Graphs Advanced Problem Solving

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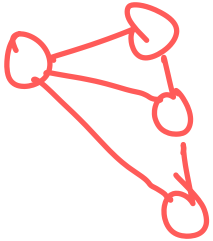
Problem 1: Second Best MST

A Minimum Spanning Tree T is a tree for the given graph G which spans over all vertices of the given graph and has the minimum weight sum of all the edges, from all the possible spanning trees. A second best MST T' is a spanning tree, that has the second minimum weight sum of all the edges, from all the possible spanning trees of the graph G.

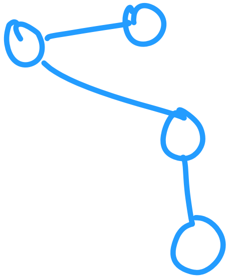
Problem Link - https://cp-algorithms.com/graph/second_best_mst.html

Observation 1

2nd best MST



⇒



2nd best MST

&

MST

MST

V-1

2nd best MST

V-2

removing 1 edge

adding 1 edge

for each edge ed: $\longrightarrow$ V-1

create MST without ed] $\longrightarrow$ E

store this MST $\rightarrow$ V

$\boxed{O(VE)}$

$$\boxed{\begin{matrix} 1 \le V \le 10^5 \\ 1 \le E \le 2 \cdot 10^5 \end{matrix}}$$

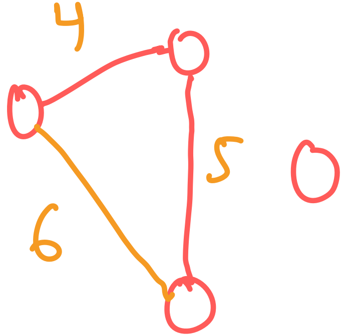
TLE

e_1 e_2 e_3 e_4

MST $\rightarrow$ e_1 e_3

V-1 edges in MST

good edges



Kruskal's Algorithm

O(E)

for each edge $e \notin E^{mst}$:

O(log V)

{ add the edge e

remove the largest edge in the cycle

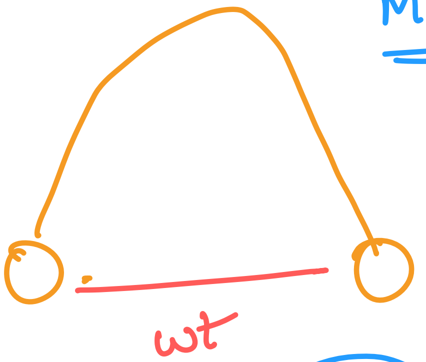
largest edge in cycle if

add the edge e

store the mst → the [wt_mst + wte − emax_path]

$e = u \xrightarrow{wt} v$

find the largest weight of any edge from u to v



MST

$\boxed{\text{MST}}$ tree

Tree Q

u v

LCA

& Binary lifting

① MST $O(E \log E)$

② for each e, find the total weight of best ST with edge e $O(E \log V)$

$O(\log V)$

③ construct an MST with edge e $O(E \log E)$

Problem 2: Fox And Names

Fox Ciel is going to publish a paper on FOCS (Foxes Operated Computer Systems, pronounce: "Fox"). She heard a rumor: the authors list on the paper is always sorted in the lexicographical order.

After checking some examples, she found out that sometimes it wasn't true. On some papers authors' names weren't sorted in lexicographical order in normal sense. But it was always true that after some modification of the order of letters in alphabet, the order of authors becomes lexicographical!

She wants to know, if there exists an order of letters in Latin alphabet such that the names on the paper she is submitting are following in the lexicographical order. If so, you should find out any such order.

Lexicographical order is defined in following way. When we compare s and t, first we find the leftmost position with differing characters: $s_i \neq t_i$. If there is no such position (i. e. s is a prefix of t or vice versa) the shortest string is less. Otherwise, we compare characters s_i and t_i according to their order in alphabet.

Problem Link - https://codeforces.com/problemset/problem/510/C january

february

march

april

may

"Impossible"

$m \rightleftarrows a$

$j < f$

$f < m$

$m < a$

$a < m$

topological ordering

nodes

$j \rightarrow f$

nodes, edges } directed graph

topo sort ↓

answer ← topo ordering

Problem 3: Ralph and Mushrooms

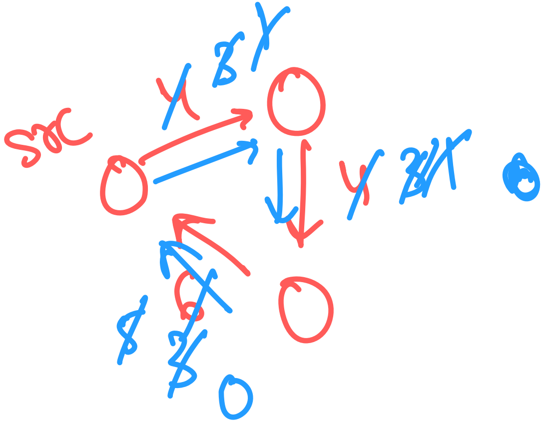
Ralph is going to collect mushrooms in the Mushroom Forest.

There are m directed paths connecting n trees in the Mushroom Forest. On each path grow some mushrooms. When Ralph passes a path, he collects all the mushrooms on the path. The Mushroom Forest has a magical fertile ground where mushrooms grow at a fantastic speed. New mushrooms regrow as soon as Ralph finishes mushroom collection on a path. More specifically, after Ralph passes a path the i-th time, there regrow i mushrooms less than there was before this pass. That is, if there is initially x mushrooms on a path, then Ralph will collect x mushrooms for the first time, x - 1 mushrooms the second time, x - 1 - 2 mushrooms the third time, and so on. However, the number of mushrooms can never be less than 0.

For example, let there be 9 mushrooms on a path initially. The number of mushrooms that can be collected from the path is 9, 8, 6 and 3 when Ralph passes by from first to fourth time. From the fifth time and later Ralph can't collect any mushrooms from the path (but still can pass it).

Ralph decided to start from the tree s. How many mushrooms can he collect using only described paths?

Problem Link - https://codeforces.com/problemset/problem/894/E



edge

directed edges

cycles

$4+4+6+3+3+5+1+1+3$

$f(x) \rightarrow$ the no. of mushrooms you can obtain, $\frac{i*(i+1)}{2} \geq x$

$x, x-1, x-3, x-6, x-10, ---, 0$

mathematic

$$\sum_{i=0}^{k} x - \frac{i*(i+1)}{2}$$

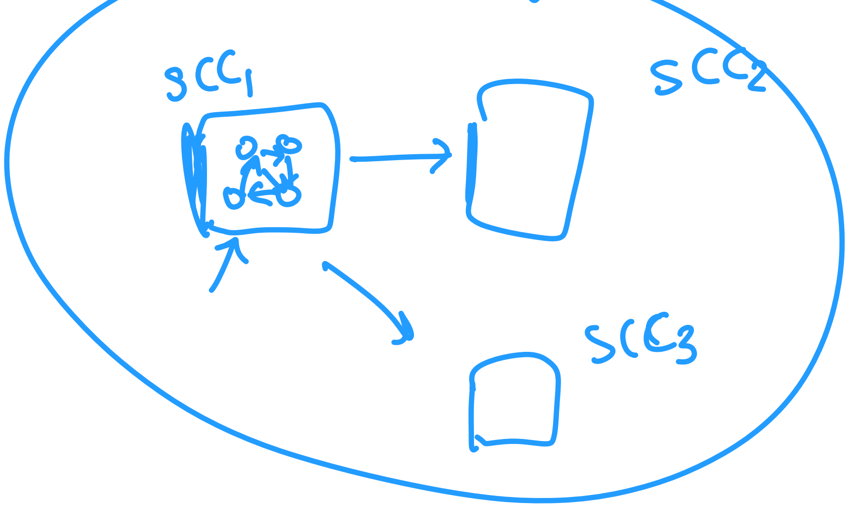
$$K * x - \frac{(K-1) * K * (K+1)}{6}$$

directed graph

cycles

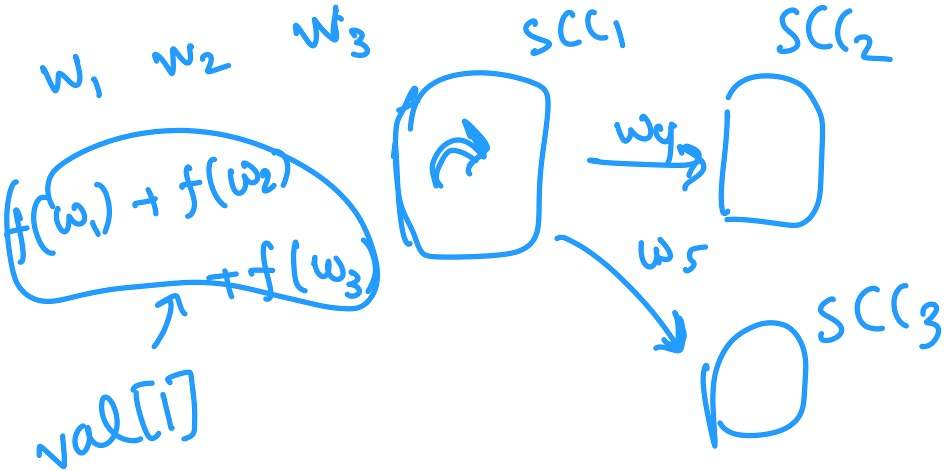
Strongly Connected Components

scc(source)



dp on SCCs

$dp[u] \rightarrow$ the no. of mushrooms I can obtain if I start from a node in the SCC u.



$$dp[1] = val[1] + \max(w_4 + dp[2], w_5 + dp[3])$$

$$\sum_{i=0}^{k} \frac{i*(i+1)}{2} \Rightarrow \frac{1}{2}\left(\sum_{i=1}^{k-1} i^2 + \sum_{i=1}^{k-1} i\right)$$

$$\sum_{i=1}^{n} i^2 \Rightarrow \frac{n*(n+1)*(2n+1)}{6} = \frac{1}{2}\left(\frac{(k-1)*k*(2k-1)}{6} + \frac{3*k*(k-1)}{3*2}\right)$$

$$\sum_{i=1}^{n} i \Rightarrow \frac{n*(n+1)}{2}$$

$$\frac{(k-1)*k*\cancel{(2k+2)}^{\,k+1}}{\cancel{2}\times 6}$$

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