CSCI 301 M3 Homework

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Collaboration statement: By submitting this assignment, I am attesting that this homework is in full compliance with the course's https://wwu.instructure.com/courses/1340003/pages/academic-dishonesty-guidelines Homework Collaboration Policy and with all the other relevant academic honesty policies of the course and university. I discussed this homework with no one and wrote this solution without input from anyone else.

1. *Proof.* Suppose x and y are odd, then xy is odd.
Then $x = 2a + 1$ and $y = 2a + 1$ for some $a \in \mathbb{Z}$, by definition of an odd number.
Thus $xy = (2a + 1) * (2a + 1) = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1$.
So $xy = 2b + 1$ *where* b *is the integer* $b = 2a^2 + 2a$.
Thus $xy = 2b + 1$ for any integer b.
Therefore xy is odd, by definition of an odd number.

2. *Proof.* Suppose if two integers have the same parity, then their sum is even.
Case 1
Let $x = 2a$ and $y = 2a$ for some $a \in \mathbb{Z}$, by definition of an even number.
Thus $x + y = 2a + 2a = 4a = 2(2a)$.
So $x + y = 2b$ *where* b *is the integer* $b = 2a$.
Thus $x + y = 2b$ for any two integers with both even parity.
Therefore $x + y$ is even when both x and y are of even parity by definition of an even number.
Case 2
Then $x = 2a + 1$ and $y = 2a + 1$ for some $a \in \mathbb{Z}$, by definition of an odd number.
Thus $x + y = 2a + 1 + 2a + 1 = 4a + 2 = 2(2a + 1)$.
So $x + y = 2b$ *where* b *is the integer* $b = 2a + 1$.
Thus $x + y = 2b$ for any two integers with both odd parity.
Therefore $x + y$ is even when both x and y are of odd parity by definition of an even number.

3. *Proof* If a and b have the same parity, then $3a + 7$ and $7b - 4$ do not.
Case 1
Let $a = 2x$ and $b = 2x$ for some $x \in \mathbb{Z}$, by definition of an even number.
Thus $3a + 7 = 3(2x) + 7 = 6x + 7 = 2(3x + 3) + 1$ and $7b - 4 = 7(2x) - 4 = 14x - 4 = 2(7x - 2)$.
So $3a + 7 = 2x + 1$ *where* x *is the integer* $x = 3x + 3$ and $7b - 4 = 2x$ *where* x *is the integer* $x = 7x - 2$.
Thus a and b have the same parity of even, while $3a + 7$ and $7b - 4$ have opposing parities by definitions of odd and even numbers.
Therefore $3a + 7$ and $7b - 4$ do not have the same parity while a and b are even parity by definitions of both odd and even numbers.
Case 2
Let $a = 2x + 1$ and $b = 2x + 1$ for some $x \in \mathbb{Z}$, by definition of an odd number.
Thus $3a + 7 = 3(2x + 1) + 7 = 6x + 3 + 7 = 6x + 10 = 2(3x + 5)$ and $7b - 4 = 7(2x + 1) - 4 = 14x - 7 = 2(7x - 3) + 1$.
So $3a + 7 = 2x$ *where* x *is the integer* $x = 3x + 5$ and $7b - 4 = 2x + 1$ *where* x *is the integer* $x = 7x - 3$.
Thus a and b have the same parity of odd, while $3a + 7$ and $7b - 4$ have opposing parities by definitions of odd and even numbers.
Therefore $3a + 7$ and $7b - 4$ do not have the same parity while a and b are odd parity by definitions of both odd and even numbers.

4. **Proposition** If n^2 is odd, then n is odd.
Proof (Contrapositive) If n is even, then n^2 is even.
Then let $n = 2a$ for some $a \in \mathbb{Z}$, by definition of an even number.
Thus $n^2 = (2a) * (2a) = 4a^2 = 2(2a)$.
So $n^2 = 2b$ *where* b *is the integer* $b = 2a$.
Thus $n^2 = 2b$ for any integer n.
Therefore n^2 is even for any even integer n by definition of an even number.
Consequently If n is even, then n^2 is even has been proving true by the contrapositive, indication the original proposition of: if n^2 is odd, then n is odd as also true.

5. **Proposition** If $x^3 - x > 0$, then $x > -1$.
Proof (Contrapositive) If $x \leq -1$, then $x^3 - x \leq 0$.
Then let $x = -1$.
Thus $-1^3 - -1 = -1 - -1 = -1 + 1 = 0$.
So when $x \leq -1$, $x = 0$ and $0 \leq 0$.
Therefore By proving the contrapositive of our original proposition if $x^3 - x > 0$, then $x > -1$ to be true, the original statement also holds true by definition of contrapositive where $P \to Q = \neg Q \to \neg P$.

6. **Proposition** If $x + y$ is even, then x and y have the same parity.
Proof (Contrapositive) If x and y have the opposite parity, then $x + y$ is odd.
Case 1
Then let $x = 2a$ for some $a \in \mathbb{Z}$, by definition of an even number and $y = 2a + 1$ for some $a \in \mathbb{Z}$, by definition of an odd number.
Thus $x + y = 2a + 2a + 1 = 4a + 1 = 2(2a) + 1$
So $x + y = 2(b) + 1$ *where* b *is the integer* $b = 2a$.
Thus $x + y$ is odd by definition of an odd number.
Therefore Case 1 proves the contrapositive true for when x and y are of opposite parity that $x + y$ is odd.
Case 2
Then let $x = 2a + 1$ for some $a \in \mathbb{Z}$, by definition of an odd number and $y = 2a$ for some $a \in \mathbb{Z}$, by definition of an even number.
Thus $x + y = 2a + 1 + 2a = 4a + 1 = 2(2a) + 1$
So $x + y = 2(b) + 1$ *where* b *is the integer* $b = 2a$.
Thus $x + y$ is odd by definition of an odd number.
Therefore Case 1 proves the contrapositive true for when x and y are of opposite parity that $x + y$ is odd.
Consequently Case 1 and 2 prove true by contrapositive indicating the original statement that if $x + y$ is even, then x and y have the same parity and is true.

7. (a) If x is positive and y is greater than x, then the square roots of y and x will also be positive.
   (b) Every integer can be written as the sum of two other integers.
   (c) The equations $x + 5$ equals 8 when x is equal to 5.
   (d) If we assume that the integer n is even, then the result of $n * m$ is also even.