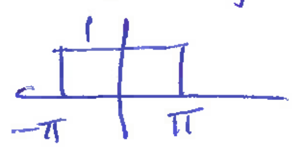
Fall 18

EC516 HW#03 Solutions

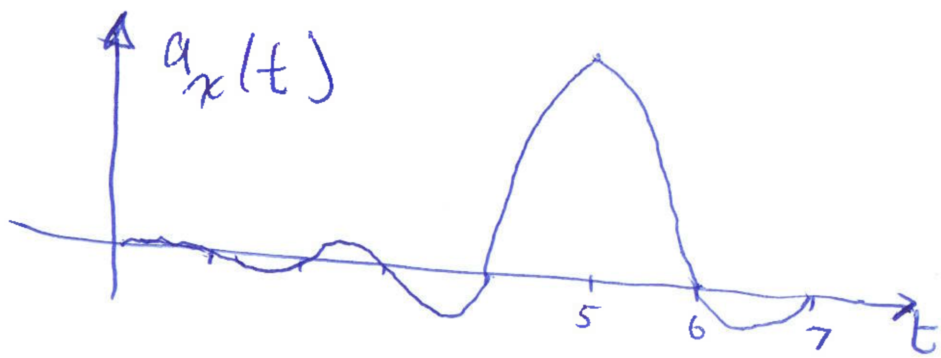
3.1 $x[n] = \delta[n-5] \Longleftrightarrow X(e^{j\omega}) = e^{-j5\omega}$

Let $a_x(t)$ be analog envelope of $x[n]$.

$\therefore A_x(j\omega) = e^{-j5\omega}$.



$$\therefore a_x(t) = \frac{\sin(\pi(t-5))}{\pi(t-5)}$$



3.2

$$x[n] = \frac{\sin(2.25\pi n)}{\pi n} = \frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}$$

$\therefore$ analog envelope of $x[n] = a_x(t) = \dfrac{\sin\left(\frac{\pi}{4}t\right)}{\pi t}$

zero crossings: $\frac{\pi}{4}t = \pi k \Rightarrow t = 4k$

except $k = 0$.

3.3

$$x_b(t) = \frac{\sin(0.5\pi(t-3.5))}{\pi(t-3.5)}$$

$$\therefore x_b[n] = \frac{\sin(0.5\pi(n-3.5))}{\pi(n-3.5)}$$

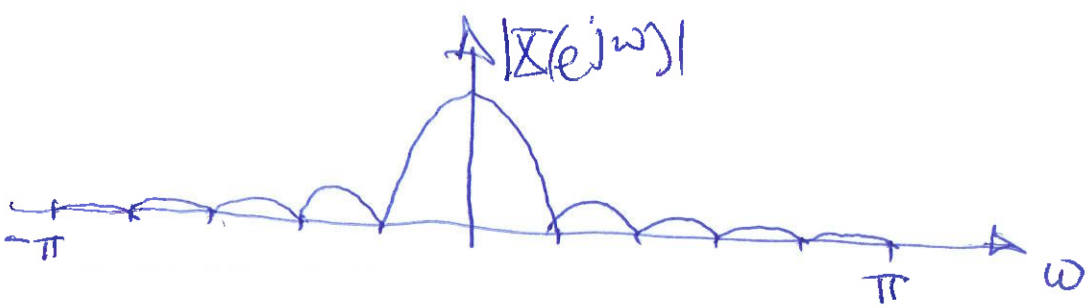
zero crossings of $\sin(\frac{\pi}{2}t)$ are $t=2k$,

But $(n-3.5)$ can never be an integer. $\Rightarrow$ No zero crossings in $x_b[n]$.

3.4

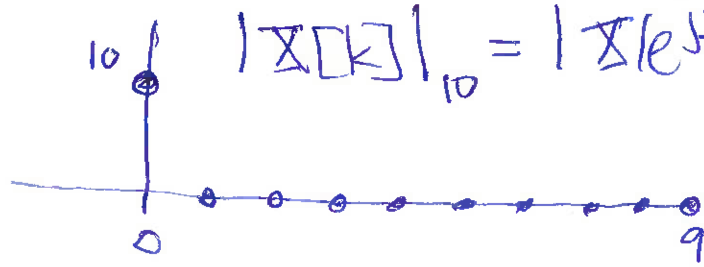
$$X(e^{j\omega}) = \left(\frac{\sin 5\omega}{\sin \omega/2}\right) e^{-j9\omega/2}$$

(a)

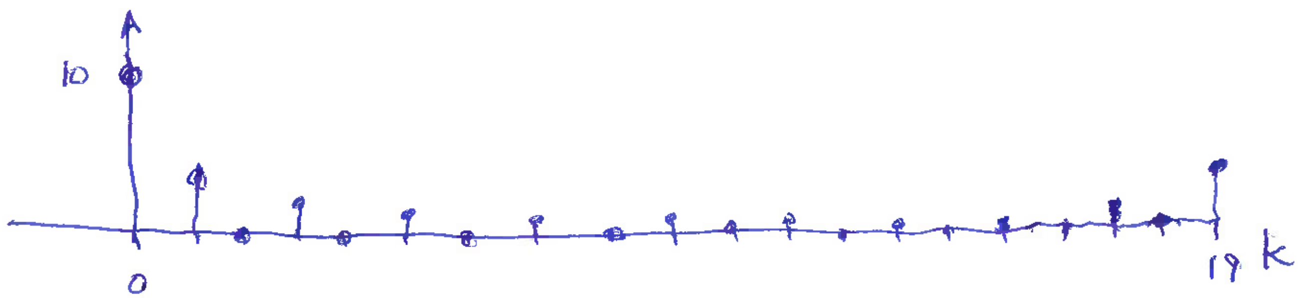


(b)

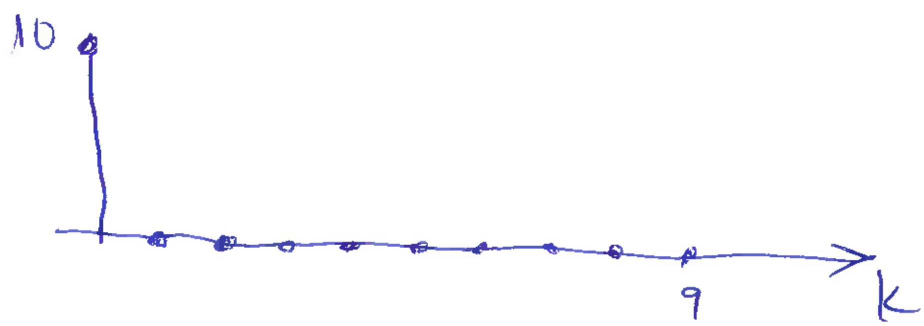
$|X[k]|_{10} = |X(e^{j\frac{2\pi k}{10}})|$ for $0 \le k \le 9$



(c) $X[k]_{20} = X(e^{j\frac{2\pi k}{20}})$ for $0 \le k \le 19$



(d) $X[k]_{20}$ compressed by 2 gives $X[2k]_{20}$



3.5

(a) $X[k]_2 = \sum_{n=0}^{1} x[n] e^{-j\frac{2\pi nk}{2}} \quad 0 \le k \le 1$

$\therefore X[0]_2 = x[0] + x[1] \longrightarrow A$

$\therefore X[1]_2 = x[0] - x[1] \longrightarrow B$

$A + B \Longrightarrow x[0] = \frac{1}{2}[X[0]_2 + X[1]_2]$

$A - B \Longrightarrow x[1] = \frac{1}{2}[X[0]_2 - X[1]_2]$

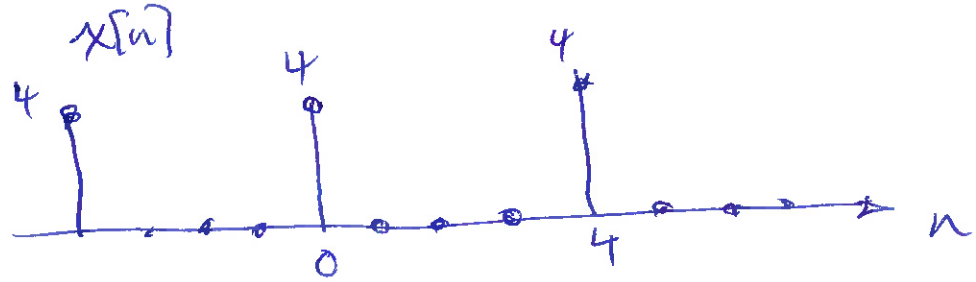
(b)
$$X[k_0]_6 = \sum_{n=0}^{5} x[n]\, e^{-j\frac{2\pi k_0}{6}n}$$

$$\frac{2\pi k_0}{6} = \pi \Rightarrow k_0 = 3$$

(c)
$$\frac{2\pi k_0}{5} = \pi \Rightarrow k_0 = \frac{5}{2}$$

$\therefore$ No integer k_0 will work.

3.6



$$x[n-4] = 1 + e^{j\frac{\pi}{2}(n-4)} + e^{j\pi(n-4)} + e^{j\frac{3\pi}{2}(n-4)}$$
$$= 1 + e^{j\frac{\pi}{2}n} \cdot e^{-j2\pi} + e^{j\pi n} e^{-j4\pi} + e^{j\frac{3\pi}{2}n} \cdot e^{-j6\pi}$$
$$= 1 + e^{j\frac{\pi}{2}n} + e^{j\pi n} + e^{j\frac{3\pi}{2}n}$$
$$= x[n].$$