CSCI 301 M2 Homework

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Collaboration statement: By submitting this assignment, I am attesting that this homework is in full compliance with the course's https://wwu.instructure.com/courses/1340003/pages/academic-dishonesty-guidelines Homework Collaboration Policy and with all the other relevant academic honesty policies of the course and university. I discussed this homework with no one and wrote this solution without input from anyone else.

1. (a) $x < y$ is not a statement because we do not know the values of x or y thus making it impossible to establish if it is true or false.

   (b) $x < y$ or $x > y$ is not a statement for the same reason as (a). We do not know the values of x or y thus establishing true or false would be impossible.

   (c) All... course can be equated to either true or false, so this is a statement.

   (d) Every... one student can also be equated to either true or false, thus it is a statement.

   (e) There exists $\ldots cos(x) > 2$ is a statement in which we can pick or find an x value to satisfy $cos(x) > 2$, making this a statement.

   (f) Every $\ldots$ than x. This is not a statement as we do not know the value of x to establish if any $y > 0$ is larger than it.

2. (a) $(P \vee Q) \to (Q \wedge R)$

   (b) $P \to \neg R$

   (c) $P \wedge (Q \vee R)$

   (d) $(P \wedge Q) \to R$

3. (a)

p	q	$\neg p \to q$
T	T	F
T	F	T
F	T	F
F	F	F

(b)

p	q	r	$(p \rightarrow q) \vee r$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

(c)

p	q	r	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

(d)

p	q	r	$(p \rightarrow q) \leftrightarrow (p \rightarrow r$
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

4.

$\neg(p \rightarrow q)$	$p \wedge \neg q$	$\neg(\neg p \vee q)$	$\neg q \rightarrow \neg p$
F	F	F	T
T	T	T	F
F	F	F	T
F	F	F	T

As you can see $\neg q \rightarrow \neg p$ is different per truth tables.

5. (a) There exists a person x for every person y where x teaches y is true.

   (b) For every person y, there exists a person x where x teaches y is true.

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(c) For every person, a professor is sufficient for some person x to teach one another person y.

(d) For every person y there exists a person x where it is not the case that the person is a professor and person x teaches person y.

6. When examining multiple quantifiers we must work through the negations carefully.

1
$\neg(\forall_{x \in P} \forall_{y \in P} T(x,y))$
$\downarrow$
$\exists_{x \in P} \neg(\forall_{y \in P} T(x,y))$
$\downarrow$
$\exists_{x \in P} \exists_{y \in P} \neg T(x,y)$
2
$\neg(\exists_{x \in P} \exists_{y \in P} T(x,y))$
$\downarrow$
$\forall_{x \in P} \neg \exists_{y \in P} T(x,y)$
$\downarrow$
$\forall_{x \in P} \forall_{y \in P} \neg T(x,y)$
3
$\exists_{x \in P} \exists_{y \in P} \neg T(x,y)$
4
$\exists_{x \in P} \neg(\forall_{y \in P} T(x,y))$
$\downarrow$
$\exists_{x \in P} \exists_{y \in P} \neg T(x,y)$

So you will see that the nested quantifier statements from $1, 3$, and 4 all result in the same ending through negation. Nested quantifier statement 2 is the outlier that is not like the others. The down arrows indicate steps in negation.