

Fluxions, Forces, and Fields

An overview of the mathematisation of physics in Europe through the modern period

Zella Baig

February 7, 2021

A Look Back

Mathematics in Modern Physics

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$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

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$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = \hat{H} |\psi\rangle$$

Mathematics in Modern Physics

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Mathematics in Modern Physics

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- The *expectation* (or even requirement) that a physicist be mathematically adept only arose \sim C20

Mathematics in Modern Physics

- Vector notation had only been around for ~ 50 years!

Tracing Back

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- Much of the groundwork had been laid by Lorentz, with his *Theory of Corresponding States*¹ . . . which generalised length-contraction theory to Maxwell's equations
- Interestingly, much of the work on length contraction was done by George Fitzgerald, who was also influenced greatly by Maxwell

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- Physics in Thomson's day had been centred around energy (and thus dynamics)...
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- ...Who worked using methods derived from Newton's work on celestial motion

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- Baconian ideals,
- Collaborative bodies such as the Royal Institute,
- And (again from Newton) *hypotheses non fingo*ⁱ

ⁱTo be discussed later on

Pre-Newtonianism

Natural Philosophy Pre Early Modern Period

- 'Physics' (or Natural Philosophy) focused largely on astronomy
 - the “noblest of all” mathematical disciplines³

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Isaac Newton (1642-1727)

Newtonian Mathematical Ideals

- Saw God as mathematical, with a fondness for geometryⁱⁱ

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- Would have grown up with Cartesianism (deductive reasoning), but clashed with it later in life
- Sought 'elegance' in mathematics⁵

⁵Westfall, 1981.

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- Important to note the ramifications of geometric arguments

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 - *The Development of Newtonian Calculus*⁸
 - *Dot-Age*⁹
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 - *The Development of Newtonian Calculus*⁸
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 - *The History of Calculus*¹⁰
- Regardless, we are interested in the **physical** influences of calculus

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The Dynamical Age: Continental Physics

Putting Calculus to Use

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 - Clairaut's work on the three-body problem particularly important
- Development of Lagrangian mechanics & applications to further contexts (such as the motion of sound)

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- Competitions set-up with e.g. the Society of Arcueil to promote mathematical collaboration
- Development of light, heat, and electromagnetic theory with various contestants (e.g. Fourier) - via Laplacian methods

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Energy Physics

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- Heat a “dynamical form of mechanical effect”, with there being “absolute numerical relations” between heat and power¹⁵

¹⁴Smith, 1978.

¹⁵Thomson, 1851.

- Together, they publish *Treatise on Natural Philosophy*:¹⁶ the first high-level **mathematically-inclined** physics textbookⁱⁱⁱ, as well as a synthesis of their work on energy

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- Intended to “give in three moderate volumes a far more complete course of Physics, Experimental and Mathematical, than exists”¹⁷

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James Maxwell (1831-1879)

- Both mathematically inclined Scottish physicists (contrasting to e.g. Faraday)¹⁸

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 - “The discussion of the various forms of energy . . . constitutes the whole of physical science”¹⁹
- On electromagnetism, pondered the nature of the 'store' of energy, e.g. in his *Dynamical Theory*²⁰

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²⁰Maxwell, 1865.

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- Work culminates in his *Treatise*²³

²¹Maxwell, 1856.

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²³Maxwell, 1873.

Vortex Model

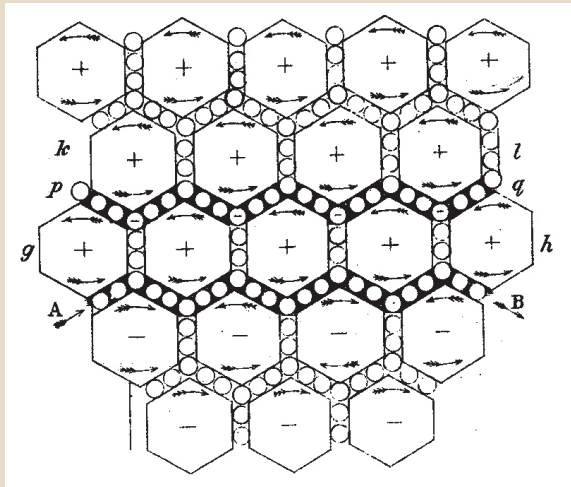


Figure 1: Maxwell's 'vortex & idle-wheel' model, in *Physical Lines*

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- Greatly develops ~~vector~~ calculus out of necessity for his mechanical models - Maxwell didn't have vector notation at his disposal!

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- Greatly develops vector calculus out of necessity for his mechanical models
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 - From 20 equations to 4
- Vector notation itself only introduced in 1843, by Hamilton, with a 'recognisable' form appearing later that century via Clifford, Gibbs, and Heaviside²⁵

²⁴Hunt, 2012.

²⁵Crowe, 1994.

Aether Theory & Special Relativity

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 - A **purely** mathematical construction²⁷

²⁶Hunt, 1994.

²⁷Brown, 2005.

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Henri Poincaré (1854-1912)

- Willing to ignore aether hypotheses, as mathematically unnecessary²⁸

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- Develops theory to be (functionally) identical to modern Lorentz transformations
 - Unwilling to assign physicality to ideas: “Of hypotheses there is never lack”³⁰
- Einstein would soon go on to have his *annus mirabilis* and completely shift away from aether theory

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²⁹Darrigol, 1995.

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Conclusion

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- Initial shift with Newton's development of *experimental philosophy* and quantisation of nature

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- Development of continental force-based physics
- Shift towards disciplinary rigour with Thomson and others
- Maxwellian development of electromagnetic theory
- The final steps away from the aether - after thousands of years

Thank You

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