DONRS HW2

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1- Robot Description:

- Wide range of ambient conditions
- Low maintenance
- Extreme precision
- Convincing in any position
- High speed
- Optimal work envelope

KR 10 R1100-2 has six axis and:

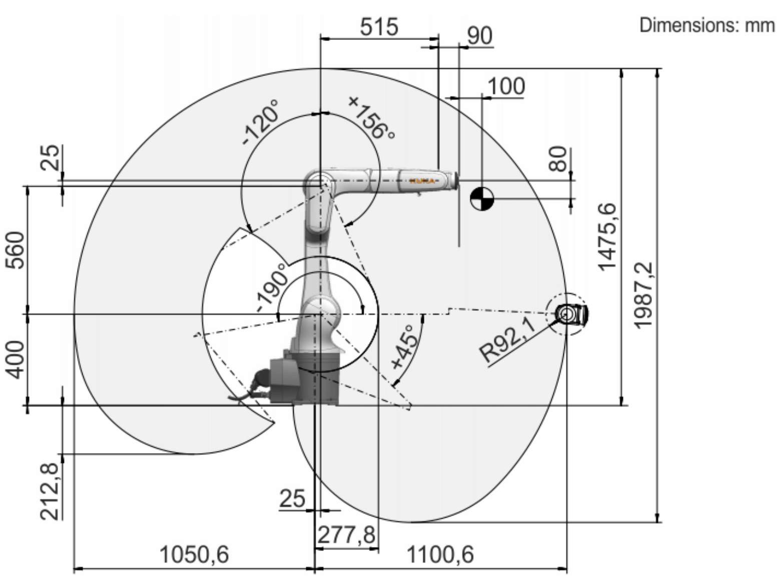
- Minimum cycle times. The KR AGILUS-2 has six axes and is consistently rated for particularly high working speeds. At the same time, it offers extreme precision.
- Space-saving integration. Low space requirements and the installation in any position make the KR AGILUS-2 extremely adaptable.
- Integrated energy supply system. Thanks to the integrated energy supply system, the KR AGILUS-2 impresses with reduced disruptive contours and the reliable supply of energy to tools.
- Two protection ratings. The KR AGILUS-2 conforms to protection rating IP 65 and, with pressurization, even meets the requirements of the higher protection rating IP 67.
- KR C4 architecture and functionalities. KUKA small robots are every bit as versatile as their larger relatives. They are operated via the KR C4 compact and KR C4 smallsize-2 controllers, with the same range of functions as the service-proven KR C4 controller.
- KUKA.SafeOperation. KUKA small robots set standards in safety. Only they offer the KUKA.SafeOperation functionality, which radically simplifies the effective cooperation of humans and machines.

KR AGILUS-2	KR 10 R1100-2
Max. reach	1,100 mm
Rated payload	10 kg
Pose repeatability	±0.01 mm
Number of axes	6
Mounting position	Floor, ceiling, wall, angle
Variant	–
Robot footprint	208 mm x 208 mm
Weight (excluding controller), approx.	55 kg

Axis data /
Range of motion

Axis 1 (A1)	+ / –170º
Axis 2 (A2)	+45º / –190º
Axis 3 (A3)	+156º / –120º
Axis 4 (A4)	+ / –185º
Axis 5 (A5)	+ / –120º
Axis 6 (A6)	+ / –350º

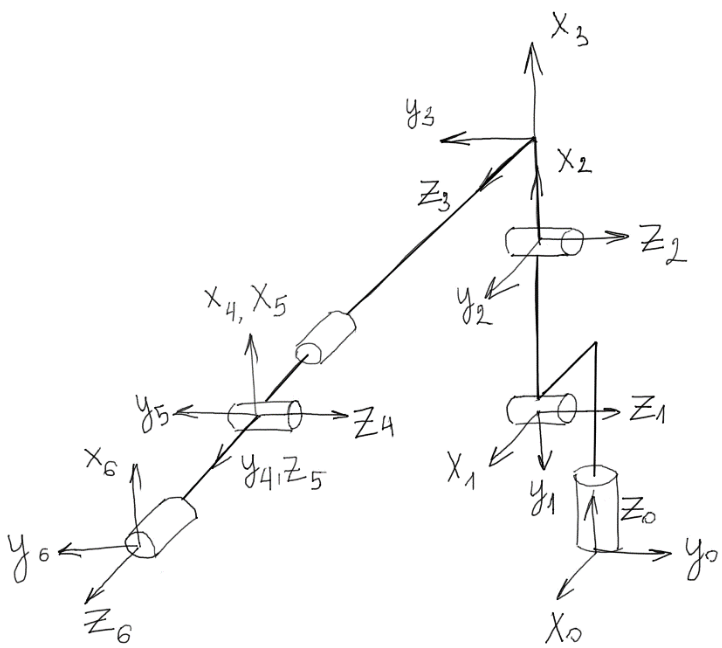
Workspace graphic

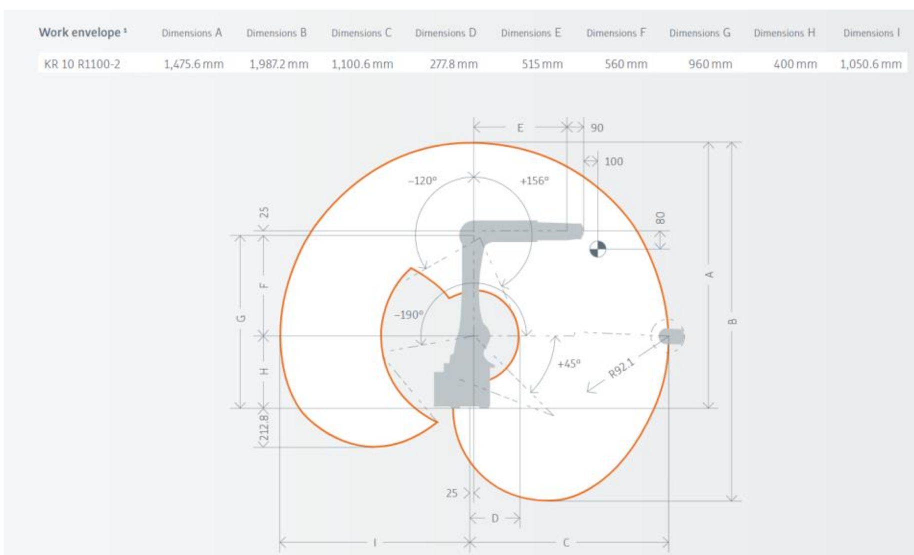


Technical data

Maximum reach	1101 mm
Maximum payload	11.1 kg
Pose repeatability (ISO 9283)	± 0.02 mm
Number of axes	6
Mounting position	Floor; Ceiling; Wall; Desired angle
Footprint	208 mm x 208 mm
Weight	approx. 55 kg

The kinematic Model





Work envelope [1]	Dimensions A	Dimensions B	Dimensions C	Dimensions D	Dimensions E	Dimensions F	Dimensions G	Dimensions H	Dimensions I
KR 10 R1100-2	1,475.6 mm	1,987.2 mm	1,100.6 mm	277.8 mm	515 mm	560 mm	960 mm	400 mm	1,050.6 mm

The Forward kinematics:

Get the forward kinematics

```
%% Define the forward kinematics

syms q1 q2 q3 q4 q5 q6 a1 a2 a4
H =  Rot_z(q1)*Tr_z(a1)*Rot_y(q2)*Tr_x(a2)*Rot_y(q3)*Tr_x(a4)*Rot_x(q4) * Rot_y(q5) * Rot_x(q6)
h = simplify (H)
```

h =

$$
\begin{pmatrix}
\cos(q_5)\,\sigma_{10} - \sin(q_5)\,\sigma_9 & \sin(q_6)\,\sigma_4 - \cos(q_6)\,\sigma_6 & \sin(q_6)\,\sigma_6 + \cos(q_6)\,\sigma_4 & \cos(q_1)\,\sigma_2 \\
\sin(q_5)\,\sigma_7 - \cos(q_5)\,\sigma_8 & \cos(q_6)\,\sigma_5 - \sin(q_6)\,\sigma_3 & -\sin(q_6)\,\sigma_5 - \cos(q_6)\,\sigma_3 & \sin(q_1)\,\sigma_2 \\
-\sin(q_2+q_3)\cos(q_5) - \cos(q_2+q_3)\cos(q_4)\sin(q_5) & \cos(q_2+q_3)\cos(q_6)\sin(q_4) - \sin(q_6)\,\sigma_1 & -\cos(q_6)\,\sigma_1 - \cos(q_2+q_3)\sin(q_4)\sin(q_6) & a_1 - a_4\sin(q_2+q_3) - a_2\sin(q_2) \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

Rotation part

```
T4_5_6 =

[          cos(q5),                                   sin(q5)*sin(q6),                            cos(q6)*sin(q5), 0]
[  sin(q4)*sin(q5), cos(q4)*cos(q6) - cos(q5)*sin(q4)*sin(q6), - cos(q4)*sin(q6) - cos(q5)*cos(q6)*sin(q4), 0]
[ -cos(q4)*sin(q5), cos(q6)*sin(q4) + cos(q4)*cos(q5)*sin(q6),   cos(q4)*cos(q5)*cos(q6) - sin(q4)*sin(q6), 0]
[                0,                                          0,                                           0, 1]
```

Translation part

```
T0_3 =

[ cos(q2 + q3)*cos(q1), -sin(q1), sin(q2 + q3)*cos(q1), cos(q1)*(a4*cos(q2 + q3) + a2*cos(q2))]
[ cos(q2 + q3)*sin(q1),  cos(q1), sin(q2 + q3)*sin(q1), sin(q1)*(a4*cos(q2 + q3) + a2*cos(q2))]
[        -sin(q2 + q3),        0,         cos(q2 + q3),      a1 - a4*sin(q2 + q3) - a2*sin(q2)]
[                    0,        0,                    0,                                      1]
```

To use the matrix as possible as I can I will assume it like this:

$$
T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}
$$

Total forward kinematics matrix build with DH parameters:

$$a_{11} = s_6\ (c_4\, s_1 + s_4\ (c_1\, s_2\, s_3 - c_1\, c_2\, c_3)) + c_6\ (c_5\ (s_1\, s_4 - c_4\ (c_1\, s_2\, s_3 - c_1\, c_2\, c_3)) - s_5\ (c_1\, c_2\, s_3 + c_1\, c_3\, s_2))$$

$$a_{12} = c_6\ (c_4\, s_1 + s_4\ (c_1\, s_2\, s_3 - c_1\, c_2\, c_3)) - s_6\ (c_5\ (s_1\, s_4 - c_4\ (c_1\, s_2\, s_3 - c_1\, c_2\, c_3)) - s_5\ (c_1\, c_2\, s_3 + c_1\, c_3\, s_2))$$

$$a_{13} = -s_5\ (s_1\, s_4 - c_4\ (c_1\, s_2\, s_3 - c_1\, c_2\, c_3)) - c_5\ (c_1\, c_2\, s_3 + c_1\, c_3\, s_2)$$

$$a_{14} = \frac{c_1}{40} + \frac{14\, c_1\, c_2}{25} - \frac{c_1\, s_2\, s_3}{40} + \frac{9\, s_1\, s_4\, s_5}{100} + \frac{9\, s_{2,3}\, c_1\, c_5}{100} + \frac{c_1\, c_2\, c_3}{40} - \frac{103\, c_1\, c_2\, s_3}{200} - \frac{103\, c_1\, c_3\, s_2}{200} + \frac{9\, c_1\, c_2\, c_3\, c_4\, s_5}{100} - \frac{9\, c_1\, c_4\, s_2\, s_3\, s_5}{100}$$

$$a_{21} = -s_6\ (c_1\, c_4 - s_4\ (s_1\, s_2\, s_3 - c_2\, c_3\, s_1)) - c_6\ (c_5\ (c_1\, s_4 + c_4\ (s_1\, s_2\, s_3 - c_2\, c_3\, s_1)) + s_5\ (c_2\, s_1\, s_3 + c_3\, s_1\, s_2)$$

$$a_{22} = s_6\ (c_5\ (c_1\, s_4 + c_4\ (s_1\, s_2\, s_3 - c_2\, c_3\, s_1)) + s_5\ (c_2\, s_1\, s_3 + c_3\, s_1\, s_2)) - c_6\ (c_1\, c_4 - s_4\ (s_1\, s_2\, s_3 - c_2\, c_3\, s_1))$$

$$a_{23} = s_5\ (c_1\, s_4 + c_4\ (s_1\, s_2\, s_3 - c_2\, c_3\, s_1)) - c_5\ (c_2\, s_1\, s_3 + c_3\, s_1\, s_2)$$

$$a_{24} = \frac{s_1}{40} + \frac{14\, c_2\, s_1}{25} - \frac{103\, c_2\, s_1\, s_3}{200} - \frac{103\, c_3\, s_1\, s_2}{200} - \frac{9\, c_1\, s_4\, s_5}{100} - \frac{s_1\, s_2\, s_3}{40} + \frac{9\, s_{2,3}\, c_5\, s_1}{100} + \frac{c_2\, c_3\, s_1}{40} + \frac{9\, c_2\, c_3\, c_4\, s_1\, s_5}{100} - \frac{9\, c_4\, s_1\, s_2\, s_3\, s_5}{100}$$

$$a_{31} = s_{2,3}\, s_4\, s_6 - c_6\ (s_{2,3}\, s_5 + s_{2,3}\, c_4\, c_5)$$

$$a_{32} = s_6\ (s_{2,3}\, s_5 + s_{2,3}\, c_4\, c_5) + s_{2,3}\, c_6\, s_4$$

$$a_{33} = s_{2,3}\, c_4\, s_5 - s_{2,3}\, c_5$$

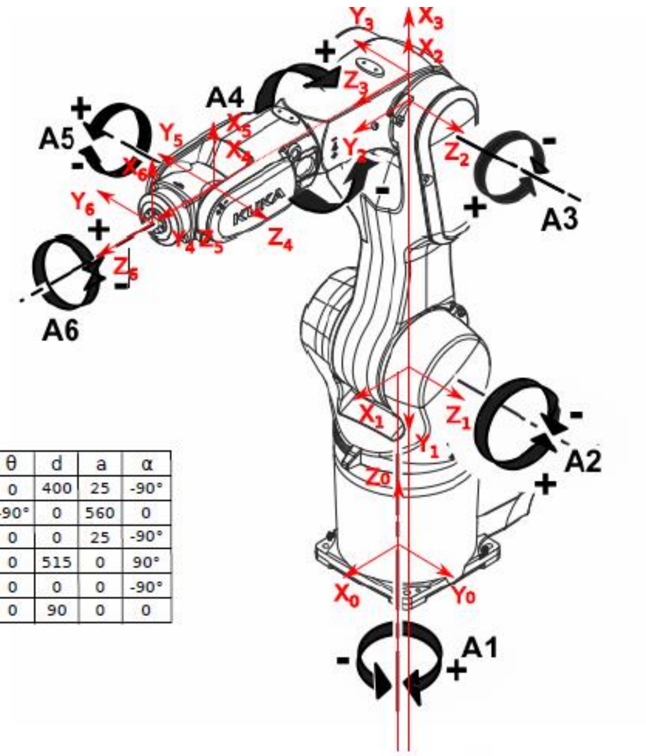
$$a_{34} = \frac{9\, \sin(\theta_4 - \theta_5)\, s_{2,3}}{200} - \frac{s_{2,3}}{40} - \frac{14\, s_2}{25} - \frac{103\, s_{2,3}}{200} - \frac{9\, s_{2,3}\, s_{4,5}}{200} + \frac{9\, s_{2,3}\, c_5}{100} + \frac{2}{5}$$

$$a_{41} = 0$$

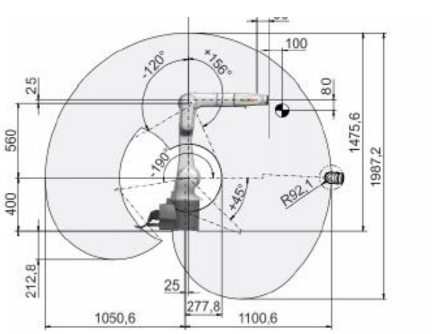
$$a_{42} = 0$$

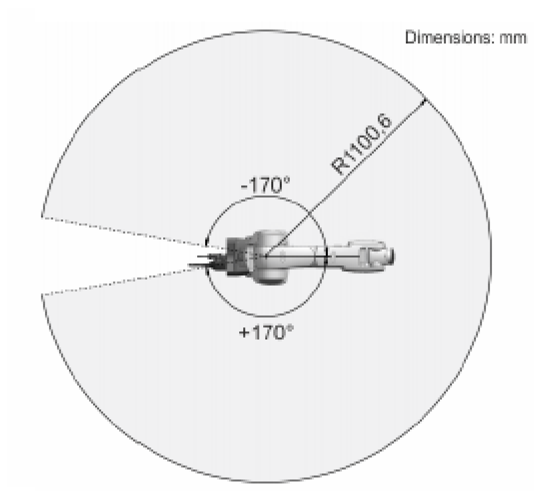
$$a_{43} = 0$$

$$a_{44} = 1$$



	θ	d	a	α
1	0	400	25	-90°
2	-90°	0	560	0
3	0	0	25	-90°
4	0	515	0	90°
5	0	0	0	-90°
6	0	90	0	0





Inverse Kinematics:

First Method:

1- **We have to know the Robot position.**

- **get the first 3 angles**
- **Put them in the FK matrix to get the first 3 joints angles to know the full Orientation.**

```
%% position solve

% x = cos(q1) * (l2 cos(q2) + l3 cos(q2+q3))
% y = sin(q1) * (l2 cos(q2) + l3 cos(q2+q3))
% z = l1 - l2 sin(q2) - l3 sin(q2+q3))

% q1 = atan2(y,x), y = T0_3(2,4), x = T0_3(1,4)

% sqrt(x^2 + y^2) = l2 cos(q2) + l3 cos(q2+q3) = x_new
%          -z+ l1 = l2 sin(q2) + l3 sin(q2+q3))= y_new

%q3 = acosd( (x_new^2 + y_new^2 - l2^2 - l3^2) / (2 * l2 *l3) )

%q2 = m * atand( l3*sin(q3) / (l2 + l3*cos(q3) ))+ atand( y_new/x_new )
```

2- **Orientation :**

```
%% Rotation solve
% if nx ~= 1, q4 = atan2( ny, -nz)
% q6 = atan2( sx, ax)

% if ax ~= 0 q5 = atan2 ( ax/ cosq6, nx)
% else q5 = atan2 ( ax/ sinq6, nx)

% if nx == 1 then q5 = acos(nx)
```

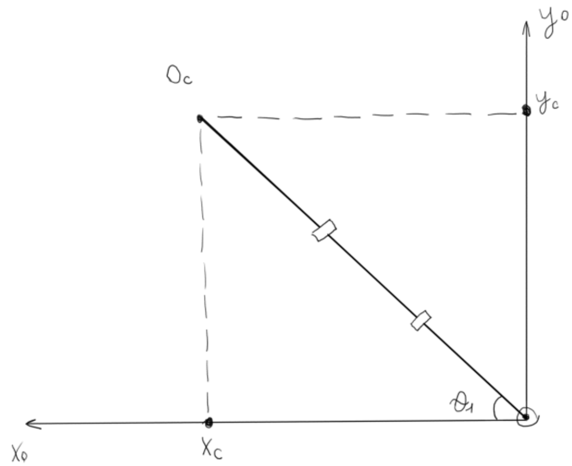
Know the last 3 angles.

```
%Now we have our 3 position angles we plug them back into the forward
%kinematics to get R0_3
T0_3 = FK_kuka([q1,q2,q3]);
%Replace inv(A)*b with A\b
T3_6 = T0_3 \ T;
% Extract and inspect the first component
% if the first element is 1 or -1 there is no unique solution
% so we have to assume one of the 2 angles
% q4 + q6 = atan2(T3_6(2,3),T3_6(2,2))
if T3_6(1,1) == 1
    q5 = acos(T3_6(1,1));
    % let q6 = pi/8
    q6 = pi/8;
    q4 = (atan2(T3_6(2,3),T3_6(2,2)) -  q6) ;
% q4 - q6 = atan2(T3_6(2,3),T3_6(2,2))
elseif T3_6(1,1) == -1
    q5 = acos(T3_6(1,1));
    % let q6 = pi/6
    q6 = pi/6;
    q4 = q6 + atan2(T3_6(2,3),T3_6(2,2));
%Else we have 2 pairs of solutions
else
    q4 = atan2( T3_6(2,1), -T3_6(3,1));
    q6 = atan2( T3_6(1,2), T3_6(1,3));
    if T3_6(3,1) ~= 0
       q5 =  atan2( (T3_6(1,3)/(cos(q6))), T3_6(1,1));
    else
        q5 =  atan2( (T3_6(1,3)/(sin(q6))), T3_6(1,1));
    end
end
Q1 = [q1 q2 q3 q4 q5 q6];
End
```

Second method " Using Euler angles"

Arm (position)

1- **get the first 3 angles**

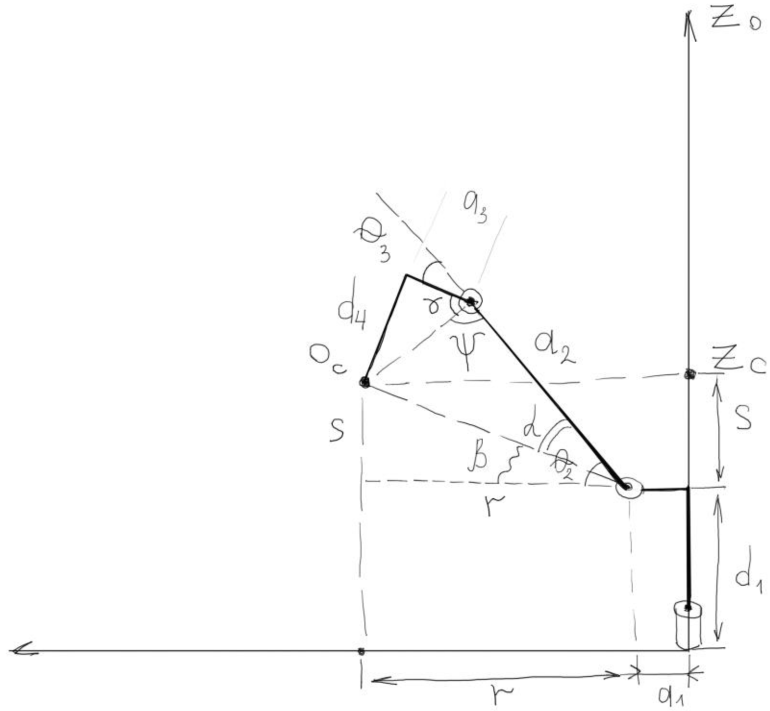


Arm top view

Oc - center of the spherical wrist.

$$O_c = O - d_6 R\left[\ 001\ \right]$$

$$\theta_1 = atan2(x_c, y_c) \text{ or } \theta_1 = \pi + atan2(x_c, y_c), \text{ where } x_c = y_c = 0 \text{ - singularity point}$$



Arm side view

$$r = \sqrt{x_c^2 - y_c^2} - a_1$$

$$s = z_c - d_1$$

$$\varphi = \theta_3 + \sigma$$

$$\gamma = atan2(d_4, a_3)$$

$$\psi = 180 - \varphi$$

$$\cos\varphi = \cos(180 - \varphi) = -\cos(\varphi)$$

Via cosine theorem:

$$\cos\varphi = \frac{r^2 + s^2 - a_2^2 - d_4^2 - a_3^2}{2a_2\sqrt{d_4^2 + a_3^2}} = D$$

$$\varphi = atan2(\pm\sqrt{1-D^2}, D)$$

$$\theta_3 = \varphi - \gamma = atan2(\pm\sqrt{1-D^2}, D) - atan2(d_4, a_3)$$

Via sine theorem:

$$\sin\alpha = \frac{\sin\varphi\sqrt{d_4^2 + a_3^2}}{\sqrt{r^2+s^2}}$$

$$\beta = atan2(s, r)$$

$$\sin\alpha = \frac{\tan\varphi\sqrt{d_4^2 + a_3^2}\cos\varphi}{\sqrt{r^2+s^2}} = \frac{\sqrt{1-D^2}}{D}\frac{d_4^2 + a_3^2 D}{\sqrt{r^2+s^2}} =$$

$$= \sqrt{1-D^2}\sqrt{\frac{d_4^2+a_3^2}{r^2+s^2}} = B$$

$$\alpha = atan2(B, \pm\sqrt{1-B^2})$$

Since $\alpha < 90 degree$

$$\theta_2 = \beta + \alpha = atan2(s, r) + atan2(B, \sqrt{1-B^2})$$

2-2- Wrist (orientation)

Total Rotation matrix:

$$R = R_3^0 R_6^3$$

As we know θ_1, θ_2, θ_3, we know R_3^0. So

$$R_6^3 = (R_3^0)^T R$$

$$R_3^0 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 \\ s_1c_{23} & -s_1s_{23} & -c_1 \\ s_{23} & c_{23} & 0 \end{bmatrix}$$

$$R_6^3 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s_4c_6 & c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & s_4s_5 \\ -s_5c_6 & s_5s_6 & c_5 \end{bmatrix}$$

in spherical wrist θ_4, θ_5, θ_6 - Euler angles. This mean that Euler angle solution can be applied to this equation.

```
%% create inverse kinematics function
```

$$c_4 s_5 = c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33}$$

$$s_4 s_5 = -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c23r33$$

$$c_5 = S_1 r13 - c_1 r23$$

For example, in case $c_4 s_5 = s_4 s_5 = 0$,

$$\theta_5 = atan2(s_1 r_{13} - c_1 r_{23}, \pm\sqrt{1 - (s_1 r_{13} - c_1 r_{23})^2})$$

for positive square root

$$\theta_4 = atan2(c_1 c_{23} r_{13} + s_1 c_{23} r_{23} + s_{23} r_{33}, -c_1 s_{23} r_{13} - s_1 s_{23} r_{23} + c_{23} r_{33})$$

$$\theta_6 = atan2(-s_1 r_{11} + c_1 r_{21}, s_1 r_{12} - c_1 r_{22})$$

Analogously for negative. If $s_5 = 0$, z_3 collinear with z_5. It is singular case. One colution to choose θ_4 arbitrary and then find θ_6.

```
% d - link length
% Coordinates
x = 0.8575;
y = 0;
z = 0.3859;
O = [x; y; z];
% RPY/xyz
roll = deg2rad(90);
pitch = deg2rad(90);
yaw = deg2rad(90);
R = rotx(roll) * roty(pitch) * rotz(yaw)

O_c = O - d(6)*R*[0; 0; 1]

x_c = O_c(1);
y_c = O_c(2);
z_c = O_c(3);

q1_front = atan2(y_c, x_c);
q1_behind = pi + atan2(y_c, x_c);

% front case
r = sqrt(x_c^2 + y_c^2) - a(1);
s = z_c - d(1);
D = (r^2 + s^2 - d(4)^2 - a(3)^2 - a(2)^2) / (2 * a(2) * sqrt(d(4)^2 +
a(3)^2)); % cos(phi)
gamma = atan2(d(4), a(3));
```

```
q3_up = atan2(sqrt(1 - D^2), D) - gamma;
q3_down = atan2(-sqrt(1 - D^2), D) - gamma;


B = sqrt(1 - D^2) * sqrt((d(4)^2 + a(3)^2 ) / (r^2 + s^2)); %
sin(alpha) - can be positive (q3_up) or negative (q3_down)
beta = atan2(s, r);

q2_up = -(atan2(B, sqrt(1 - B^2)) + beta);
q2_down = -(atan2(-B, sqrt(1 - B^2)) + beta);


q4 = atan2(R(2,3), R(1,3));
q5 = atan2(R(2,3), -R(3,3) * sin(q4));
q6 = atan2( -R(3,2), R(3,1));



q_fu = [q1_front, q2_up, q3_up q4 q5 q6]
q_fd = [q1_front, q2_down, q3_down q4 q5 q6]
```

link to github :

https://github.com/sulimanbadour1/DONRSHW2.git