1D Kalman Filter for Radio-Controlled Toy Car

1 Introduction

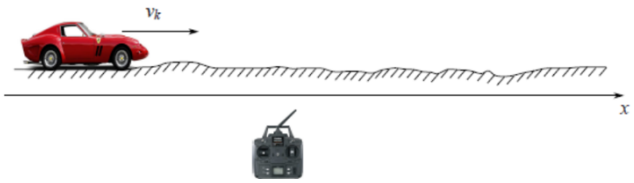

Figure 1: Radio control toy car used in simulations.

This document provides a comprehensive introduction to the implementation of a 1D Kalman Filter, aimed to estimate the position of a radio-controlled toy car moving along a one-dimensional track. The Kalman Filter combines noisy sensor data with a predictive model to enhance the accuracy of position estimates over time, making it an essential tool in control systems and robotics.

2 Problem Definition

The state of the radio-controlled toy car at any time step k is defined by its position x_k on a linear track. The dynamics and measurement models are given as follows:

2.1 State Transition Model

The car's state is updated based on its velocity v_k and the time interval dt:

$$x_{k+1} = x_k + v_k \cdot dt \tag{1}$$

Here, v_k represents the constant or slowly varying velocity of the car.

2.2 Observation Model

The observed position z_k of the car includes Gaussian noise, representing measurement errors:

$$z_k = x_k + \text{noise} \tag{2}$$

The noise is assumed to follow a normal distribution with zero mean and a variance that is known a priori.

3 Kalman Filter Implementation

The implementation of the Kalman Filter occurs in two phases: prediction and update. This process iteratively refines the estimates of the car's position.

3.1 Initialization

Set the initial estimates for the car's position $\hat{x}_0$ and estimation error covariance P_0. The parameters for process noise covariance Q and measurement noise covariance R are also established based on the expected level of noise in the system.

3.2 Prediction

- Predict the state: $\hat{x}_{k|k-1} = \hat{x}_{k-1} + v_{k-1} \cdot dt$
- Predict the error covariance: $P_{k|k-1} = P_{k-1} + Q$

3.3 Update

- Compute Kalman Gain: $K_k = \frac{P_{k|k-1}}{P_{k|k-1}+R}$
- Update the estimate using the new measurement: $\hat{x}_k = \hat{x}_{k|k-1} + K_k(z_k - \hat{x}_{k|k-1})$
- Adjust the error covariance: $P_k = (1 - K_k)P_{k|k-1}$