High Frequency Data-Driven Dynamic Portfolio Optimization for Cryptocurrencies

Abstract-Recently there has been a growing interest in constructing portfolios with different types of assets as a risk diversification strategy. As cryptocurrency prices increase over the years, there is an uprise in investing in cryptocurrencies, along with diversifying portfolios by adding multiple cryptocurrencies to the existing portfolios. Even though investing in cryptocurrency leads to high returns, it also leads to high risk due to the high uncertainty of cryptocurrency price changes. Thus, more robust risk measures have been introduced to capture market risk and avoid investment loss, along with different types of portfolios to mitigate risks. Many portfolio techniques assume asset returns are normally distributed with constant variance. However, these assumptions violate in many cases. This study investigates the recently proposed data-driven exponentially weighted moving average (DDEWMA) volatility model to estimate the variancecovariance matrix of asset returns in Markowitz portfolio optimization considering hourly adjusted closing prices of cryptocurrencies. The experimental results show that for high-frequency data, the DDEWMA approach outperforms the existing portfolio optimization model that uses the empirical variance-covariance matrix. Improvements have been identified in terms of the Sharpe ratio as well as risks (volatility, mean absolute deviation (MAD), Value-at-Risk (VaR), and Expected shortfall (ES)).

Index Terms—Cryptocurrencies, Portfolio Optimization, Sharpe ratio, Volatility

I. Introduction

Numerous data sources follow a sequential pattern and necessitate distinctive handling while constructing predictive models. The task of prediction becomes challenging, especially when dealing with financial time series data like trading volumes, stock and bond prices, and exchange rates. Despite the complexities, certain indices can still be reasonably forecasted with a degree of accuracy [1].

There is a growing interest in investing in different types of portfolios as a risk diversification strategy. Following the financial crisis in 2008, the governing authorities have imposed on financial institutions to use more robust risk measures incorporating market risk to reduce the potential loss of investments [2] in the future. Employing risk measures such as volatility, value-at-risk (VaR), and Expected Shortfall (ES) in the decision-making process helps to avoid potential risks of financial failures, and various studies have been carried out on how these different risk measures that could lead to efficient capital allocation and improved risk management.

In 2008, the digital currency was introduced with the introduction of Bitcoin [3]. A cryptocurrency is a digital form of money that derives its value purely from the trust placed in them. Digital currencies are changing the traditional financial system too rapidly due to a lack of financial intermediaries, no transactional delays, no paperwork, and so on. As Bitcoin

prices increase over the years, there is an uprise in investing in cryptocurrencies along with diversifying portfolios by adding multiple cryptocurrencies to the existing portfolios [4], [5].

Markowitz introduced modern portfolio theory in 1952 with a framework to calculate the optimal weights of assets in an investment portfolio [6]. The initial model finds the optimal weights (fund allocation of initial investment) for a given level of risk, and different versions of the model have been studied since then with complicated modifications. Recently [7] studied the machine learning approach with the Markowitz model, and the idea of graphical Lasso was investigated in minimizing portfolio risk by [8]. Moreover, researchers have also investigated different metrics in portfolio optimization, such as Naïve portfolio, Mean-variance portfolio, Sharpe ratio (SR), and it have been shown that a cryptocurrency diversified portfolio provides better returns compared to a portfolio without cryptocurrencies [9]. However, many of the approaches/studies assume normality and are required to estimate/calculate the inverse covariance matrix of asset price return. [10] showed that the normality assumption violates for the selected stocks and the existing portfolio risk measures are affected by the larger skewness and kurtosis of the portfolio returns. Thus, [10] proposed new risk measures for portfolio optimization that incorporate high skewness and kurtosis, and recently [11] has investigated the performance of new risk measures for high-frequency cryptocurrency data.

[12] presents a portfolio optimization approach based on a data-driven exponentially weighted moving average (DDEWMA) model for volatility. This study extends the work of [12] by proposing a dynamic portfolio optimization with DD volatility for cryptocurrencies. Regular stock markets generally operate five days per week with eight-hour shifts, and at the end of the day, a daily adjusted price is reported for each stock. Therefore most studies consider daily data in the analysis. However, cryptocurrency transactions happen twenty-four hours a day and seven days per week. Given its nature, it is important to investigate how a portfolio can be affected by this high-moving market commodity. Thus, this study considers high-frequency data (hourly adjusted closing prices) for cryptocurrencies in the analysis.

The remainder of the paper consists of two sections and the conclusions. Section 2 discusses data-driven variance-covariance matrices and a performance evaluation metric for the new approach. Section 3 provides a detailed summary of the numerical experiment(s) for a portfolio considering six different cryptocurrencies.

II. METHODOLOGY

A. Data-Driven Exponentially Weighted Moving Average Model for Volatility

It can be observed that the variance of asset returns changes over time [13]. Thus, models which can incorporate time-varying volatility may provide better forecasts of risks. In 2020, for time-varying volatility of centered log returns, Thavaneswaran et al. [14] introduced a novel Data-Driven Exponentially Weighted Moving Average model for volatility. The model can be extended to portfolio returns (R_t) by simply replacing asset log returns with portfolio returns. The model can be written as:

$$\sigma_{t+1} = \frac{\alpha}{\rho} |R_t - \bar{R}| + (1 - \alpha)\sigma_t \quad 0 < \alpha < 1$$

where $sigma_t$ is volatility at time t and α is the smoothing constant obtained by minimizing the one step ahead forecast error sum of squares. ρ is the sign-correlation of the return,

$$\rho = Corr(R - \bar{R}, sign(R - \bar{R})).$$

If the asset returns are believed to be Student-t distributed with ν degrees of freedom (DF), ν can be estimated by solving the equation 1 (see [14] for more details).

$$2\sqrt{\nu - 2} = \rho(\nu - 1)Beta\left(\frac{\nu}{2}, \frac{1}{2}\right) \tag{1}$$

Let P_t be the adjusted closing price at time t. Then simple return at time t (R_t) can be calculated by $R_t = (P_t - P_{t-1})/P_{t-1}$. The weighted average of asset returns ($R_{i,t}$) is known as Portfolio return ($R_{p,t}$). When new observation comes, the portfolio needs to be rebalanced in high-frequency trading. This need for frequent updates on smoothed values can be handled efficiently by DDEWMA.

The training period window for the study is $[1, T_1]$. If the last data point is collected at time T_2 , the test period is denoted by $[T_1 + 1, T_2]$. Once the optimal smoothing constant is determined using training data, it will be used to forecast the volatility in the testing period. The DDEWMA algorithm for computing a one-step-ahead forecast of volatility and residuals proposed in [12], [15] is summarized in algorithm 1.

B. Optimal Portfolio Weights

As assets with high risks yield high returns in general, maximizing return leads to high risk and lower risk resulting in low return in the simplest setup in portfolio optimization. Thus, it is important to have a measure that captures both return and risk, and the Sharpe ratio (SR) is the most commonly used and popular measure that captures both risks and returns. The Portfolio Sharpe ratio (equation 2) is the risk-adjusted portfolio return, and it enabled us to compare the performance of different portfolios considering both risk and return.

$$SR = \frac{E[R_{p,t}] - \frac{r_f}{N}}{\sqrt{Var(R_{p,t})}} = \frac{\mu_p - \frac{r_f}{N}}{\sigma_p}$$
 (2)

where r_f is the annual risk-free rate and N is the number of tradings in one year. Then, annualized Sharpe ratio can be calculated by $\sqrt{N} * SR$.

Algorithm 1 DDEWMA volatility forecasts of returns

Require: Data: Adjusted closing asset prices $P_t, t = 0, \ldots, k, \ldots, T_1, \ldots T_2$.

1: $R_{i,t} \leftarrow \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}, t = 1, \ldots, T_1, \ldots T_2$ 2: $\hat{\rho} \leftarrow \operatorname{Corr}(R - \bar{R}, \operatorname{sign}(R - \bar{R}))$ 3: $Z_t \leftarrow |R_t - \bar{R}|/\hat{\rho}$ 4: $S_0 \leftarrow \frac{\sum_{t=1}^k Z_t}{k}$ 5: $\alpha \leftarrow (0,1)$ 6: $S_t \leftarrow \alpha Z_t + (1 - \alpha)S_{t-1}, t = 1, \ldots, T_1$ 7: $\alpha_{opt} \leftarrow \min_{\alpha} \sum_{t=k+1}^{T_1} (Z_t - S_{t-1})^2$ 8: $\mathbf{for} \ t \leftarrow 1, T_2 \ \mathbf{do}$ $S_t \leftarrow \alpha_{opt} Z_t + (1 - \alpha_{opt})S_{t-1}$ $res_t \leftarrow \frac{R_t - \bar{R}}{S_{t-1}}$ 9: $\mathbf{return} \ S_t, t = T_1, \ldots T_2$ 10: $\mathbf{return} \ res_t, t = 1, 2, \ldots T_2$

Finding the tangency portfolio, known as the optimal risky portfolio, maximizes the Sharpe ratio given in equation 2. The constraint for this optimization problem is that the sum of portfolio weights (ω) (fund allocation from initial investment) is equal to one. Thus, the entire initial investment will be invested in the given portfolio, and $\omega_i * 100\%$ gives the percentage of funds allocated from the initial investment in asset i. Portfolio mean return is calculated as $\mu_p = \omega^T \mu$ and portfolio standard deviation is computed by $\sigma_p = \sqrt{\omega^T \Sigma \omega}$. Here Σ is the covariance matrix of asset returns. The optimal weights for the tangency portfolio are given by

$$w_{Optimal} = \frac{\Sigma^{-1}(\mu - r_f)}{\mathbf{1}^T \Sigma^{-1}(\mu - r_f)}.$$

C. Data-Driven and Empirical Covariance Matrices

The correlation matrix is derived from the covariance matrix of the standardized residuals. The variance-covariance matrix of returns contains squared values of data-driven EWMA volatilities on its diagonal, while its off-diagonal elements represent the correlations between DDEWMA residuals, multiplied by the corresponding DDEWMA volatilities. Thus, an element in the data-driven covariance matrix can be written as

$$\Sigma^{dd}[t-1]_{i,j} \leftarrow cor^{dd}[t-1]_{i,j} \hat{\sigma}_{i,t} \hat{\sigma}_{j,t}$$

. Following the equation II-B, obtaining optimal weights for the portfolio using a data-driven covariance matrix is shown in the algorithm 2. Once the optimal weights are determined, realized return for the portfolio using the testing sample is calculated by

$$R_{p,t} = \sum w_{i,t-1} R_{i,t}.$$

In this study, in addition to the Sharpe ratio, we also forecast risks such as mean absolute deviation (MAD), Value-at-Risk (VaR), and Expected Shortfall (ES). The computational formulas for Sharpe ratio, MAD, VaR, and ES using one-step-ahead volatility forecast $(\hat{\sigma}_{T+1})$ are given in equations 3, 4, 5, and 6, respectively.

Algorithm 2 Portfolio Weights from Data-Driven EWMA Covariance Matrix

Require: Data: Adjusted Closing Price of assets $P_{i,t}$, t = $0,\ldots,k,\ldots T_1,\ldots T_2, i=1,\ldots,n$ $1: R_{i,t} \leftarrow \frac{P_{i,t}-P_{i,t-1}}{P_{i,t-1}}, t=1,\ldots,T_2$ $2: \hat{\sigma}_{i,t} \text{ is the } S_{t-1} \text{ for } R_i \text{ in Algorithm 1, } t=T_1+1,\ldots T_2$ $3: \sum^{dd}[t-1]_{i,i} \leftarrow \hat{\sigma_{i,t}}^2$ $4: \sum^{dd}[t-1]_{i,j} \leftarrow cor^{dd}[t-1]_{i,j}\hat{\sigma_{i,t}}\hat{\sigma_{j,t}}$ 5: Check whether the matrix is positive definite.

5: Check whether the matrix is positive definite

 $\begin{array}{l} \text{6: } Z \leftarrow \Sigma^{-1}(\mu - R_f) \\ \text{7: } w_{i,t-1}^{dd} \leftarrow \sum\limits_{j=1}^{Z_i} \\ \text{8: } \mathbf{return} \quad w_{i,t-1}^{dd} \end{array}$

$$SR_{T+1} = \frac{E[R_{T+1}|R_T, R_{T-1}, \dots R_1] - \frac{R_f}{N}}{\sigma(R_{T+1}|R_T, R_{T-1}, \dots R_1)},$$

$$= \frac{\hat{\mu}_p - \frac{R_f}{N}}{\hat{\sigma}_{T+1}},$$
(3)

$$MAD_{T+1} = \hat{\sigma}_{T+1}\hat{\rho},\tag{4}$$

$$VaR_{T+1} = (-1000)(\hat{\sigma}_{T+1}t_{\nu}^{-1}(p)\sqrt{\frac{\nu-2}{\nu}}), \tag{5}$$

$$ES_{T+1} = 1000 * \hat{\sigma}_{T+1} * \left(\frac{f_{\nu}(t_{\nu}^{-1}(p))}{p} \right) \left(\frac{\nu + (t_{\nu}^{-1}(p))^{2}}{\nu - 1} \right) \sqrt{\frac{\nu - 2}{\nu}},$$
 (6)

where $\hat{\mu}_p$ is the expected portfolio return and f_{ν} is the PDF of a student-t distribution with ν DF.

D. Utility of Portfolio Optimization

The study by Zimo Zhu et al. [12] proposed a new metric to quantify the performance increase of the data-driven approach against the empirical approach in terms of the Sharpe ratio. The newly introduced measure quantifies the percentage increase in utility achieved by transitioning from the empirical variance-covariance matrix to the data-driven exponentially weighted moving average variance-covariance matrix.

$$\%\Delta U^* = \frac{SR_{DD}^2 - SR_{EMP}^2}{SR_{EMP}^2}.$$
 (7)

III. EXPERIMENTAL RESULTS

The study considers hourly adjusted closing price data from six cryptocurrencies (Bitcoin (BTC), Ethereum (ETH), Binance Coin (BNC), Ripple (XRP), Dogecoin (DOGE), and Cardano (ADA)). Hourly adjusted closing prices (USD) are downloaded from Yahoo! Finance (http://www.finance.yahoo. com) from November 2020 to May 2021. The selection of cryptocurrencies is based on their market cap (current Price x circulating supply) according to Coinmarketcap (http://www. coinmarketcap.com). Note that in Coinmarketcap, Tether has the third-highest market cap, and USD Coin has the fifthhighest market cap. However, they have not been included in this study as Tether and USD Coin are stablecoins, and they would be less attractive to be included in a portfolio.

Figure 1 and Table I provide hourly adjusted closing prices (in USD) and summary statistics for all the assets during the study period. Note that during the three months, hourly adjusted closing prices for cryptocurrencies gradually increase. However, hourly adjusted prices also show sudden jumps and fall, indicating high uncertainty of cryptocurrency prices. DOGE has the highest mean return, and ETH has the lowest mean return among the selected cryptocurrencies. However, DOGE has the highest standard deviation for mean returns. The Skewness of mean returns indicates that data are not significantly skewed except for DOGE. Nonetheless, for all the cryptocurrencies, kurtosis is significantly high, indicating heavy tail distributions for mean returns. The table also provides the auto-correlation function values of log returns, and since the values are not equal to zero, the series is significantly autocorrelated, indicating volatility clustering. Based on the estimated sign correlation value $(\hat{\rho})$, the corresponding DF (ν) for student-t distribution is determined. For all the assets, simple returns indicate heavy tails with lower DF.

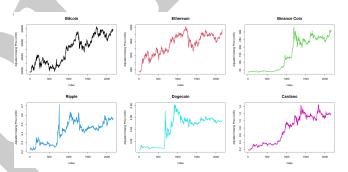


Fig. 1. Hourly Adjusted Closing Prices (USD) from January 2021 to March 2021)

TABLE I SUMMARY STATISTICS OF HOURLY RETURNS

Asset	Mean	SD	Skewness	Kurtosis	ACF	$\hat{ ho}$	ν
BTC	0.0004	0.0108	0.3685	10.4182	0.0361	0.6913	3.66
ETH	0.0005	0.0126	-0.1244	7.3092	0.0288	0.6926	3.69
BNC	0.0011	0.0172	0.6385	12.4631	-0.0217	0.6467	3.09
XRP	0.0006	0.0192	0.1596	24.8716	-0.0294	0.5899	2.70
DOGE	0.0017	0.0347	3.5342	49.1230	0.0077	0.4759	2.33
ADA	0.0010	0.0179	0.4961	8.4593	-0.0650	0.6957	3.75

In contrast to regular stock markets, cryptocurrency markets are not physically operated markets. Thus, transactions occur every day of the week and even every second. Here in this study, one month of adjusted closing prices of assets (24 hours per day * 30 days per month = 720 data points per month) are used as the training data, and portfolio forecasts are obtained using the data-driven and empirical approaches for one day (24 hours/24 data points). More importantly, we consider a dynamic setup for obtaining the portfolio forecasts, and it enables us to avoid the seasonality and cyclic effect of cryptocurrency price data. In the upcoming subsections, SR rolling forecasts and rolling risks forecasts of portfolios are summarized using 2021 January, February, and March monthly data. Once the models are trained, one day of hourly forecasts of SR, MAD, VaR, and ES are obtained with both empirical and data-driven variance-covariance matrices. The risk-free rate for this study is the average treasury bill rate (T-bill rate), and for each month, we consider the mean treasury bill rate of the month as the risk-free rate. The treasury bill rates are obtained from Bloomberg (https://www.bloomberg.com/canada).

A. Forecasts of SR and risks using January 2021

In this subsection summary of the forecasts using January 2021 data are provided for both data-driven and empirical approaches. Table II summarizes means and standard deviations of SR and risks. It is interesting to see that the mean SR using the data-driven variance-covariance matrix is higher with a lower standard deviation compared to the empirical approach. Moreover, All the risks are higher using the empirical covariance matrix, and the forecasts have higher standard derivations compared to the data-driven approach.

TABLE II
SUMMARY OF FORECASTS USING JANUARY 2021 DATA

		$\hat{\sigma}$	SR	MAD	VaR	ES
EMP	mean	0.0120	0.0960	0.0088	30.4175	42.3860
	SD	0.0020	0.0135	0.0014	5.3154	7.3874
DD	mean	0.0068	0.1430	0.0051	16.3438	22.0146
	SD	0.0005	0.0112	0.0004	1.2623	1.6178

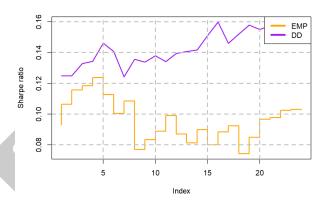


Fig. 2. SR Rolling Forecasts of Portfolios (January 2021)

Figures 2 and 3 visualize the rolling forecasts of SR and risk for January 2021 data. SR is higher with the data-driven variance-covariance matrix, and risks are lower for each hour. This indicates that using the empirical variance-covariance matrix leads to portfolios with high risks and lower SR compared to the data-driven approach.

Figure 4 further summarizes and visualizes the risks of the portfolio using comparison boxplots for the data-driven and empirical approaches for January 2021 data. Observe that

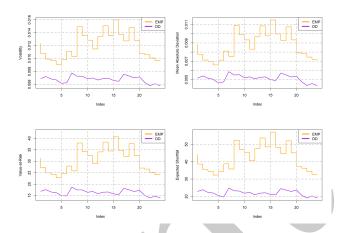


Fig. 3. Rolling Risk Forecasts of Portfolios (January 2021)

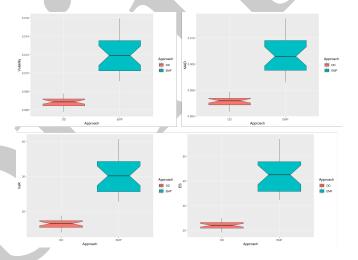


Fig. 4. Comparison Boxplots of Risks (January 2021)

risks using empirical covariance matrix are always high with high variability. Even though it is visible from the figures that portfolios constructed with data-driven covariance matrix outperform, it is also important to quantify the improvement for further clarification. The estimated percentage improvement in utility for switching from the empirical covariance matrix to the DDEWMA variance-covariance matrix is 138.80% for January 2021 data. It is a significant improvement in SR. Figure 5 presents the histogram of the percentage change of the utility by the Sharpe ratio forecasts, and most of the percentage change of the utility is greater than zero.

B. Forecasts of SR and risks Using February 2021 Data

The SR and risk forecasts using data-driven and empirical variance-covariance matrices with February 2021 data are summarized in this subsection. Means and standard deviations of the forecasts are provided in Table III. Similar to January data, the SR forecast obtained with February data using the data-driven variance-covariance matrix is larger than that with the empirical variance-covariance matrix. Moreover, risks are higher with the empirical variance-covariance matrix indi-

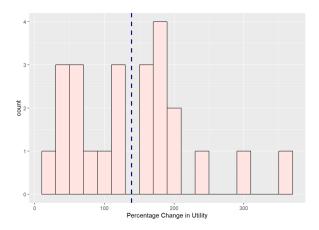


Fig. 5. Histogram of Utility Change based on the mean of Sharpe ratio rolling forecasts (January 2021)

cating the new data-driven approach is superior considering February 2021 data as well.

TABLE III
SUMMARY OF FORECASTS USING FEBRUARY 2021 DATA

		$\hat{\sigma}$	SR	MAD	VaR	ES
EMP	mean	0.0116	0.1314	0.0086	28.6721	39.4283
	SD	0.0021	0.0226	0.0016	5.4796	7.2324
DD	mean	0.0087	0.1725	0.0063	21.3815	30.7767
	SD	0.0020	0.0325	0.0015	5.2433	6.9800

Forecasts of SR and risks obtained for the testing period (one day/twenty-four hours) using February data are visualized in Figures 6 and 7, respectively. The SR rolling forecasts with data-driven covariance matrix is higher than the SR rolling forecasts with empirical covariance matrix, except for the eleventh hour. However, SR forecasts for the eleventh hour are closed using both approaches, indicating similar performances. Risk forecasts using data-driven covariance matrices are consistently lower compared to the empirical approach. It is also important to point out that the gap between the forecasts using data-driven and empirical covariance matrices is smaller compared to forecast gaps considering January data. This can be further seen in the comparison boxplots given in Figure 8. Compared to January data (section III-A) risks forecasts, whiskers of the boxplots of risks are overlapping. Nonetheless, risks obtained with the data-driven covariance matrix are lower in general compared to the risks obtained with the empirical covariance matrix.

Since the performances of the two approaches are close at certain testing hours, it is important to see the improvements in SR using the data-driven variance-covariance matrix over the empirical variance-covariance matrix. The estimated average change of the percentage increase is 76.86%. Figure 9 presents the histogram of the percentage change of the utility by the Sharpe ratio forecasts using February data, and it can be seen that most of the percentage change of the utility is greater than zero.

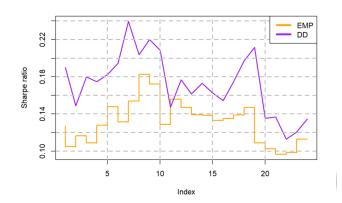


Fig. 6. SR Rolling Forecasts of Portfolios (February 2021)

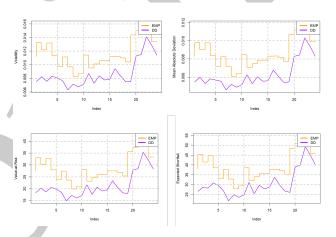


Fig. 7. Rolling Risk Forecasts of Portfolios (February 2021)

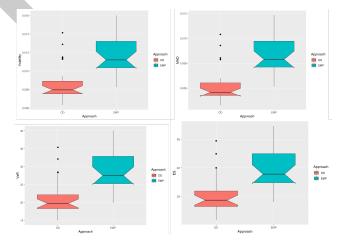


Fig. 8. Comparison Boxplots of Risks (February 2021)

C. Forecasts of SR and risks for March 2021

The SR and risks forecasts using March data are also obtained, and this subsection summarizes the results. Means and standard deviations of the SR and risks forecasts are

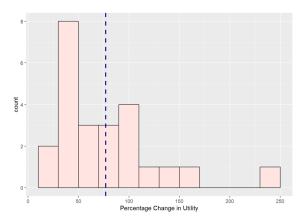


Fig. 9. Histogram of Utility Change based on the mean of Sharpe ratio rolling forecasts (February 2021)

provided in Table IV. The mean SR forecast using the data-driven variance-covariance matrix is larger than the mean SR forecast obtained with the empirical variance-covariance matrix. However, the standard deviation of the SR forecasts is larger with the data-driven covariance matrix. Mean risks forecasts are still lower with the data-driven covariance matrix, and standard deviations of the forecasts are also lower compared to the empirical approach. Thus, summary statistics of the forecasts indicate that the data-driven approach outperforms the empirical approach using March 2021 data in terms of SR and risks.

TABLE IV
SUMMARY OF FORECASTS USING MARCH 2021 DATA

		$\hat{\sigma}$	SR	MAD	VaR	ES
EMP	mean	0.0040	0.0845	0.0031	9.7942	12.5759
	SD	0.0005	0.0060	0.0004	1.1315	1.4449
DD	mean	0.0036	0.0922	0.0027	8.8411	11.6315
	SD	0.0003	0.0142	0.0002	0.7745	1.0986

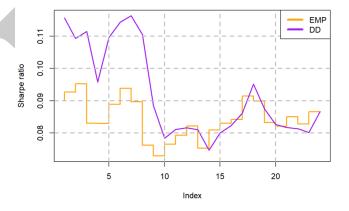


Fig. 10. SR Rolling Forecasts of Portfolios (March 2021)

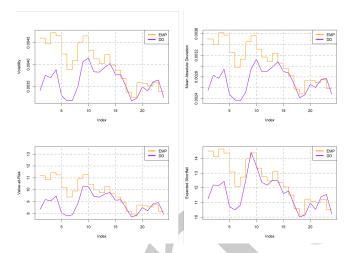


Fig. 11. Rolling Risk Forecasts of Portfolios (March 2021)

From Figure 1, we can observe that for some cryptocurrencies adjusted closing prices are decreasing towards the end of the study period. This will bring a negative effect on portfolio returns, ultimately leading to a lower Sharpe ratio when the risk is fixed. Also, we see sudden price increases and decreases during March 2021 for some cryptocurrencies. Therefore, during this volatile period (March), we expect somewhat different results in terms of SR and risk forecasts compared to the forecasts obtained with January and February data. The SR and risk forecasts for the testing period (one day/twentyfour hours) using March 2021 data are given in Figure 10 and Figure 11, respectively. During the testing period, SR forecasts using the data-driven variance-covariance matrix are higher or closer to the SR forecasts obtained with the empirical variancecovariance matrix. Similar observations can also be made for the risk forecasts, and in most instances, risks using the datadriven covariance matrix are lower. This closed performance of the empirical approach and data-driven approach can be seen with comparison boxplots given in Figure 12 for risks. In contrast to January and February comparison boxplots for risks, here, boxplots for the two approaches overlap. However, boxplots indicate that risk forecasts obtained with the datadriven variance-covariance matrix have lower median and lower variability compared to the risk forecasts obtained with the empirical variance-covariance matrix. This ambiguity of the performances of the two approaches can be addressed by measuring the gain in utility when switching to the data-driven approach. Using the equation 7, the percentage gain has been calculated, and for March 2021 data 19.54\% gain in utility can be obtained when switching from the empirical variancecovariance matrix to the DDEWMA variance-covariance matrix. Figure 13 presents the histogram of the percentage change of the utility by the Sharpe ratio forecasts, and most of the percentage change of the utility is greater than zero.

IV. CONCLUSION

This paper extends the portfolio optimization based empirical covariance matrix to dynamic portfolio optimization

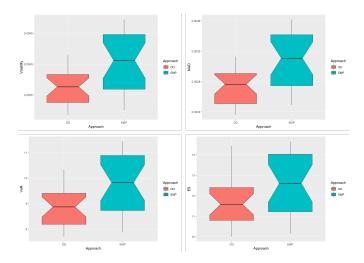


Fig. 12. Comparison Boxplots of Risks (March 2021)

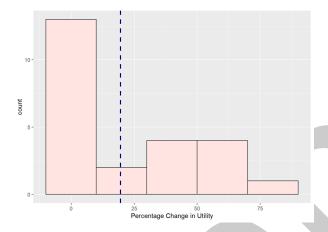


Fig. 13. Histogram of Utility Change based on the mean of Sharpe ratio rolling forecasts (March 2021)

with high frequency (hourly) cryptocurrency data. The driving idea, unlike the existing work, is studying dynamic portfolio optimization without assuming normality for portfolio returns. Our experimental results show that the new approach outperforms (i.e. larger Sharpe ratio and smaller risks) existing commonly used portfolio optimization based on the empirical variance-covariance matrix when portfolios are constructed with cryptocurrencies.

REFERENCES

- [1] James, G., Witten, D., Hastie, T., Tibshirani, R. (2013). An introduction to statistical learning (Vol. 112, p. 18). New York: springer.
- [2] "Basel III monitoring report," 2021. [Online]. Available: https://www.bis.org/bcbs/publ/d524.pdf (Accessed on 06/20/2023).
- [3] Nakamoto, S., & Bitcoin, A. (2008). A peer-to-peer electronic cash system. Bitcoin.—URL: https://bitcoin. org/bitcoin. pdf, 4(2).
- [4] Kajtazi, A., & Moro, A. (2019). The role of bitcoin in well diversified portfolios: A comparative global study. International Review of Financial Analysis, 61, 143-157.
- [5] Borri, N. (2019). Conditional tail-risk in cryptocurrency markets. Journal of Empirical Finance, 50, 1-19.
- [6] Markowitz, H. (1952). Portfolio Selection. The Journal of Finance, 7(1), 77–91. https://doi.org/10.2307/2975974

- [7] Awoye, O. A. (2016). Markowitz minimum variance portfolio optimization using new machine learning methods (Doctoral dissertation, (UCL) University College London).
- [8] Millington, T., & Niranjan, M. (2017). Robust portfolio risk minimization using the graphical lasso. In Neural Information Processing: 24th International Conference, ICONIP 2017, Guangzhou, China, November 14-18, 2017, Proceedings, Part II 24 (pp. 863-872). Springer International Publishing.
- [9] Ma, Y., Ahmad, F., Liu, M., & Wang, Z. (2020). Portfolio optimization in the era of digital financialization using cryptocurrencies. Technological forecasting and social change, 161, 120265.
- [10] Thavaneswaran, A., Liang, Y., Yu, N., Paseka, A., & Thulasiram, R. K. (2021, July). Novel Data-Driven Resilient Portfolio Risk Measures Using Sign and Volatility Correlations. In 2021 IEEE 45th Annual Computers, Software, and Applications Conference (COMPSAC) (pp. 1742-1747). IEEE.
- [11] Bowala, S., & Singh, J. (2022). Optimizing Portfolio Risk of Cryptocurrencies Using Data-Driven Risk Measures. Journal of Risk and Financial Management, 15(10), 427.
- [12] Zhu, Z., Thavaneswaran, A., Paseka, A., Frank, J., & Thulasiram, R. (2020, July). Portfolio optimization using a novel data-driven EWMA covariance model with big data. In 2020 IEEE 44th Annual Computers, Software, and Applications Conference (COMPSAC) (pp. 1308-1313). IEEE.
- [13] Hao, J., & Zhang, J. E. (2013). GARCH option pricing models, the CBOE VIX, and variance risk premium. Journal of Financial Econometrics, 11(3), 556-580.
- [14] Thavaneswaran, A., Paseka, A., & Frank, J. (2020). Generalized value at risk forecasting. Communications in Statistics-Theory and Methods, 49(20), 4988-4995.
- [15] Zhu, Z. (2020). Dynamic data science applications in finance (Master's thesis).