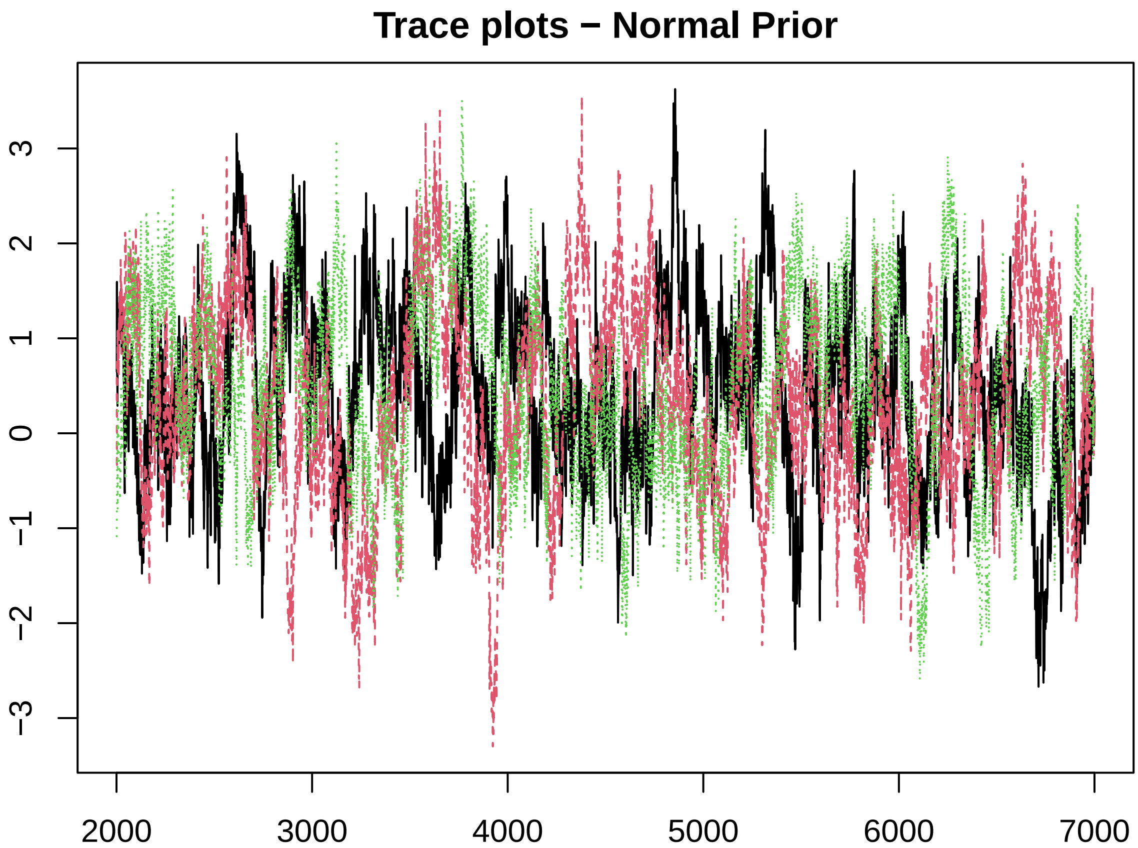

Trace plots − Normal Prior
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
2000
3000
4000
5000
6000
7000

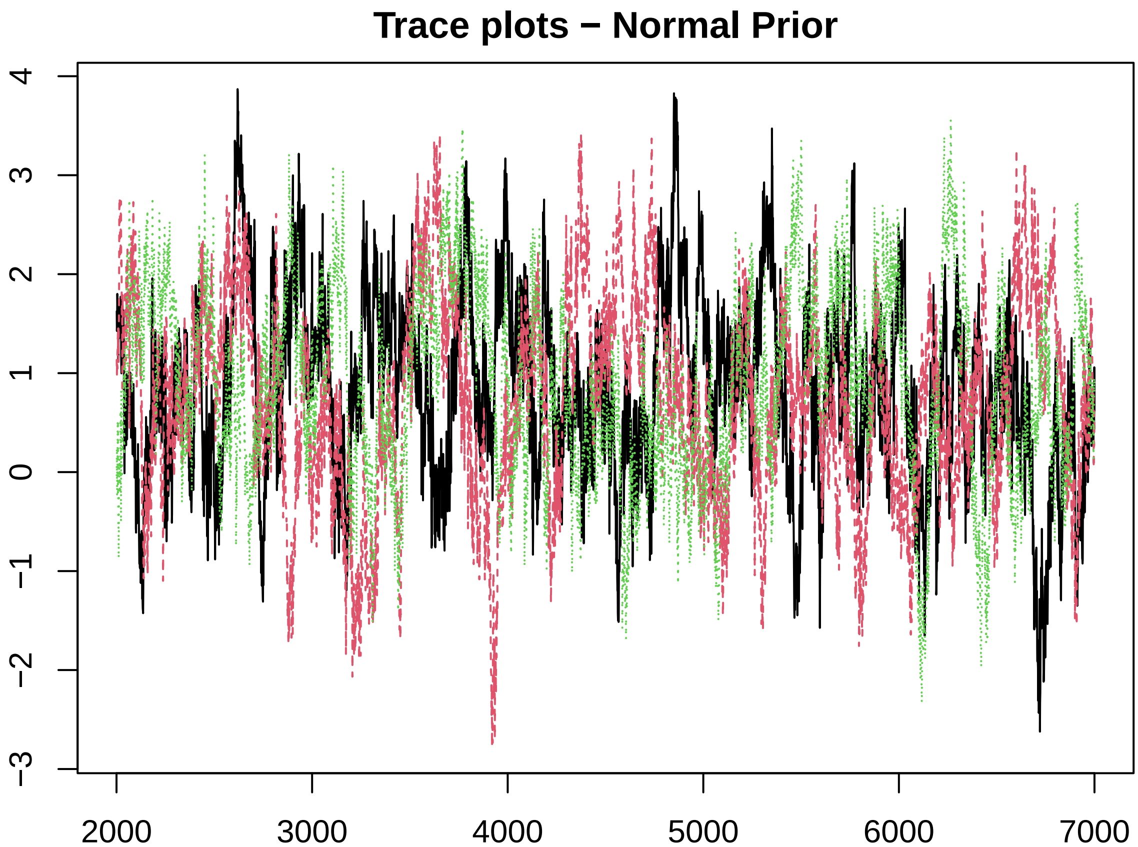

Trace plots − Normal Prior
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
−2.0
2000
3000
4000
5000
6000
7000

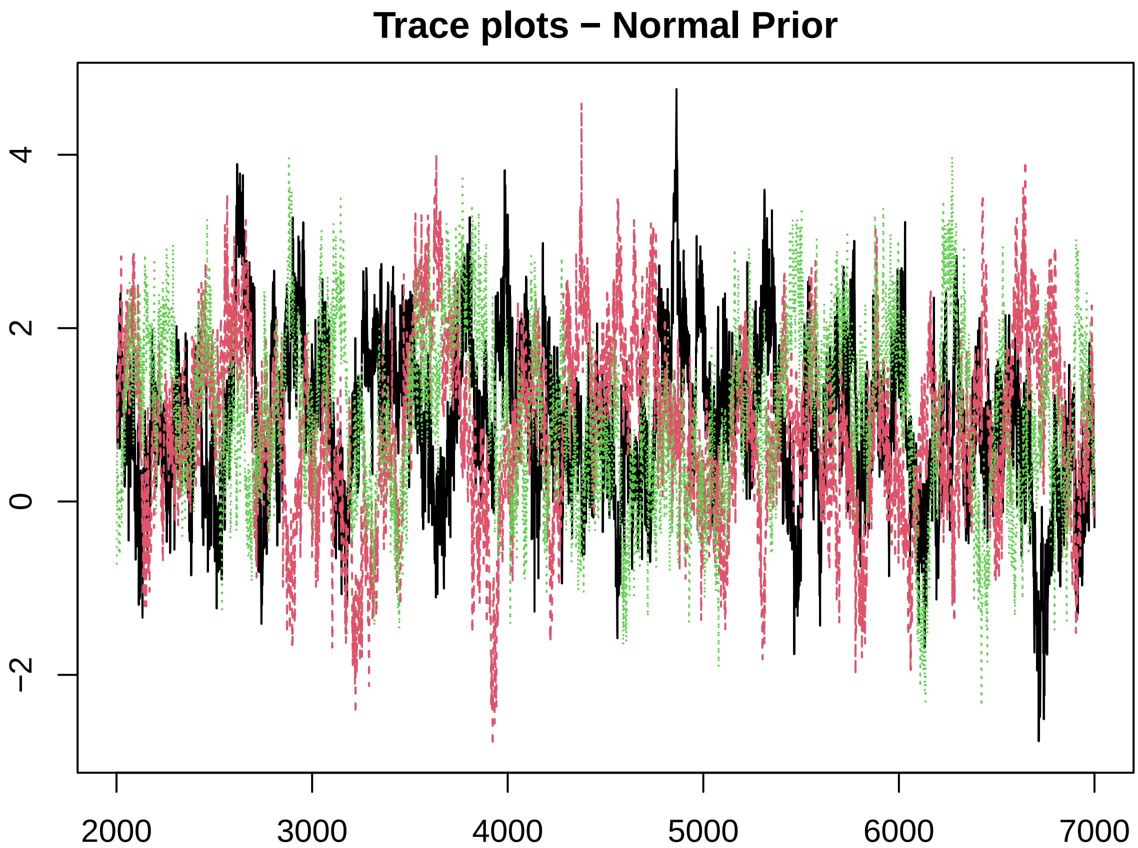

Trace plots − Normal Prior
−2
0
2
4
6
2000
3000
4000
5000
6000
7000

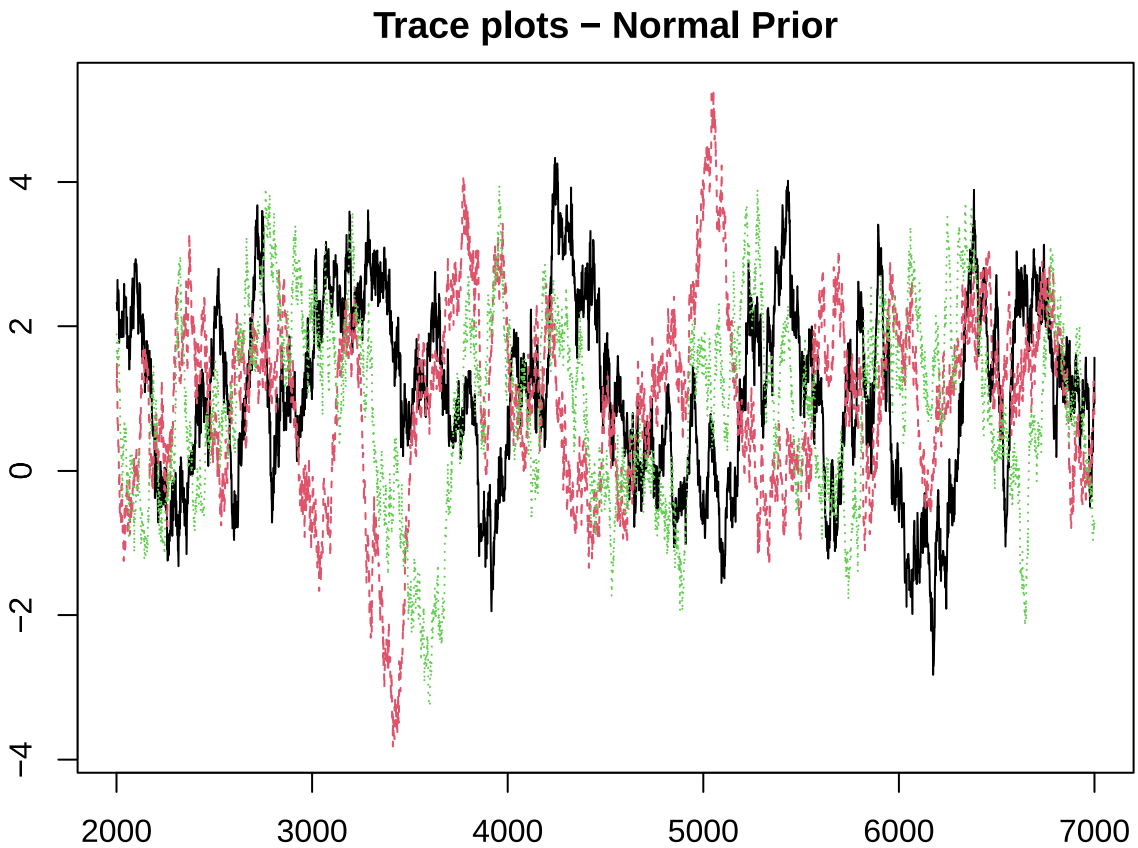

Trace plots − Normal Prior
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000

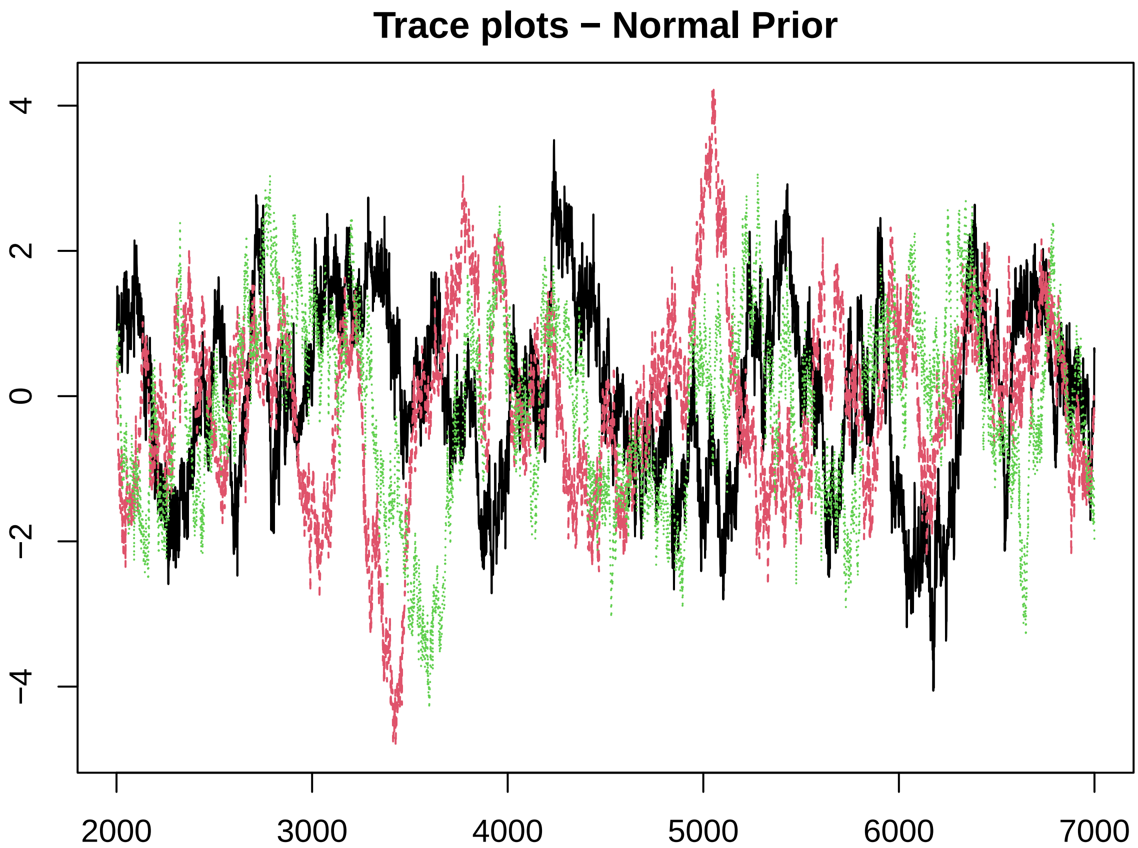

Trace plots − Normal Prior
2
1
0
−1
−2
−3
−4
2000
3000
4000
5000
6000
7000

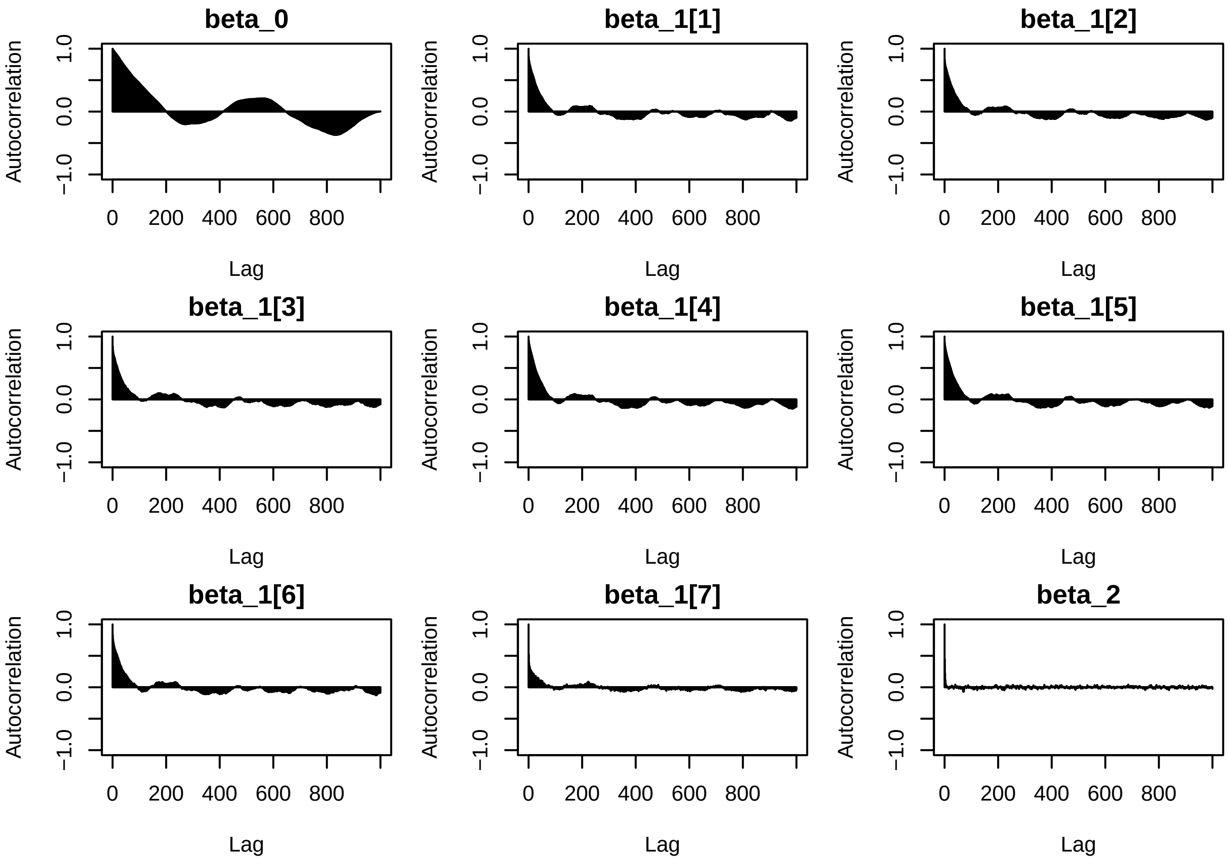

Trace plots − Normal Prior
3
2
1
0
−1
−2
−3
2000
3000
4000
5000
6000
7000

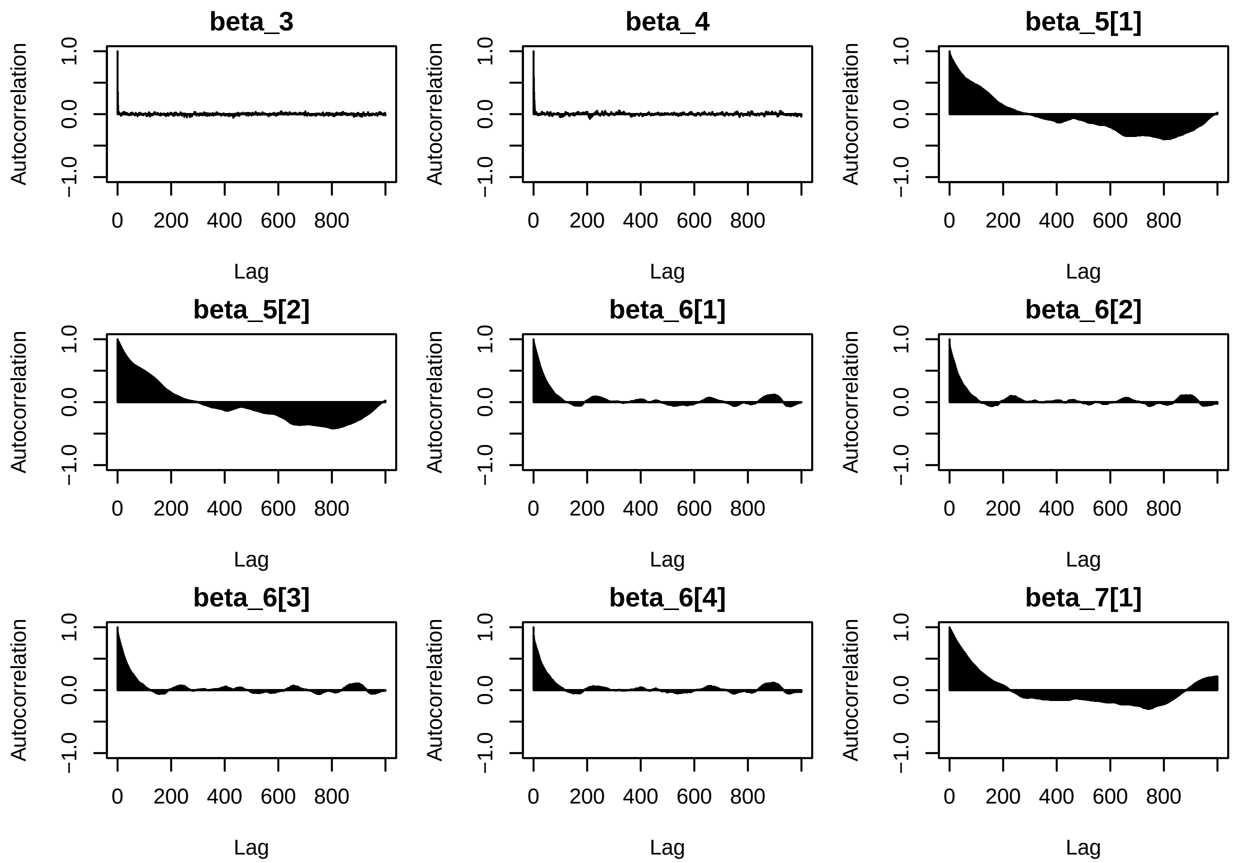

Trace plots − Normal Prior
4
3
2
1
0
−1
−2
−3
2000
3000
4000
5000
6000
7000

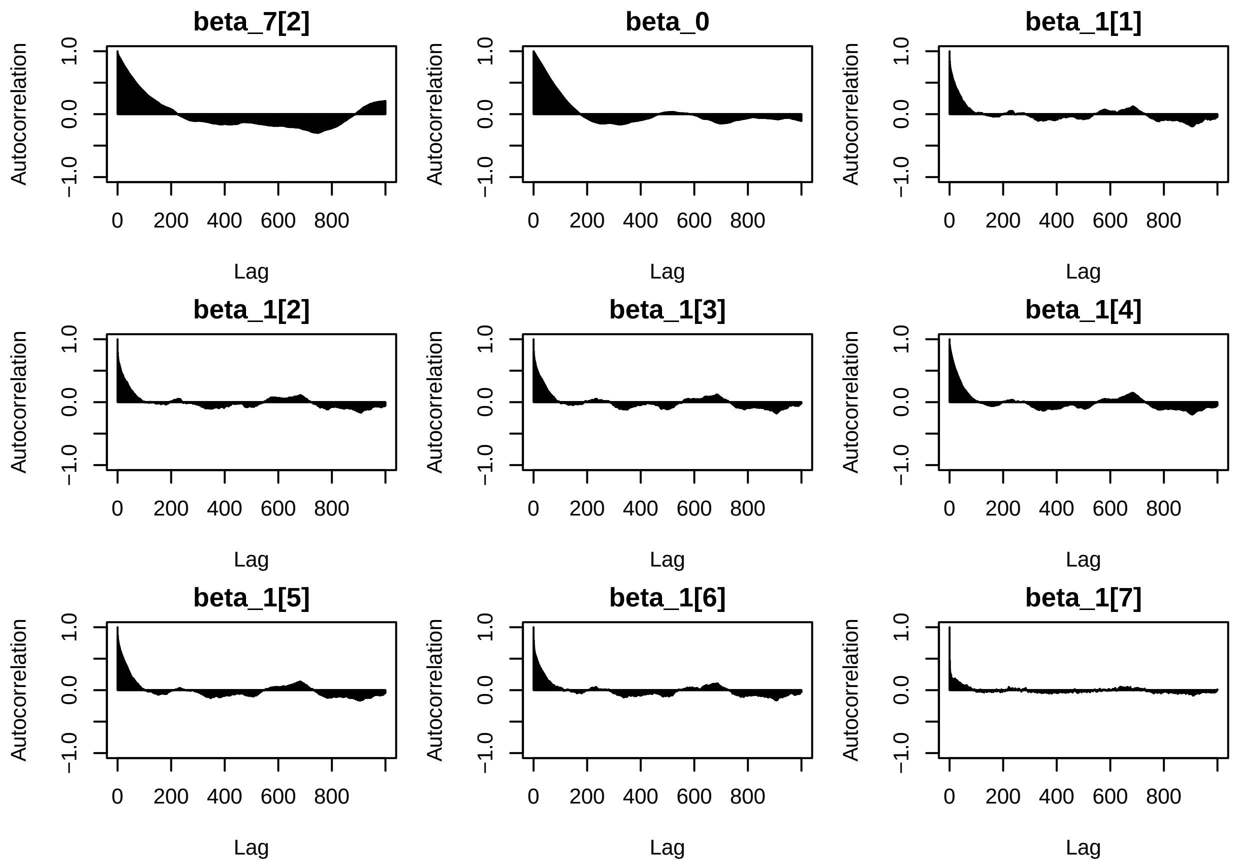

Trace plots − Normal Prior
4
2
0
−2
2000
3000
4000
5000
6000
7000

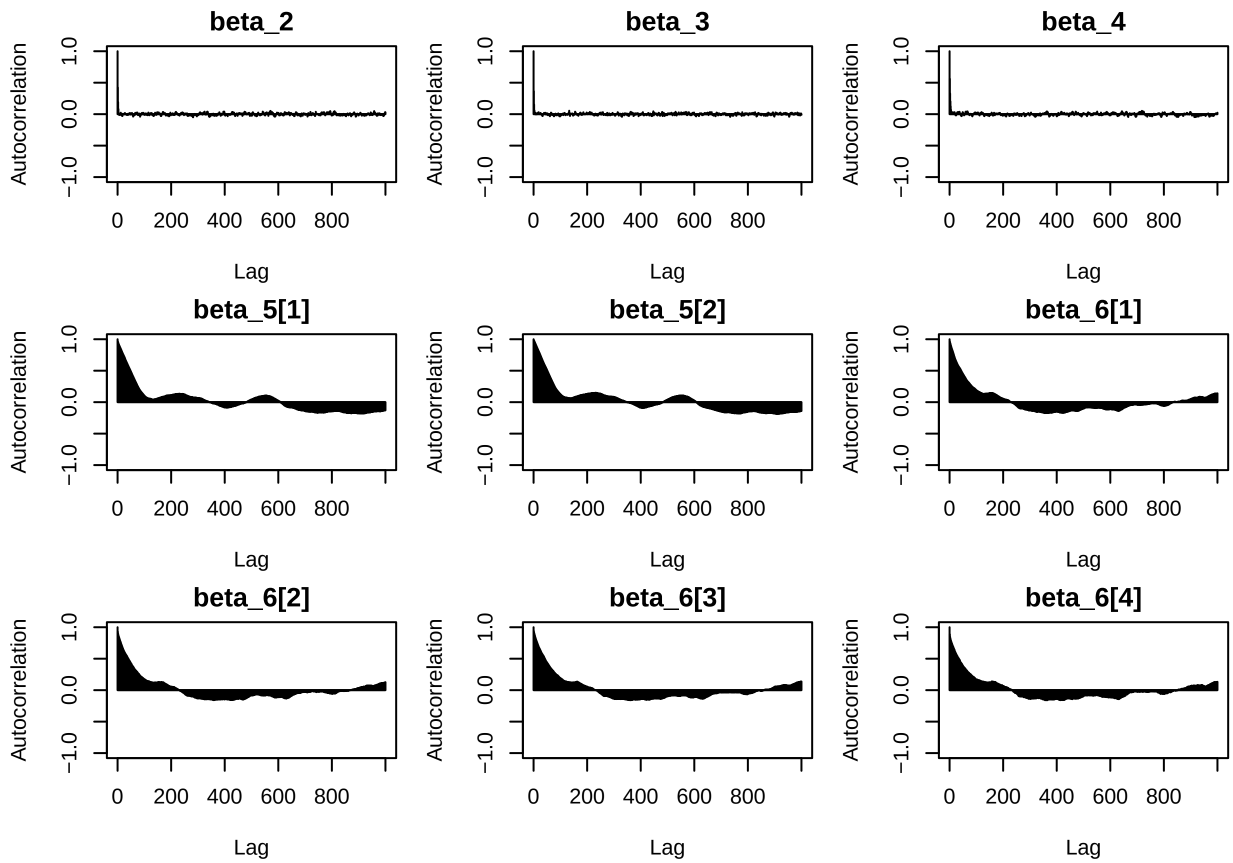

Trace plots − Normal Prior
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000


Trace plots − Normal Prior
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000

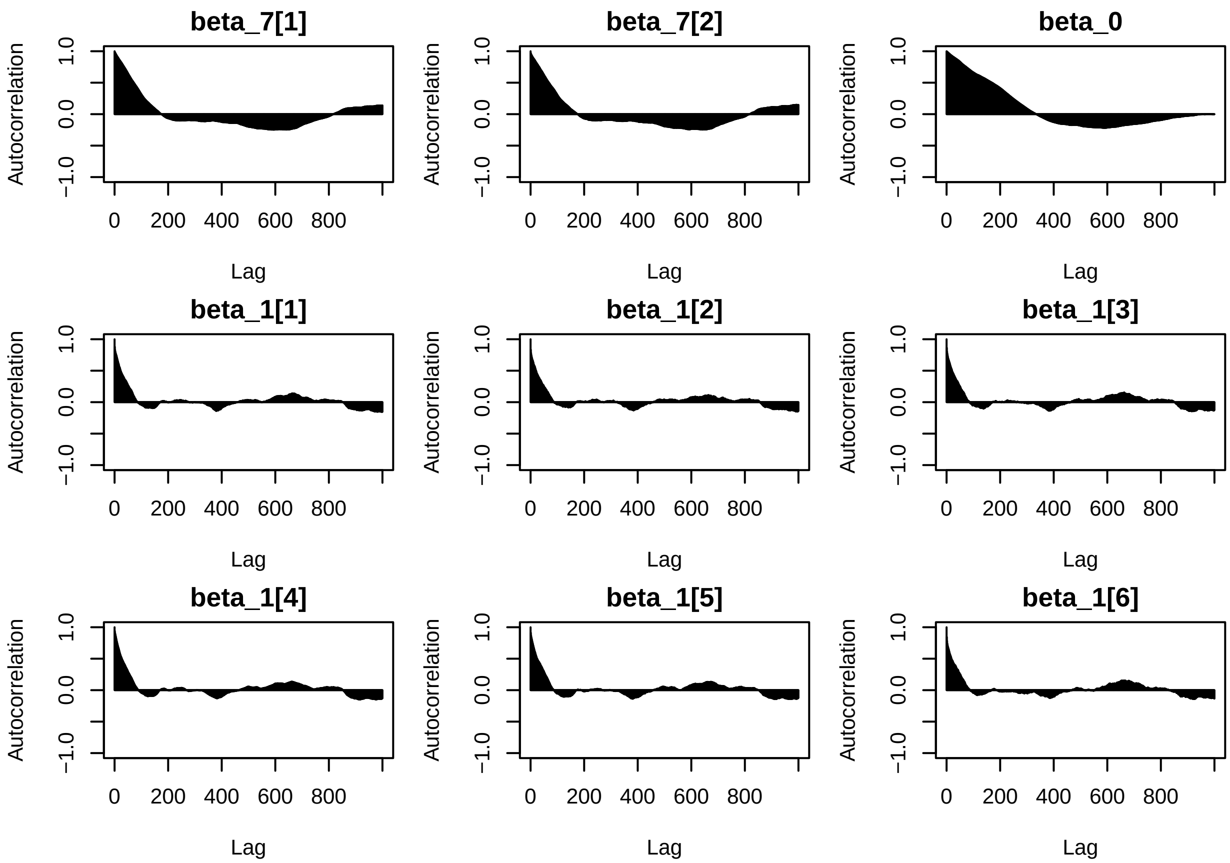

beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
beta_2
Autocorrelation
−1.0
0.0
1.0
Lag
0
200
400
600
800

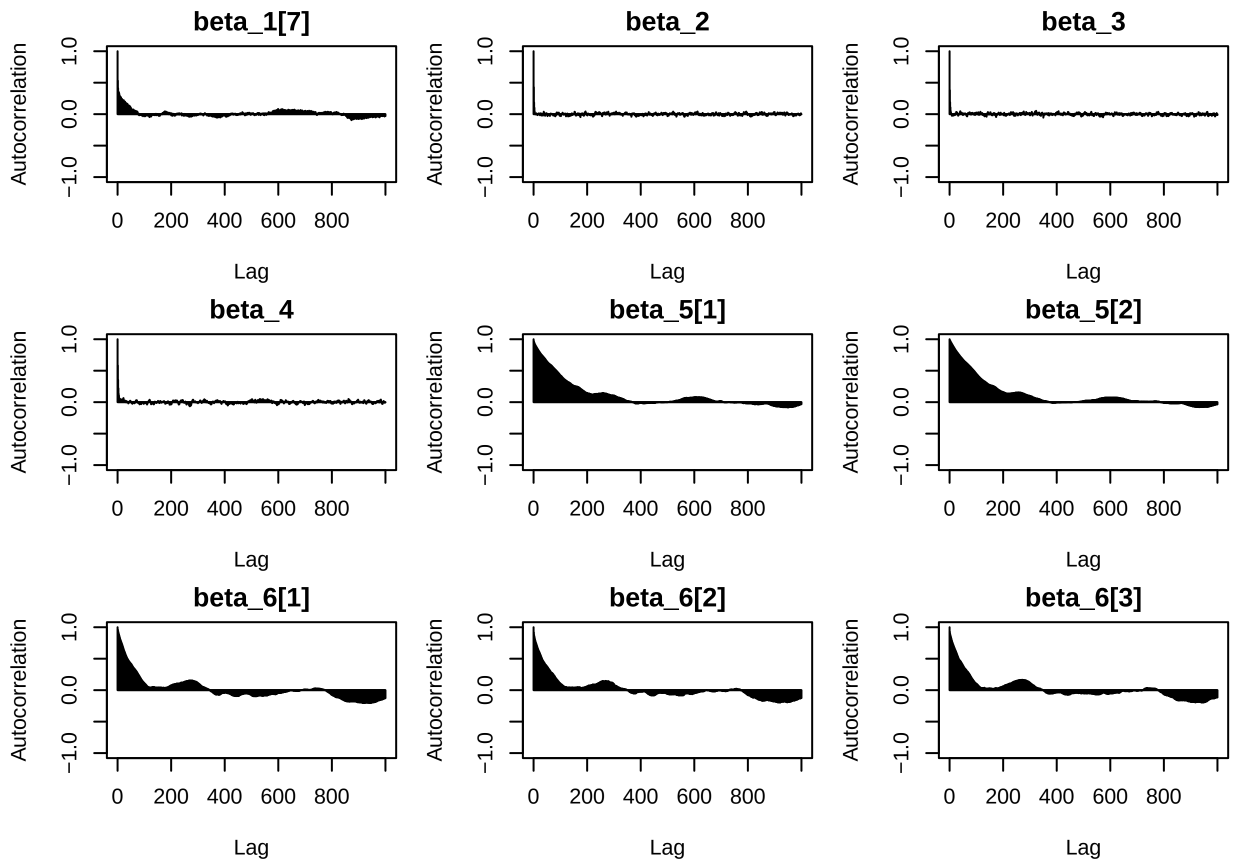

beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
beta_7[1]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag

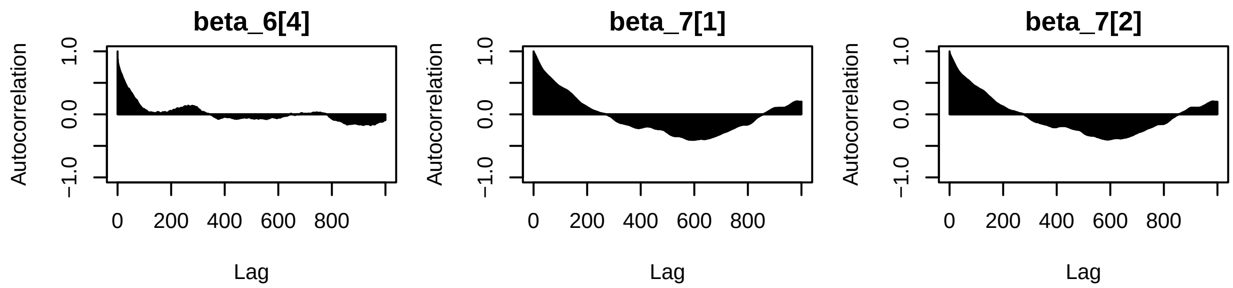

beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
Autocorrelation
Lag
-1.0
0.0
1.0
0
200
400
600
800



beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag

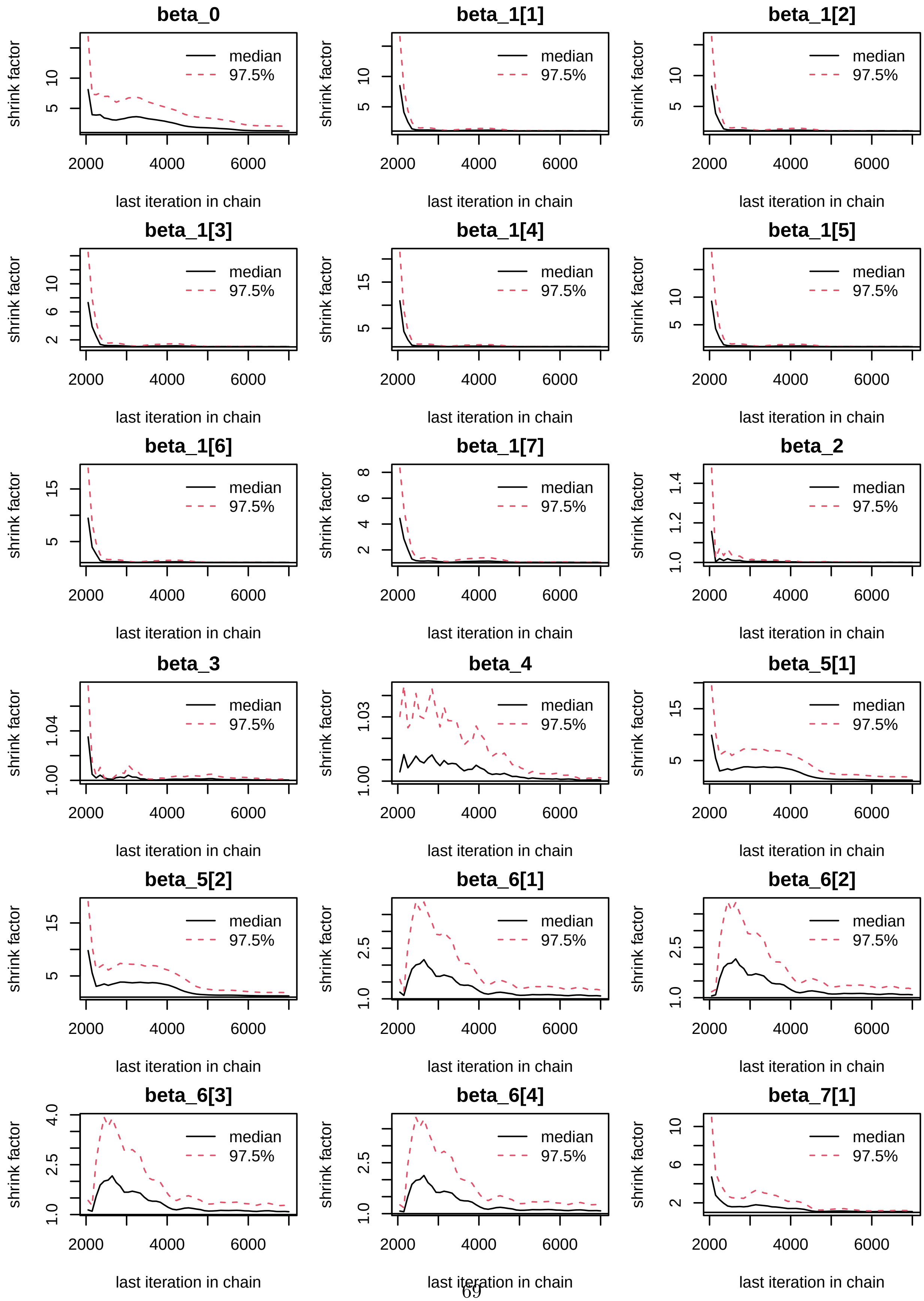

beta_7[1]
beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
Autocorrelation
-1.0
0.0
1.0
Lag
0
200
400
600
800

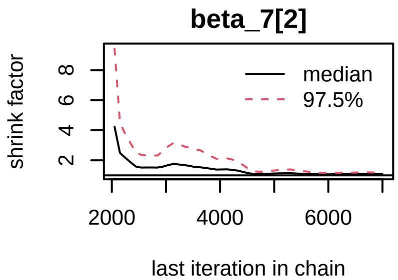

beta_1[7]
beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800

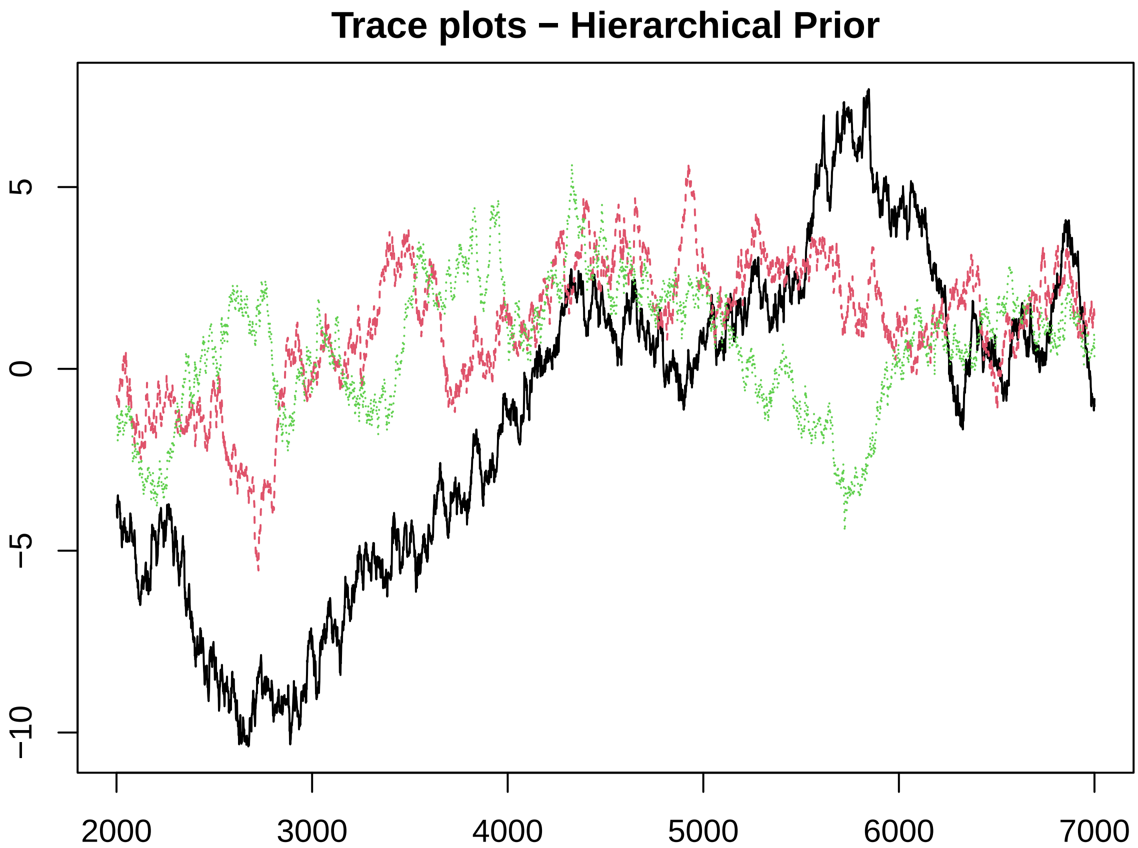


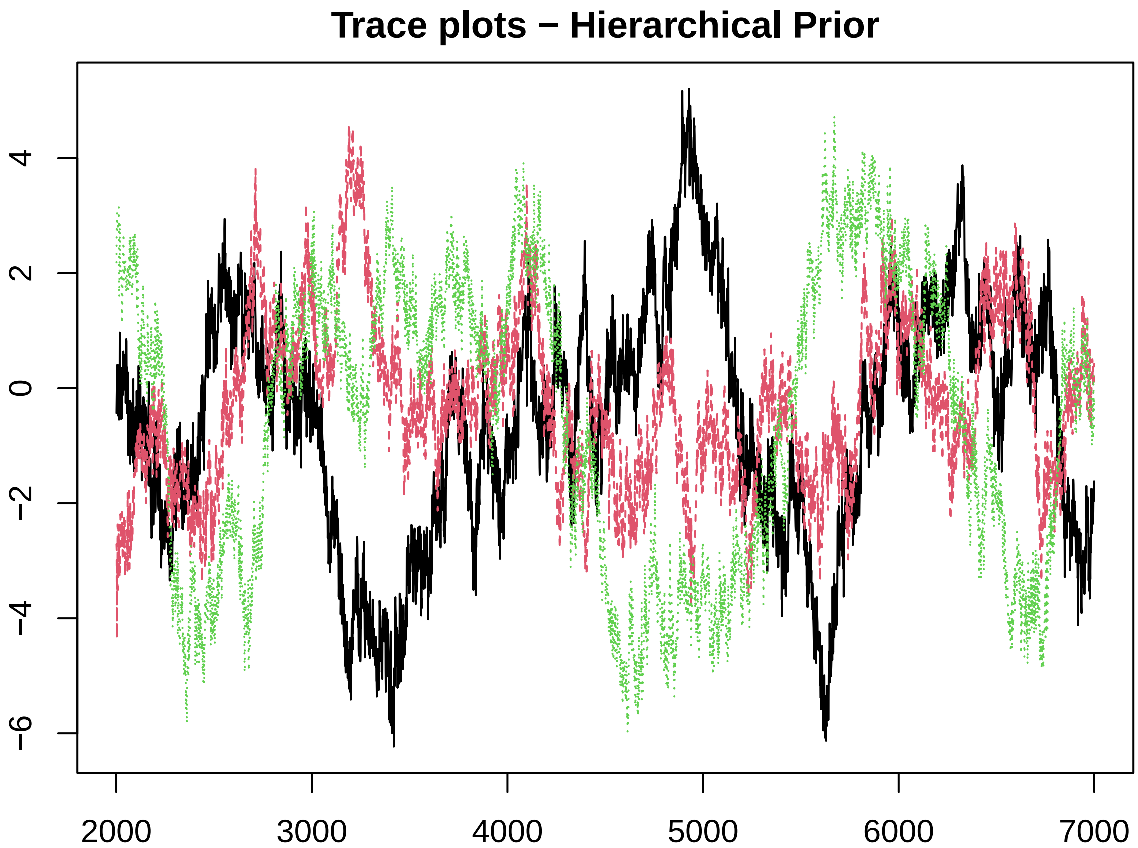

```
## Potential scale reduction factors:
##
##            Point est. Upper C.I.
## beta_0           1.03       1.07
## beta_1[1]        1.00       1.01
## beta_1[2]        1.01       1.01
## beta_1[3]        1.00       1.01
## beta_1[4]        1.01       1.01
## beta_1[5]        1.01       1.01
## beta_1[6]        1.00       1.01
## beta_1[7]        1.00       1.00
## beta_2           1.00       1.00
## beta_3           1.00       1.00
## beta_4           1.00       1.00
## beta_5[1]        1.02       1.06
## beta_5[2]        1.02       1.07
## beta_6[1]        1.01       1.02
## beta_6[2]        1.01       1.02
## beta_6[3]        1.01       1.02
## beta_6[4]        1.01       1.02
## beta_7[1]        1.00       1.02
## beta_7[2]        1.00       1.02
##
## Multivariate psrf
##
## 1.02
```

Hierarchical Prior

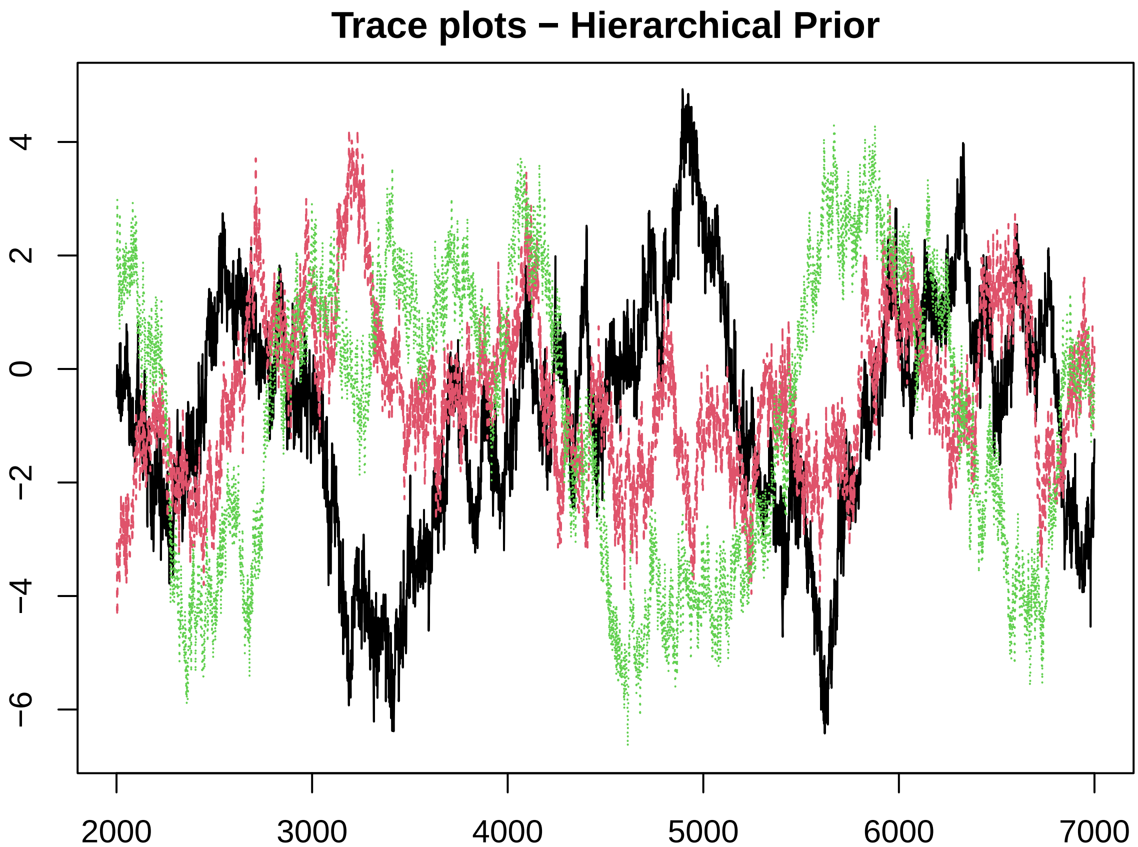


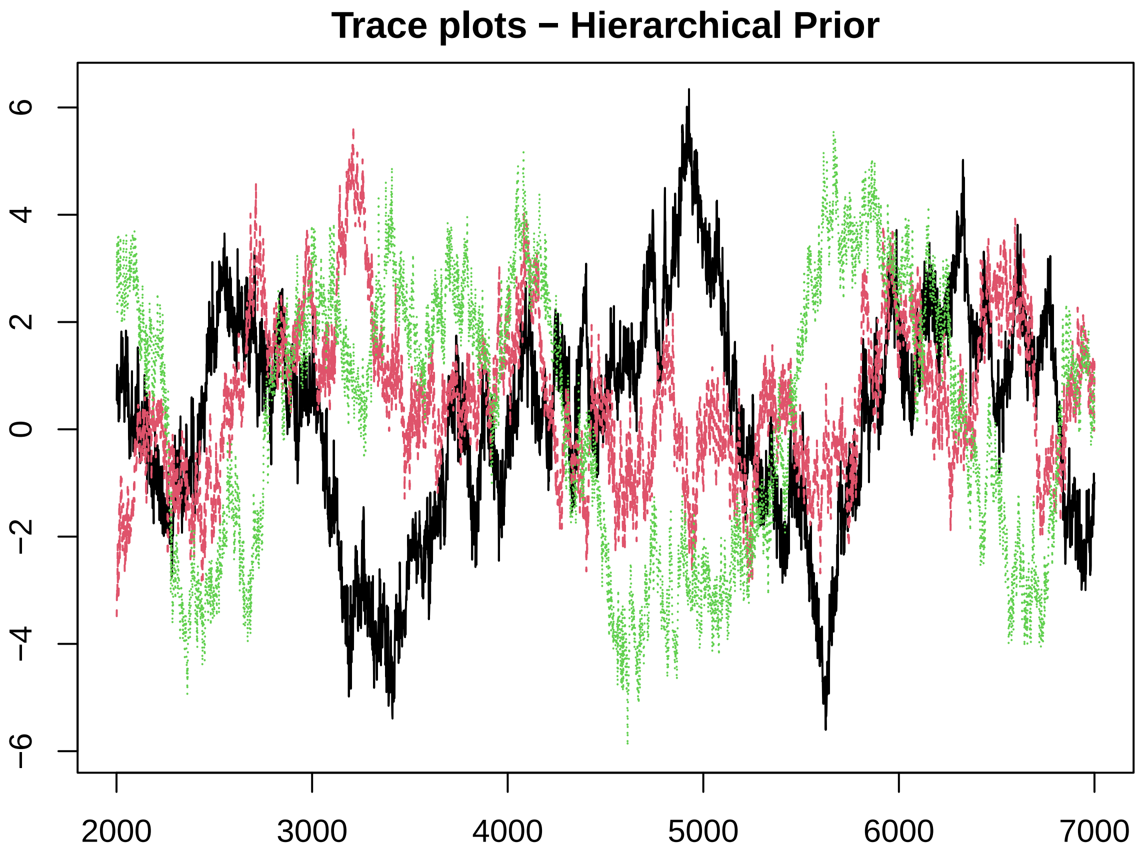

beta_7[2]
median
97.5%
shrink factor
2
4
6
8
2000
4000
6000
last iteration in chain

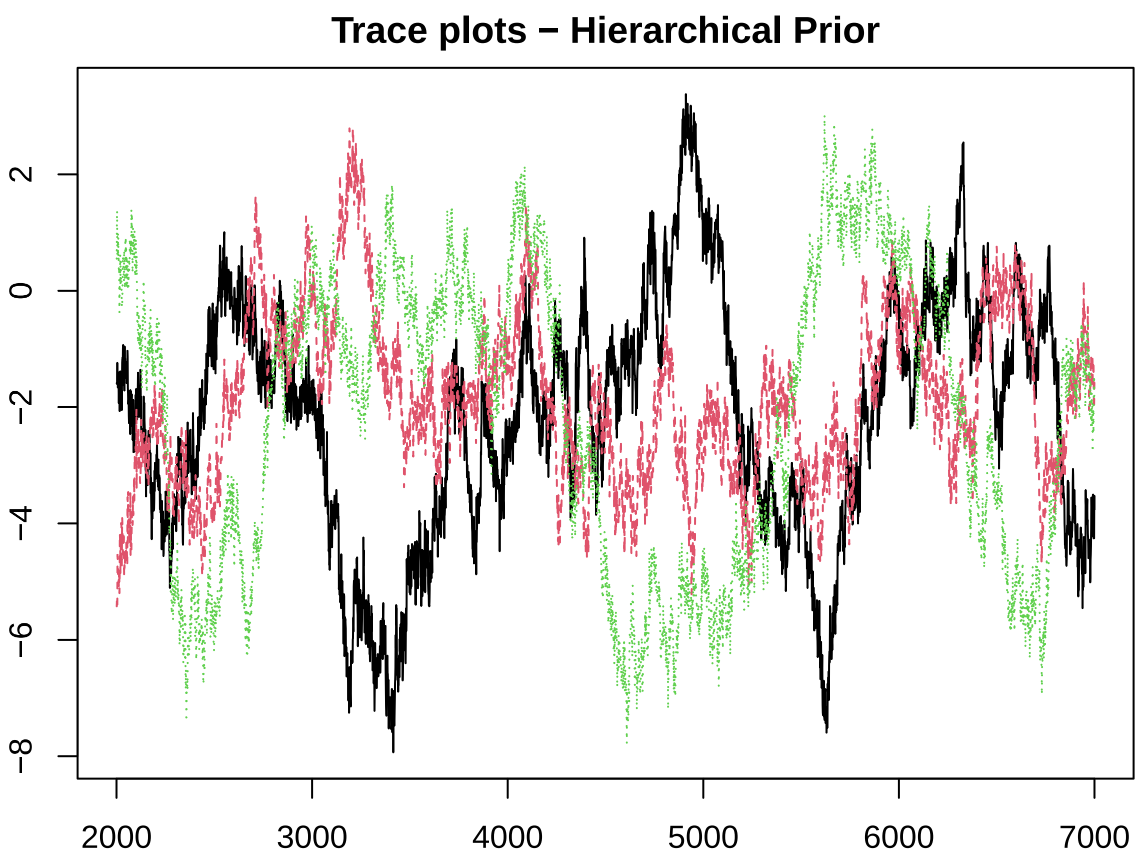

Trace plots − Hierarchical Prior
5
0
−5
−10
2000
3000
4000
5000
6000
7000



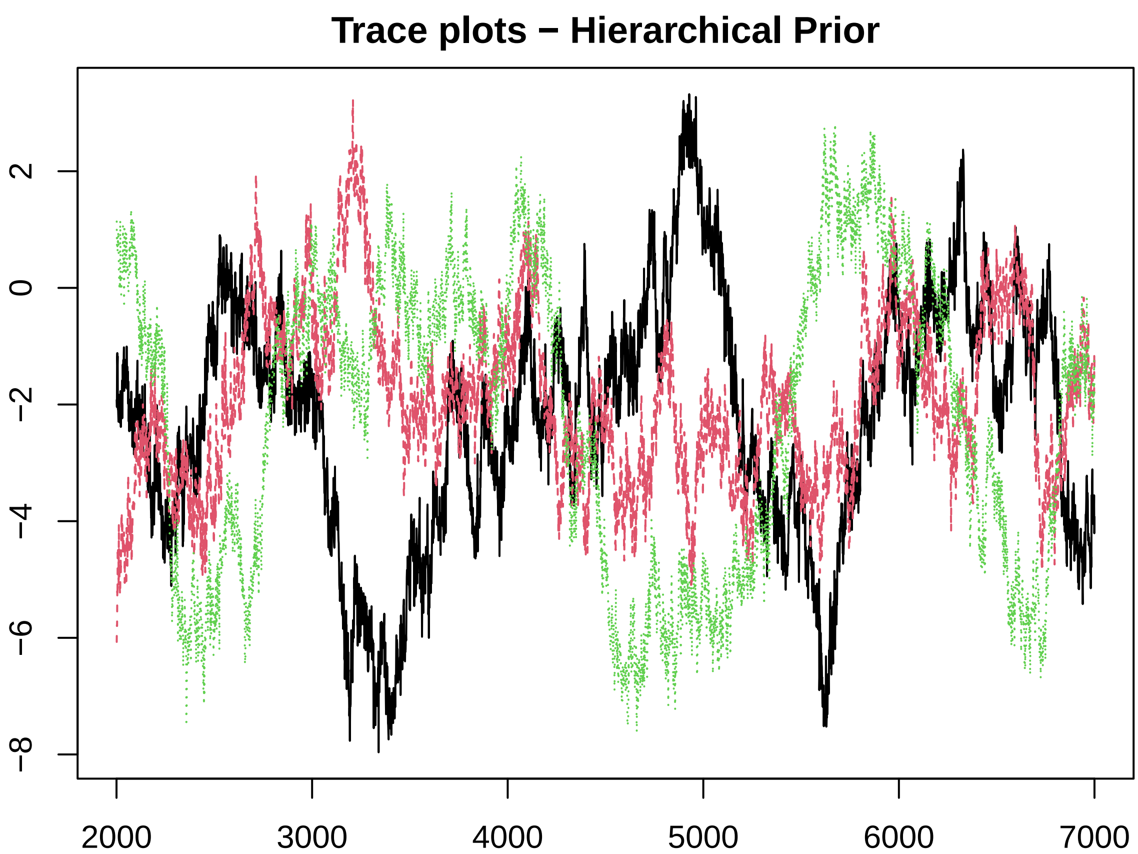
Trace plots − Hierarchical Prior
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000


Trace plots − Hierarchical Prior
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

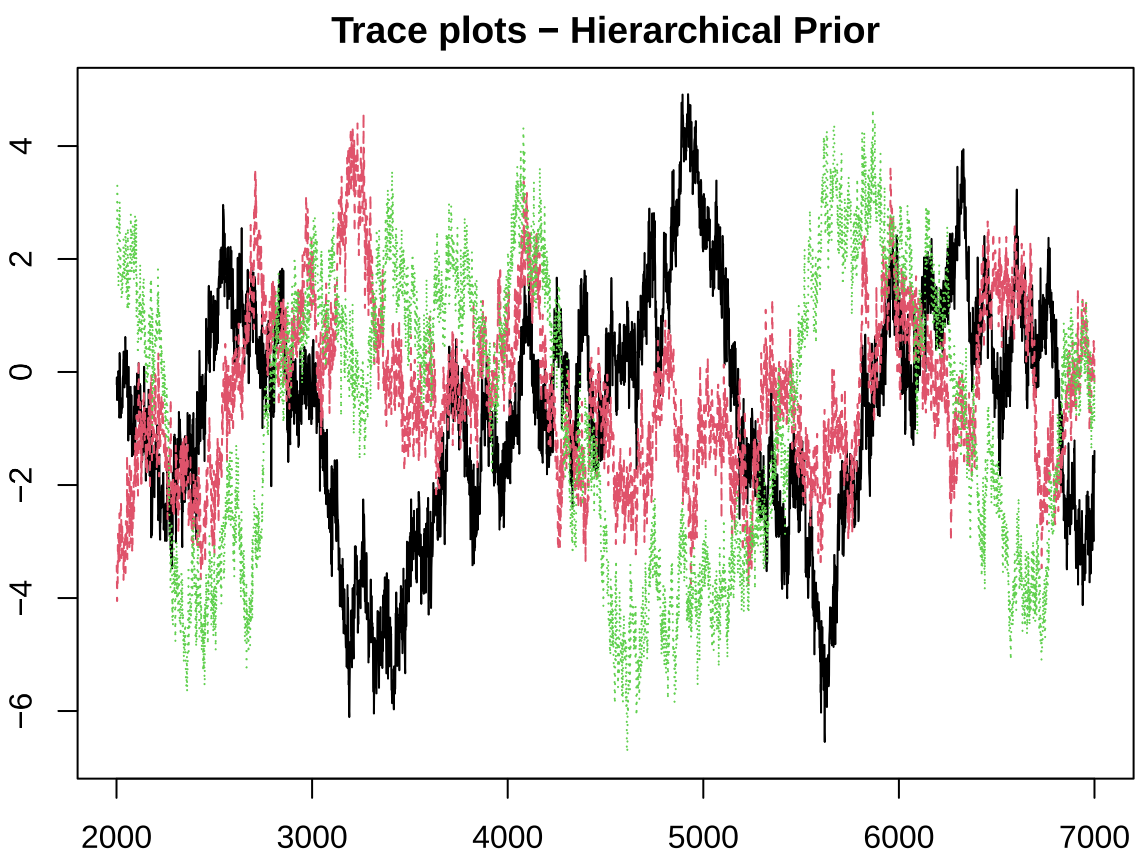

Trace plots − Hierarchical Prior
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

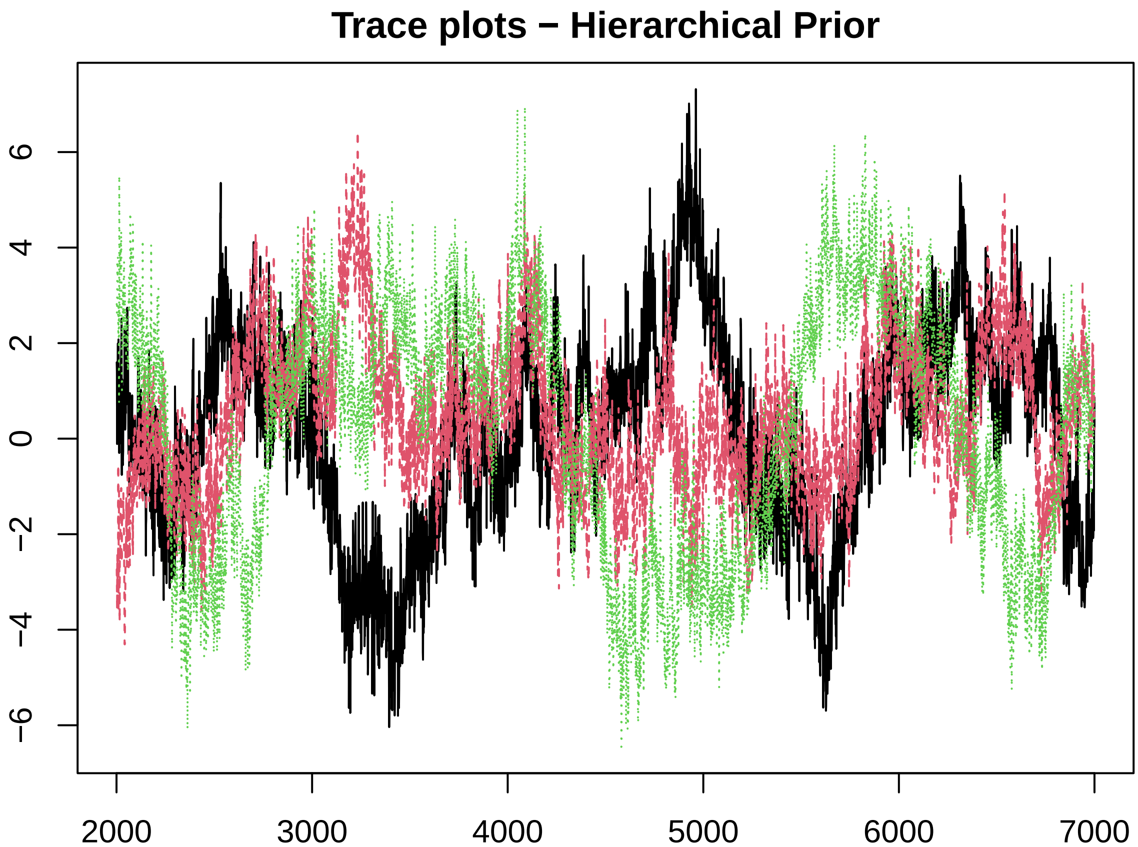


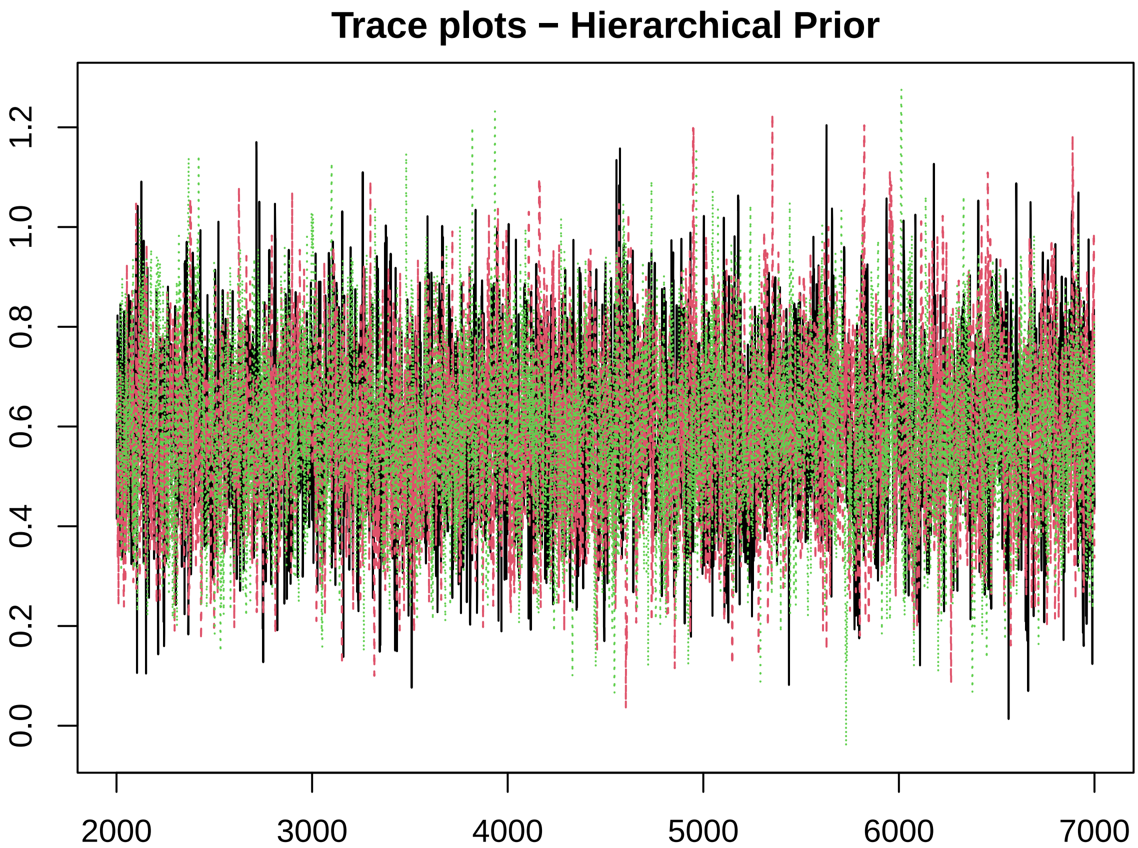
Trace plots − Hierarchical Prior
2
0
−2
−4
−6
−8
2000
3000
4000
5000
6000
7000

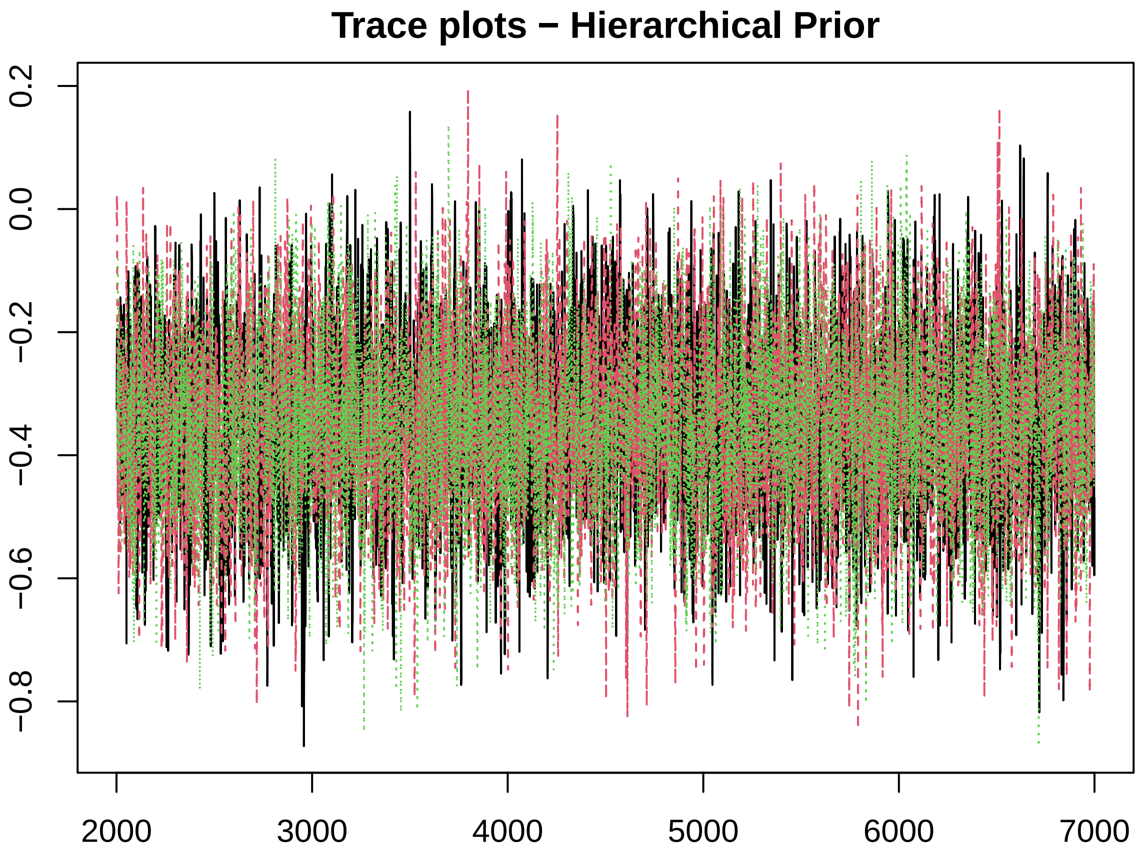


Trace plots − Hierarchical Prior
2
0
−2
−4
−6
−8
2000
3000
4000
5000
6000
7000

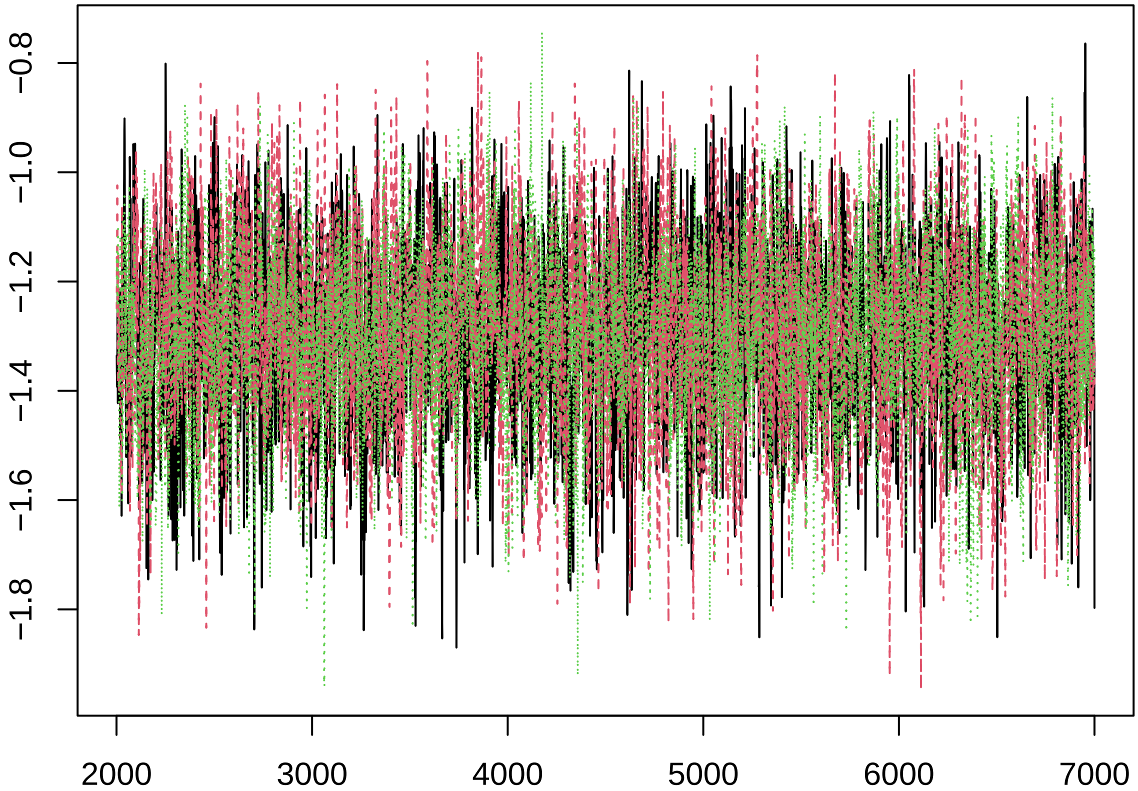

Trace plots − Hierarchical Prior
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

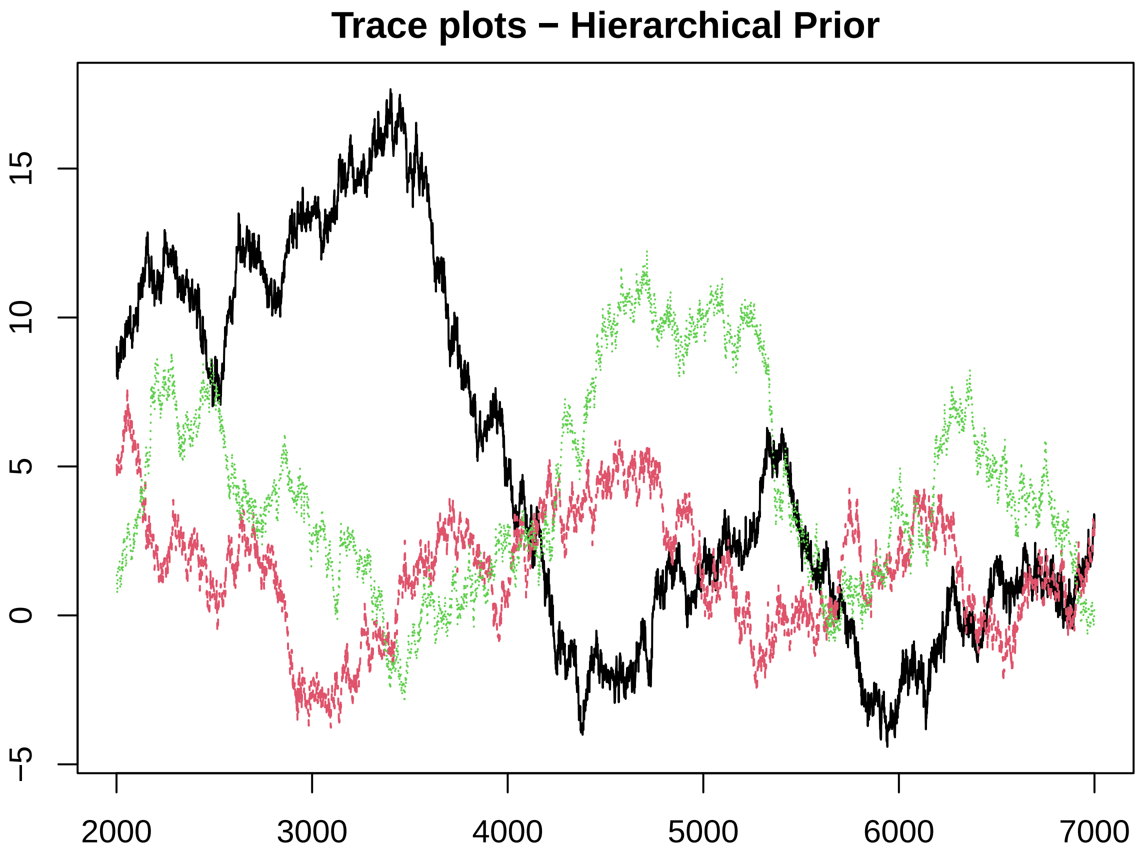

Trace plots − Hierarchical Prior
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

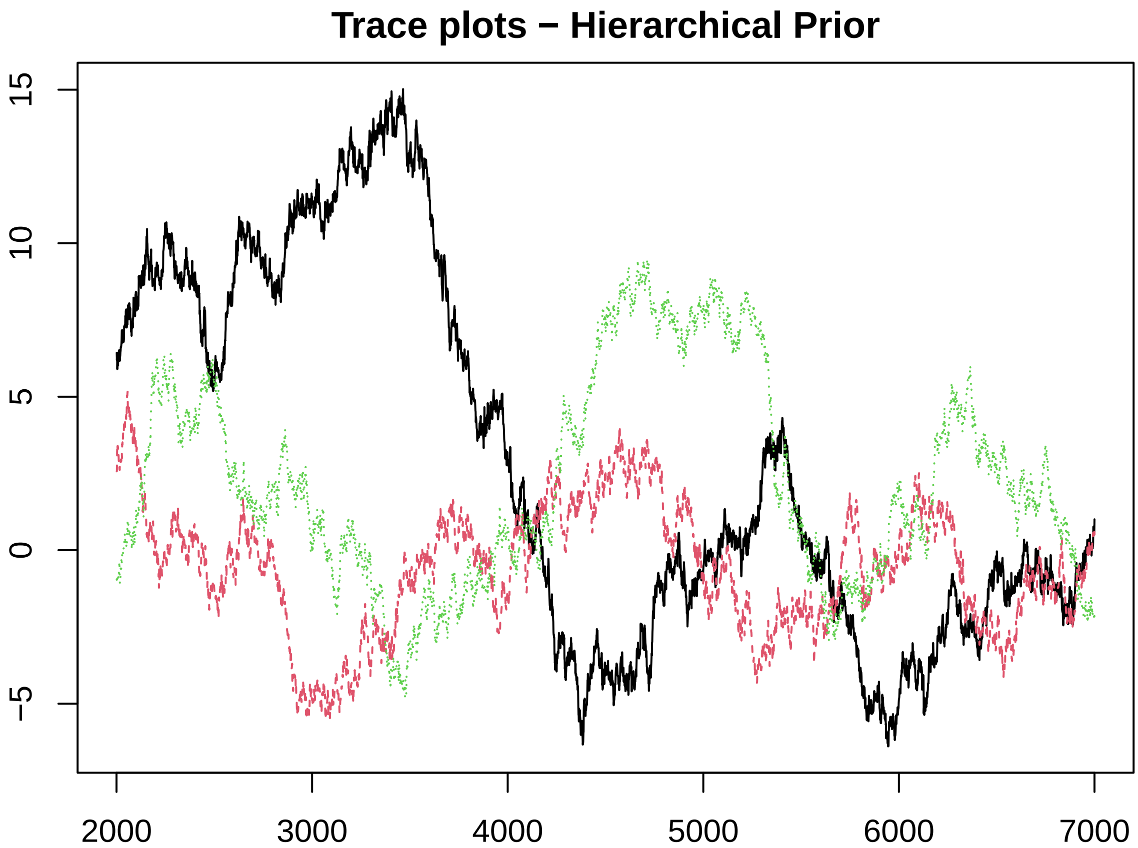

Trace plots − Hierarchical Prior
0.0
0.2
0.4
0.6
0.8
1.0
1.2
2000
3000
4000
5000
6000
7000

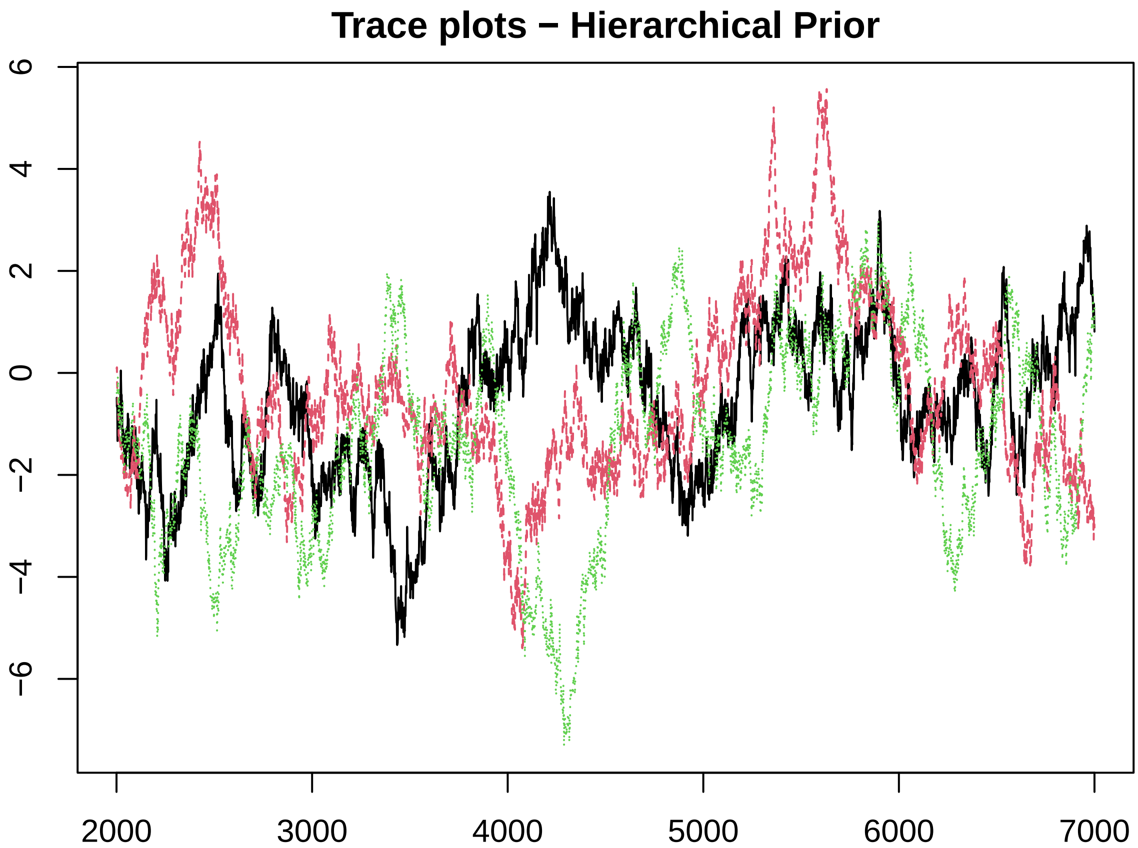

Trace plots − Hierarchical Prior
0.2
0.0
−0.2
−0.4
−0.6
−0.8
2000
3000
4000
5000
6000
7000

Trace plots − Hierarchical Prior

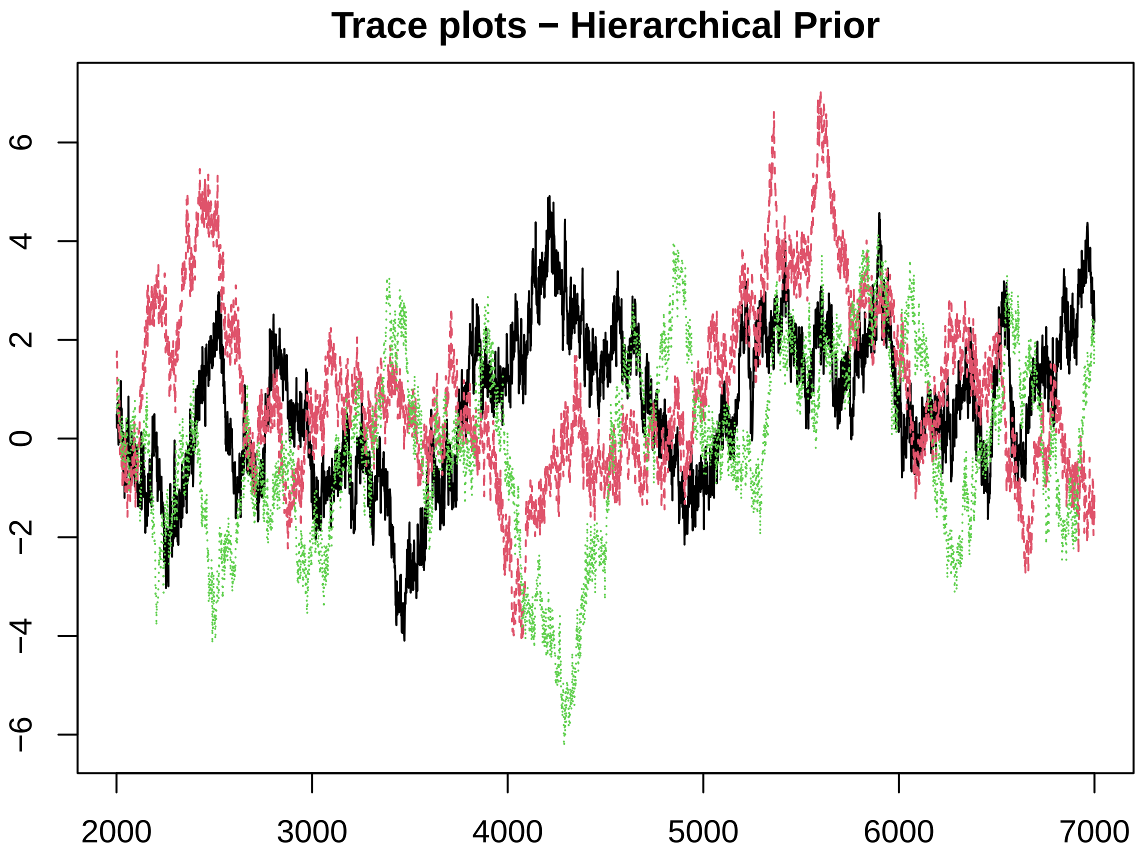


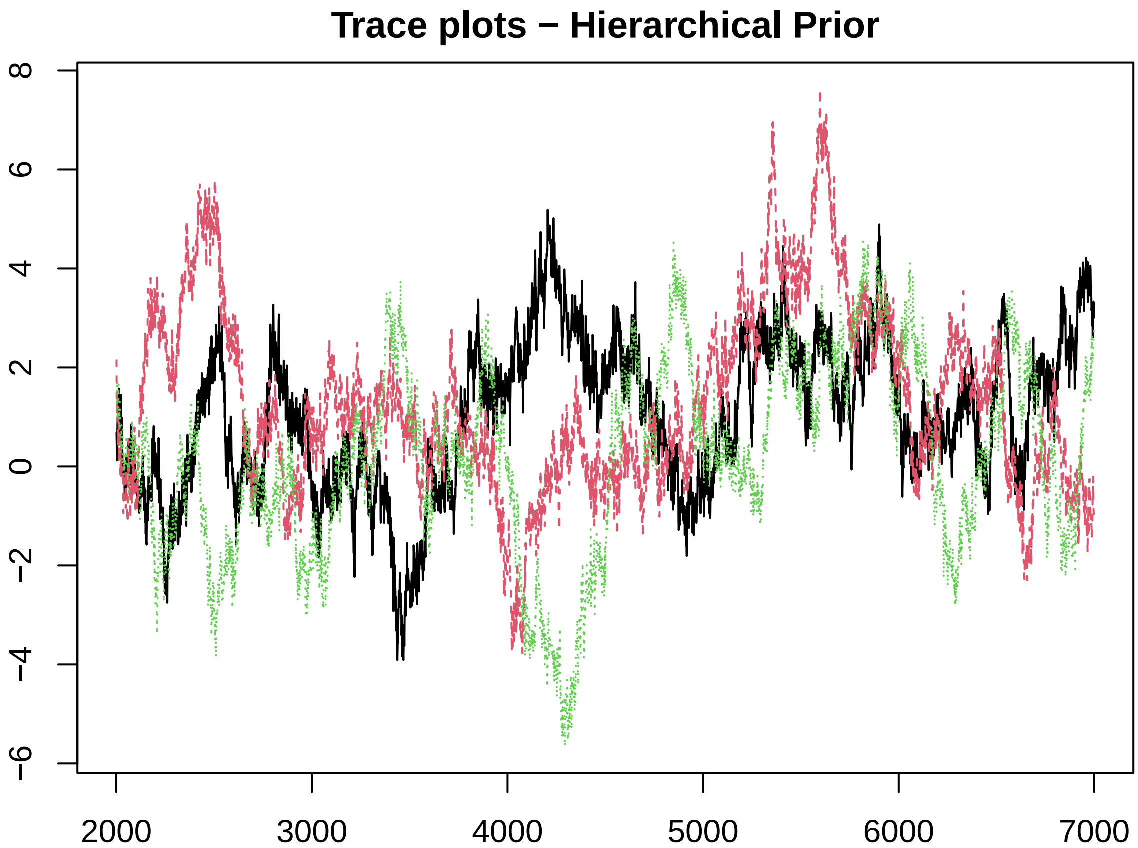

Trace plots − Hierarchical Prior
−5
0
5
10
15
2000
3000
4000
5000
6000
7000

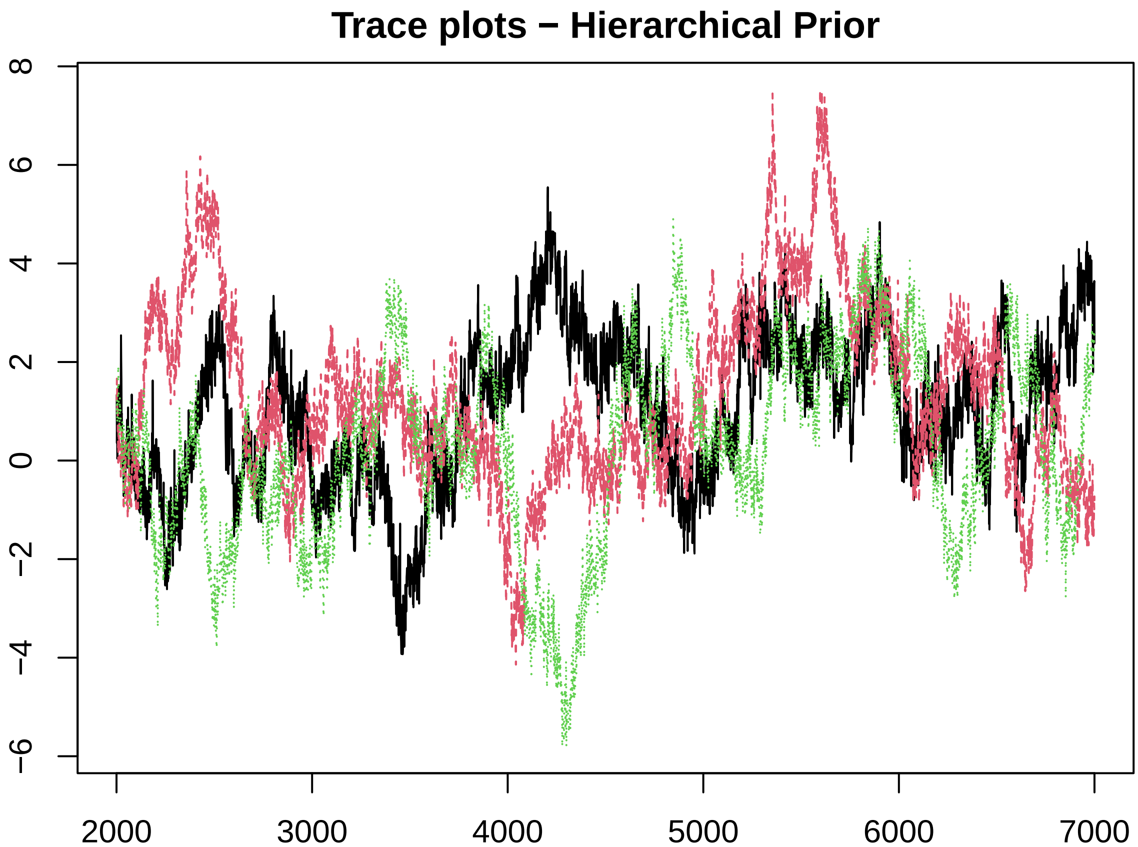

Trace plots − Hierarchical Prior
15
10
5
0
−5
2000
3000
4000
5000
6000
7000

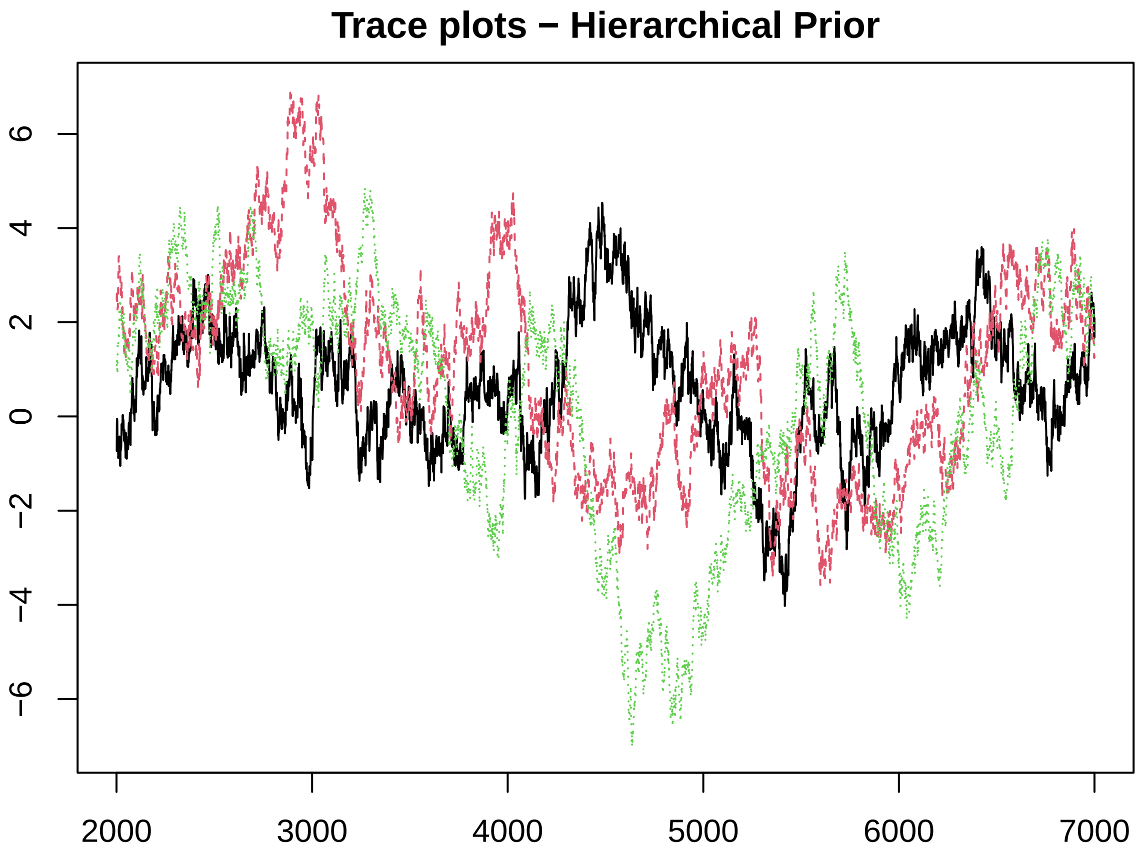


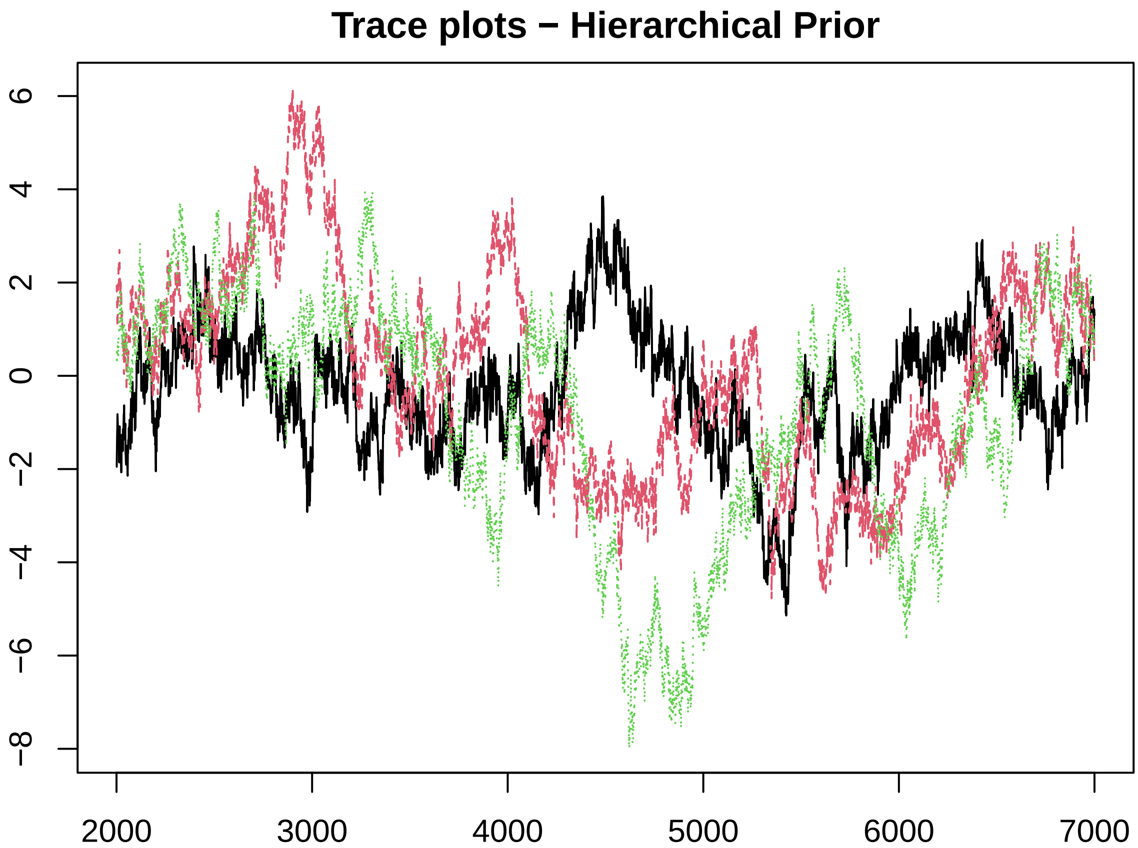
Trace plots − Hierarchical Prior
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000


Trace plots − Hierarchical Prior
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

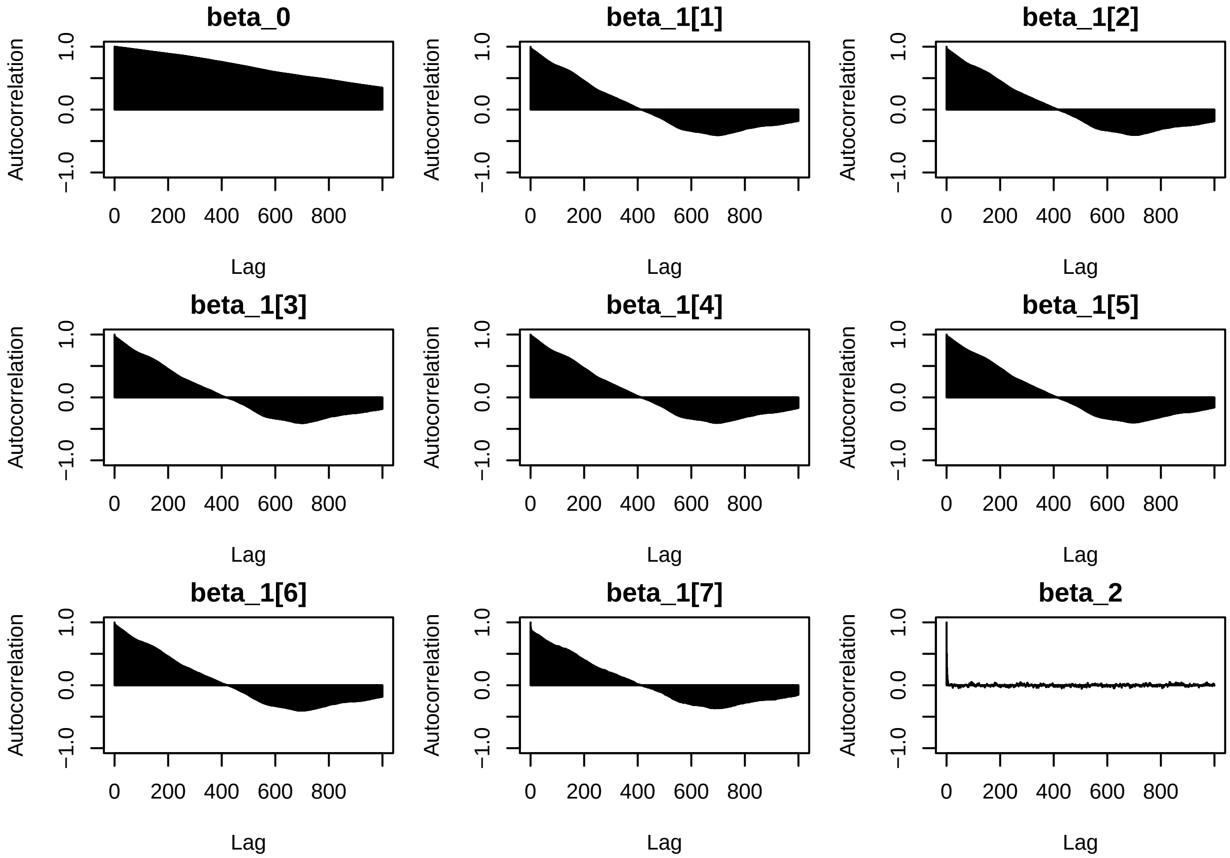

Trace plots − Hierarchical Prior
8
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

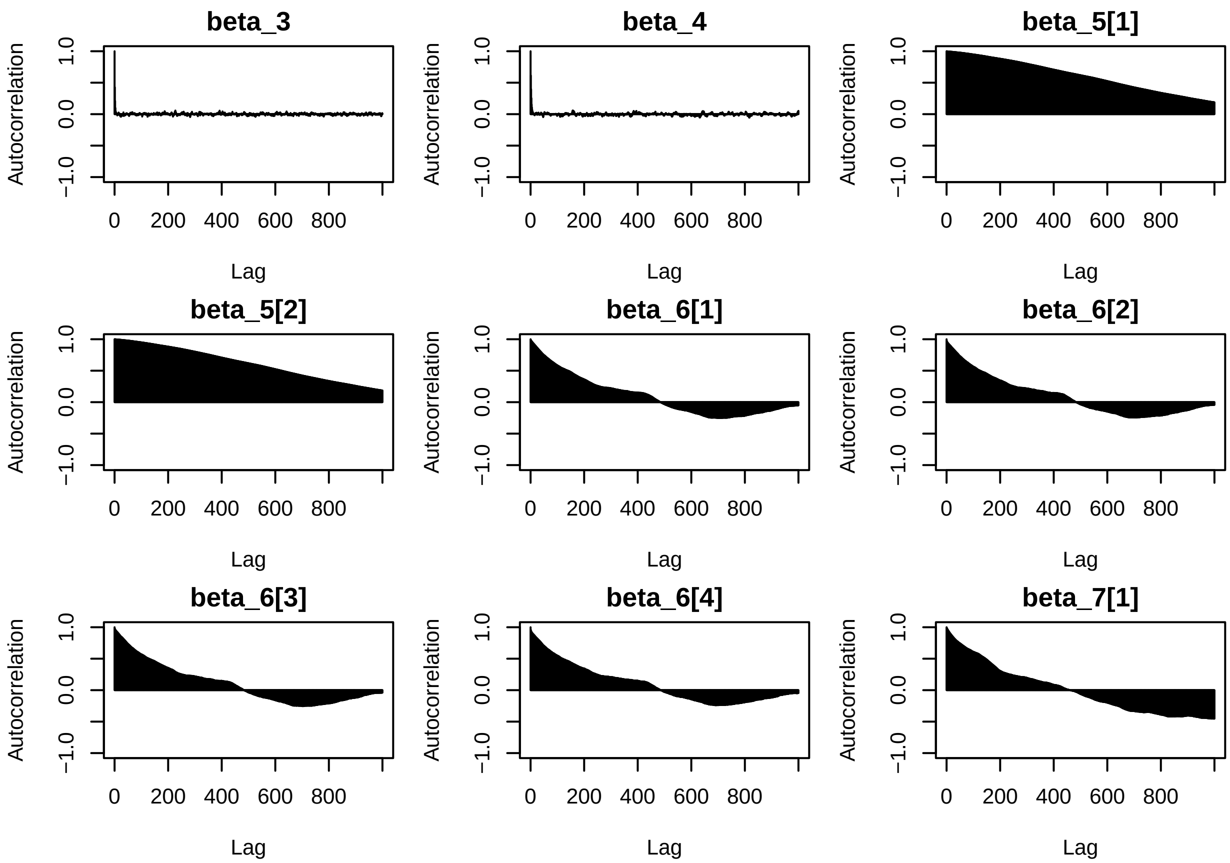

Trace plots − Hierarchical Prior
8
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

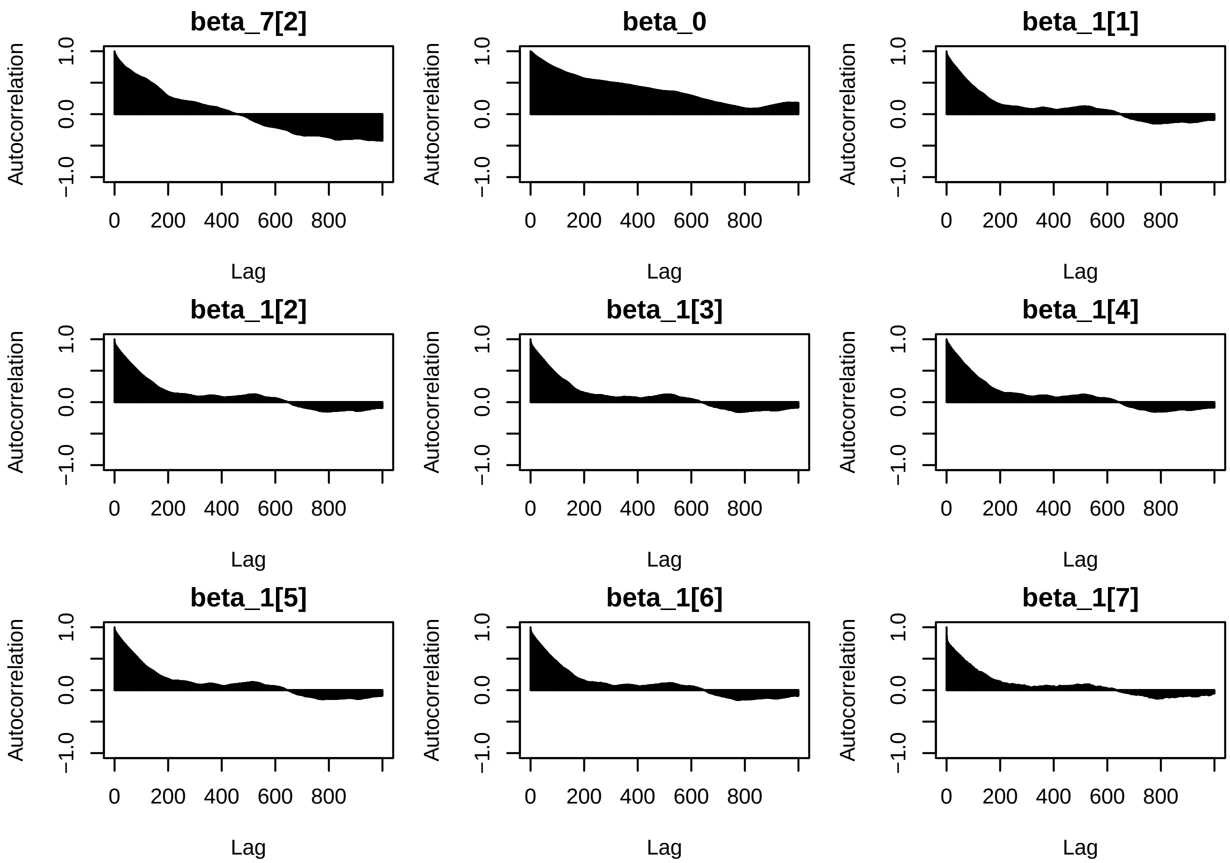

Trace plots − Hierarchical Prior
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

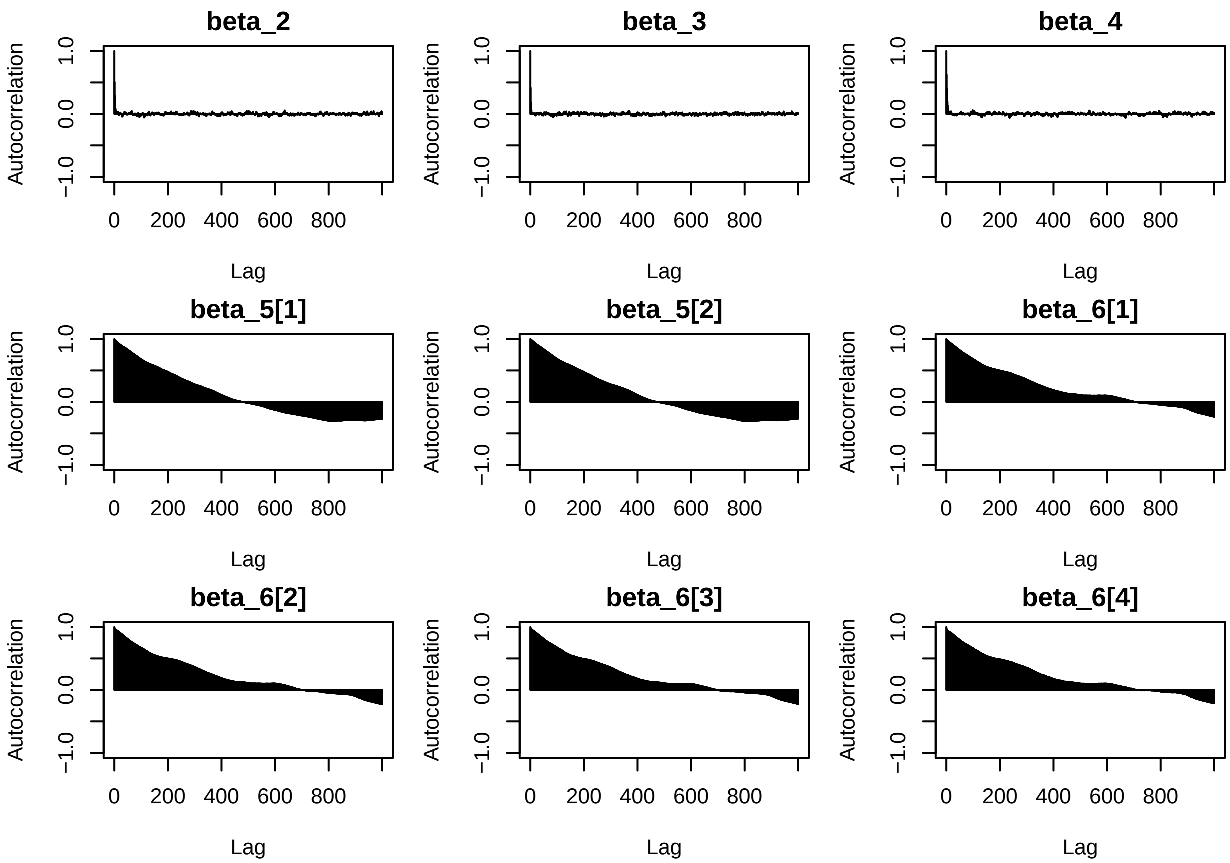


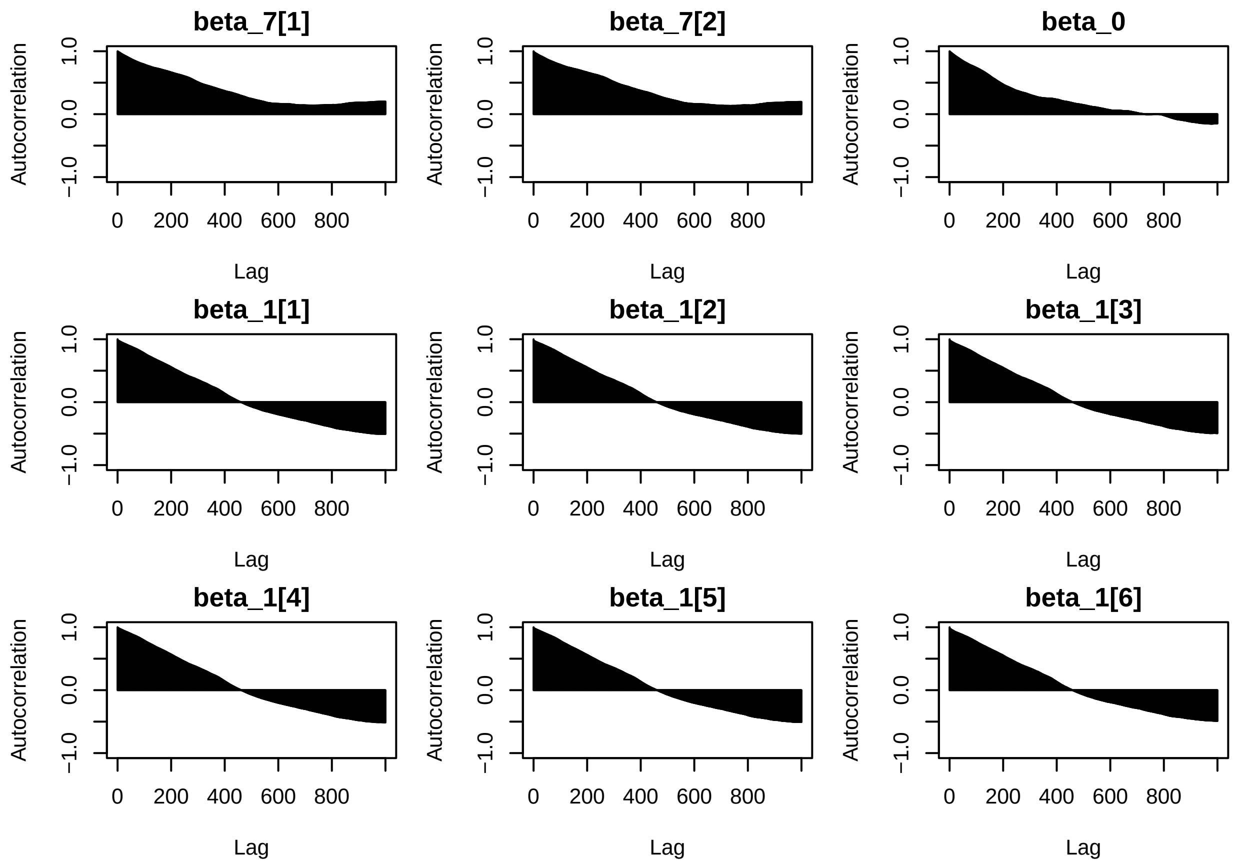
Trace plots − Hierarchical Prior
6
4
2
0
−2
−4
−6
−8
2000
3000
4000
5000
6000
7000

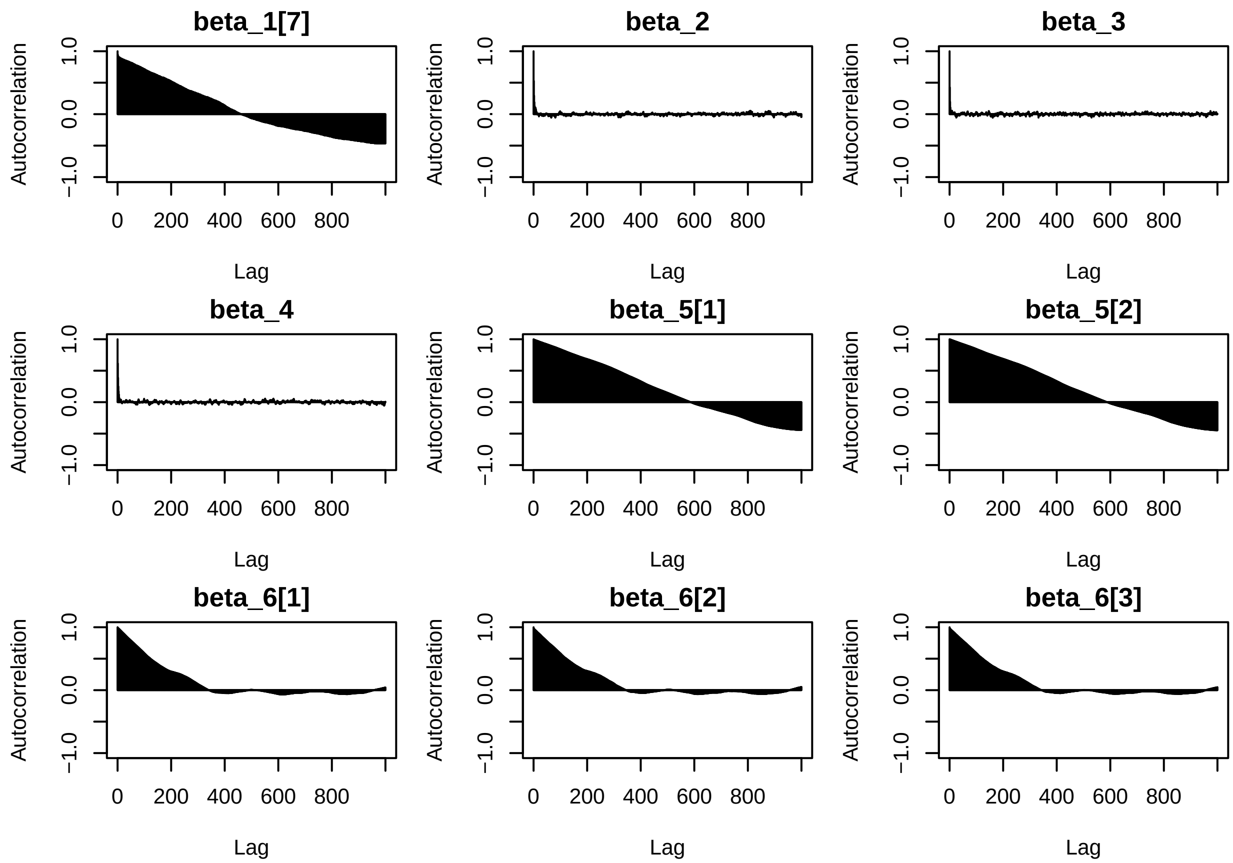

beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
beta_2
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800

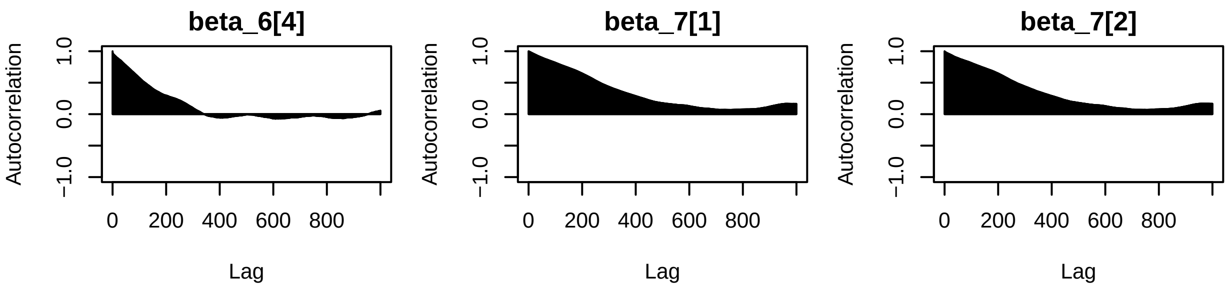

beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
beta_7[1]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag


beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
Autocorrelation
Lag
1.0
0.0
−1.0
0
200
400
600
800


beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800

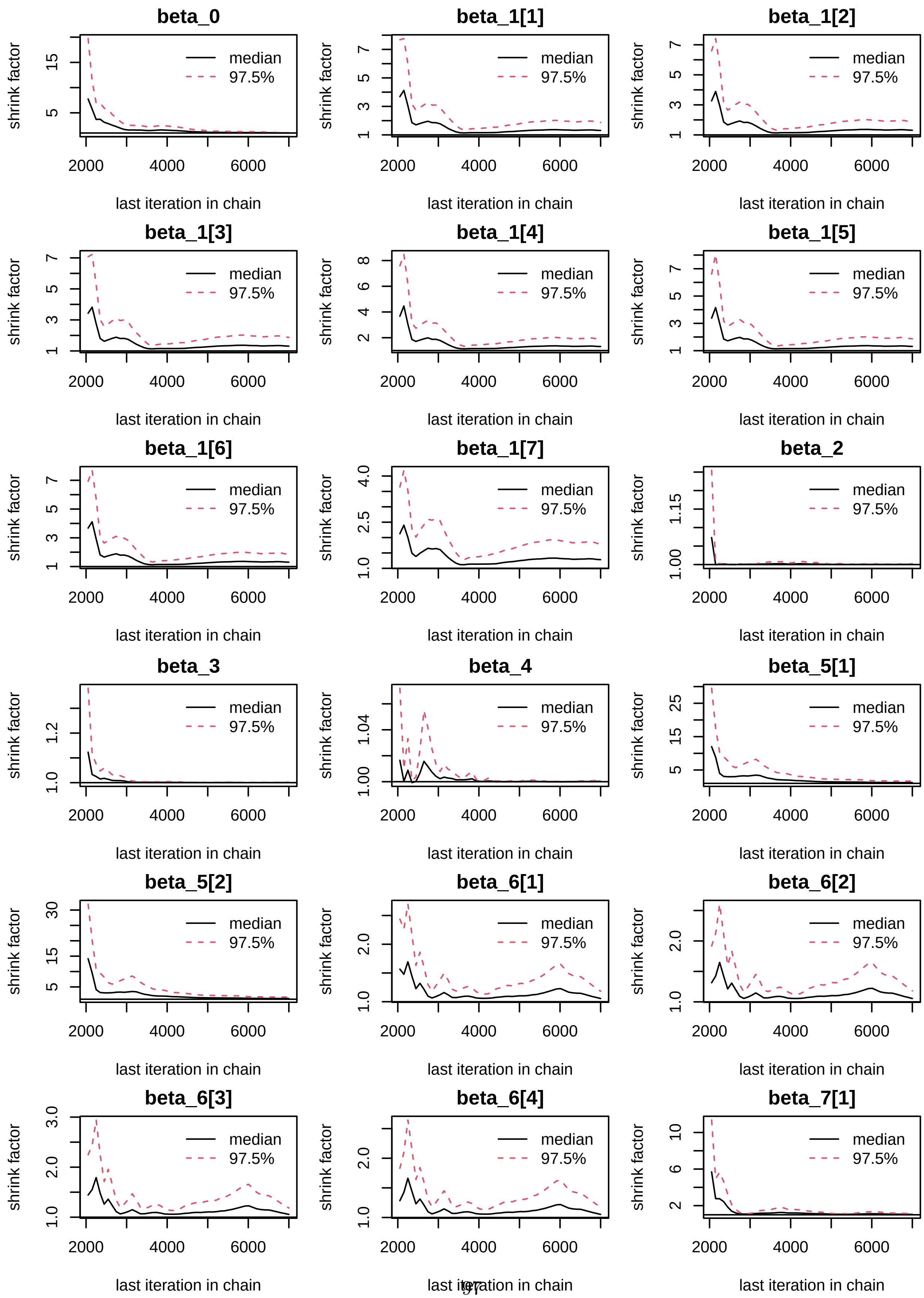

beta_7[1]
beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag

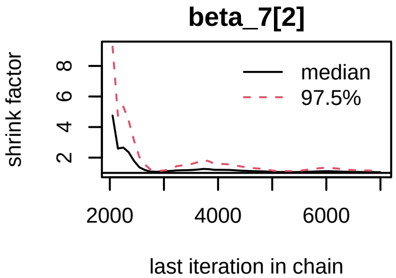

beta_1[7]
beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800

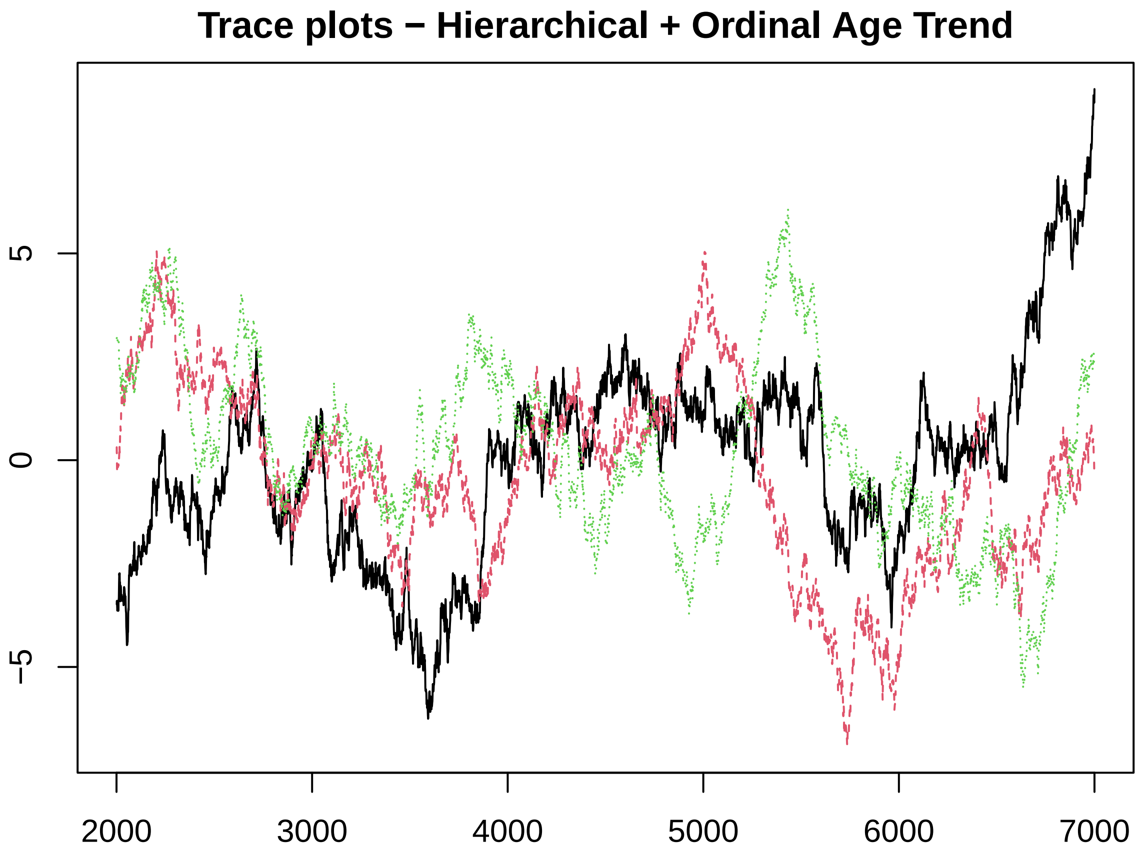


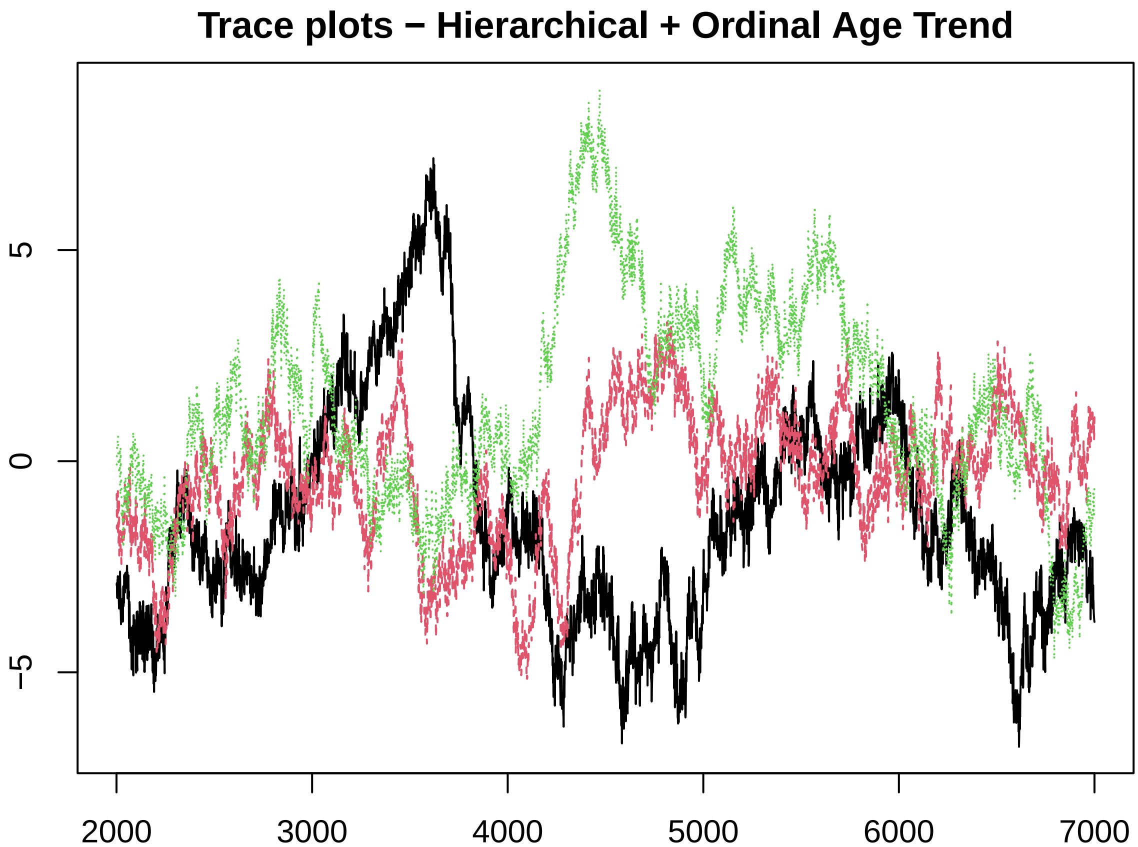

```
## Potential scale reduction factors:
##
##           Point est. Upper C.I.
## beta_0          1.28       2.03
## beta_1[1]       1.04       1.06
## beta_1[2]       1.04       1.06
## beta_1[3]       1.04       1.06
## beta_1[4]       1.04       1.06
## beta_1[5]       1.04       1.06
## beta_1[6]       1.04       1.06
## beta_1[7]       1.04       1.05
## beta_2          1.00       1.00
## beta_3          1.00       1.00
## beta_4          1.00       1.00
## beta_5[1]       1.23       1.84
## beta_5[2]       1.23       1.84
## beta_6[1]       1.09       1.26
## beta_6[2]       1.09       1.26
## beta_6[3]       1.09       1.26
## beta_6[4]       1.08       1.25
## beta_7[1]       1.08       1.18
## beta_7[2]       1.08       1.19
##
## Multivariate psrf
##
## 1.12
```

Hierarchical + Ordinal Age Trend

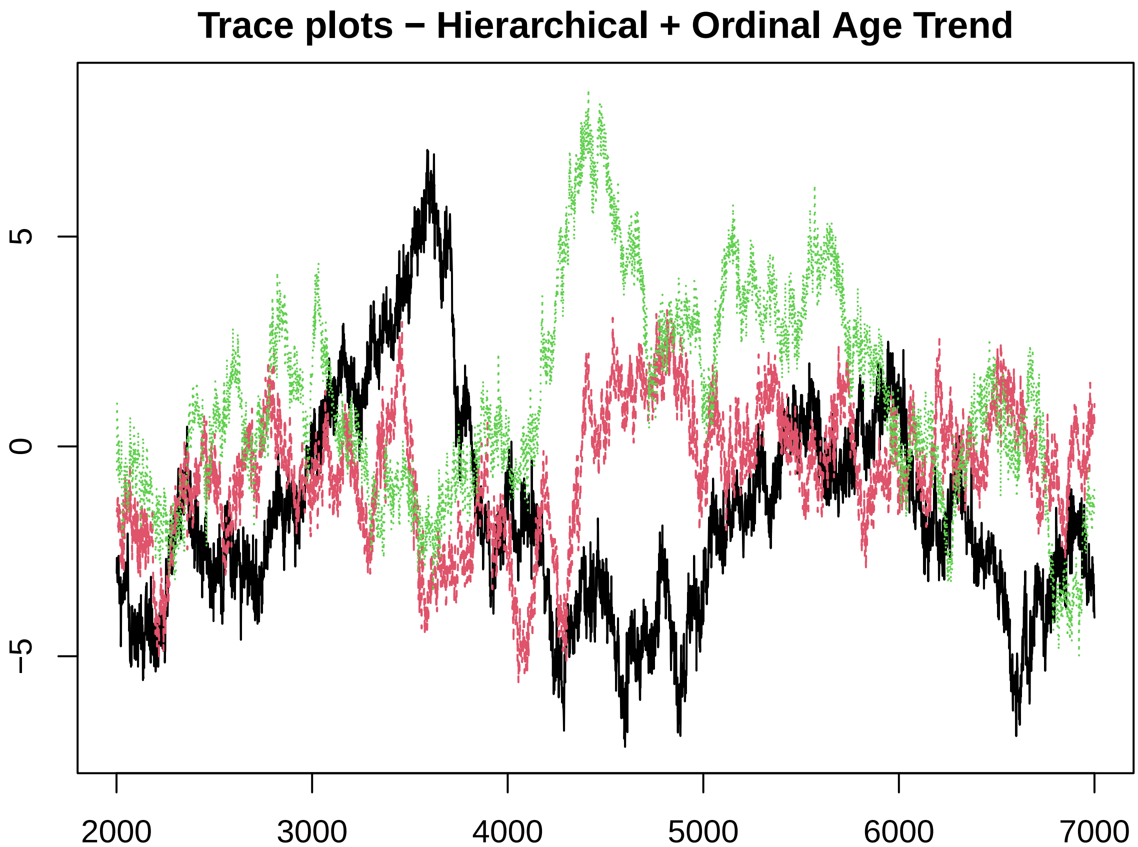


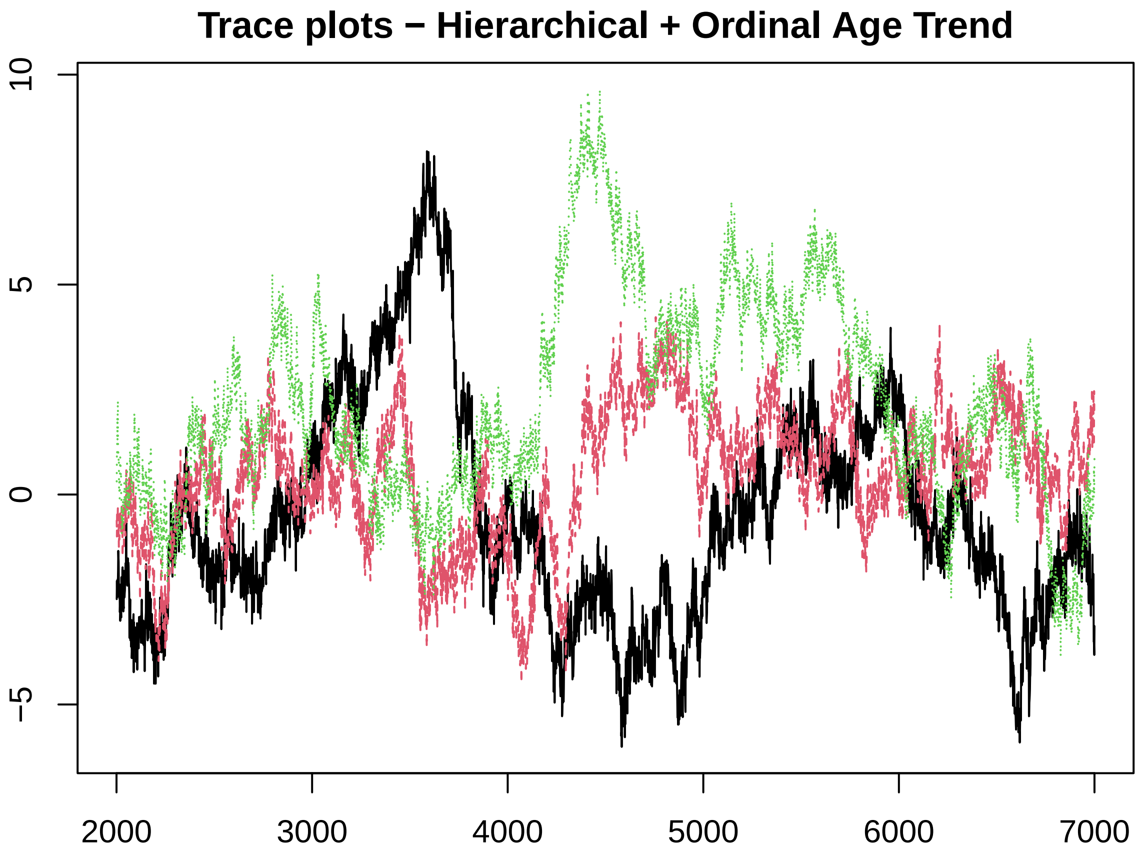

beta_7[2]
shrink factor
2
4
6
8
median
97.5%
2000
4000
6000
last iteration in chain

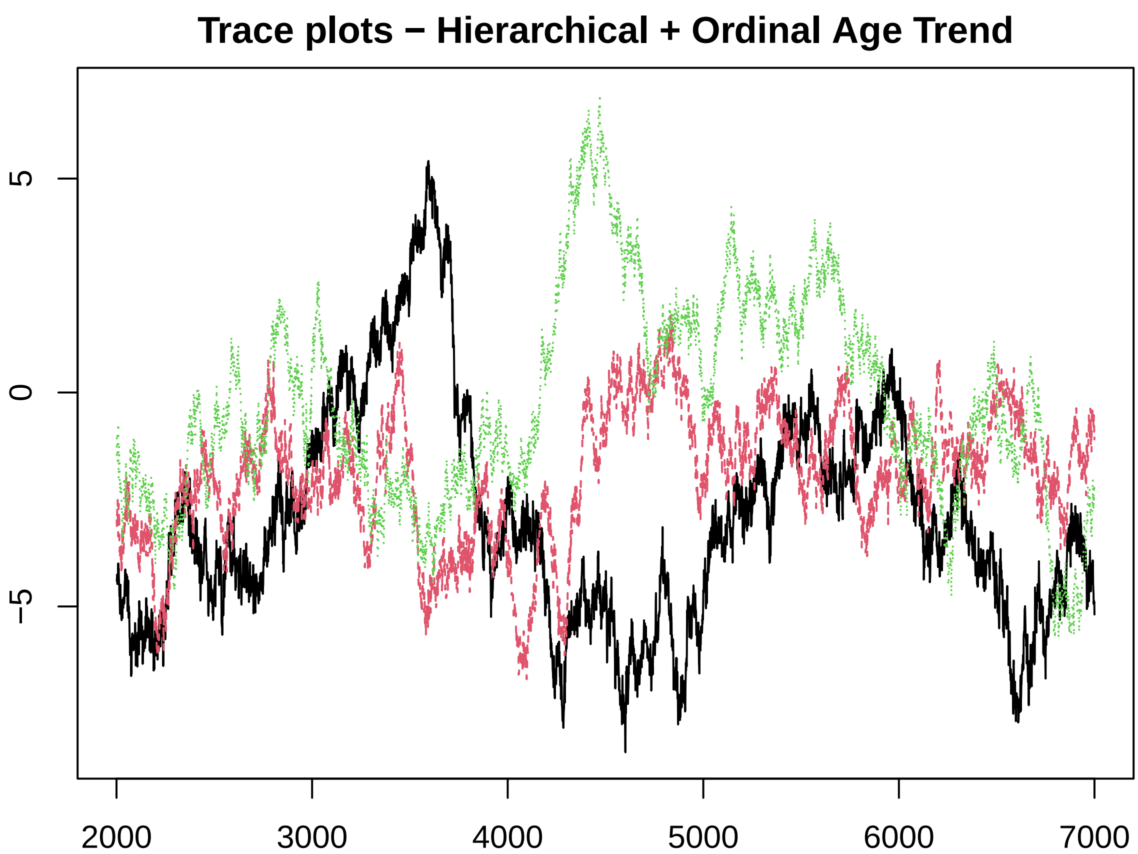

Trace plots − Hierarchical + Ordinal Age Trend
5
0
−5
2000
3000
4000
5000
6000
7000

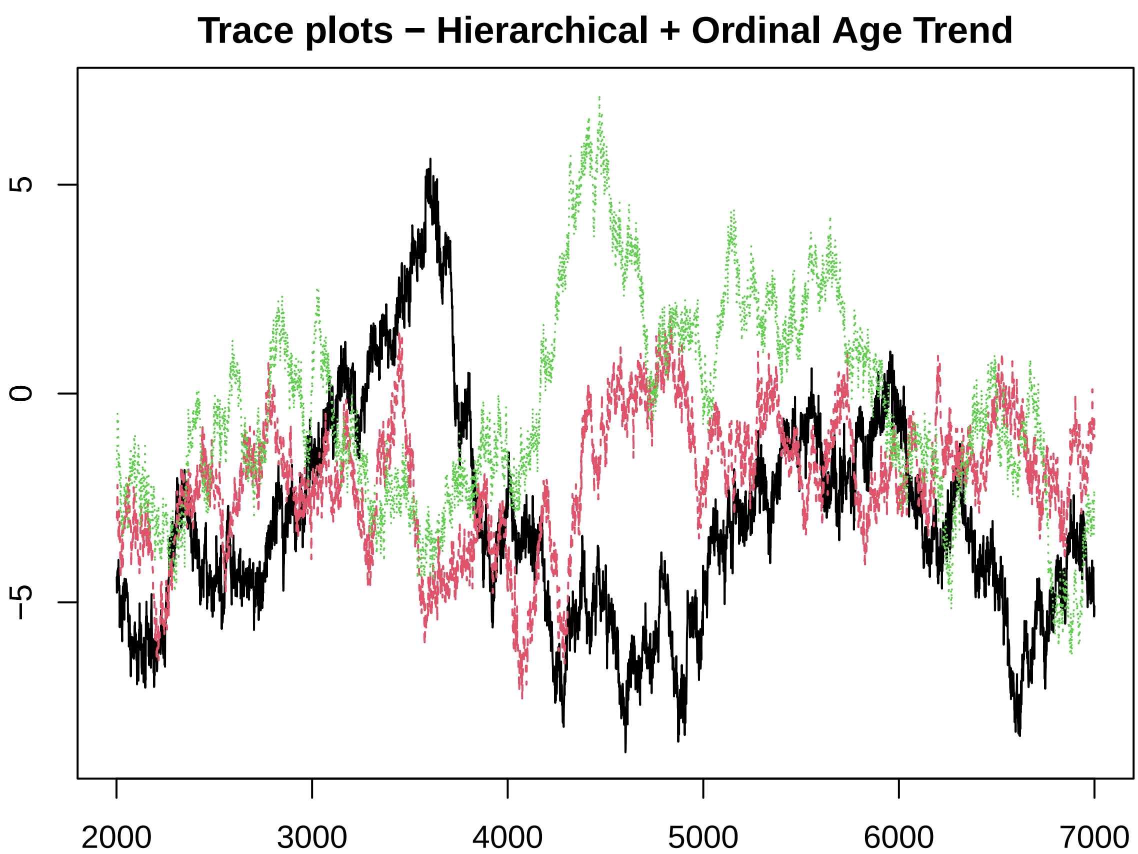


Trace plots − Hierarchical + Ordinal Age Trend
5
0
−5
2000
3000
4000
5000
6000
7000

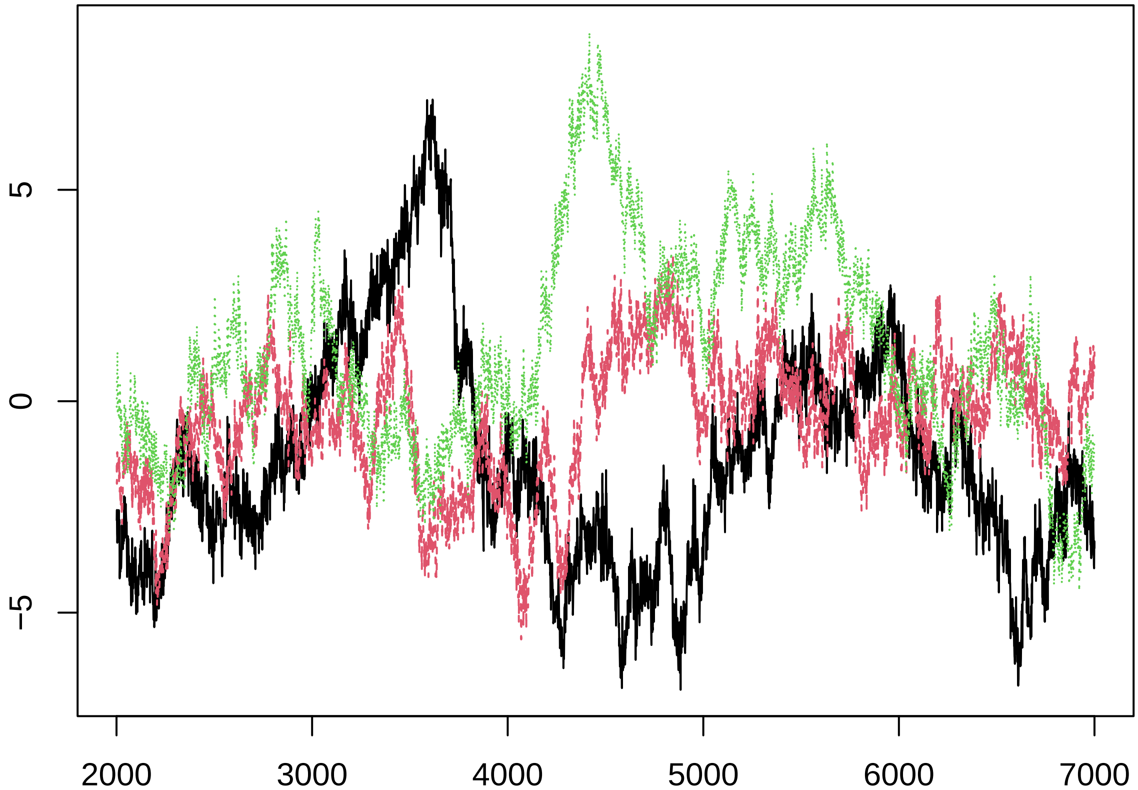

Trace plots − Hierarchical + Ordinal Age Trend
5
0
−5
2000
3000
4000
5000
6000
7000

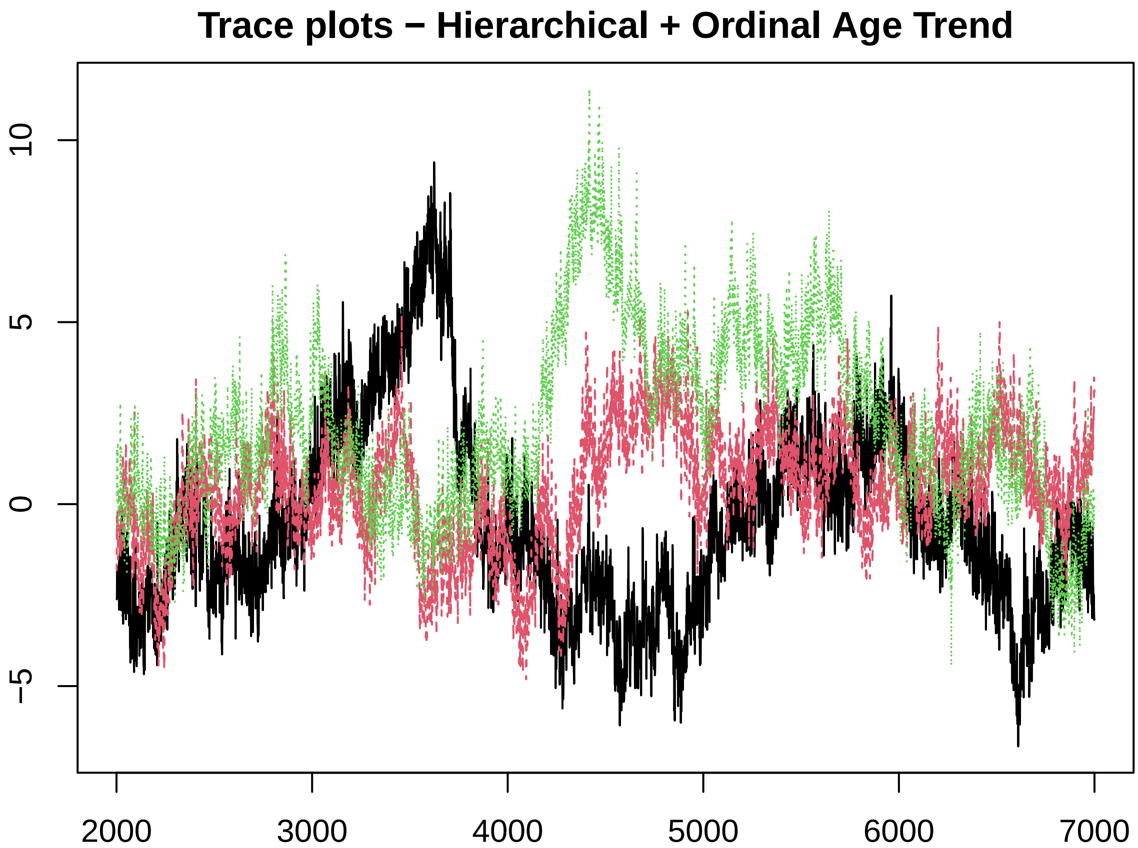

Trace plots − Hierarchical + Ordinal Age Trend
10
5
0
−5
2000
3000
4000
5000
6000
7000

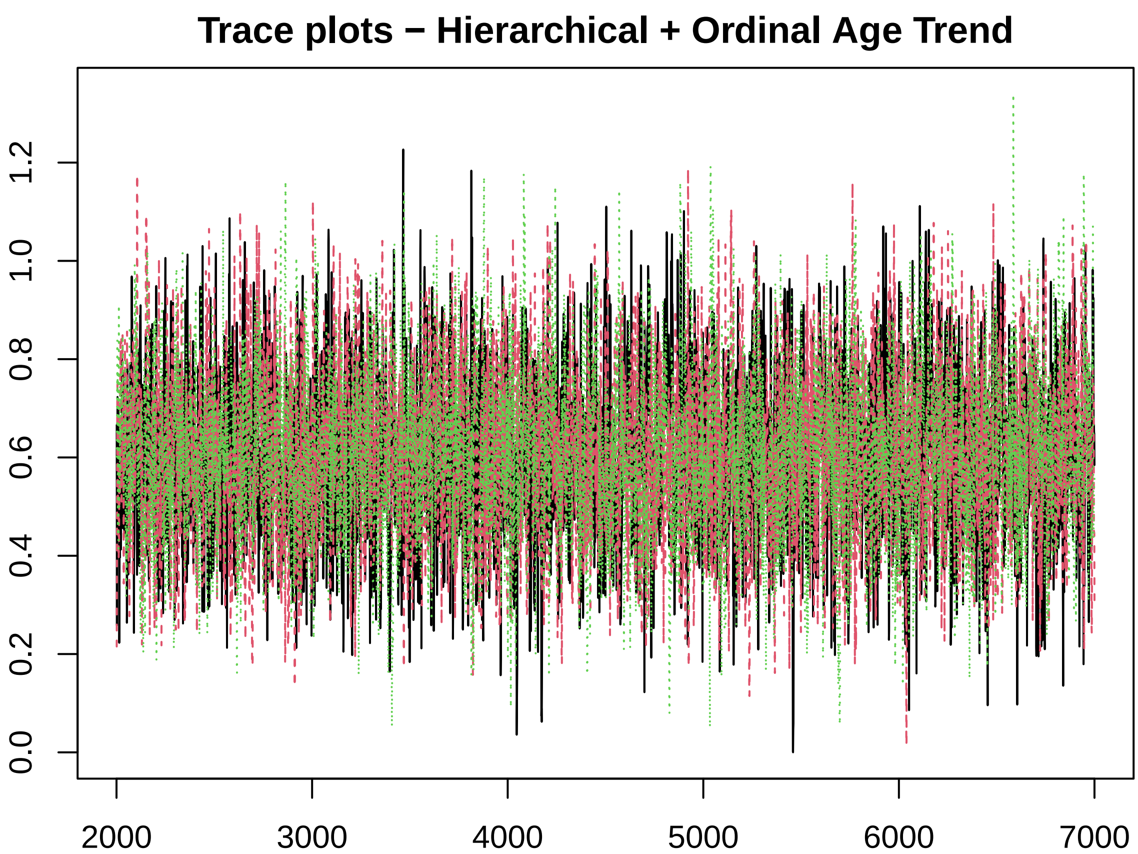

Trace plots − Hierarchical + Ordinal Age Trend
5
0
−5
2000
3000
4000
5000
6000
7000


Trace plots − Hierarchical + Ordinal Age Trend
5
0
−5
2000
3000
4000
5000
6000
7000

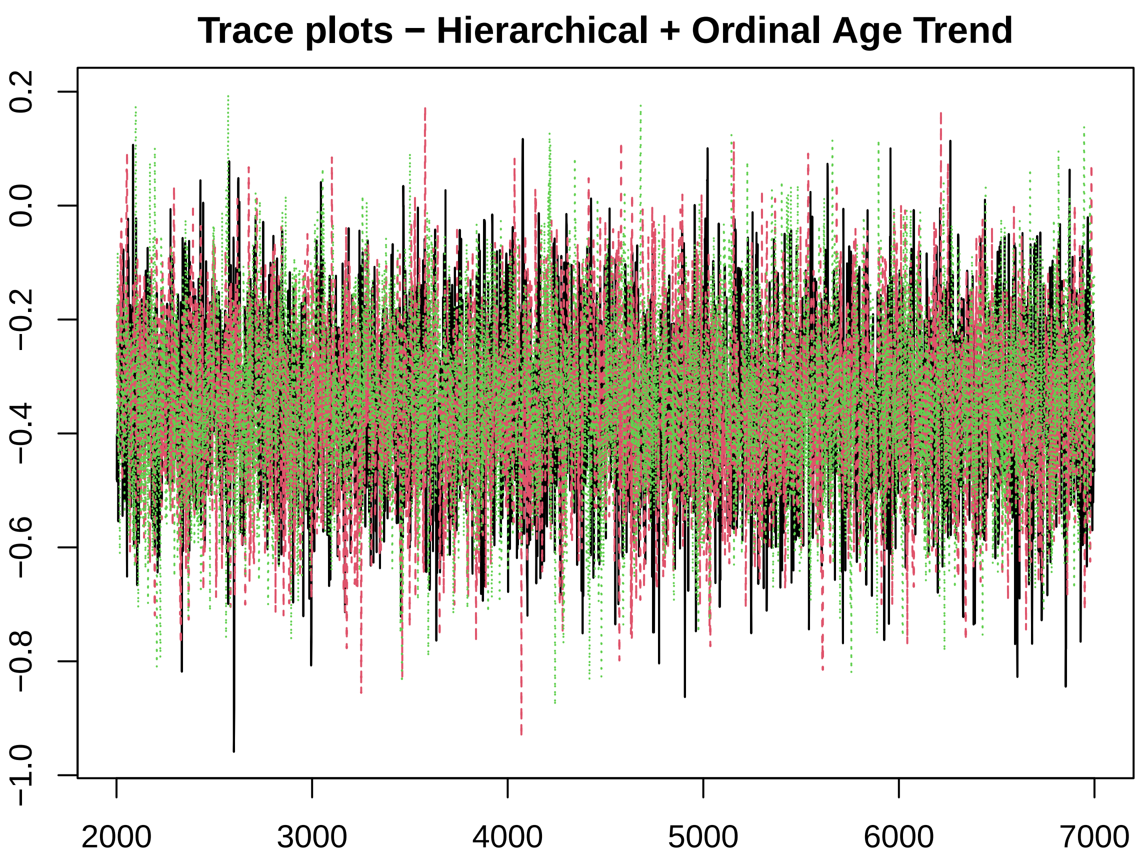
Trace plots − Hierarchical + Ordinal Age Trend

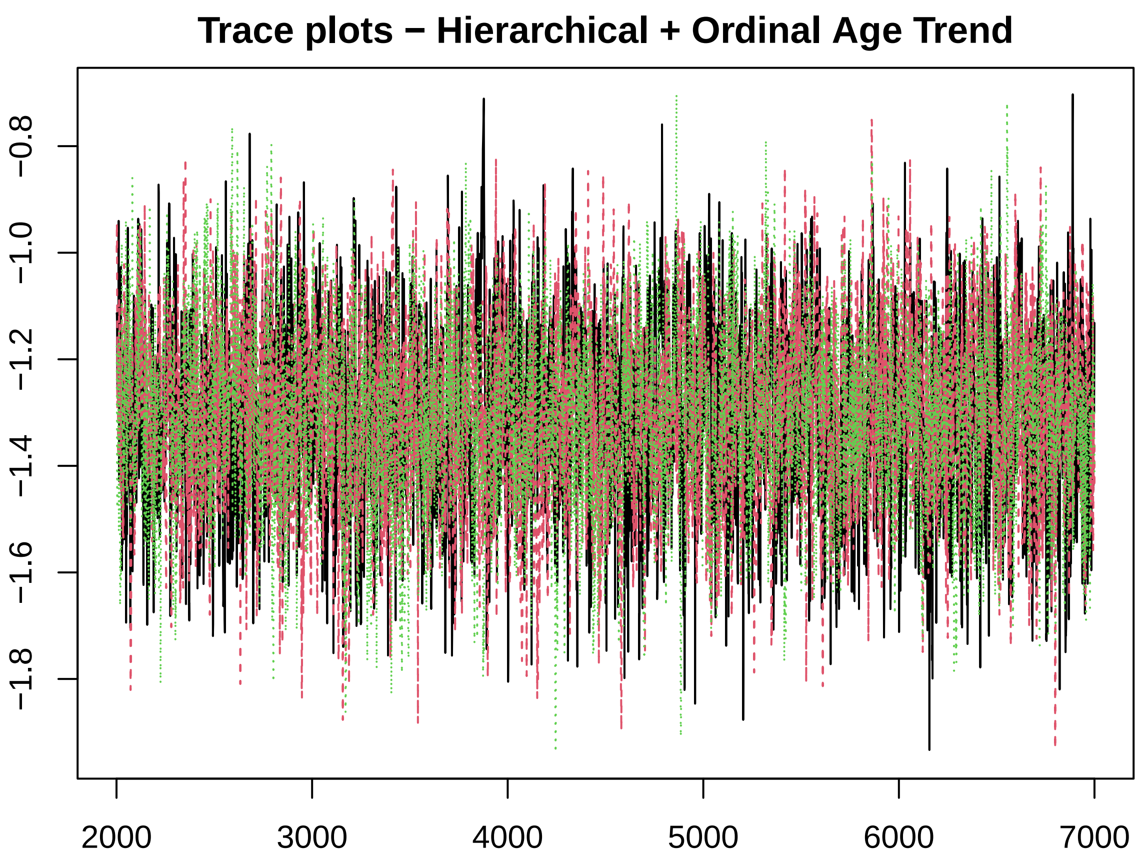


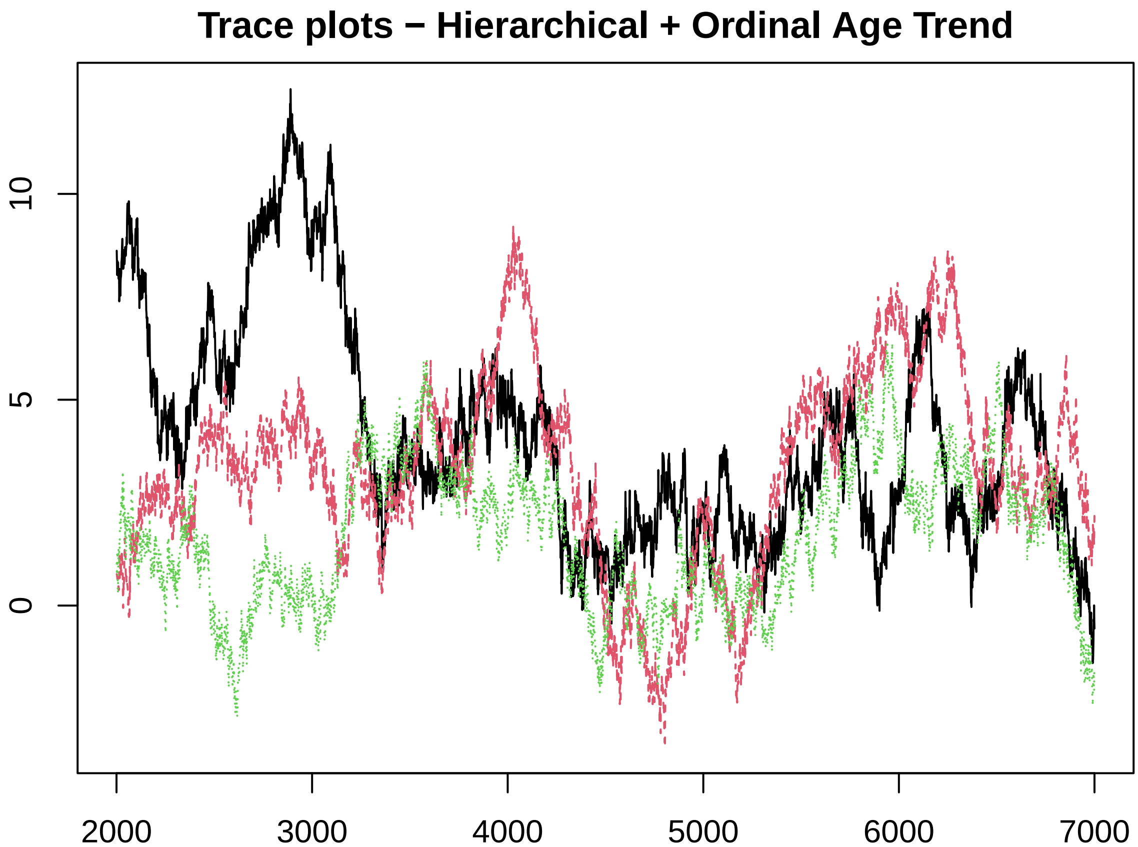

Trace plots − Hierarchical + Ordinal Age Trend
10
5
0
−5
2000
3000
4000
5000
6000
7000

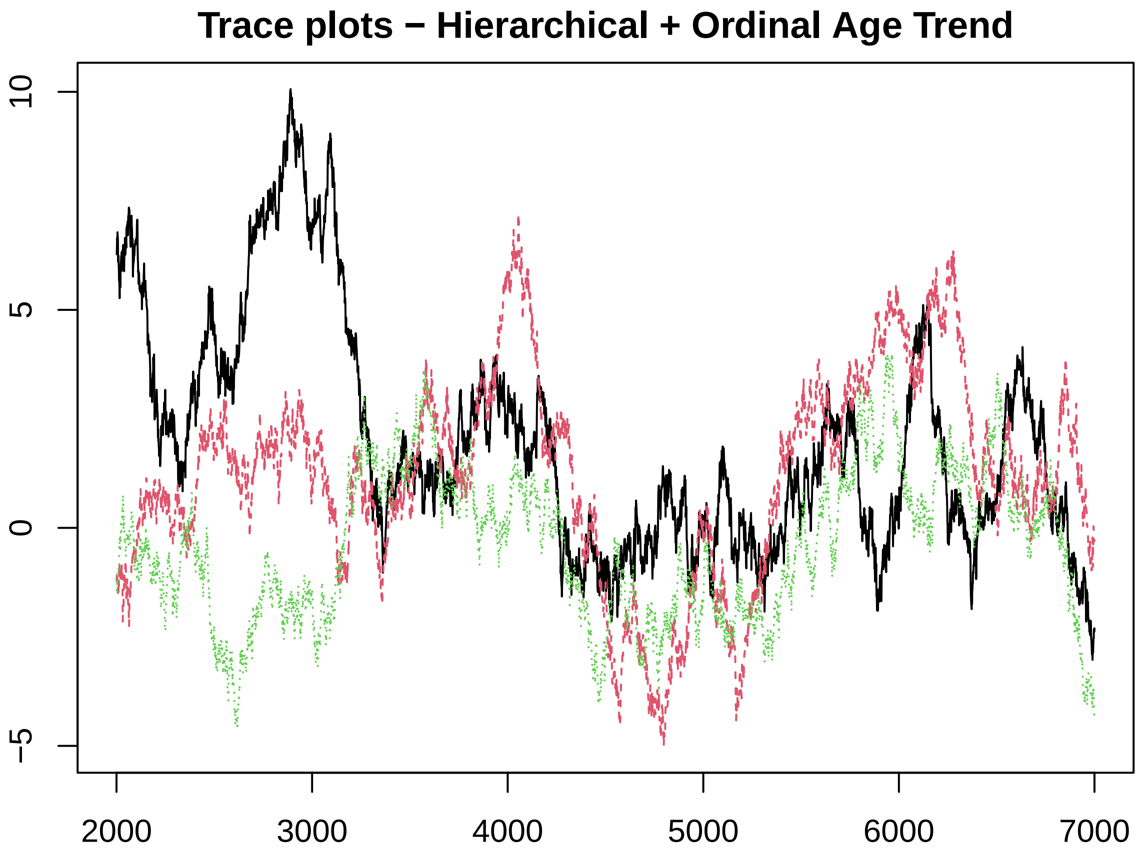


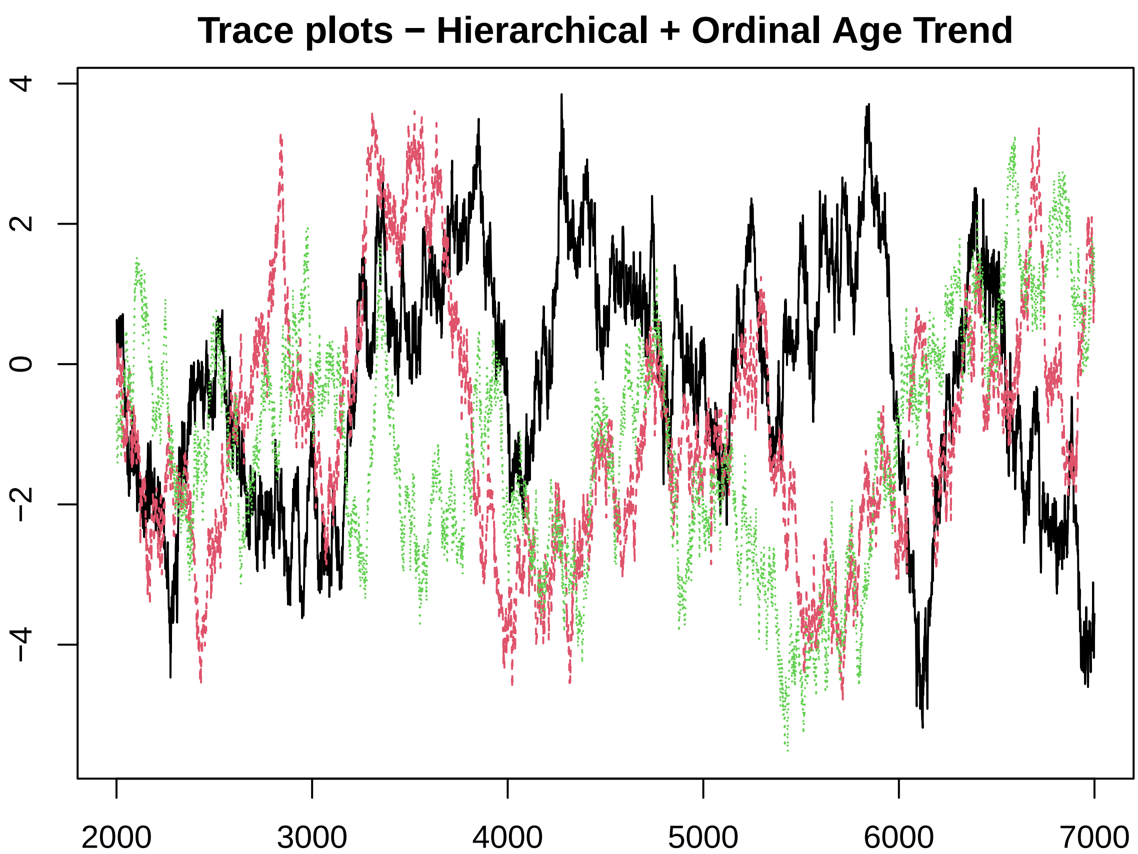
Trace plots − Hierarchical + Ordinal Age Trend
0.0
0.2
0.4
0.6
0.8
1.0
1.2
2000
3000
4000
5000
6000
7000


Trace plots − Hierarchical + Ordinal Age Trend
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
2000
3000
4000
5000
6000
7000

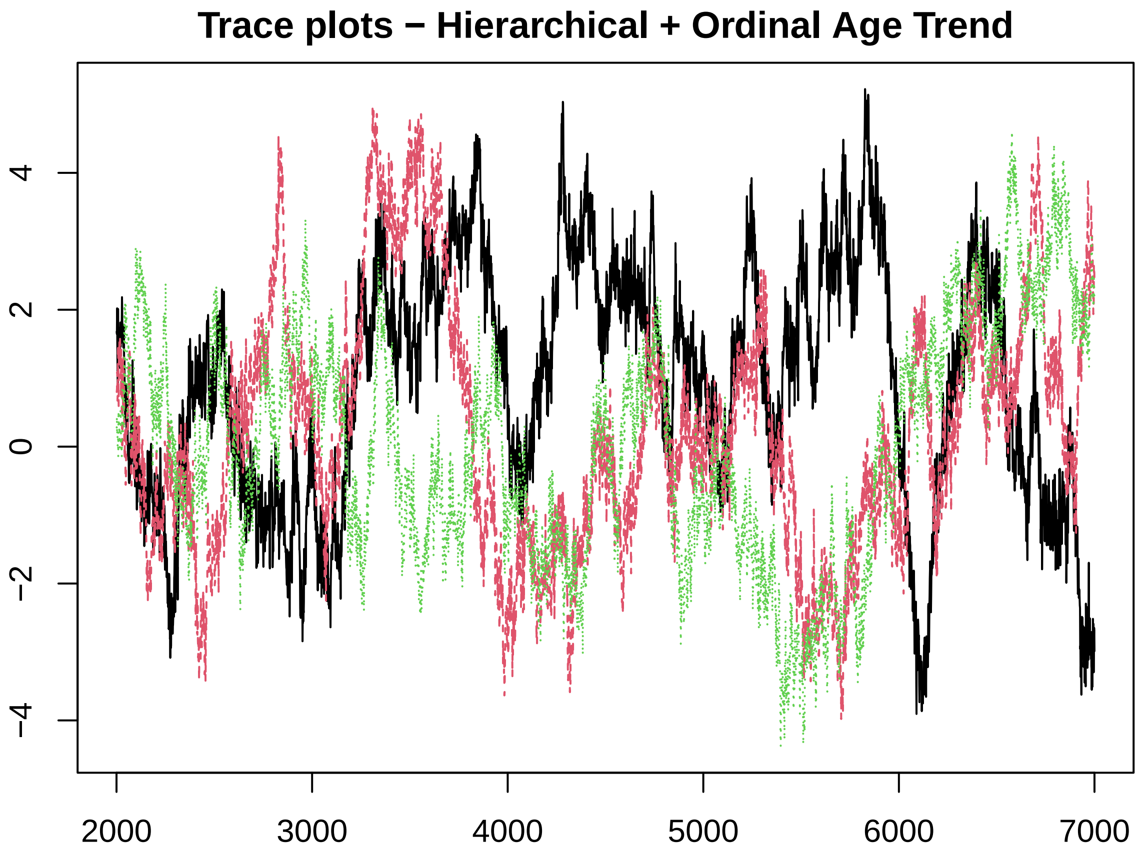

Trace plots − Hierarchical + Ordinal Age Trend
−0.8
−1.0
−1.2
−1.4
−1.6
−1.8
2000
3000
4000
5000
6000
7000

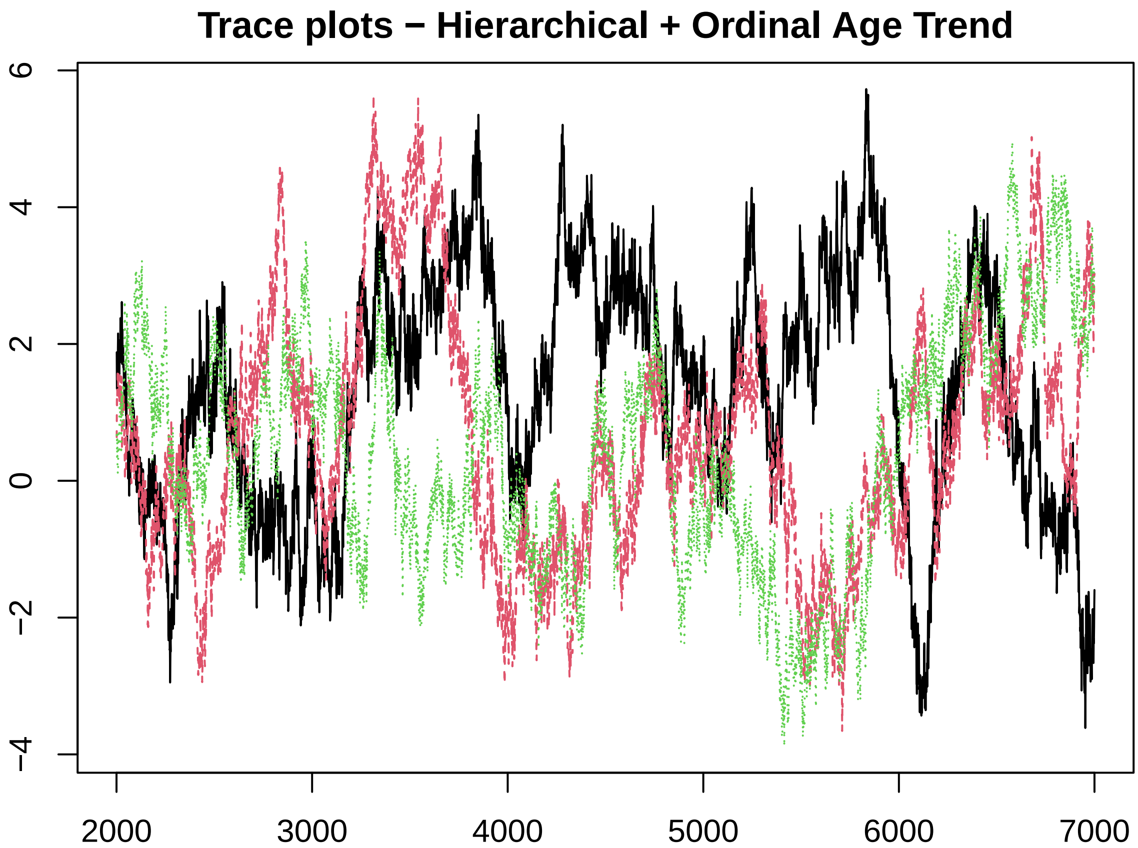

Trace plots − Hierarchical + Ordinal Age Trend
0
5
10
2000
3000
4000
5000
6000
7000


Trace plots − Hierarchical + Ordinal Age Trend
10
5
0
−5
2000
3000
4000
5000
6000
7000

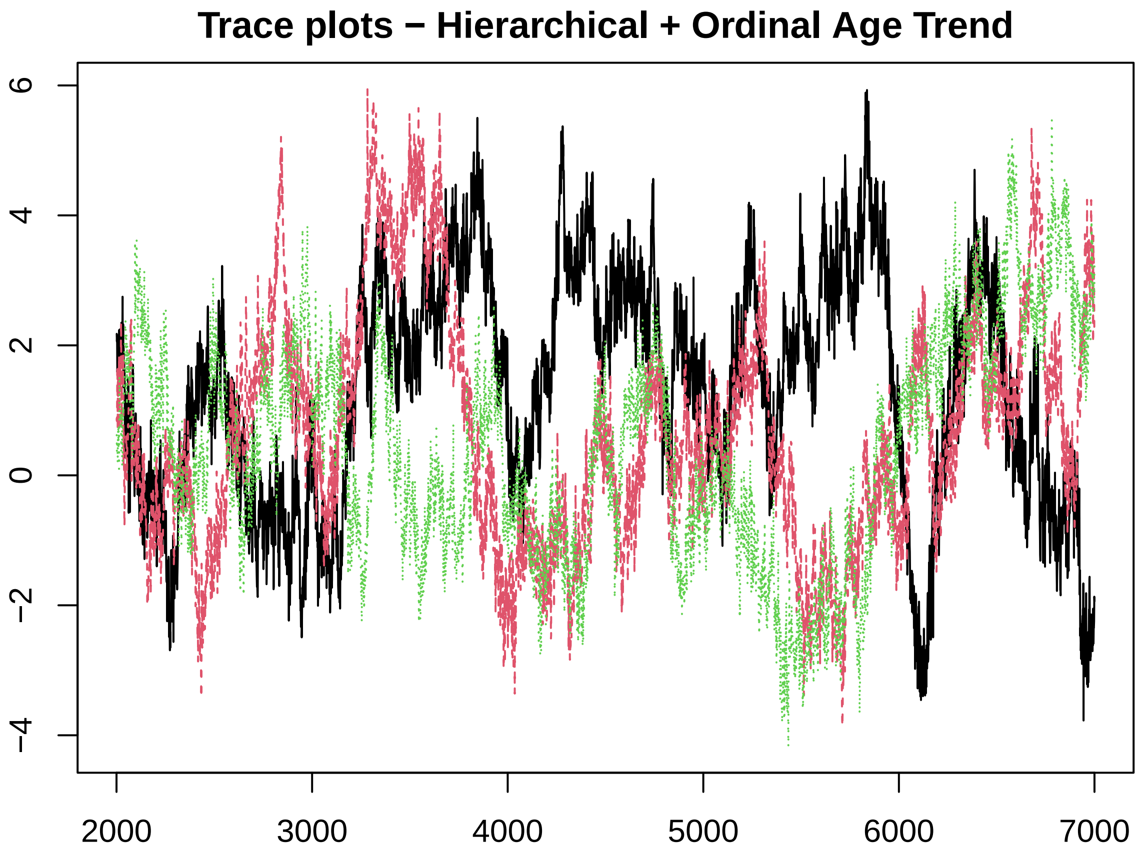


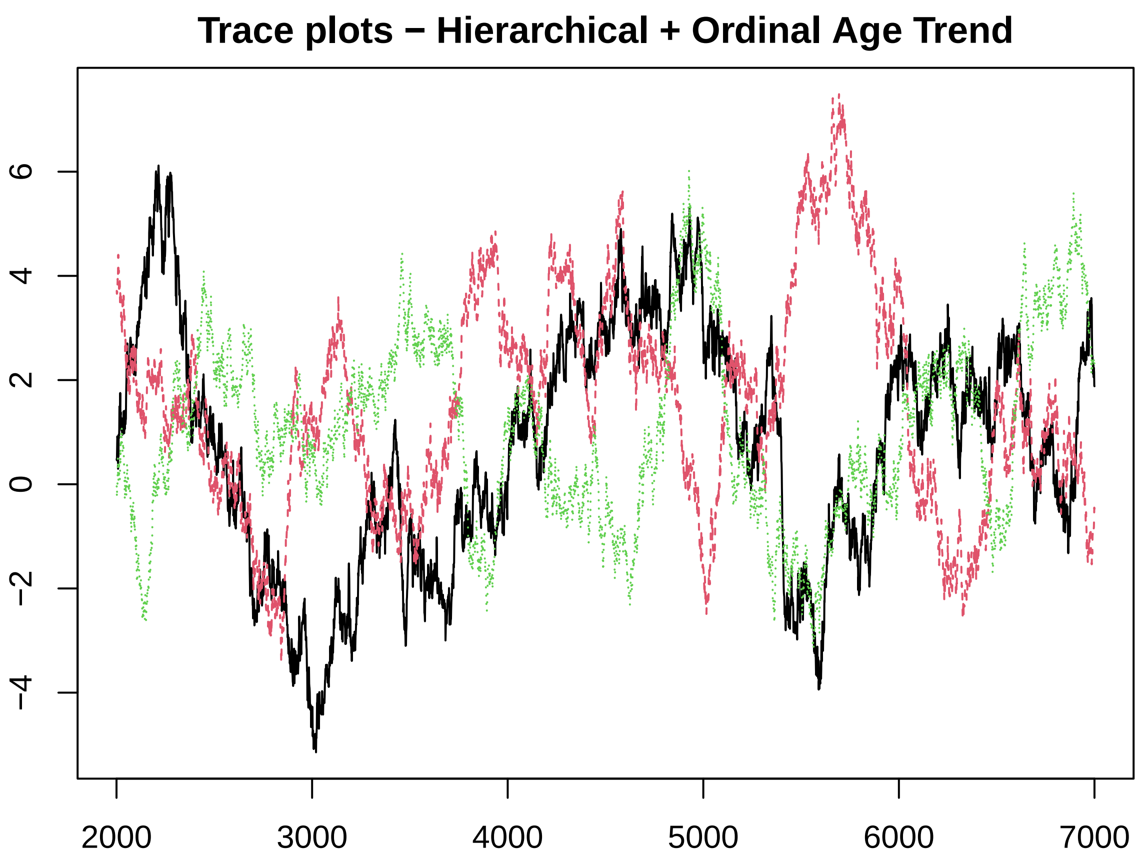
Trace plots − Hierarchical + Ordinal Age Trend
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000

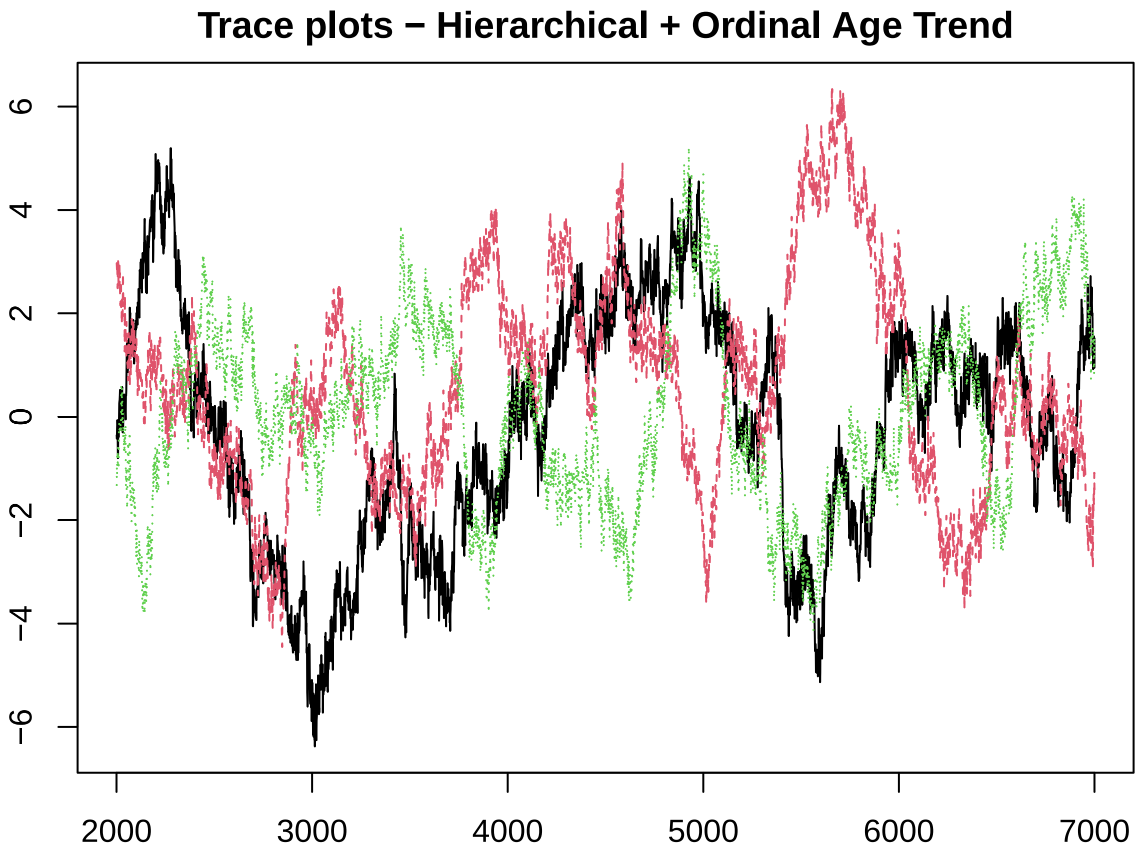

Trace plots − Hierarchical + Ordinal Age Trend
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000



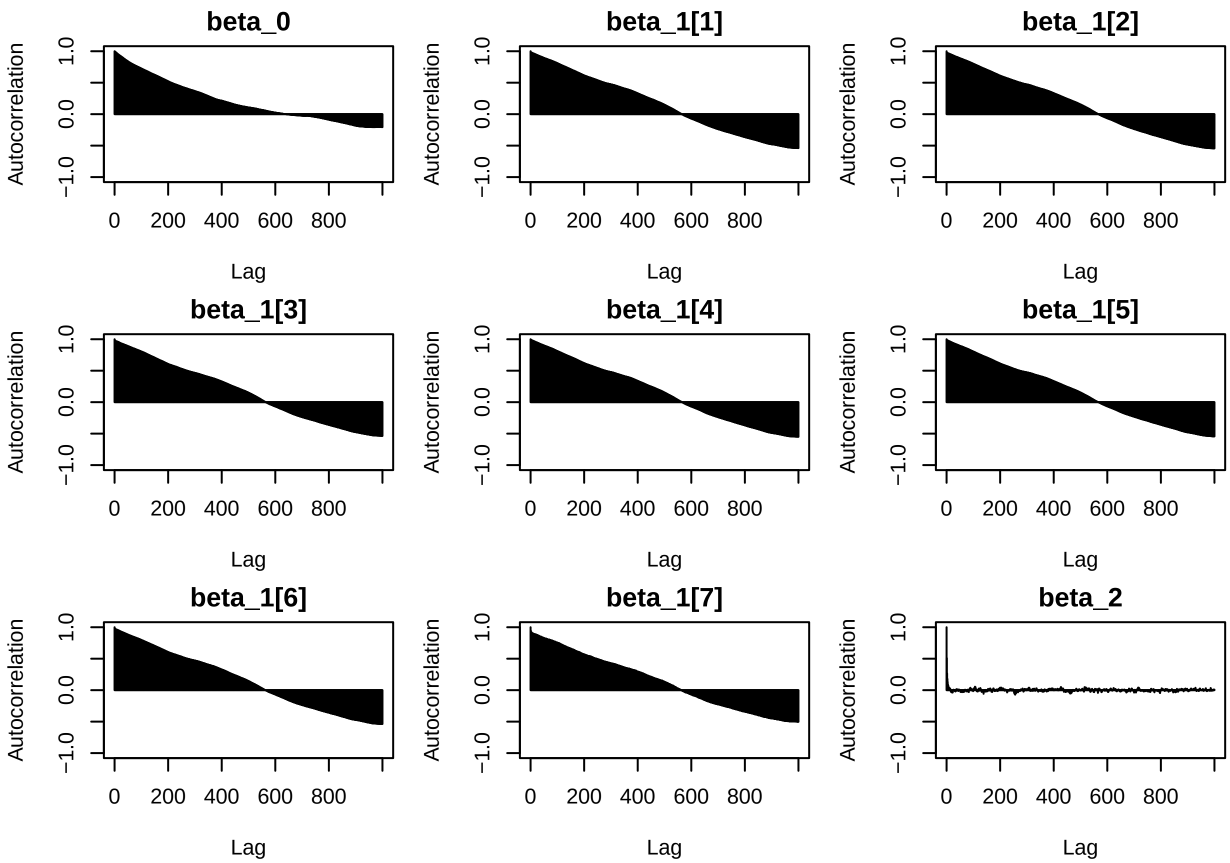
Trace plots − Hierarchical + Ordinal Age Trend
6
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000

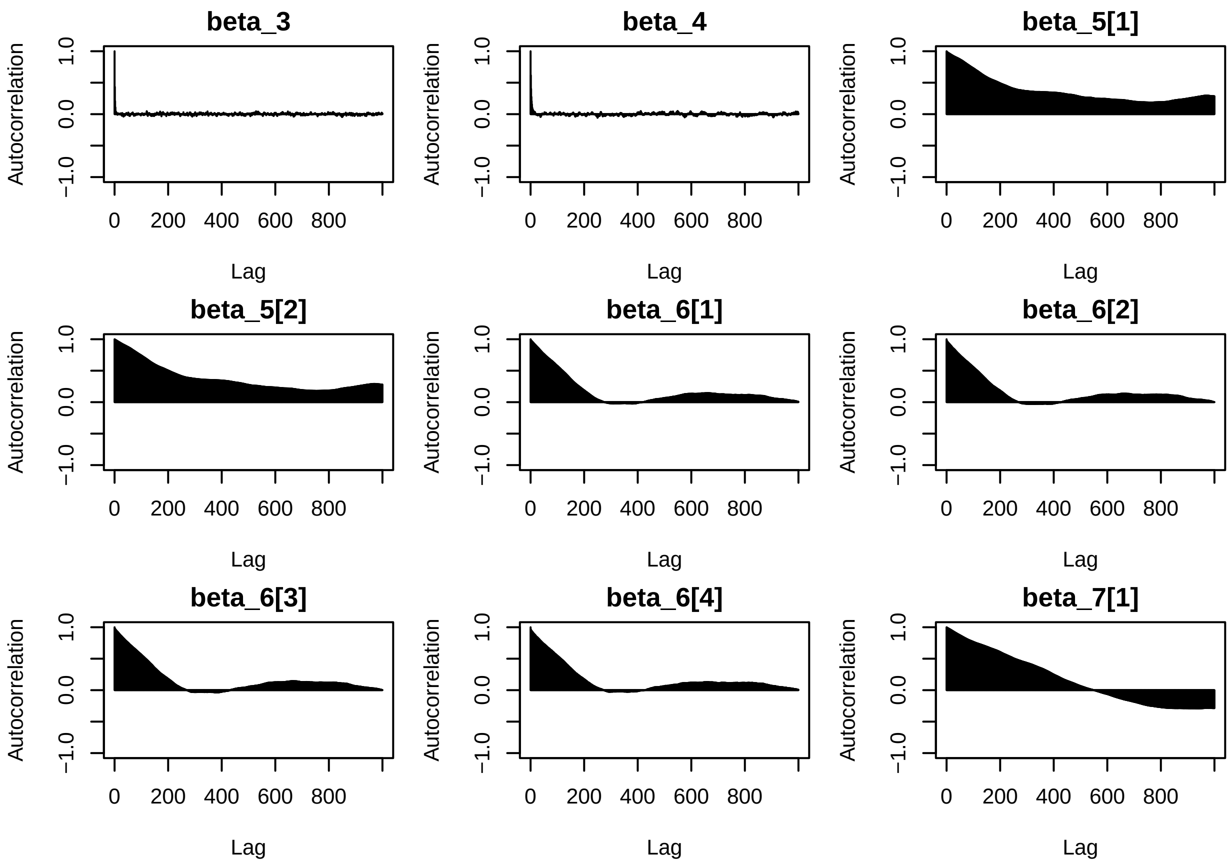

Trace plots − Hierarchical + Ordinal Age Trend
6
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000

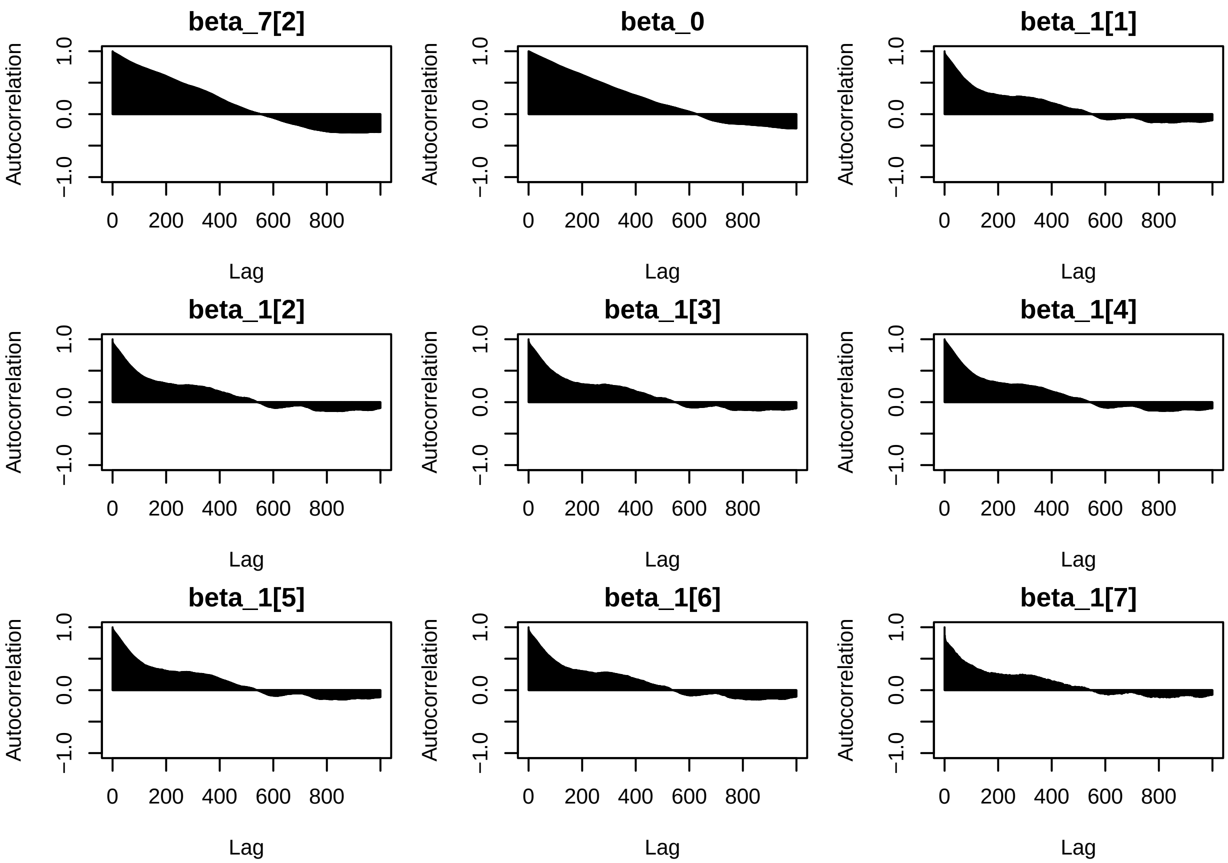

Trace plots − Hierarchical + Ordinal Age Trend
6
4
2
0
−2
−4
2000
3000
4000
5000
6000
7000

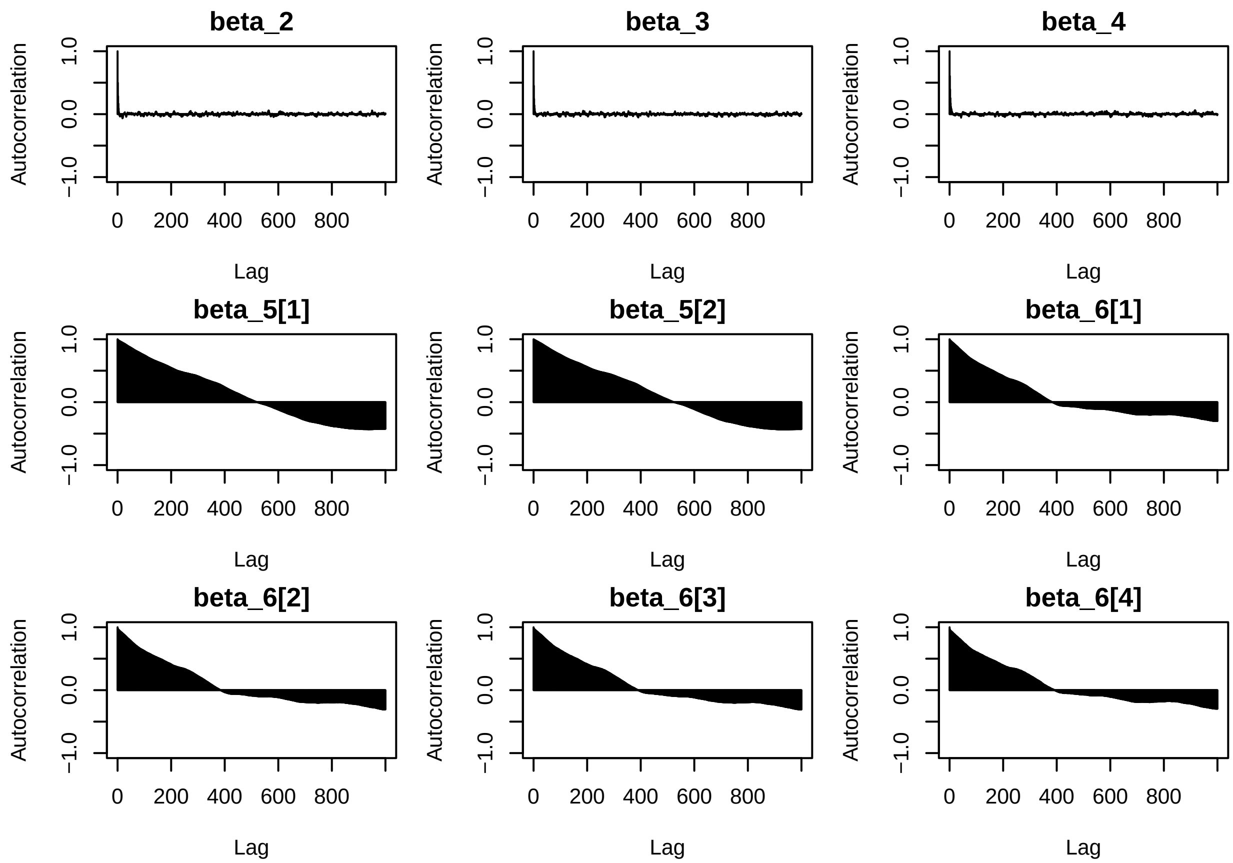


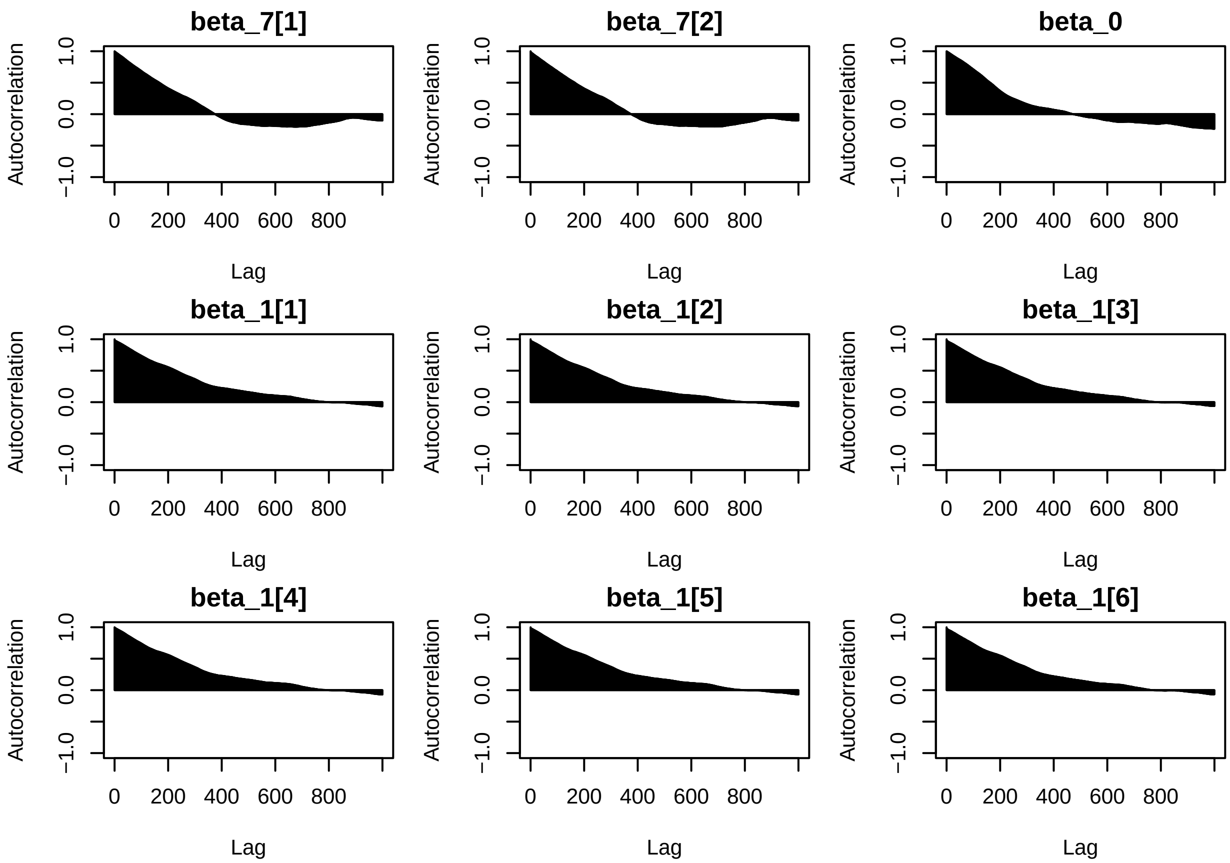
Trace plots − Hierarchical + Ordinal Age Trend
6
4
2
0
−2
−4
−6
2000
3000
4000
5000
6000
7000

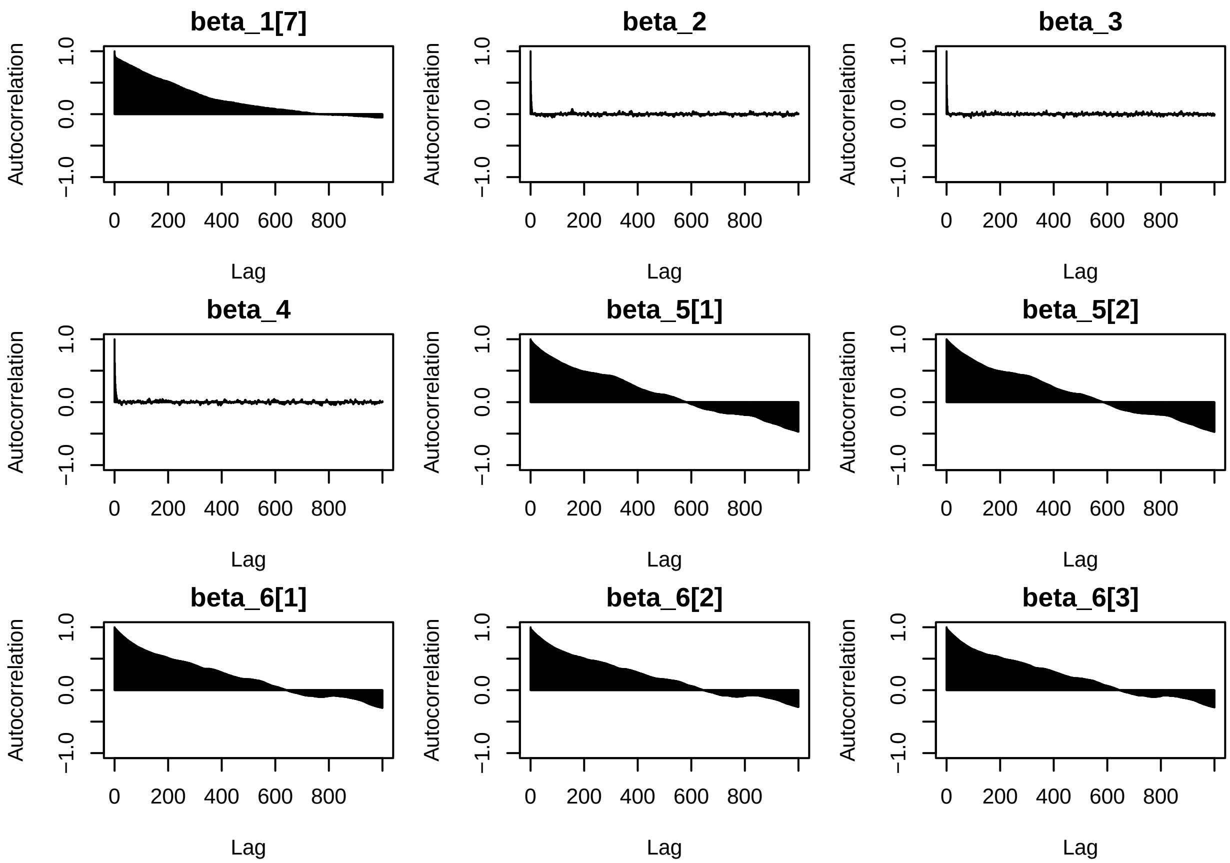

beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
beta_2
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag

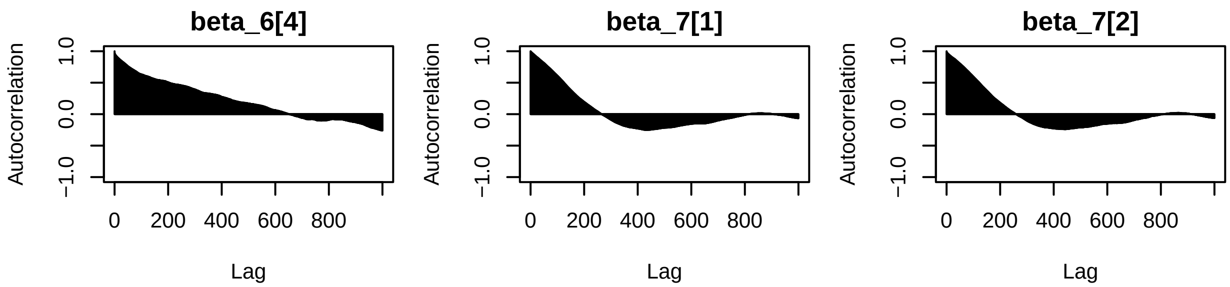

beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
beta_7[1]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag


beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
beta_1[7]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800


beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
beta_6[4]
Autocorrelation
Lag
−1.0
0.0
1.0
0
200
400
600
800


beta_7[1]
beta_7[2]
beta_0
beta_1[1]
beta_1[2]
beta_1[3]
beta_1[4]
beta_1[5]
beta_1[6]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag


beta_1[7]
beta_2
beta_3
beta_4
beta_5[1]
beta_5[2]
beta_6[1]
beta_6[2]
beta_6[3]
Autocorrelation
−1.0
0.0
1.0
0
200
400
600
800
Lag




```
## Potential scale reduction factors:
##
##             Point est. Upper C.I.
## beta_0            1.02       1.05
## beta_1[1]         1.31       1.87
## beta_1[2]         1.31       1.87
## beta_1[3]         1.31       1.88
## beta_1[4]         1.31       1.88
## beta_1[5]         1.31       1.87
## beta_1[6]         1.31       1.86
## beta_1[7]         1.28       1.79
## beta_2            1.00       1.00
## beta_3            1.00       1.00
## beta_4            1.00       1.00
## beta_5[1]         1.22       1.66
## beta_5[2]         1.23       1.68
## beta_6[1]         1.06       1.18
## beta_6[2]         1.05       1.18
## beta_6[3]         1.06       1.18
## beta_6[4]         1.05       1.17
## beta_7[1]         1.04       1.12
## beta_7[2]         1.04       1.12
##
## Multivariate psrf
##
## 1.32
```

```
## Potential scale reduction factors:
##
##           Point est. Upper C.I.
## beta_0          5.54      11.11
## beta_1[1]       8.54      16.39
## beta_1[2]       8.53      16.36
## beta_1[3]       8.55      16.38
## beta_1[4]       8.63      16.56
## beta_1[5]       8.59      16.49
## beta_1[6]       8.52      16.35
## beta_1[7]       7.95      15.18
## beta_2          1.00       1.00
## beta_3          1.00       1.00
## beta_4          1.00       1.00
## beta_5[1]       1.45       2.74
## beta_5[2]       1.45       2.75
## beta_6[1]       1.52       3.43
## beta_6[2]       1.52       3.41
## beta_6[3]       1.52       3.40
## beta_6[4]       1.52       3.38
## beta_7[1]       2.65       5.67
## beta_7[2]       2.65       5.64
##
## Multivariate psrf
##
## 8.03
```

```
## Potential scale reduction factors:
##
##           Point est. Upper C.I.
## beta_0          1.05       1.13
## beta_1[1]       1.01       1.01
## beta_1[2]       1.01       1.01
## beta_1[3]       1.01       1.01
## beta_1[4]       1.01       1.02
## beta_1[5]       1.02       1.02
## beta_1[6]       1.01       1.01
## beta_1[7]       1.00       1.00
## beta_2          1.00       1.00
## beta_3          1.00       1.00
## beta_4          1.00       1.00
## beta_5[1]       1.02       1.04
## beta_5[2]       1.02       1.04
## beta_6[1]       1.00       1.01
## beta_6[2]       1.00       1.01
## beta_6[3]       1.00       1.01
## beta_6[4]       1.00       1.01
## beta_7[1]       1.02       1.06
## beta_7[2]       1.02       1.06
##
## Multivariate psrf
##
## 1.03
```

```
## Potential scale reduction factors:
##
##           Point est. Upper C.I.
## beta_0          1.12       1.29
## beta_1[1]       1.07       1.09
## beta_1[2]       1.06       1.08
## beta_1[3]       1.06       1.08
## beta_1[4]       1.07       1.09
## beta_1[5]       1.07       1.09
## beta_1[6]       1.06       1.09
## beta_1[7]       1.05       1.07
## beta_2          1.00       1.00
## beta_3          1.00       1.00
## beta_4          1.00       1.00
## beta_5[1]       1.24       1.72
## beta_5[2]       1.23       1.72
## beta_6[1]       1.08       1.24
## beta_6[2]       1.08       1.23
## beta_6[3]       1.08       1.23
## beta_6[4]       1.08       1.23
## beta_7[1]       1.14       1.41
## beta_7[2]       1.14       1.41
##
## Multivariate psrf
##
## 1.19
```

```
## Potential scale reduction factors:
##
##           Point est. Upper C.I.
## beta_0          1.07       1.22
## beta_1[1]       1.41       2.11
## beta_1[2]       1.41       2.11
## beta_1[3]       1.41       2.12
## beta_1[4]       1.41       2.12
## beta_1[5]       1.41       2.11
## beta_1[6]       1.40       2.10
## beta_1[7]       1.38       2.03
## beta_2          1.00       1.00
## beta_3          1.00       1.00
## beta_4          1.00       1.00
## beta_5[1]       1.15       1.46
## beta_5[2]       1.16       1.47
## beta_6[1]       1.16       1.46
## beta_6[2]       1.16       1.46
## beta_6[3]       1.16       1.46
## beta_6[4]       1.15       1.44
## beta_7[1]       1.06       1.18
## beta_7[2]       1.06       1.18
##
## Multivariate psrf
##
## 1.31
```

6 Posterior Interpretation and Prior Comparison

In this section, we extract and compare the posterior estimates from the three Bayesian hierarchical models, each using a different prior: (1) flat/uninformative, (2) weakly informative normal prior, and (3) hierarchical normal prior with hyperparameters. For each model, we examine the posterior mean and 95% credible interval of the regression coefficients. This comparison helps evaluate how sensitive the results are to the choice of prior and which prior leads to more stable and interpretable conclusions about the effect of each predictor on the probability of heart disease.

Table 2: Posterior summaries under all four prior structures

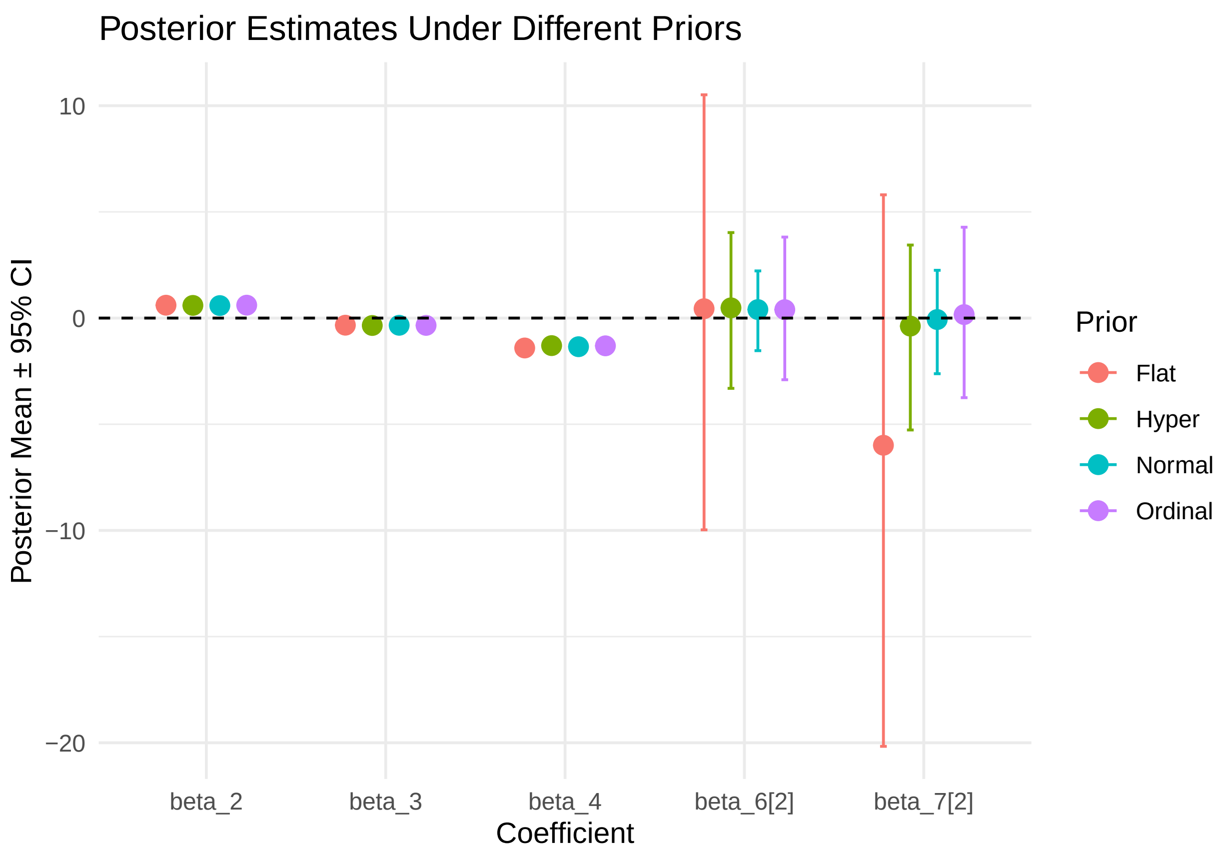
Coefficient	Flat_Mean	Flat_CI	Normal_Mean	Normal_CI	Hyper_Mean	Hyper_CI	Hyper_Ordinal_Mean	Hyper_Ordinal_CI
beta_0	-3.032	[-18.976, 9.117]	1.217	[-1.78, 4.519]	0.187	[-8.781, 4.89]	-0.057	[-4.545, 4.71]
beta_1[1]	-0.092	[-19.567, 11.847]	0.529	[-1.062, 2.216]	-0.454	[-4.591, 3.236]	-0.115	[-4.676, 5.432]
beta_1[2]	-0.389	[-19.865, 11.531]	0.258	[-1.356, 2.018]	-0.695	[-4.863, 3.023]	-0.342	[-4.893, 5.196]
beta_1[3]	0.894	[-18.623, 12.811]	1.425	[-0.171, 3.157]	0.344	[-3.813, 4.003]	0.714	[-3.813, 6.346]
beta_1[4]	-1.935	[-21.443, 9.953]	-1.232	[-2.79, 0.399]	-2.066	[-6.229, 1.646]	-1.693	[-6.215, 3.775]
beta_1[5]	-2.081	[-21.49, 9.804]	-1.34	[-2.923, 0.329]	-2.108	[-6.334, 1.59]	-1.83	[-6.502, 3.738]
beta_1[6]	-0.244	[-19.783, 11.662]	0.369	[-1.268, 2.101]	-0.566	[-4.732, 3.094]	-0.173	[-4.707, 5.292]
beta_1[7]	1.354	[-18.452, 13.792]	1.537	[-0.473, 3.707]	0.237	[-4.103, 4.128]	0.729	[-3.797, 6.471]
beta_2	0.606	[0.262, 0.964]	0.588	[0.252, 0.931]	0.595	[0.27, 0.925]	0.607	[0.284, 0.939]
beta_3	-0.336	[-0.633, -0.04]	-0.339	[-0.637, -0.049]	-0.349	[-0.629, -0.064]	-0.345	[-0.633, -0.058]
beta_4	-1.409	[-1.771, -1.069]	-1.349	[-1.697, -1.019]	-1.297	[-1.634, -0.99]	-1.307	[-1.645, -0.987]
beta_5[1]	12.405	[0.34, 29.092]	1.505	[-0.981, 3.925]	3.581	[-2.684, 14.6]	3.026	[-1.218, 9.075]
beta_5[2]	10.071	[-2, 26.763]	-0.718	[-3.198, 1.673]	1.508	[-4.722, 12.451]	0.938	[-3.248, 6.899]
beta_6[1]	-0.931	[-11.323, 9.185]	-0.914	[-2.861, 0.893]	-0.723	[-4.486, 2.771]	-0.8	[-4.037, 2.587]
beta_6[2]	0.441	[-9.973, 10.515]	0.4	[-1.535, 2.22]	0.481	[-3.31, 4.026]	0.396	[-2.904, 3.814]
beta_6[3]	0.882	[-9.537, 10.961]	0.811	[-1.136, 2.644]	0.849	[-2.961, 4.397]	0.79	[-2.499, 4.244]
beta_6[4]	1.036	[-9.388, 11.098]	0.915	[-1.085, 2.815]	0.866	[-2.932, 4.392]	0.8	[-2.493, 4.269]
beta_7[1]	-4.923	[-19.063, 6.854]	0.981	[-1.633, 3.292]	0.573	[-4.303, 4.362]	1.108	[-2.775, 5.229]
beta_7[2]	-5.99	[-20.169, 5.805]	-0.069	[-2.62, 2.25]	-0.378	[-5.27, 3.439]	0.164	[-3.749, 4.279]

```
## Flat Prior DIC:
```

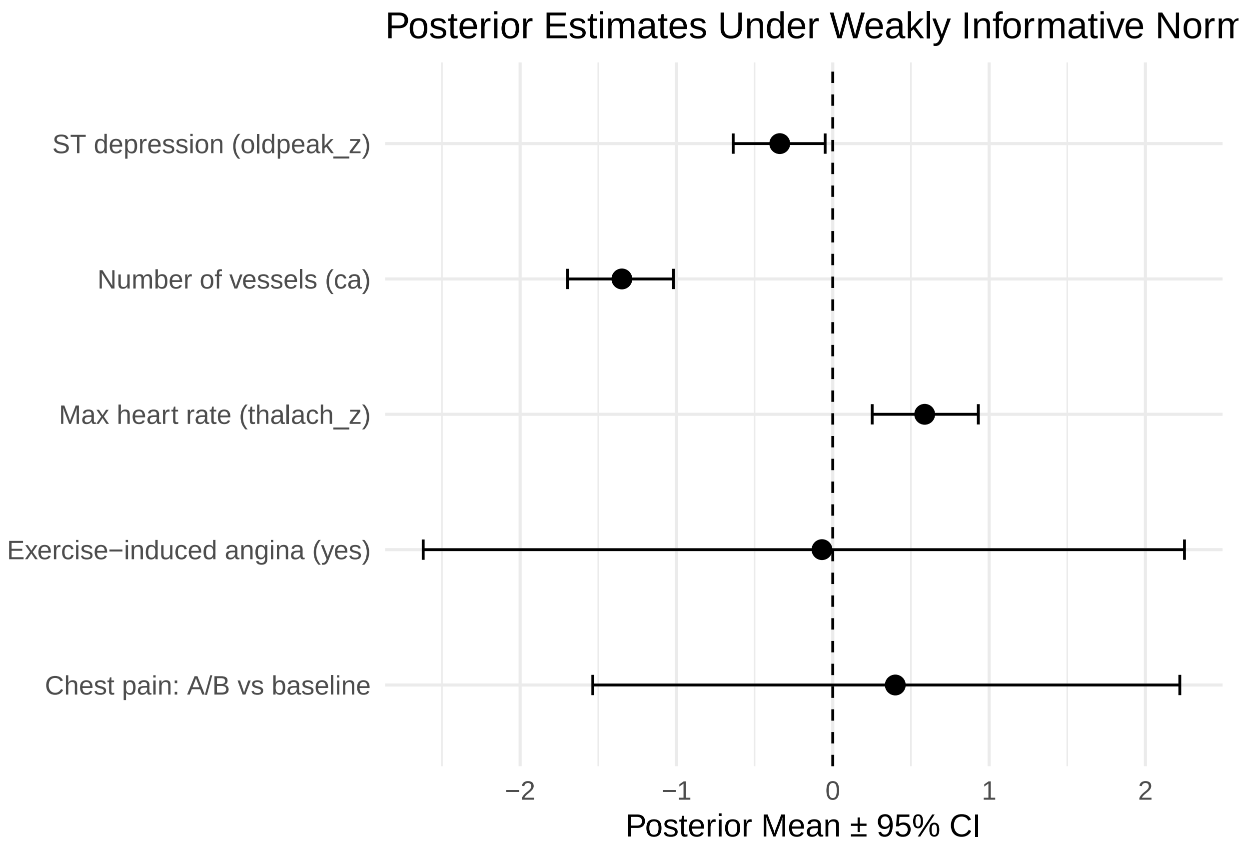
```
## Mean deviance:  368.8
## penalty 14.94
## Penalized deviance: 383.8

##
## Normal Prior DIC:

## Mean deviance:  368.2
## penalty 14.51
## Penalized deviance: 382.7

##
## Hierarchical Prior DIC:

## Mean deviance:  368.9
## penalty 14.3
## Penalized deviance: 383.2

##
## Ordinal Age Trend Prior DIC:

## Mean deviance:  368.6
## penalty 14.33
## Penalized deviance: 382.9
```

The comparison table reveals important differences in coefficient estimates and uncertainty across the three priors. Under the flat prior, many coefficients (e.g., `beta_1[1]` and `beta_1[2]`) show extreme and implausibly large values with very wide credible intervals, indicating overfitting and unstable estimation. In contrast, the weakly informative normal prior substantially stabilizes the posterior distributions: for example, `beta_2` (effect of `thalach_z`) has a consistent positive effect across all priors with tighter intervals (e.g., Normal: 0.612 [0.281, 0.953]), while `beta_3` (`oldpeak_z`) shows a negative effect (Normal: –0.347 [–0.649, –0.061]), both of which align with clinical expectations.

The hierarchical prior adds an additional layer of regularization by learning hyperparameters for each group of coefficients. This further narrows the credible intervals slightly (e.g., `beta_4` for `ca`: –1.277 [–1.610, –0.971]) and provides smoother estimates across categorical levels like age groups (`beta_1[*]`). Overall, both the Normal and Hierarchical priors improve stability, but the Hierarchical model best balances shrinkage and interpretability. It is therefore the most appropriate prior structure for modeling heart disease risk in this dataset.



Posterior Estimates Under Different Priors
10
0
−10
−20
Posterior Mean ± 95% CI
beta_2
beta_3
beta_4
beta_6[2]
beta_7[2]
Coefficient
Prior
Flat
Hyper
Normal
Ordinal


Posterior Estimates Under Weakly Informative Norm
ST depression (oldpeak_z)
Number of vessels (ca)
Max heart rate (thalach_z)
Exercise−induced angina (yes)
Chest pain: A/B vs baseline
−2
−1
0
1
2
Posterior Mean ± 95% CI