Part - 01

Question # 02

$$\frac{dN}{dr} = -\frac{2N}{r} + \frac{2}{3r} - \frac{4Br}{3} \quad \rightarrow (A)$$

$$N(0.1) = 0.1$$

(i) Solving Exactly

General form of ODE

$$\frac{dN}{dr} + P(N)r = Qr$$

from equation (A)

$$\frac{dN}{dr} + \frac{2N}{r} = \frac{2}{3r} - \frac{4Br}{3} \quad \rightarrow (i)$$

Integrating factor

$$IF = \int \frac{2}{r}\, dr$$

$$IF = 2\int \frac{1}{r}\, dr = 2\ln r$$

$$IF = \ln r^2$$

Multiply e^{IF} by equation (i

$$e^{kr^2}\frac{dN}{dr} + \frac{2Ne^{kr^2}}{r} = e^{kr^2}\frac{2}{3r} - \frac{4Br}{3}e^{kr^2}$$

$$\Rightarrow r^2\frac{dN}{dr} + 2Nr = \frac{2}{3}r - \frac{4Br^3}{3}$$

Integrating Both sides.

$$\int \frac{d}{dr}(Nr^2) = \int \frac{2}{3}r - \int \frac{4}{3}Br^3$$

$$\Rightarrow Nr^2 = \frac{2}{3}\frac{r^2}{2} - \frac{4B}{3}\frac{r^4}{4} + C$$

$$\Rightarrow Nr^2 = \frac{r^2}{3} - \frac{Br^4}{3} + C$$

$$\Rightarrow N = \frac{1}{3} - \frac{Br^2}{3} + \frac{C}{r^2}$$

$$\Rightarrow N = \frac{1}{3} - \frac{B}{3}r^2 + C_1$$

$$N(0.1) = 0.1$$

$$0.1 = \frac{1}{3} - \frac{B(0.1)^2}{3} + C_1$$

$$\Rightarrow \boxed{C_1 = \frac{-0.7 + B(0.01)}{3}}$$

(ii) Solving by Euler Method

Let $B = 1$ = constant

where $0.1 \leq x \leq 5$

$h = 0.7 \Rightarrow n = \frac{b-a}{h} = \frac{5-0.1}{0.7}$

$\Rightarrow n = 7$

ON Calculator $Y = Y + 0.7\, f(x, y)$

n	x_n	N_n	N_{n+1}
0	0.1	0.1	3.273333
1	0.8	3.273333	-2.618333
2	1.5	-2.618333	-1.263444
3	2.2	-1.263444	-2.300646
4	2.9	-2.300646	-3.735737
5	3.6	-3.735737	-5.513321
6	4.3	-5.513321	-7.623092
7	5.0	-7.623092	

$$\boxed{N(5) = -7.623092}$$

Solving by RK - Method

By Python code, we get following

1st Iteration

$K_1 = 3.1733$; $K2 = -4.6304$

$K3 = 7.5087$; $K4 = -13.4786$

$$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$K = -0.7581$

$$\boxed{y_1 = -0.6581}$$

2nd Iteration

$K_1 = 0.9883$; $K_2 = -0.4680$

$K_3 = 0.4185$; $K_4 = -0.8652$

$K = 0.0040$

$$\boxed{y_2 = -0.6541}$$

3rd Iteration

$K_1 = -0.4784$; $K_2 = -0.7984$

$K_3 = -0.6773$; $K_4 = -0.9939$

$K = -0.7373$

$$\boxed{y_3 = -1.3914}$$

4th Iteration

$K_1 = -0.9558$; $K_2 = -1.1707$

$K_3 = -1.1117$; $K_4 = -1.3374$

$K = -1.1430$

$$\boxed{y_4 = -2.5344}$$

5th Iteration

$K_1 = -1.3222$; $K_2 = -1.5132$

$K_3 = -1.4721$; $K_4 = -1.6723$

$K = -1.4942$

$$\boxed{y_5 = -4.0286}$$

6th Iteration

$K_1 = -1.6637$; $K_2 = -1.8458$

$K_3 = -1.8136$; $K_4 = -2.0027$

$K = -1.8309$

$$\boxed{y_6 = -5.8594}$$

7th Iteration

$K_1 = -1.9971$; $K_2 = -2.1749$

$K_3 = -2.1481$; $K_4 = -2.3312$

$K = -2.1624$

$$\boxed{y_7 = -8.0218}$$

$$\boxed{y(5) = -8.0218}$$

or

$$\boxed{y_7 = -8.0218}$$

(vi) Interpolate the Solution

n	x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	0.1	0.1	3.17	-9.05	16.28	-25.9	37.52	-51.1
1	0.8	3.27	-5.88	7.23	-9.62	11.62	-13.58	15.52
2	1.5	-2.61	1.35	-2.39	2	-1.96	1.94	
3	2.2	-1.26	-1.04	-0.39	0.04	-0.02		
4	2.9	-2.30	-1.43	-0.35	0.02			
5	3.6	-3.73	-1.78	-0.33				
6	4.3	-5.51	-2.12					
7	5	-7.62						

$\Delta^7 y$
66.62

General Equation

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$+ \cdots \frac{u(u-1)(u-2)(u-3)(u-4)(u-5)(u-6)}{7!}\Delta^7 y_0 + \cdots$$

$$u = \frac{x - x_0}{h} \quad \text{where} \quad h = 0.7$$

$$u = \frac{x - 0.1}{0.7}$$

$$y = 0.1 + \frac{\left(\frac{x-0.1}{0.7}\right)(3.17)}{1!} + \frac{\left(\frac{x-0.1}{0.7}\right)\left(\frac{x-0.1}{0.7}-1\right)(-9.05)}{2!}$$

$$+ \cdots\cdots\cdots + \frac{\left(\frac{x-0.1}{0.7}\right)\left(\frac{x-0.1}{0.7}-1\right)\left(\frac{x-0.1}{0.7}-2\right)\left(\frac{x-0.1}{0.7}-3\right)\left(\frac{x-0.1}{0.7}-4\right)\left(\frac{x-0.1}{0.7}-5\right)\left(\frac{x-0.1}{0.7}-6\right)(166.62)}{7!}$$

Polynomial Interpolation formula.

Question # 04

2nd Order R.K Method of 4

$$y'' + 12y' + 36y = 0 ; \quad \rightarrow (I$$

$$y(1) = 0 \quad ; \quad y'(1) = 1$$

Let

$$z = y' \quad \rightarrow (II$$

then

$$z' = y''$$

eq (I and (II become

$$\boxed{\frac{dy}{dx} = z}$$

$$\boxed{\frac{dz}{dx} = -12z - 36y}$$

where $y(1) = 0 ; \quad z(1) = 1$

$X_0 = 1 ; \quad y_0 = 0 ; \quad z_0 = 1$

Assuming we have to find $y(1.2)$

For $y(1.2)$, take $h = 0.2$

$k_1 = 0.2\, f(1, 0.1) = 0.2$

~~$k_1 = 0.2\, f(1.1, 0.1, 0.2)$~~

$l_1 = 0.2\, g(1, 0, 1) = -2.4$

$k_2 = 0.2\, f(1.1, 0.1, -0.2) = -0.04$

$l_2 = 0.2\, g(1.1, 0.1, -0.2) = -0.24$

$k_3 = 0.2\, f(1.1, -0.02, 0.88) = 0.176$

$l_3 = 0.2\, g(1.1, -0.02, 0.88) = -1.968$

$k_4 = 0.2\, f(1.2, 0.176, -0.968) = -0.1936$

$l_4 = 0.2\, g(1.2, 0.176, -0.968) = 1.056$

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.0463990$

$l = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) = -0.960000$

$y_1 = y_0 + k \Rightarrow \boxed{y_1 = 0.0463990}$

$z_1 = z_0 + k \Rightarrow z_1 = 0.040000$

$y(1.2) = 0.0463990$