CS685 - Homework 1, *Jana Košecká*, due September 15
Be as concise as possible.

1. (4) Read the CARLA paper (link provided on the webpage). What are the three different architectures/approaches outlined and compared in the paper ? What is the best performing approach across all tasks when driving in new town and when driving in new town and new weather condition ?

2. (5) Consider rigid body transformations in the plane. Draw a right triangle defined by three points $A = (0, 0), B = (2, 0), C = (0, 3)$.

   - Consider a rotation matrix
   $$T_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

   a. What is the determinant of the matrix ?

   - Consider transformation matrix
   $$T_2 = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$$

   a. Is the matrix orthonormal ? What is the determinant of the matrix ?

   c. Is T_2 rigid body transformation ? What is the difference between T_1 and T_2, how are the results different?

3. (5) Point $P_A = [2, 0, 3]^T$ expressed in coordinate frame A. Frame A is rotated about axis Z_A by θ degrees and then rotated around axis X_A by ϕ degrees. Give a rotation matrix that accomplishes these two rotations. What is the coordinate of this point in the new coordinate frame A' obtained by rotation of frame A ? Write a small program in Python or Matlab which computes the final rotation matrix for values $\theta = 30°$ and $\phi = 45°$ and the coordinates of the point $P_{A'}$ in the new rotated frame.

4. (3) *Properties of Rotation Matrices*: Let $R \in SO(3)$ be a rotation matrix generated by rotating about a unit vector ω by θ radians that satisfies $R = exp(\hat{\omega}\theta)$.

   Consider following rotation matrix:
   $$R = \begin{bmatrix} 0.1729 & -0.1468 & 0.9739 \\ 0.9739 & 0.1729 & -0.1468 \\ -0.1468 & 0.9739 & 0.1729 \end{bmatrix}$$

   a) Use the formulas given in class to compute the rotation axis and the associated angle.
   b) Use Matlab/Python function for computing the eigenvalues and eigenvectors of the above rotation matrix R. What is the eigenvector associated with unit eigenvalue ? Give its form and explain its meaning ?

5. (3) Consider and example of a single two link manipulator in the figure. The forward kinematics equations of the leg is:
   $$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \quad (1)$$
   $$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \quad (2)$$

   Compute the determinant of the Jacobian and determine if it is singular.

6. (3) Write a Python program to simulate the motion a differential drive robot using first order integration model.

- The function should take as an input vector specifying the initial pose $[x_0, y_0, z_0]$ and velocities v, ω and time t denotes number of time steps and δt the length of the time step. You should return resulting path as three vectors each $1 \times n$ long where n is the number of time steps. .

  `[x,y,theta] = diffDrive([x0, y0, theta0], v, omega, t, delta)`

- For the following example assume that at time t = 0 the configuration (pose) of the robot is $\xi = [x, y, \theta] = [100, 50, 45°]$. Robot starts moving with some angular and linear velocity $\omega = 2°/s$ and $v = 1m/s$. Discuss how is the path affected by the choice of δt ? Hand in the plot of the code and the plot of the path.