LE06: Networks and graph theory

Menger and Flows

Be $D = (V, E)$ a digraph and let q and s be vertices in D. A set E' of edges in D is called a q and s **separating** edge set in D, if every directed path from q to s contains at least one edge from E'.

The theorem 22.18, known as the Theorem of **Menger** (1927), establishes an **equality** : The maximum number of **edge-disjoint** directed paths in D from q to s is equal to the cardinality of a **minimal** q and s separating edge set in D.

One can prove this by considering a **flow network** on D, in which every edge has the capacity 1. The value of a maximum flow on this network corresponds to the maximum number of edge-disjoint paths.

For non-adjacent vertices q and s, the vertex version of Menger's Theorem (Theorem 22.20) states that the maximum number **of internally vertex-disjoint** directed paths in D from q to s is equal to the cardinality of a minimal q and s separating **vertex set**.

König-Egerváry and Matchings

Let G be a graph. A set of edges P is called a **matching** in G, if the edges in P have no common endpoint.

A set U of vertices in G is called a **vertex cover** of G, if for every edge uv in G, $u \in U$ or $v \in U$ holds.

The theorem 22.22 (König-Egerváry, 1931) applies to **bipartite** graphs and states that the cardinality of a **maximum** matching in G is equal to the cardinality of a **minimum vertex** vertex cover of G.

Hall and Systems of Representatives

The Theorem of **Hall** (1935), which is also known as the **Marriage Theorem**, deals with the existence of a perfect matching in a bipartite graph.

Let G be a bipartite graph with 2-partition $\{V_1, V_2\}$, where $|V_1| = |V_2|$. A perfect matching exists if and only if for every subset U of V_1: $|U^+| \geq |U|$. In this context, U^+ consists of all vertices of V_2 that are **adjacent** with any vertex from U.