LE10: Combinatorial Optimization and graph theory

A system of sets M on a base set A is called a **filter** if it is closed with respects to inclusion. Such a system is called a **matroid** if it also satisfies the so-called **exchange property**. The independant sets with the maximum number of elements within a matroid are called **bases**. An important property of finite matroids is that all these sets are always **of same cardinality**.

A significant result of optimization states that a **greedy algorithm** for a filter M finds an optimal solution for any cost function if and only if M is a matroid. A classic example of this is the **graphic matroid** matroid of a graph, where the independant sets are the edge sets that spann a **forest**. Applying the greedy principle to this always yields a **minimum spanning tree**.

In graph theory, the topic of vertex coloring is studied. The minimum number of colors required to color the vertices of a graph such that adjacent vertices have different colors is called **the chromatic number**. A graph is **2-colorable** if it is bipartite. While determining this number exactly is an **NP-hard problem** greedy method is often used. However, the effectiveness of this method depends strongly on the chosen **order** of the vertices.

Mathematically, the number of colors can be limited by other graph parameters. It is at least as large as the number of nodes in a maximal **clique**. It is limited from above by the **maximum degree**. $\Delta(G)$ plus one is bounded. For connected, non-regular graphs, this bound can even be improved to $\chi(G) \leq \Delta(G)$.