Comparison of Fuzzy Risk Forecast Intervals for Cryptocurrencies

***Abstract*—Data-driven volatility models and neuro-volatility models have the potential to revolutionize the area of Computational Finance. *Volatility* measures the variation of a time series data, and thus it is also a driving factor for the risk forecasting of returns from investment in cryptocurrencies. A cryptocurrency is a decentralized medium of exchange that relies on cryptographic primitives to facilitate the trustless transfer of value between different parties. Instead of being physical money, cryptocurrency payments exist purely as digital entries on an online ledger called blockchain that describe specific transactions.**

Many commonly used risk forecasting models do not take into account the uncertainty associated with the volatility of an underlying asset to obtain the risk forecasts. Some tools from the fuzzy set theory can be incorporated in the forecasting models to account for this uncertainty. Interest in the use of hybrid models for fuzzy volatility forecasts is growing. However, a major drawback is that the fuzzy coefficient hybrid models used in fuzzy volatility forecasts are not data-driven. This paper uses fuzzy set theory with data-driven volatility and data-driven neuro-volatility forecasts to study the fuzzy risk forecasts. Simple yet effective models incorporating fuzziness to obtain fuzzy risk volatility forecasts and fuzzy VaR forecasts are presented. The key underlying idea, unlike the existing risk forecasting, is the use of hybrid nonlinear adaptive fuzzy model for volatility.

***Index Terms*—Fuzzy risk forecasting, Asymteric Nonlinear Adaptive fuzzy number, Data-driven volatility**

I. INTRODUCTION

Modeling and forecasting volatility is crucial for risk management [1]. After the Basel Accord of 1996, financial risk management has become a top priority of the financial services industry. From the Basel III monitoring report by the Basel Committee on Banking Supervision in 2021 [2] financial institutions are required to use more complex credit scoring models for efficient capital allocation and enhanced risk management.

Following the 2008 global financial crisis, a new alternative to central bank controlled financial systems was proposed in the form of cryptocurrencies. Cryptocurrency is a decentralized medium of exchange that relies on cryptographic primitives to facilitate the trustless transfer of value between different parties. Instead of being physical money, cryptocurrency payments exist purely as digital entries on an online ledger called blockchain that describe specific transactions. Cryptocurrencies derive their value purely from the trust that is placed on them, and they are not backed by any commodity, such as gold or silver. Many consider Bitcoin [3] to be the first cryptocurrency, and it was created in 2008 . Some advantages that cryptocurrencies offer over the traditional payment methods (such as credit cards) include high liquidity, lower transaction costs, and anonymity [4].

Recently, there is a significant growth in research taking place in the area of risk forecasting for financial time series data. Three key metrics that characterize risk are volatility, Value at Risk (VaR), and Expected Shortfall (ES). In financial econometrics, *volatility* is often defined as the measure of variation in price of the series over time and mathematically stated by the standard deviation of logarithmic returns. *VaR* is a widely used risk metric which represents the financial loss that can be incurred by an investment for a given level of confidence and over a certain time. *ES* is a risk metric that complements VaR, and is calculated by taking a conditional expectation of losses beyond the VaR cut-off.

Different volatility forecasting models can be used to calculate VaR and ES. Some widely used models used in practice include Historical Simulation (HS), Moving Average (MA), Exponentially Weighted Moving Average (EWMA), Generalized Autoregressive Conditional Heteroscedasticity (GARCH) as well as student-t GARCH (tGarch). In these models the distribution of logarithmic returns on daily closing prices is generally assumed to be either Normal or Student's t with fixed degrees of freedom (d.f.) [5]. However, It has been shown in empirical studies that sometime the logarithmic returns may have heavy-tailed distributions, such as Student's t having less than four degrees of freedom, and theoretically infinite kurtosis [6].

In regular regression and volatility forecasting, more complicated models have been proposed to capture variations in the real world. However, a more complex and flexible model leads to the risk of over-fitting. This can be overcome by regularized estimates, which are widely used in different machine learning models to prevent over-fitting. These regularized methods are also known as shrinkage methods, in which data values are shrunk towards a central point, such as the mean. A recent study [7] explored the application of three different regularization methods i.e., Ridge, Lasso, and elastic net in financial risk forecasting on the stock prices of some large technology companies. They found that the regularized versions of data-driven models improved the stability of risk forecasts, as measured by the model risk ratio. Our study also explores the application of these regularized method for risk forecasting of cryptocurrencies.

We use the concepts of the fuzzy interval from the fuzzy set theory to study the efficiency of different forecasting models. The use of fuzzy set theory to model certain financial problems [8], [9], [10] is of particular interest to several researchers due to its ability to quantitatively and qualitatively model the problems, which involve vagueness and imprecision. Recent

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studies have shown that a fuzzy random variable can be considered as a measurable mapping from a probability space to a set of fuzzy variables [11]. Fuzzy time series models provide a new avenue to deal with subjectivity observed in most financial time series models. Most of the fuzzy financial models developed so far have generally, been confined to modeling parameters through some form of defuzzification or linear type of fuzzy numbers such as Trapezoidal Fuzzy Number (Tr.F.N.) or Triangular Fuzzy Number (T.F.N.). The main reason for using a linear membership function is to avoid complex nonlinear computations [12], [13].

Fuzzy methods remain difficult to use in practice, and hence, there is a need for data-driven approaches to pragmatically use the fuzzy models for real-world financial models. In this study, we use the fuzzy approach in conjunction with the data-driven volatility forecasting models.

The data for this study includes the price data of five stocks/indexes and six cryptocurrencies. The first set includes Apple (APPL), Amazon (AMZN), Facebook (FB), Google (GOOG), and CBOE Volatility Index (VIX), while the second set includes Bitcoin (BTC), Ether (ETH), Binance Coin (BNB), Rippe (XRP), Dogecoin (DOGE), and Cardano (ADA) cryptocurrencies. The selection of cryptocurrencies is based on their market capital as per Coinmarketcap. Even though Tether, Solana, and Polkadot are among the top six cryptocurrencies by market capitalization, they are not included in the study. Tether (USDT) is a stable coin and exhibits negligible volatility, while Solana (SOL) and Polkadot (DOT) were recently launched, and their price data is unavailable for the period of study.

We collect all our data from the Yahoo! Finance, which is a leading source of financial data. Downloadable data include opening, high, low, closing, and adjusted prices as well as volume for numerous stocks, indexes, and cryptocurrencies. In this study, we specifically use the daily adjusted price, which is an amended version of the closing price obtained by nullifying the impact of certain corporate actions which can affect the price post market closing. The window of our study period is from 2017-10-01 to 2021-11-26, to accommodate most of the cryptocurrencies with large market capitalization, as many of them are fairly new.

II. Related Work

A key part of risk management involves forecasting the volatility of financial markets. The review by Poon and Granger [14] studies various volatility forecasting models, defining volatility as a sample standard deviation of logarithmic returns. The most widely used models for volatility forecasting are the ARCH class of models introduced by Engle in his seminal work [15] and reviewed by Bera and Higgins [16]. A commonly used metric to quantify market risk is Value at Risk (VaR), which is the expected value of loss incurred in a given time interval and given confidence level [17]. Recently, some interesting work has been reported by Thavaneswarana et al. [6] in the area of data-driven statistical models for financial time series. One such method is the novel data-driven generalized Exponentially Weighted Moving Average (DD-EWMA) model to estimate VaR forecasts for returns of stocks and indexes with larger kurtosis. This data-driven model was further improved by Liang et al. [7] by introducing regularization to estimate the optimal model parameters. Another promising area is integrating the core concepts of fuzzy logic with the volatility forecasting models to get better estimates [18]. This approach has been demonstrated in [19] for applications in option pricing, and in [20] for applications in portfolio optimization. A large portion of the literature on volatility forecasting targets the returns from stocks or indexes. However, an entirely new financial asset called cryptocurrency has emerged with introduction of Bitcoin - a decentralized digital currency [21]. Some key features of cryptocurrencies can be found in a technical survey by Tschorsch and Scheuermann [22]. Caporale and Zekokh used the Markov-switching GARCH models to model the volatility of the four most popular Cryptocurrencies at the time of its writing [23]. Recently, a significant stream of research literature is focussing on using machine learning models for predicting the future price movements in cryptocurrencies using the historical prices and other extraneous factors as inputs to models (for example [24], [25]). Liu and Tsyvinski [26] review different theoretical models used to describe cryptocurrency prices and lay down a set of tenets for benchmarking such models in the future.

III. Data-Driven Risk Forecasting

A. Value-at-Risk (VaR) and Expected Shortfall (ES)

Throughout the finance literature, risk forecasts are calculated using a time series of logarithmic returns $r_t = \log P_t - \log P_{t-1}$ as an input to the forecasting model. In this work, the one-step ahead forecasts of VaR and ES are computed using given equations (1) and (2):

$$\mathrm{VaR}_{t+1}(p) = -\hat{\sigma}_{t+1} F_r^{-1}(p), \tag{1}$$

$$\mathrm{ES}_{t+1}(p) = -\frac{\hat{\sigma}_{t+1}}{p} \int_{\infty}^{F_r^{-1}(p)} x f(x) dx. \tag{2}$$

Here $f(x)$ is the density function for the conditional distribution of logarithmic returns r_t, and $F_r^{-1}(p)$ is the inverse of Cumulative Density Function (CDF) of r_t evaluated at the tail probability p, while $\hat{\sigma}_{t+1}$ represent the volatility forecast at time $t+1$ given the past t observations. Thus, the process for forecasting the VaR and ES involves forecasting the volatility, and identifying a suitable distribution for log returns.

B. Data-Driven Volatility Forecasting

As shown in [7] and [9], if a random variable follows a Student's t distribution with d.f. greater than two, we can compute the optimal value of d.f. ν by finding solutions of the following equation:

$$2\sqrt{\nu - 2} = (\nu - 1)\rho_X \mathrm{Beta}\left[\frac{\nu}{2}, \frac{1}{2}\right]. \tag{3}$$

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Here ρ_X denotes the sign correlation of a random variable with its mean μ, and can be computed using below equation:

$$\rho_X = \text{Corr}(X-\mu, \text{sgn}(X-\mu)) = \frac{E|X-\mu|}{2\sigma\sqrt{F(\mu)(1-F(\mu))}} \tag{4}$$

where $F(\mu)$ is the Cumulative Density Function (CDF) of X evaluated at the mean, and σ^2 is the finite variance of X.

The method to find the conditional distribution of log returns for VaR forecasts has been discussed in [6]. Furthermore, the d.f. of student's t or generalized t distribution can be computed by using the sample estimate of sign correlation in equation (3). The observed volatility based on data-driven sign correlation incorporates skewness and non-normality.

C. Regularized Risk Forecasting

A further improvement to the data-driven volatility forecasts described above was suggested in [7] by introducing regularization to the volatility estimation model. After calculating the volatility forecasts $\hat{\sigma}_{t+1}$ using the data-driven approach, these forecasts are regularized with penalties using the elastic net regularization technique (5). The explicit form of the regularized volatility forecasts, $\tilde{\sigma}^{en}_{t+1}$, based on the elastic net penalties is given by:

$$\tilde{\sigma}^{en}_{t+1} = \frac{\text{sgn}(\hat{\sigma}_{t+1}-s)(\mid \hat{\sigma}_{t+1}-s \mid -\omega^{en}\lambda)_+}{1+(1-\omega^{en})\lambda} + s, \tag{5}$$

where $\lambda, \omega^{en} \in [0,1]$ are the tuning parameters of the model, and s represents the sample standard deviation of the time series of volatility estimates.

D. Neuro-Volatility Forecasting

A neural network (NN) is a powerful prediction model which can approximate any nonlinear real function on a bounded domain to a very high accuracy. The motivation for the design of neural networks comes from the interconnected neurons in the animal brain. A neural network fundamentally consists of the layers of nodes called *perceptrons*, and each node functions as a binary classifier. The simplest form of NN is a feed-forward neural network. It contains an input unit that reads the input variables, followed by an arbitrary number of interconnected hidden layers followed by an output layer. The transformations between two consecutive layers can be represented by the equations 6 and 7.

$$z^k_l = w^{(k-1)}_{l0} + \sum_{j=1}^{p_k-1} w^{(k-1)}_{lj} a^{(k-1)}_j \tag{6}$$

$$a^{(k)}_l = g^{(k)}\left(z^{(k)}_l\right) \tag{7}$$

Here, z^k_l is the linear transformation for l^{th} unit of the k^{th} layer of a feed forward NN, $w^{(k-1)}_{l0}$ is the bias term in $(k-1)^{th}$ layer, while $w^{(k-1)}_{lj}$ represent the weights of $(k-1)_{th}$ layer. $g^{(k)}$ is any non-linear function (eg: Sigmoid function).

NNs differ from traditional time series forecasting models used in finance by the number of parameters that be tuned, which is much higher in NNs. Also, unlike traditional models, all the parameters do not need to be optimized in a NN to get a universal approximate solution. Most of the NN models studied in the finance literature deal with stock price prediction. Thavaneswarana et al. [18] were the first to study the volatility and VaR forecasting using a generalized neuro-volatility model. They trained their neuro-volatility model, a feed forward neural network, on the p lagged values of the centered absolute log returns $|r^*_{t-1}|$, $|r^*_{t-2}|$, $|r^*_{t-3}|$, ... , $|r^*_{t-p}|$ to predict the target variable $|r^*_t|$, which is defined as $r^*_t = \frac{r_t-\bar{r}}{\rho}$, as a prediction from the output layer of neural network. In our neuro-volatility model, we use the p-lagged values of the volatility of absolute log returns V_{t-1}, V_{t-2}, V_{t-3}, ... , V_{t-p} as inputs and predict the target variable V_t as an output of the model (Fig. 1).

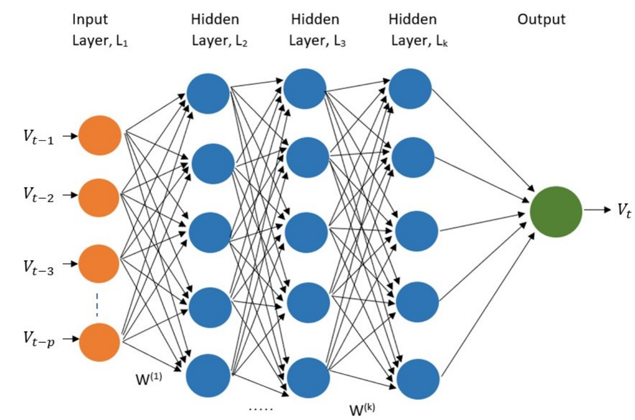

Fig. 1. Illustration of feed-forward neuro-volatility network

To fit this neuro-volatility model, we use the `nnetar` function of the R Package *forecast* [27].

E. Data-Driven Fuzzy Volatility Using Nonlinear Adaptive Fuzzy Numbers

If R be the set of all real numbers, a fuzzy number $A(x)$, $x \in R$ is of the following form

$$A(x) = \begin{cases} g(x) & when \quad [a,b) \\ 1 & when \quad [b.c) \\ h(x) & when \quad [c,d) \\ 0 & otherwise \end{cases} \tag{8}$$

where g is a real, right continuous, and increasing function while h is a real, left continuous, and decreasing function. Also, a, b, c, d are real numbers and $a < b < c < d$. It should be noticed that $A(x)$ can be classified as an LR fuzzy number having strictly monotone shape functions as mentioned by Bodjanova [10]. Refer Dubois and Prade [28] and Zimmermann [29] for more details on fuzzy numbers. A fuzzy number A with shape functions g and h is denoted by $A = [a,b,c,d]_{m,n}$. Here g and h are defined as:

$$g(x) = \left(\frac{x-a}{b-a}\right)^m \tag{9}$$

$$h(x) = \left(\frac{d-x}{d-c}\right)^n \tag{10}$$

Here, $m,\ n > 0$. If $m, n = 1$, we simply write $A = [a, b, c, d]$, which is known as a trapezoidal fuzzy number which is a Linear Fuzzy number. If m or $n \neq 1$, a fuzzy number $A = [a, b, c, d]_{m,n}$ is a modification of a trapezoidal fuzzy number and known as nonlinear adaptive asymmetric fuzzy number (Fig. 2). Each fuzzy number A has the following α-level sets (α-level sets), $A(\alpha) = [a(\alpha), b(\alpha)]$,$a(\alpha)$, $b(\alpha) \in R$, $\alpha \in [0, 1]$ and $A(\alpha) = [g^{-1}(\alpha), h^{-1}(\alpha)]$, $A_1 = [b, c]$, $A_0 = a, d$. If $A = [a, b, c, d]_{m,n}$ then, for all $\alpha \in [0, 1]$, $A(\alpha) = [a + \alpha^{\frac{1}{m}}(b - a), d - \alpha^{\frac{1}{n}}(d - c)]$.

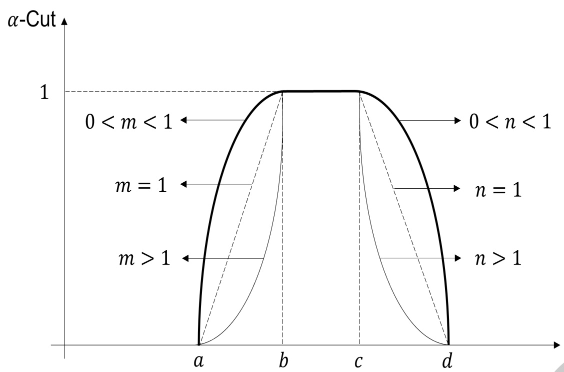

Fig. 2. Nonlinear membership function

To find the forecast interval widths of VaR and ES for stocks/indexes and cryptocurrencies, the α-cuts of the annualized volatility is calculated using different α, (a, b, c, d), m, and n values. For this study we choose (a, b, c, d) as the 0.05, 0.25, 0.75, and 0.95 quantiles respectively of the volatility forecasts.

F. Empirical study

The window of our study period is from 2017-10-01 to 2021-11-26, even though we use only the last 1000 days of adjusted closing prices to calculate log returns for both stocks and cryptocurrencies. It is important to note that the adjusted closing prices are available throughout the week for cryptocurrencies, however, for stocks, adjusted closing prices are only available for weekdays.

Also, while running the neuro-volatility model, to cover three months period for stocks and cryptocurrencies, a 63-day rolling forecasts and a 90-day rolling forecasts were considered, respectively. We use Algorithm 1 to produce our results as published in various tables.

IV. Experimental Result

A. Summary Statistics

Fig. 3 and Fig. 4 show the density plots of daily log-returns of interested stocks and cryptocurrencies. Based on the fitted normal curve, it is clear that stocks/indexes and cryptocurrency returns are more peaked at the center with a fatter tail than the normal curve. Although Fig 3 and Fig. 4 indicate distributions symmetric at a certain level, in general, log-returns of stocks and cryptocurrency have asymmetric distributions.

Table I summarized the basic descriptive statistics of the daily log returns. Except for VIX and XRP both the average and the standard deviation of the log-returns for cryptocurren-

Algorithm 1 Data-driven Risk forecasts

Require: Data: adjusted closing price of stock/index and Cryptocurrencies $P_t, t = 0, \ldots, n$

$r_t \leftarrow \log P_t - \log P_{t-1}, t = 1, \ldots, n$

$\hat{\rho} \leftarrow \text{Corr}(r_t - \bar{r}, \text{sgn}(r_t - \bar{r}))$ {Compute sample sign correlation of r_t}

$\nu \leftarrow$ Solve $2\sqrt{\nu - 2} = \hat{\rho}(\nu - 1)\text{Beta}\left[\frac{\nu}{2}, \frac{1}{2}\right]$ {Determine t d.f. of the conditional distribution of r_t}

$V_t \leftarrow |r_t - \bar{r}|/\hat{\rho}$ {Compute forecasts of volatility}

$s \leftarrow mean\,(|r_t - \bar{r}|/\hat{\rho})$ {Compute the sample mean of forecasts of volatility}

$S_0 \leftarrow \bar{V}_k$ {Initial volatility forecast}

$\alpha \leftarrow (0, 1)$ {Set a range for the smoothing constant}

$S_t \leftarrow \alpha V_t + (1 - \alpha)S_{t-1}, t = 1, \ldots, n$ {Calculate smoothed value of volatility}

$\alpha_{opt} \leftarrow \arg\min_\alpha \sum_{t=k+1}^{n}(V_t - S_{t-1})^2$ {Determine the optimal value of α}

$S_t^{dd} \leftarrow S_t, t = 1, \ldots, n$ {Get Smoothed value}

$(\hat{\sigma}_{t+1})_{dd} \leftarrow S_n^{dd}$ {Get the data-driven estimate of volatility}

$(\hat{\sigma}_{t+1})_{en} \leftarrow S_n^{en}$ {Get the elastic-net regularized data-driven volatility estimate (refer [7]}

Pass $V_1 \ldots, V_1$ to the NN using `nnetar` function and assign the output of NN, i.e., $\hat{V}_{t+1}$ to $\hat{\sigma}_{t+1}$ {Get the neuro-volatility estimate}

Calculate the α-cuts of the annualized volatility for the data-driven and neuro-volatility forecasts {Use the non-linear fuzzy number defined in III-E }

$\hat{\text{VaR}}_{n+1} \leftarrow -\hat{\sigma}_{n+1}F_r^{-1}(p)$ {VaR forecast from volatility}

$\hat{\text{ES}}_{n+1} \leftarrow -\frac{\hat{\sigma}_{n+1}}{p}\int_{-\infty}^{F_r^{-1}(p)} xf(x)dx$ {ES forecast from volatility}

return $\hat{\text{VaR}}_{n+1}, \hat{\text{ES}}_{n+1}$

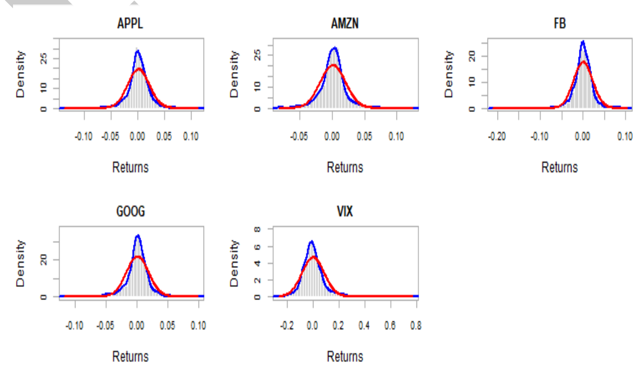

Fig. 3. Density Plots of Log Returns for Stocks

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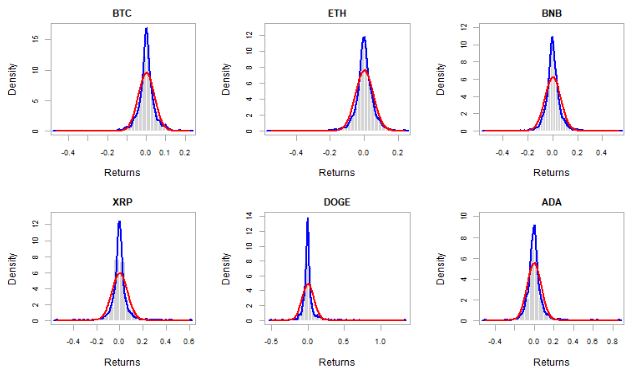

Fig. 4. Density Plots of Log Returns for Cryptocurrencies

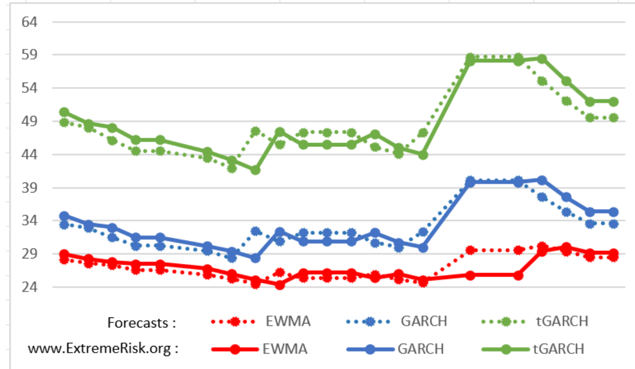

Fig. 5. VaR Forecasts for Apple (2021-11-03 to 2021-11-26)

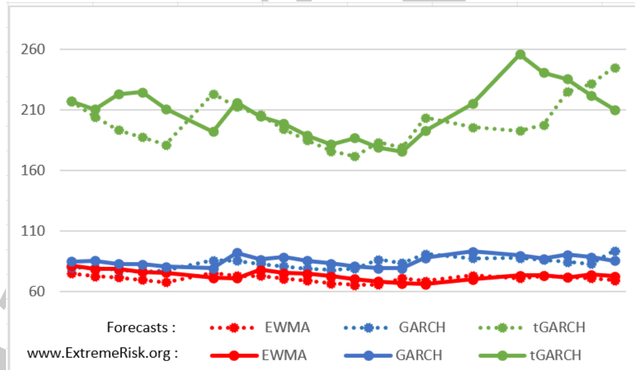

Fig. 6. VaR Forecasts for Bitcoin (2021-11-03 to 2021-11-26)

cies are high compared to stocks/indexes that we are interested in. It is important to note that even though VIX indicates the lowest average log returns, its standard deviation is the highest among the stocks. VIX has the smallest Sharpe ratio value among the stocks and cryptocurrencies. Thus, VIX gives a better risk-return trade-off compared to other stocks and cryptocurrencies. Among the cryptocurrencies, XRP has the lowest Sharpe ratio, and BNB has the highest. However, the Sharpe ratio of XRP is larger than VIX, and the Sharpe ratio of BNB is smaller than APPL, which has the highest Sharpe ratio among stocks. For Cryptocurrencies and stocks, Kurtosis is positive, and it indicates data are heavy-tailed relative to a normal distribution. Nonetheless, compared to cryptocurrencies, Kurtosis values for stocks are smaller. Among the stocks, FB has the highest Kurtosis, which is 11.34. For APPL, FB, GOOG, BTC, and ETH, returns are negatively skewed, whereas returns of AMZN, BNB, XRP, DOGE, and ADA are positively skewed. It is important to note that the Skewness of the AMZN is the smallest among all the stocks and cryptocurrencies.

TABLE I
DESCRIPTIVE STATISTICS OF LOG RETURNS

	Mean	SD	Sharpe	Kurtosis	Skewness
APPL	0.14%	2.03%	0.07	6.33	-0.36
AMZN	0.13%	1.97%	0.06	3.84	0.07
FB	0.07%	2.25%	0.03	11.34	-0.99
GOOG	0.11%	1.81%	0.06	5.52	-0.27
VIX	0.06%	8.56%	0.01	8.87	1.60
BTC	0.17%	4.18%	0.04	12.33	-0.86
ETH	0.17%	5.26%	0.03	10.49	-1.00
BNB	0.39%	6.39%	0.06	12.88	0.37
XRP	0.10%	6.76%	0.01	15.60	0.85
DOGE	0.35%	8.15%	0.04	56.19	3.87
ADA	0.27%	7.25%	0.04	22.14	1.90

B. VaR and ES Using Traditional Models

We report VaR forecasts using volatility forecasting models including HS, MA, EWMA, Garch, tGarch for all the stocks and cryptocurrencies. We also validate our VaR and ES forecasts against the corresponding forecasts reported in www.ExtremeRisk.org (Fig. 5, Fig. 6).

Forecasts of VaR and ES are reported in Table II and Table III. Except for APPL and VIX, model risks under VaR for stocks are less compared to cryptocurrencies. Among cryptocurrencies, under VaR, XRP has the highest models risk, and ETH has the lowest. In contrast, DOGE has the highest, and BNB has the lowest model risk under ES. APPL has the highest models risk under VaR and ES among stocks. It is also important to point out that the model risk of APPL is higher than ETH, BNB, and ADA under VaR, and under ES, it is higher than BTC, ETH, BNB, and ADA. Except for APPL, model risks for stocks under both VaR and ES vary between 1.5 and 2.0. However, for cryptocurrencies, model risks are grater than two under all the volatility forecasting models. This is a clear indication of a need of using modified and improved forecast models for both stocks and cryptocurrencies. Table II and III also report degrees of freedom for each stock and cryptocurrency. Our results highlight that even though some studies use fixed degrees of freedom, this may not be appropriate for all data.

C. Data-Driven Regularized Risk Forecasts

In this section, we report *VaR* and *ES* using data-driven regularized risk forecasting methods. Reported model risks are based on four forecasting models and the superiority of the forecast model can be observed through model risks. Forecasting models include normal Garch, tGarch, data-driven

TABLE II
FORECASTS OF VAR AND MODEL RISK

Stock	HS	MA	EWMA	Garch	tGarch	df	MR
APPL	59.88	47.98	28.54	33.60	49.54	4.95	2.10
AMZN	56.00	45.58	40.58	42.03	61.33	5.15	1.51
FB	65.18	53.05	42.00	45.02	69.86	4.08	1.66
GOOG	51.90	42.67	31.83	32.74	52.63	3.84	1.65
VIX	184.18	201.96	120.14	158.23	233.96	4.29	1.95
BTC	108.02	93.25	69.45	93.63	244.92	2.52	3.53
ETH	142.85	118.46	90.19	111.62	200.29	3.49	2.22
BNB	146.76	135.84	103.92	113.53	188.94	3.72	1.82
XRP	169.04	145.64	81.56	176.25	508.62	2.30	6.24
DOGE	196.25	199.21	104.43	150.89	396.66	2.33	3.80
ADA	134.35	136.77	84.16	122.81	210.10	3.70	2.50

TABLE III
FORECASTS OF ES AND MODEL RISK

Stock	HS	MA	EWMA	Garch	tGarch	df	MR
APPL	84.38	54.97	32.69	38.49	50.71	4.95	2.58
AMZN	68.66	52.22	46.50	48.15	63.11	5.15	1.48
FB	96.82	60.77	48.11	51.58	69.14	4.08	2.01
GOOG	68.67	48.89	36.46	37.51	51.37	3.84	1.88
VIX	211.39	231.38	137.64	181.28	233.90	4.29	1.70
BTC	165.53	106.83	79.56	107.26	188.48	2.52	2.37
ETH	220.90	135.72	103.32	127.88	190.15	3.49	2.14
BNB	248.91	155.62	119.06	130.06	182.81	3.72	2.09
XRP	279.00	166.85	93.44	201.92	328.29	2.30	3.51
DOGE	307.90	228.23	119.64	172.87	265.01	2.33	2.57
ADA	214.63	156.69	96.42	140.70	202.94	3.70	2.23

generalized EWMA, and data-driven EWMA, and summarized results are given in Table IV. Observe that compared to regular models reported in the previous section, data-driven models provide more stable VaR and ES forecasts in general. Also, further improvement of the risk forecasting can be noted due to regularization as VaR and ES forecasts get closer to 1.

TABLE IV
MODEL RISK FOR VAR AND ES USING DDRRF

	Non-Regularized		Regularized	
	VaR	ES	VaR	ES
APPL	2.09	1.72	1.82	1.53
AMZN	1.07	1.24	1.09	1.37
FB	2.30	1.81	2.17	1.79
GOOG	1.52	1.51	1.17	1.37
VIX	1.45	1.28	1.36	1.20
BTC	1.18	1.60	1.24	1.63
ETH	1.17	1.48	1.23	1.56
BNB	1.80	1.45	1.80	1.45
XRP	1.17	1.72	1.25	1.72
DOGE	1.20	1.77	1.24	1.77
ADA	1.11	1.47	1.19	1.58

D. Comparison of fuzzy forecast intervals

All the experiments were executed on a computer with an Intel Core(TM) i5-8265U CPU, running 4 Cores (8 logical cores) at a clock speed of 1.60GHz and 8GB of RAM on Windows 11 operating system. First, we investigate computation time for both Data-Driven Volatility Forecast (DDVF) and neuro-volatility Forecasts (NVF) for stocks and cryptocurrencies. It can be seen from Tables V and VI that NVF takes more computation time compared to DDVF. This is due to maintaining higher level of accuracy that results from NVF, which uses multiple hidden layers and nodes. Thus, it is important to further investigate if the accuracy of the risk measures could be improved using DDVF and NVF for stocks and cryptocurrencies.

TABLE V
COMPUTATION TIME (SECONDS) FOR STOCKS

	AAPL	AMZN	FB	GOOG	VIX
DDVF	0.21	0.4	0.47	0.17	0.44
NVF	181.03	139.98	232.36	172.95	225.26

TABLE VI
COMPUTATION TIME (SECONDS) FOR CRYPTOCURRENCIES

	BTC	ETH	BNB	XRP	DOGE	ADA
DDVF	0.46	0.23	0.52	0.44	0.67	0.48
NVF	419.01	191.19	350.28	272.78	1535.5	283.64

As we develop α-cuts using DDVF and NVF to calculate VaR and ES, there are several cases to be investigated depending on values of m and n. First we set m to be 0.25 and n to be 0.25, 0.75, and 1.0. forecasts of VaR and ES interval widths for APPL, BTC, and ADA are reported in Table VII, Table VIII, and Table IX respectively. Note that for APPL, VaR interval widths are smaller using NVF for $n = 0.25$, $n = 0.75$, and $n = 1.0$, whereas for BTC, VaR interval widths using DDVF are smaller. Except for ADA, for all the cryptocurrencies, VaR interval widths using DDVF are smaller, and for stocks, VaR interval widths using NVF are smaller. Thus, we can conclude with one exception that VaR forecasts using DDVF are much stable for cryptocurrencies, and VaR forecasts using NVF are more stable for stocks. It is important to investigate interval widths when m changes through its parameter space. Our experimental results suggest that except for ADA VaR interval widths using DDVF for crypto are smaller compared to VaR interval widths using NVF considering $m = 0.75$ and $m = 1.0$. The results also suggest that VaR interval widths using NVF for stocks are smaller compared to VaR interval widths using DDVF when $m = 0.75$ and $m = 1.0$. Further, the same conclusions can be drawn for ES interval widths.

Thus, the narrowest α-cuts of the annualized volatility are provided using the data-driven volatility forecast for cryptocurrencies, whereas for regular stocks, the narrowest α-cuts of the annualized volatility are provided using the neural volatility forecast. This hold when n and m (tuning parameters) be chosen in between 0 and 1.

V. CONCLUSION

Despite claiming to have sophisticated risk management techniques, many large financial service firms incurred heavy losses during the 2008 global financial crisis. This led to reassessment of the commonly used risk models for their robustness and accuracy. Volatility forecasting is an integral component of commonly used risk management models, and

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TABLE VII
α-CUTS OF VaR INTERVAL WIDTHS FOR APPLE USING DDVF AND NVF ($m = 0.25$)

	DDVF			NVF		
$\alpha \setminus n$	0.25	0.75	1.0	0.25	0.75	1.0
0	240.08	240.08	240.08	205.41	205.41	205.41
0.1	240.07	237.68	234.93	205.40	200.99	195.90
0.2	239.89	233.95	229.68	205.21	194.24	186.35
0.3	239.13	229.20	224.10	204.38	186.06	176.63
0.4	237.06	223.21	217.79	202.14	176.56	166.55
0.5	232.71	215.49	210.18	197.44	165.65	155.85
0.6	224.79	205.41	200.57	188.87	153.09	144.16
0.7	211.75	192.11	188.07	174.78	138.51	131.05
0.8	191.75	174.60	171.65	153.15	121.49	116.03
0.9	162.66	151.70	150.10	121.70	101.46	98.51
1.0	122.08	122.08	122.08	77.82	77.82	77.82

TABLE VIII
α-CUTS OF VaR INTERVAL WIDTHS FOR BITCOIN USING DDVF AND NVF ($m = 0.25$)

	DDVF			NVF		
$\alpha \setminus n$	0.25	0.75	1.0	0.25	0.75	1.0
0	655.12	655.12	655.12	1130.77	1130.77	1130.77
0.1	655.08	645.32	634.03	1130.70	1106.57	1078.66
0.2	654.54	630.23	612.72	1129.61	1069.51	1026.26
0.3	652.17	611.55	590.65	1124.88	1024.48	972.82
0.4	645.80	589.09	566.90	1112.15	971.96	917.12
0.5	632.37	561.91	540.17	1085.32	911.14	857.41
0.6	607.95	528.61	508.82	1036.51	840.40	791.47
0.7	567.73	487.35	470.81	956.15	757.45	716.57
0.8	506.04	435.85	423.77	832.87	659.37	629.50
0.9	416.33	371.47	364.93	653.59	542.71	526.53
1.0	291.16	291.16	291.16	403.46	403.46	403.46

TABLE IX
α-CUTS OF VaR INTERVAL WIDTHS FOR ADA USING DDVF AND NVF ($m = 0.25$)

	DDVF			NVF		
$\alpha \setminus n$	0.25	0.75	1.0	0.25	0.75	1.0
0	1907.15	1907.15	1907.15	1842.45	1842.45	1842.45
0.1	1907.08	1888.80	1867.66	1842.31	1786.29	1721.48
0.2	1906.00	1860.48	1827.72	1840.14	1700.61	1600.17
0.3	1901.32	1825.28	1786.15	1830.77	1597.65	1477.70
0.4	1888.72	1782.54	1741.00	1805.52	1480.01	1352.67
0.5	1862.15	1730.24	1689.54	1752.28	1347.88	1223.11
0.6	1813.85	1665.32	1628.26	1655.48	1200.14	1086.52
0.7	1734.29	1583.81	1552.85	1496.06	1034.71	939.80
0.8	1612.27	1480.87	1458.24	1251.53	848.69	779.33
0.9	1434.81	1350.83	1338.58	895.91	638.47	600.90
1.0	1187.22	1187.22	1187.22	399.77	399.77	399.77

hence the quality of volatility forecasts greatly impact the robustness of risk models. Also, in last few years, the interest in Cryptocurrencies as an alternative financial asset has exploded, with the value locked in them growing exponentially. In this work, we obtained the forecasts of Value at Risk (VaR) and Expected Shortfall (ES) for the top six Cryptocurrency by market capitalization. We used both the traditional time series models as well as relatively recent data-driven, Regularized, and Neuro-volatility methods to get the volatility forecasts used in our models. We also compare the stability of our forecasts using the Model Risk metric. Also, to compare the quality of forecasts between data-driven and neuro-volatility models, we obtain fuzzy confidence intervals for the forecasts using a trapezoidal membership function. The narrower fuzzy intervals imply a better forecast quality. We observed that the data-driven models produced better forecasts for cryptocurrencies, while for the regular stocks and indexes, neuro-volatility model gave better forecasts. Also, the data-driven models are much efficient in terms of computational complexity as the running time of neuro-volatility model is significantly higher than that of data-driven model.

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