CSCI 301 M6 Homework

Bo Sullivan

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Collaboration statement: By submitting this assignment, I am attesting that this homework is in full compliance with the course's https://wwu.instructure.com/courses/1340003/pages/academic-dishonesty-guidelines Homework Collaboration Policy and with all the other relevant academic honesty policies of the course and university. I discussed this homework with no one and wrote this solution without input from anyone else.

1. (a) f is injective because there exists a one-to-one of inputs of A to outputs in B. $f(a) = x$, $f(b) = x$ etc.
   (b) f is NOT surjective because not all three elements of the codomain are images(the range of) elements of the domain. If there was an $f(a) = z$, it would be surjective.
   (c) f does NOT have an inverse because there is no one-to-one correspondence. If, for example, $f(b) = z$, there would exist an inverse.
2. (a) f is injective because for any input of a $\mathbb{N}$, a horizontal line would never intersect at two or more points, horizontal line test.
   (b) f is **not** surjective as for any input of $\mathbb{N}$ only corresponds to it's own element of $\mathbb{N}$.
   (c) $g \circ f = (n + 1)(n - 1) = n^2 - 1$.
   (d) $g \circ f$ is surjective as the one-to-one mapping for all $\mathbb{N}$.
   (e) $g \circ f$ is **not** injective as the union of 0 will not be mapped onto.
   (f) The range of $g \circ f$ is $\mathbb{N}$.
3. (a) f^{-1} is not invertible because f^{-1} is not surjective, a requirement to be invertible.
   (b) $h \circ g = \{(x, 2), (y, 2), (z, 3)\}$.
   (c) $g \circ f = \{(a, \alpha), (b, \beta)\}$.
   (d) $h \circ (g \circ f)$ and $(h \circ g) \circ f$ are the **same** per Theorem 12.1 Book of Proofs, stating composition of functions is associative.
   (e) $h \circ g \circ f$ and $f \circ g \circ h$ are **not** the same functions as composition of functions are not commutative.
4. (a) The cardinality will be equal as a countable infinity minus a countable infinity is still a countable infinite set and $\aleph_0$ is countably infinite.
   (b) The cardinality will be strictly greater as the set of all real numbers is an uncountable infinite set and $\aleph_0$ is an countable infinite set. Uncountable infinite sets are larger than countable infinite sets.
   (c) This will be strictly less than $\aleph_0$, as the cardinality of a fixed size set will be 2^n which is clearly less than all natural numbers as 2^{10} is less than infinity.
   (d) The power set of what seems to be all positive integers would be an uncountable infinite set which is strictly greater than $\aleph_0$. The $|\mathcal{P}(\mathbb{N})|$ is greater than the $|\mathbb{N}|$ per Book of Proofs example 14.2.

5. Let $A = \{2^0, 2^1, 2^2, 2^3, \ldots\}$, let $B = \{2n : n \in \mathbb{N}\}$ and let $C = \{2n+1 : n \in \mathbb{N}\}$. **Prove** that $|A| = |B \times C|$.
Since A is a set of countably infinite elements and both B and C are elements of $\mathbb{N}$ which is also countably infinite...
Then we know that Theorem 14.5 from the Book of Proofs says that if two sets are countably infinite, so is their product.
Thus the cardinality of countably infinite sets A and B and C is $\aleph_0$ as is their product.

6. **Prove** that the set of irrational numbers $\mathbb{I} = (\mathbb{R} - \mathbb{Q})$ is uncountable.
Suppose for the sake of **contradiction** that the set of irrational numbers $\mathbb{I} = (\mathbb{R} - \mathbb{Q})$ is countable.
Then since $\mathbb{I}$ is countable we also know that $\mathbb{R} - \mathbb{Q}$ is countable as the $A \cup B$ is countably infinite per Theorem 14.6 Book of Proofs.
Thus we could suppose that the $\mathbb{I}$ must be countable because which **contradicts** our initial proof.