Data-Driven and Neuro Volatility Fuzzy Forecasts for Cryptocurrencies

Abstract—**The forecasting problems in Computational Finance involve modelling the vagueness and imprecision inherent to the financial markets. Fuzzy set theory has a unique ability to quantitatively and qualitatively model and analyze such problems. Volatility forecasting plays an important role in financial risk management and in option pricing. Recently, there has been a growing interest in data-driven volatility models and neuro-volatility models for risk forecasting of stocks and index funds. However, even these state-of-the-art models do not take into account the fuzzy volatility in their risk forecasts.**

***Cryptocurrencies* are a novel financial asset class based on the *Blockchain* technology. Cryptocurrencies have gained popularity among retail investors as a financial asset with high risks and high returns. The extremely volatile nature of cryptocurrencies (compared to traditional assets) makes forecasting their volatility more challenging. This paper provides fuzzy forecasts of the volatility of *Algo returns* for six major cryptocurrencies. A simple algorithmic trading approach, *SMA crossover strategy*, is used to calculate the Algo returns. We also compute and compare fuzzy volatility forecasts of four major tech stocks and CBOE volatility index (VIX). Our experimental results show that the data-driven models produce better forecasts for cryptocurrencies, while for the regular stocks and indexes, no such definitive conclusion could be drawn.**

Index Terms—**Fuzzy intervals, Cryptocurrency, Volatility Forecasting, Data-driven volatility, Neuro-volatility, Sharpe ratio, Algo return, Simple Moving Average (SMA)**

I. Introduction

A large portion of the capital investment in today's economy is handled by a few very large financial institutions, which makes the system overly centralized. However, following the global financial crisis of 2008, an interesting development took place in the world of finance with the launch of Bitcoin payment network [1]. Bitcoin is a decentralized digital currency system, which relies on cryptographic primitives to facilitate trust-free value transfer among the participating nodes, instead of relying on a trusted central authority like a bank. The underlying core technology of Bitcoin, a distributed ledger called Blockchain, gained significant traction and many more such currencies followed, and the term *cryptocurrency* was coined. Cryptocurrencies derive their value purely from the trust that is placed on them, and they are not backed by any commodity, such as gold or silver or by the state guarantee. This makes the price of cryptocurrencies highly volatile and hard to forecast.

Modelling and forecasting volatility is extremely crucial for the risk management arms of all major financial institutions [2]. After the Basel Accord of 1996, financial risk management has become a top priority of the financial services industry. From the Basel III monitoring report by the Basel Committee on Banking Supervision in 2021 [3], financial institutions are required to use more complex models for efficient capital allocation and enhanced risk management. Due to strict risk management requirements, financial institutions at large do not invest in cryptocurrencies, as existing risk management modelling is not sufficient to capture the uncertainty associated with cryptocurrency volatility forecasts. Fuzzy confidence intervals [4] can prove effective in incorporating inherent uncertainty in the forecast of cryptocurrency returns.

For any financial asset, the most common way to quantify return is to capture the day-to-day changes in its *adjusted closing price*. The adjusted closing price is an asset's closing price amended to reflect the stock's value after accounting for any corporate action(s). The return metric used in this paper is called *Algo Return*, which is calculated by algorithmically simulating long and short positions at different points of time. The Simple Moving Average (SMA) of the asset price, which is an average of the past adjusted closing prices P_t, $t = 1, 2, ..., D$ is a key parameter to simulate the trading strategy. The short-term and long-term SMA trend indicators are calculated to decide whether to buy or sell an asset at a given time point. The trend-following position indicator takes the values of 1 and -1 at each trending time t. If the short-term SMA $\geq$ long-term SMA, then the position is selected as long position (1), and if the short-term SMA $<$ long-term SMA, then the position is selected as short position (-1).

The daily adjusted price can be converted to simple returns as $R_t = (P_t - P_{t-1})/P_{t-1}$. When the return is multiplied by the corresponding position for each t, the resulting quantity is called Algo return (A_t). If μ_A and σ_A are the mean and standard deviation of daily algo returns respectively, then the daily Sharpe ratio (SR) is computed as

$$Daily.SR = \frac{E(A_t - r_f/N)}{\sqrt{Var(A_t - r_f/N)}} = \frac{\mu_A - r_f/N}{\sigma_A} \tag{1}$$

where r_f is the annual risk-free rate, and N is the number of trading periods in a year. Sharpe ratio helps investors to understand the return of an investment compared to its risk. A higher SR is indicative of higher risk-adjusted returns for an investment, and hence it can also be used as a measure of the quality of an algorithmic trading strategy. A strategy that gives high returns and low volatility is always preferred.

The data for this study is collected from the Yahoo! Finance, which is a leading source of financial data. The data API includes the opening, high, low, closing, and adjusted prices as well as volume for numerous stocks, indexes, and cryptocurrencies. Collected data includes price data of five stocks/indexes and six cryptocurrencies. Apple (AAPL), Amazon (AMZN), Facebook (FB), Google (GOOG), and CBOE Volatility Index (VIX) have been considered under stocks/indexes, and Bitcoin (BTC), Ether (ETH), Binance Coin (BNB), Rippe (XRP), Dogecoin (DOGE), and Cardano (ADA) are studied under cryptocurrencies. The selection of cryptocurrencies is based on their market capital as per Coinmarketcap. Even though Tether, Solana, and Polkadot are also among the top six cryptocurrencies by market capitalization, they are not included in the study. Tether (USDT) is a stable coin and exhibits negligible volatility, while Solana (SOL) and Polkadot (DOT) were recently launched, and their price data is unavailable for the period of study. Two different study periods are considered for stocks/indexes and cryptocurrencies due to different daily trading periods for a given year. For stocks/indexes, weekend adjusted closing prices are not available. However, for cryptocurrencies, weekend adjusted closing prices are available. Thus, for a given year, cryptocurrencies have 365 daily trading periods ($N = 365$), and stocks/indexes have only 252 daily trading periods ($N = 252$). Therefore, to accommodate most of the cryptocurrencies with large market capitalization and most recent data, the study periods of 2017-01-01 to 2021-12-31 and from 2018-01-01 to 2021-12-31 have been chosen for stocks/indexes and cryptocurrencies, respectively.

II. Related Work

Neuro-fuzzy volatility forecasting models can prove highly instrumental in the area of computational finance as they can be applied in problems across risk management, hedging and pricing of financial instruments. The review by Poon and Granger [5] studies various volatility forecasting models, defining volatility as a sample standard deviation of logarithmic returns. The most widely used models for volatility forecasting are the ARCH class of models introduced by Engle in his seminal work [6] and reviewed by Bera and Higgins [7]. Several previous works [8], [9] found hybrid of GARCH and neural network based model to outperform the vanilla GARCH models in diverse set of assets, such as gold and NASDAQ indices. A recent paper by Rahimikia and Poon [10] describes a Long Short term Memory (LSTM) neural network based volatility forecasting model for some popular NASDAQ stocks. The model has been primarily trained on features extracted from the LOBSTER limit order book dataset [11]. In addition, features extracted from the news articles of the major financial media outlets were incorporated to augment the model performance. Recently, the generalized risk forecasting methods have been reported by Thavaneswaran et al. [12] in the area of data-driven volatility models. One such method is the novel data-driven generalized Exponentially Weighted Moving Average (DD-EWMA) model to estimate volatility forecasts for logarithmic returns of stocks and indexes with larger kurtosis. This data-driven model was further improved by Liang et al. [13] by incorporating the elastic net regularization while estimating the optimal model parameters. Another promising area is integrating the core concepts of neuro-fuzzy prediction with the volatility forecasting models to get better estimates [14]. This approach has been demonstrated in [15] for applications in option pricing, and in [16] for applications in portfolio optimization. The majority of research literature on volatility forecasting targets traditional financial assets stocks or indexes. However, an entirely new financial asset called cryptocurrency has emerged with introduction of Bitcoin - a decentralized digital currency [17]. Some key features of cryptocurrencies can be found in a technical survey by Tschorsch and Scheuermann [18]. Caporale and Zekokh used the Markov-switching GARCH models to model the volatility of the four most popular Cryptocurrencies at the time of its writing [19]. Recently, a significant stream of research literature is focussing on using machine learning models for predicting the future price movements in cryptocurrencies using the historical prices and other extraneous factors as inputs to models (for example [20], [21]). Liu and Tsyvinski [22] review different theoretical models used to describe cryptocurrency prices and lay down a set of tenets for benchmarking such models in the future.

III. Approach

A. Volatility Forecasts

The forecast of SR is carried out using the data-driven volatility forecasting method and data-driven exponentially weighted moving average using the *sign correlation* ([23]).

The sign correlation of a random variable X with mean μ is defined as

$$\rho_X = \mathrm{Corr}(X-\mu, \mathrm{sign}(X-\mu)). \tag{2}$$

For any symmetric distribution with finite mean μ and variance σ^2, the sign correlation ρ_X is given by

$$\rho_X = \frac{\mathrm{E}|X-\mu|}{\sigma}.$$

If X follows a Student's t distribution with sign correlation ρ_X and finite variance, the corresponding degrees of freedom (d.f.) ν can be computed by solving the following equation:

$$2\sqrt{\nu-2} = (\nu-1)\rho_X \mathrm{Beta}\left[\frac{\nu}{2}, \frac{1}{2}\right]. \tag{3}$$

The data-driven algo volatility estimator, in terms of algo returns $A_1, \cdots, A_n$, is given as

$$\hat{\sigma}_A = \frac{1}{n}\sum_{t=1}^{n} \frac{|A_t - \bar{A}|}{\hat{\rho}_A}, \tag{4}$$

where $\hat{\rho}_A$ is the sample sign correlation of A_t which can be calculated using 2. The asymptotic variance of the data-driven algo volatility estimator $\hat{\sigma}_A$ is

$$\left(\frac{1-\rho_A^2}{\rho_A^2}\right)\frac{\sigma_A^2}{n}, \tag{5}$$

The most commonly used volatility estimator in practice is the sample standard deviation s_n, and its asymptotic variance is given by

$$\frac{(\kappa+2)}{4}\frac{\sigma_A^2}{n},$$

here κ is the excess kurtosis. As reported in (see [23]), investment returns follow Student's t distribution with degrees of freedom, d.f., less than four, and hence have an infinite kurtosis as per theory. Thus, the sample standard deviation estimator has a very large or infinite asymptotic variance for the returns with heavy-tailed distributions, such as returns from the technology stocks. For such distributions, [23] propose an alternative data-driven volatility estimator $\hat{\sigma}_A$, which has a smaller and finite asymptotic variance.

Volatility forecasting has been widely used in risk management and asset pricing, but its use in algorithmic trading is not common. However, very recently, [23] has discussed usage of SR fuzzy forecasts based on volatility forecasting models.

For algo return A_t, the conditional mean and conditional variance is defined as

$$E(A_t|\mathcal{F}_{t-1}) = \mu_t, Var(A_t|\mathcal{F}_{t-1}) = \sigma_t^2, t = 1, \cdots, n,$$

where $\mathcal{F}_{t-1}$ is the data until time $t-1$. Further, as per [23], the volatility forecasting model for algo returns can be written as

$$\hat{\sigma}_{t+1} = (1-\omega)\,\hat{\sigma}_t + \omega\frac{|A_t - \bar{A}|}{\hat{\rho}_A}, \quad 0 < \omega < 1. \tag{6}$$

B. Asymmetric Adaptive Nonlinear Fuzzy Numbers

A fuzzy number $A(x)$, $x \in R$, a set of all real numbers, is of the following form

$$A(x) = \begin{cases} g(x) & when \quad [a,b) \\ 1 & when \quad [b.c) \\ h(x) & when \quad [c,d) \\ 0 & otherwise \end{cases} \tag{7}$$

where g is a real, right continuous, and increasing function while h is a real, left continuous, and decreasing function. Also, a, b, c, d are real numbers and $a < b < c < d$. It should be noticed that $A(x)$ can be classified as an LR fuzzy number having strictly monotone shape functions as mentioned by Bodjanova [24]. Refer Dubois and Prade [25] and Zimmermann [26] for more details on fuzzy numbers. A fuzzy number A with shape functions g and h is denoted by $A = [a,b,c,d]_{m,n}$. Here g and h are defined as:

$$g(x) = \left(\frac{x-a}{b-a}\right)^m \tag{8}$$

$$h(x) = \left(\frac{d-x}{d-c}\right)^n \tag{9}$$

Here, m, $n > 0$. If $m, n = 1$, we simply write $A = [a,b,c,d]$, which is known as a trapezoidal fuzzy number which is a Linear Fuzzy number. If m or $n \neq 1$, a fuzzy number $A = [a,b,c,d]_{m,n}$ is a modification of a trapezoidal fuzzy number and known as nonlinear adaptive asymmetric fuzzy number (Figure 1). Each fuzzy number A has the following α-level sets (α-level sets), $A(\alpha) = [a(\alpha), b(\alpha)]$,$a(\alpha)$, $b(\alpha) \in R$, $\alpha \in [0,1]$ and $A(\alpha) = [g^{-1}(\alpha), h^{-1}(\alpha)]$, $A_1 = [b,c]$, $A_0 = a, d$. If $A = [a,b,c,d]_{m,n}$ then, for all $\alpha \in [0,1]$, $A(\alpha) = [a + \alpha^{\frac{1}{m}}(b-a), d - \alpha^{\frac{1}{n}}(d-c)]$.

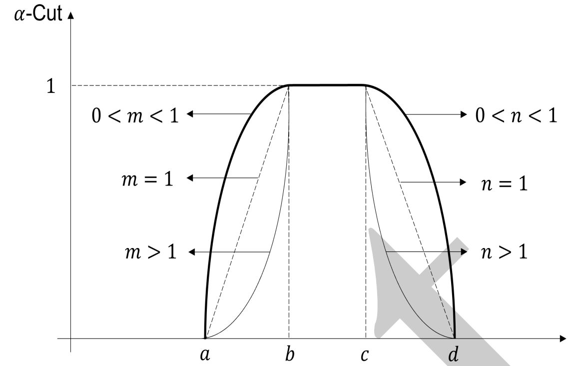

Fig. 1. Nonlinear membership function

In order to find the forecast interval widths of algo volatility for stocks/indexes and cryptocurrencies, the α-cuts of the annualized volatility is calculated using different values of α, (a,b,c,d), m, and n values. For this study we choose (a,b,c,d) as the 0.05, 0.25, 0.75, and 0.95 quantiles respectively of the volatility forecasts.

C. Annualized Sharpe Ratio

The data-driven estimates for SR of the algo returns are computed using the SMA crossover strategy as described in [23]. We further compute the annualized SR (equation 10) from daily SR as it provides a better metric for comparing different assets.

$$Annualized.SR = \sqrt{N} Daily.SR \tag{10}$$

The α-cuts for σ_A can be written as

$$\overline{\sigma_A(\alpha)} = \hat{\sigma}_A \pm cv_\alpha \sqrt{\frac{(1-\hat{\rho}_A^2)\hat{\sigma}_A^2}{n\hat{\rho}_A^2}}, \tag{11}$$

where cv_α is the critical value of level α and $\hat{\sigma}_A$ is the data-driven volatility estimate (DDVE) of algo returns. Then, the α-cuts of annualized SR can be written as

$$\left(\frac{\sqrt{N}(\bar{A}_t - r_f/N)}{UL^{\sigma_A}}, \frac{\sqrt{N}(\bar{A}_t - r_f/N)}{LL^{\sigma_A}}\right), \tag{12}$$

where LL^{σ_A} denotes lower limit of σ_A and UL^{σ_A} denotes upper limit of σ_A.

D. Dynamic Data-Driven Rolling Sharpe Ratio Forecasts

Time-varying volatility models would be more appropriate if there is an indication of significant sample autocorrelation in the absolute values of the also returns. The first step towards designing such a model is to compute the daily DD-EWMA volatility forecasts and daily data-driven neuro volatility forecasts. Then, the annualized SR forecasts can be computed from the two daily rolling volatility forecasts using Equation 1 and Equation 10.

The sample sign correlation $\hat{\rho}_A$ and observed algo volatility $Z_t = |A_t - \bar{A}|/\hat{\rho}_A$ can be computed based on the past observations of algo returns. The optimal smoothing constant ω is determined by minimizing the one-step ahead forecast error sum of squares (FESS). Using the optimal ω, smoothed value S_t ($S_t = \omega Z_t + (1-\omega)S_{t-1},\ t = 1, \ldots, k$) can be calculated recursively. The initial smoothed valued is calculated using the first l observations, and the last smoothed value of S_t is the one-day-ahead volatility forecast. The root mean square error (RMSE) of volatility forecasts can be computed as

$$\sum_{t=l+1}^{k} (Z_t - S_{t-1})^2/(k-l).$$

We use the rolling window approach in order to calculate both the daily and the annualized SR forecasts using the algo DD-EWMA volatility method and neuro volatility forecasts. The neuro volatility forecast in this paper has been computed by using the *nnetar* function from R package *forecast* [27]. Also, the α-cuts of the SR forecasts are computed using below equation.

$$\begin{aligned}\overline{Daily.SR(\alpha)} &= (LL^{Daily.SR}, UL^{Daily.SR}) \\ &= mean(Daily.SR_i) \pm cv_\alpha sd(Daily.SR_i).\end{aligned}$$

The α-cuts of annualized SR forecasts can be calculated by daily SR forecasts as

$$\overline{Annualized.SR(\alpha)} = (\sqrt{N}\, LL^{Daily.SR}, \sqrt{N}\, UL^{Daily.SR}).$$

IV. Experimental Result

The moving average crossover strategy calculates two separate simple moving averages (SMAs) with different window sizes. The crossing of these two SMAs signal a change in strategy (buy/sell). Based on the strategy, one would sell a unit of share of an asset if the SMA with shorter window size crosses below the SMA with longer window size (green shaded area in Figures 2 and 3). Contrarily one would buy an asset if SMA with shorter window size crosses above the SMA with longer window size (pink shaded area in Figures 2 and 3). Considering adjusted closing prices of AAPL, the SMA crossover strategy with a short term window size of 10 days and long-term window size of 20 days is visualized in Figure 2. Figure 3 visualizes the SMA crossover strategy with short term window size 10 and long-term window size 30 considering adjusted closing prices of BTC.

In order to select the optimal SMA window sizes for different assets, the annualized SR estimates are used. We use the *brute force* technique to find the optimal short term window size by fixing the long term window size to values in 20, 30, 60. The corresponding short-term window is selected from a range of 1 to the long-term window size. Figure 4 shows the progression of SR for AAPL. It can be seen that the SR stays relatively stable after the short-term window size of 10, and after the window size of 20, SR becomes almost constant, and hence it is chosen as the long-term window size. Similarly, for BTC, if we choose a long-term window size of 30, then 10 can be selected as a short-term window size (Figure 5).

As observed empirically, for all the stocks/indexes, the short-term window size can be chosen as 10 with the long-term window size of 20. For cryptocurrencies, short-term window sizes vary between 10 and 20, while 30 is an appropriate long-term window size. Our observation suggests short-term window sizes of 10 for ETH, 15 for BNB and XRP, and 20 for DOGE and ADA are appropriate.

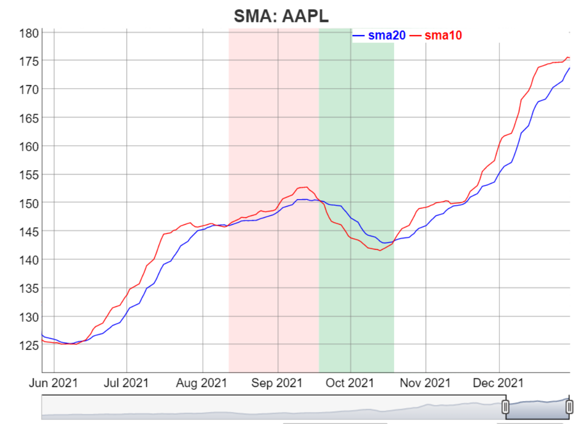

Fig. 2. SMA crossover strategy - AAPL

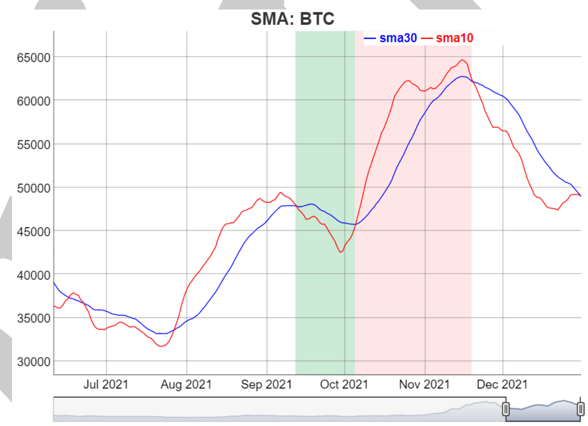

Fig. 3. SMA crossover strategy - BTC

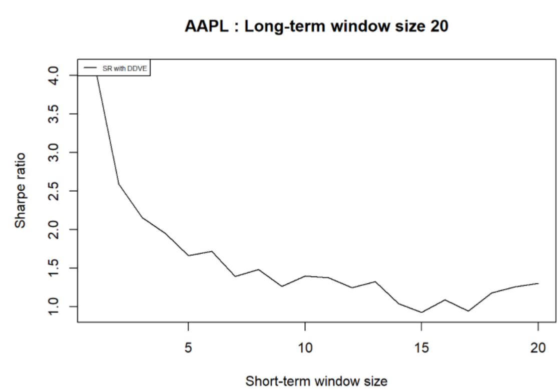

Fig. 4. Estimated Annualized SR - AAPL

Considering the adjusted closing prices of the assets, the respective daily algo returns (A_t) can be calculated using Algorithm 1. For all the stocks/indexes, the short term and

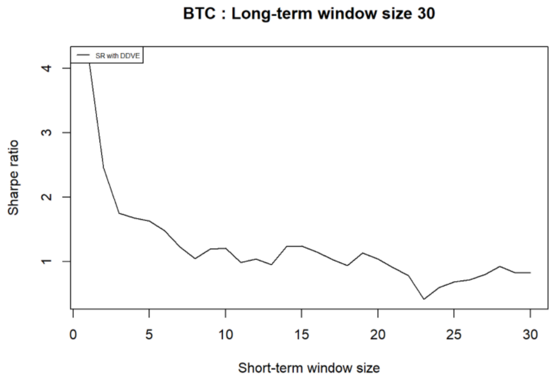

Fig. 5. Estimated Annualized SR - BTC

Algorithm 1 SMA crossover trading strategy

Require: $P_t, t = 0, \dots, n$ (Adjusted closing price of the asset), S (short-term window size), L (long-term window size)

$R_t \leftarrow (P_t - P_{t-1})/P_{t-1}, t = 1, \cdots, n$ {Calculate returns}

$SSMA_t \leftarrow SMA(P_t, S)$ {Short-term SMA}
$LSMA_t \leftarrow SMA(P_t, L)$ {Long-term SMA}
if $SSMA_t \geq LSMA_t$ **then**
  $Position_t \leftarrow 1$
else
  $Position_t \leftarrow -1$
end if
$AlgoReturn \leftarrow R_t * Position_t$ for each t {Algo returns}

return $AlgoReturn$

the long term window sizes are chosen to be 10 and 20, respectively. Even though different short-term window sizes seem appropriate for different cryptocurrencies, for simplicity, the short-term windows size of 20 is chosen for all the cryptocurrencies. Nonetheless, the long-term window size of 30 remains fixed for all the cryptocurrencies. The summary statistics of A_t for all the assets are listed in Table I. For stocks/indexes, the sample sign correlation of algo returns ($\hat{\rho}_A$) vary between 0.64 and 0.70. For all the cryptocurrencies, the $\hat{\rho}_A$ is less than 0.71 and greater than 0.36. Except for ADA, For all the assets, the algo returns have a t-distribution with d.f less than 4. However, note that the corresponding d.f. for ADA is very close to 4. Moreover, the absolute algo returns $|A_t|$ are significantly autocorrelated for all the assets, which indicates the volatility clustering. Figure 6 and Figure 7 visualize volatility clustering plots for AAPl and BTC, respectively. There is a clear indication that within the study periods for AAPL and BTC, there are periods with low volatility and periods with high volatility. Volatilities indicate their highest values during March and April in 2021 (peak period of COVID-19 pandemic) for both assets.

Table II summarizes α-CUTs of annualized SR estimates using risk as DDVE. DDVE are calculated using equation

TABLE I
SUMMARY STATISTICS OF DAILY ALGO RETURNS FOR ALL ASSETS

Assets	Mean	SD	kurtosis	A_t	$\lvert A_t \rvert$	$\lvert A_t^2 \rvert$	$\hat{\rho}_A$	Est df
AAPL	0.0018	0.0193	6.4445	-0.1370	0.2420	0.3245	0.6804	3.4837
AMZN	0.0007	0.0187	4.6384	-0.0308	0.1988	0.1585	0.6965	3.7634
FB	0.0000	0.0210	9.5641	-0.1065	0.1680	0.1266	0.6782	3.4507
GOOG	0.0004	0.0171	5.9019	-0.1290	0.2378	0.2829	0.6769	3.4323
VIX	-0.0023	0.0936	24.2155	-0.0817	0.1832	0.0741	0.6401	3.0288
BTC	0.0022	0.0390	7.7319	-0.0807	0.1306	0.0844	0.6771	3.4353
ETH	0.0028	0.0504	5.7681	-0.0685	0.1127	0.1140	0.6991	3.8163
BNB	0.0028	0.0583	19.1238	-0.0424	0.2495	0.1734	0.6493	3.1121
XRP	0.0017	0.0619	14.0692	-0.0064	0.2685	0.1442	0.6112	2.8208
DOGE	0.0048	0.1207	526.1476	0.0422	0.2245	0.0136	0.3604	2.1590
ADA	0.0044	0.0599	3.8096	-0.0390	0.1586	0.1273	0.7095	4.0604

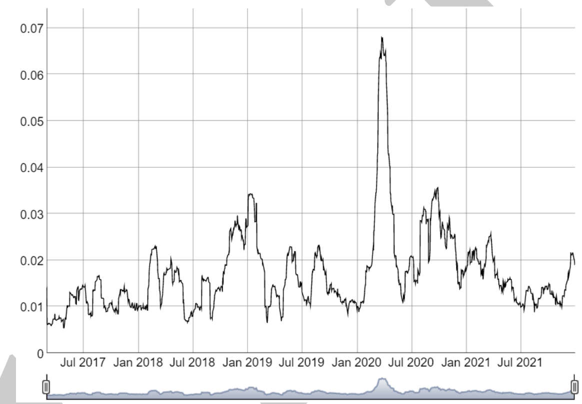

Fig. 6. Volatility Clustering - AAPL

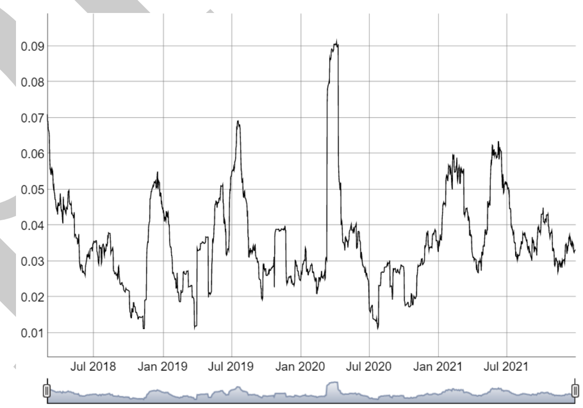

Fig. 7. Volatility Clustering - BTC

4 following computation of sample sign correlation of A_t. Moreover, using the asymptotic variance of the data-driven volatility estimator, fuzzy α-cuts of the estimates of the daily algo volatility and annualized SR (ASR) estimates are provided in Table II. Considering all the assets, AAPL has the highest ASR valued at 1.40. Among the cryptocurrencies, ADA has the highest ASR, and it is close to the ASR of AAPL. Further, VIX has the lowest ASR among the stocks/indexes, and XRP has the lowest among cryptocurrencies.

Similarly, sample standard deviation (s_n), mean absolute deviation (MAD) ($\hat{\rho}_A s_n$), and Value at Risk at $\alpha = 0.05$ ($VaR_{0.05}$) based on t distribution of A_t are used to estimate the daily volatility and annualized SR. Results are summarized in Tables III, IV, and V for all assets. It can be seen that fuzzy

TABLE II
α-CUTS FOR ANNUALIZED SR USING DDVE

Assets	DDVE	0.05-cut of DDVE	ASR	0.05-cut of ASR
AAPL	0.0193	(0.0181, 0.0204)	1.3957	(1.3167, 1.4847)
AMZN	0.0187	(0.0176, 0.0197)	0.5293	(0.5006, 0.5615)
FB	0.0210	(0.0197, 0.0222)	-0.0443	(-0.0418, -0.0472)
GOOG	0.0171	(0.0161, 0.0182)	0.2775	(0.2616, 0.2954)
VIX	0.0935	(0.0873, 0.0998)	-0.4010	(-0.3759, -0.4297)
BTC	0.0389	(0.0367, 0.0411)	1.0417	(0.9862, 1.1038)
ETH	0.0503	(0.0477, 0.0530)	1.0419	(0.9895, 1.1002)
BNB	0.0583	(0.0547, 0.0618)	0.8943	(0.8431, 0.9520)
XRP	0.0618	(0.0577, 0.0660)	0.5200	(0.4873, 0.5574)
DOGE	0.1193	(0.1033, 0.1353)	0.7629	(0.6728, 0.8810)
ADA	0.0598	(0.0567, 0.0629)	1.3778	(1.3104, 1.4525)

α-cut estimates of annualized SR using DDVE are narrower than that using sample Standard Deviation (SD) and MAD. MAD has the largest fuzzy α-cut estimates of annualized SR considering all the assets.

TABLE III
α-CUTS FOR ANNUALIZED SR USING SD

Assets	SD	0.05-cut CI of SD	ASR	0.05-cut of ASR
AAPL	0.0193	(0.0177, 0.0209)	1.3943	(1.2900, 1.5171)
AMZN	0.0187	(0.0173, 0.0200)	0.5290	(0.4936, 0.5699)
FB	0.0210	(0.0190, 0.0230)	-0.0442	(-0.0404, -0.0489)
GOOG	0.0171	(0.0158, 0.0185)	0.2774	(0.2572, 0.3009)
VIX	0.0936	(0.0803, 0.1069)	-0.4007	(-0.3507, -0.4674)
BTC	0.0390	(0.0358, 0.0421)	1.0405	(0.9627, 1.1319)
ETH	0.0504	(0.046, 0.0540)	1.0407	(0.9707, 1.1217)
BNB	0.0583	(0.0514, 0.0653)	0.8930	(0.7980, 1.0136)
XRP	0.0619	(0.0555, 0.0683)	0.5198	(0.4709, 0.5800)
DOGE	0.1207	(0.0489, 0.1926)	0.7540	(0.4727, 1.8623)
ADA	0.0599	(0.0562, 0.0637)	1.3751	(1.2943, 1.4666)

TABLE IV
α-CUTS FOR ANNUALIZED SR USING MAD

Assets	MAD	0.05-cut of MAD	ASR	0.05-cut of ASR
AAPL	0.0131	(0.0121, 0.0142)	2.0493	(1.8959, 2.2297)
AMZN	0.0130	(0.0121, 0.0139)	0.7595	(0.7087, 0.8182)
FB	0.0143	(0.0129, 0.0156)	-0.0652	(-0.0596, -0.0720)
GOOG	0.0116	(0.0107, 0.0125)	0.4098	(0.3800, 0.4446)
VIX	0.0599	(0.0514, 0.0685)	-0.6261	(-0.5480, -0.7301)
BTC	0.0264	(0.0243, 0.0285)	1.5367	(1.4218, 1.6717)
ETH	0.0352	(0.0327, 0.0378)	1.4887	(1.3885, 1.6045)
BNB	0.0379	(0.0334, 0.0424)	1.3752	(1.2289, 1.5610)
XRP	0.0378	(0.0339, 0.0417)	0.8503	(0.7704, 0.9488)
DOGE	0.0435	(0.0176, 0.0694)	2.0918	(1.3113, 5.1669)
ADA	0.0425	(0.0399, 0.0452)	1.9381	(1.8243, 2.0672)

We obtain the rolling forecasts of the daily and annualized SR. The most recent 1000 data points of algo returns are chosen to calculate the rolling forecasts of volatility. We compute the one-day ahead algo volatility forecast and RMSE using the window sizes of 60 and 90 for stocks/indexes and cryptocurrencies, respectively. The selected window sizes account for three months of consecutive observations. For each asset, we define the *daily SR forecast* as the average of one-day-ahead SR forecasts computed for each of the rolling windows. Results are summarized in Table VI and the average RMSEs of the assets are reported in the second column of Table VI. Among all the assets, DOGE has the highest RMSE.

TABLE V
α-CUTS FOR ANNUALIZED SR USING $VaR_{0.05}$

Assets	$VaR_{0.05}$	0.05-cut of $VaR_{0.05}$	ASR	0.05-cut of ASR
AAPL	0.0262	(0.0246, 0.0279)	1.0257	(0.9640, 1.0960)
AMZN	0.0270	(0.0254, 0.0286)	0.3654	(0.3451, 0.3883)
FB	0.0304	(0.0285, 0.0322)	-0.0306	(-0.0289, -0.0326)
GOOG	0.0244	(0.0229, 0.0259)	0.1950	(0.1837, 0.2078)
VIX	0.1301	(0.1215, 0.1386)	-0.2883	(-0.2706, -0.3086)
BTC	0.0541	(0.0509, 0.0573)	0.7495	(0.7080, 0.7962)
ETH	0.0722	(0.0683, 0.0762)	0.7258	(0.6879, 0.7681)
BNB	0.0780	(0.0731, 0.0829)	0.6681	(0.6285, 0.7129)
XRP	0.0789	(0.0734, 0.0843)	0.4079	(0.3817, 0.4379)
DOGE	0.0851	(0.0729, 0.0973)	1.0693	(0.9352, 1.2481)
ADA	0.0861	(0.0814, 0.0907)	0.9574	(0.9082, 1.0122)

BTC has the minimum RMSE among cryptocurrencies. However, note that the RMSE of all the stocks/indexes is smaller than BTC except for VIX.

The majority of the research literature only reports point forecasts of SR depicted by the red line in Figure 8 and 9. But, this paper also presents the fuzzy α-cuts of the forecast, which are more realistic. Table VI summarizes the results for all the assets, and Figure 8and 9 visualize the α-cuts for AAPL and BTC. The fuzzy forecast intervals are highly relevant for this forecasting problem due to the fact that volatility, and by extension Sharpe Ratio, is highly uncertain and exhibits temporal variance.

TABLE VI
SR FUZZY FORECASTS - DDVF

Assets	RMSE	DSR	ASR	0.05-cut of ASR	0.01-cut of ASR
AAPL	0.015	0.097	1.547	(-4.269, 7.363)	(-6.097, 9.191)
AMZN	0.015	0.024	0.376	(-3.957, 4.710)	(-5.319, 6.072)
FB	0.019	-0.003	-0.050	(-4.166, 4.065)	(-5.459, 5.359)
GOOG	0.015	0.029	0.467	(-4.067, 5.002)	(-5.492, 6.427)
VIX	0.074	-0.049	-0.773	(-5.070, 3.524)	(-6.420, 4.874)
BTC	0.038	0.060	1.146	(-3.031, 5.322)	(-4.343, 6.634)
ETH	0.046	0.051	0.981	(-2.940, 4.902)	(-4.172, 6.135)
BNB	0.051	0.019	0.365	(-5.186, 5.917)	(-6.931, 7.661)
XRP	0.062	0.044	0.832	(-2.999, 4.663)	(-4.203, 5.867)
DOGE	0.125	0.054	1.030	(-3.807, 5.868)	(-5.327, 7.388)
ADA	0.053	0.078	1.481	(-2.889, 5.851)	(-4.262, 7.224)

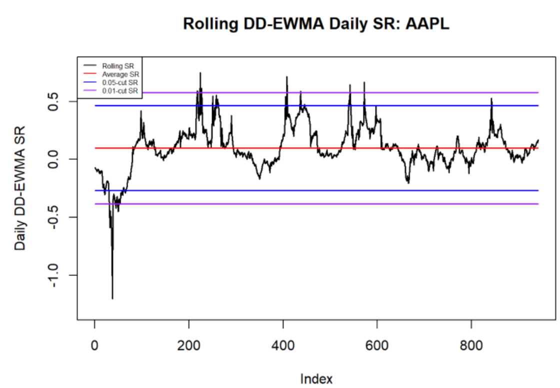

Fig. 8. Daily rolling SR forecasts using DD-EWMA - AAPL

Next, we compute daily and annualized rolling SR forecasts applying the neuro volatility model using the same window sizes as the data-driven rolling forecast described above. Table

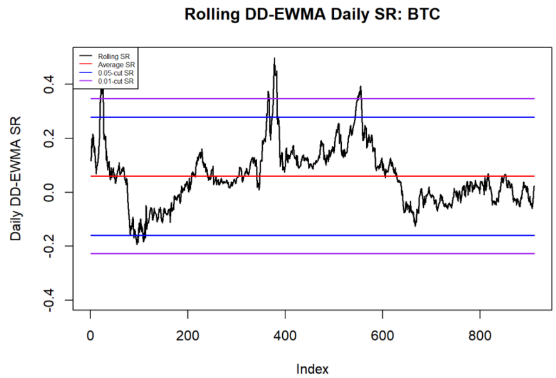

Fig. 9. Daily rolling SR forecasts using DD-EWMA - BTC

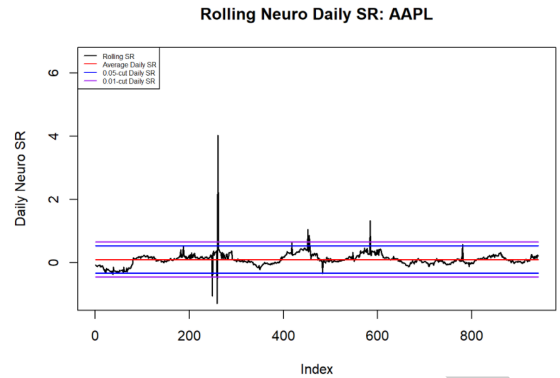

Fig. 10. Daily rolling SR forecasts using Neuro volatility model - AAPL

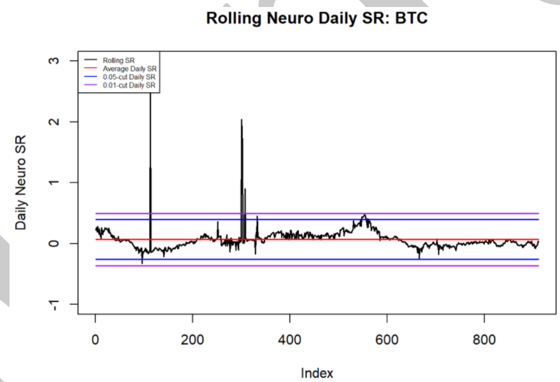

Fig. 11. Daily rolling SR forecasts using Neuro volatility model - BTC

VII summarizes the results from the neuro-volatility model. Also, daily rolling SR forecasts and their averages for AAPL and BTC are plotted in Figure 10 and Figure 11 respectively. Fuzzy α-cuts for assets are provided in Table VII, and plotted in Figure 10 and 11. Observations suggest that data-driven volatility forecast (DDVF) provides narrower α-cuts compared to neuro-volatility forecasts (NVF). Computation times for both DDVF and NVF were investigated under both stocks/indexes and cryptocurrencies. NVF takes more computation time compared to DDVF. Thus, it is important to investigate further if the accuracy of the forecasts could be improved using DDVF and NVF for stocks and cryptocurrencies. All the experiments were executed on a computer with an Intel Core(TM) i5-8265U CPU, running 4 Cores (8 logical cores) at a clock speed of 1.60GHz and 8GB of RAM on the Windows 11 operating system.

TABLE VII
SR Fuzzy Forecasts - NVF

Assets	DSR	ASR	0.05-cut of ASR	0.01-cut of ASR
AAPL	0.100	1.581	(-4.420, 7.581)	(-6.305, 9.467)
AMZN	0.025	0.403	(-3.915, 4.721)	(-5.272, 6.078)
FB	-0.006	-0.102	(-3.804, 3.600)	(-4.967, 4.763)
GOOG	0.020	0.319	(-6.300, 6.939)	(-8.380, 9.019)
VIX	-0.044	-0.697	(-4.931, 3.537)	(-6.262, 4.867)
BTC	0.067	1.271	(-4.988, 7.530)	(-6.955, 9.497)
ETH	0.068	1.297	(-10.081, 12.674)	(-13.656, 16.249)
BNB	-0.001	-0.018	(-23.444, 23.407)	(-30.805, 30.768)
XRP	0.051	0.983	(-4.656, 6.623)	(-6.428, 8.395)
DOGE	0.067	1.273	(-16.437, 18.983)	(-22.002, 24.548)
ADA	0.076	1.444	(-5.525, 8.414)	(-7.715, 10.604)

As described in our approach, we use the non-linear adaptive trapezoidal fuzzy numbers to compute forecast intervals. The membership function for such a fuzzy number is parameterized by two additional variables, m and n. We compute the fuzzy intervals using different values of m and n. We consider these six cases for both the neuro forecast and data-driven forecast : **case1**: $m = 0.25$, $n = 0.25, 0.50, 0.75$, **case2**: $m = 0.50$, $n = 0.25, 0.50, 0.75$, **case3**: $m = 0.75$, $n = 0.25, 0.50, 0.75$, **case4:** $m = 0.50$, $n = 1.0, 2.0, 3.0$, **case5**: $m = 1.0$, $n = 1.0, 2.0, 3.0$, **case6**: $m = 2.0$, $n = 1.0, 2.0, 3.0$). Table VIII reports the width of fuzzy intervals for BTC using DDVF and NVF when ($m = 0.25$, $n = 0.75$), ($m = 1.0$, $n = 1.0$), and ($m = 1.0$, $n = 2.0$). We found that for all the top six cryptocurrencies, as well as for the stocks AMZN, FB, and GOOG, the fuzzy interval widths computed for the DDVF are narrower than the corresponding intervals obtained for the NVF (Table IX). However, for the stock AAPL and the CBOE Volatility index (VIX), the fuzzy intervals obtained for NVF are smaller compared to DDVF (Table X).

TABLE VIII
α-cuts of Volatility Interval Widths for BTC using DDVF and NVF

	DDVF			NVF		
α	$m = 0.25$ $n = 0.75$	$m = 1.0$ $n = 1.0$	$m = 1.0$ $n = 2.0$	$m = 0.25$ $n = 0.75$	$m = 1.0$ $n = 1.0$	$m = 1.0$ $n = 2.0$
0.0	0.0135	0.0135	0.0135	0.6259	0.6259	0.6259
0.1	0.0132	0.0127	0.0113	0.6183	0.5779	0.5426
0.2	0.0128	0.0120	0.0104	0.6063	0.5299	0.4895
0.3	0.0122	0.0112	0.0096	0.5905	0.4819	0.4414
0.4	0.0116	0.0104	0.0089	0.5696	0.4339	0.3959
0.5	0.0109	0.0096	0.0083	0.5412	0.3859	0.3521
0.6	0.0101	0.0089	0.0077	0.5022	0.3379	0.3094
0.7	0.0092	0.0081	0.0072	0.4483	0.2900	0.2676
0.8	0.0082	0.0073	0.0067	0.3749	0.2420	0.2265
0.9	0.0071	0.0065	0.0062	0.2762	0.1940	0.1860
1.0	0.0058	0.0058	0.0058	0.1460	0.1460	0.1460

Thus, the narrowest α-cuts of the annualized volatility are provided using the data-driven volatility forecast for cryp-

TABLE IX
α-CUTS OF VOLATILITY INTERVAL WIDTHS FOR AMZN USING DDVF AND NVF

	DDVF			NVF		
	$m = 0.25$	$m = 1.0$	$m = 1.0$	$m = 0.25$	$m = 1.0$	$m = 1.0$
α	$n = 0.75$	$n = 1.0$	$n = 2.0$	$n = 0.75$	$n = 1.0$	$n = 2.0$
0.0	0.1170	0.1170	0.1170	0.1982	0.1982	0.1982
0.1	0.1158	0.1109	0.1054	0.1946	0.1834	0.1667
0.2	0.1140	0.1047	0.0985	0.1890	0.1686	0.1496
0.3	0.1116	0.0986	0.0924	0.1821	0.1539	0.1348
0.4	0.1086	0.0925	0.0867	0.1736	0.1391	0.1212
0.5	0.1047	0.0864	0.0812	0.1631	0.1243	0.1084
0.6	0.0996	0.0803	0.0759	0.1500	0.1096	0.0961
0.7	0.0927	0.0742	0.0708	0.1333	0.0948	0.0843
0.8	0.0836	0.0681	0.0657	0.1120	0.0801	0.0728
0.9	0.0715	0.0620	0.0608	0.0849	0.0653	0.0615
1.0	0.0559	0.0559	0.0559	0.0505	0.0505	0.0505

TABLE X
α-CUTS OF VOLATILITY INTERVAL WIDTHS FOR AAPL USING DDVF AND NVF

	DDVF			NVF		
	$m = 0.25$	$m = 1.0$	$m = 1.0$	$m = 0.25$	$m = 1.0$	$m = 1.0$
α	$n = 0.75$	$n = 1.0$	$n = 2.0$	$n = 0.75$	$n = 1.0$	$n = 2.0$
0.0	0.2512	0.2512	0.2512	0.1397	0.1397	0.1397
0.1	0.2478	0.2405	0.2250	0.1350	0.1272	0.1053
0.2	0.2427	0.2298	0.2121	0.1278	0.1148	0.0897
0.3	0.2365	0.2192	0.2014	0.1191	0.1024	0.0772
0.4	0.2291	0.2085	0.1918	0.1092	0.0899	0.0663
0.5	0.2205	0.1979	0.1830	0.0980	0.0775	0.0565
0.6	0.2103	0.1872	0.1747	0.0854	0.0650	0.0473
0.7	0.1982	0.1765	0.1667	0.0711	0.0526	0.0387
0.8	0.1836	0.1659	0.1591	0.0549	0.0401	0.0305
0.9	0.1659	0.1552	0.1517	0.0364	0.0277	0.0227
1.0	0.1446	0.1446	0.1446	0.0152	0.0152	0.0152

tocurrencies, whereas for regular stocks, data-driven or neuro volatility forecasts provide narrowest α-cuts depending on the stocks/indexes. This holds for any two points of n and m (tuning parameters) in their parameter space.

V. Conclusion

This paper presents simple yet effective data-driven and neuro models for fuzzy volatility forecasting of algo returns. The algo returns are calculated using an SMA crossover trading strategy based on the smoothed value of adjusted closing prices. Both the long-term and short-term window sizes used in the SMA crossover strategy are calculated in a data-driven fashion based on annualized Sharpe ratio. Also, we compute the fuzzy alpha cuts for annualized Sharpe ratio to accommodate the uncertainty associated with returns. The superiority of fuzzy forecasts over minimum mean squared forecasts had been first demonstrated in [28]. However, a drawback of this work is the use of simple triangular fuzzy numbers for modelling volatility, which is inappropriate as it makes an unrealistic assumption of the symmetric distribution of volatility. A key contribution of this work is the modelling of the volatility of algo returns as an asymmetric non-linear adaptive fuzzy number.

Also, this paper analyzes the relative effectiveness of data-driven and neuro models for the volatility forecasting of two distinct asset classes, Cryptocurrencies and traditional financial assets (stocks, and indices). In order to compare the quality of forecasts obtained by data-driven and neuro volatility models, fuzzy forecasts of volatility are obtained. The narrower fuzzy intervals imply a better forecast quality. It is observed that the data-driven models produce better forecasts for cryptocurrencies, while for the regular stocks and indices, no such definitive conclusion could be drawn. That is, for AMZN, FB, and GOOG, the data-driven model produced narrower fuzzy intervals, while for AAPL and VIX, the fuzzy intervals for the neuro volatility model are narrower.

Acknowledgment

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