

LE11: Fundamental optimization

In graph theory, the **chromatic polynomial** $p_G(k)$ is the number of ways to color a graph with k valid colors. Such a polynomial always has **integer coefficients**. For practical calculation, an iterative procedure can be used in which the graph is successively reduced to **cliques/complete graphs** (K_n).

For complex optimization problems, such as the set coverage problem or the knapsack problem, various algorithms are often used. While **backtracking** seeks an exact solution, heuristic methods such as **Hill Climb** or **Simulated Annealing** use approximate approaches.

A central concept in combinatorial optimization is the **matroid**. For a subset T of a matroid, the **rank** $\text{rg}(T)$ is defined, which, among other things, possesses the property of **monotonicity**. (from $T \subseteq S$ follows that $\text{rg}(T) \leq \text{rg}(S)$).

Linear **Programming** aims to maximize or minimize an objective function while adhering to constraints. The set of all points that satisfy the constraints $Ax \leq b$ and $x \geq 0$ is called the **feasible solutions**. For every primal linear program (LP), there exists a corresponding **dual LP**.

An important result here is Lemma 24.1: For a feasible solution x of the primal LP and a feasible solution y of the dual LP, the relation $c^T x \leq b^T y$ always holds. If, for two feasible solutions, $c^T x = b^T y$, then both solutions are **optimal** for their respective problems. Many well-known theorems of graph theory, such as the **Max-Flow Min-Cut Theorem** or König-Egerváry theorems, can be interpreted as special cases of this duality theorem.