Fall 18

EC516 HW #1

1.1 (a) Alternative patternizations of a signal are different ways of representing the signal to make it easier for desired information in the signal to become more easily accessible

(b) 1- Sampling at a rate that causes aliasing

2- Filtering to emphasize "low pass" trends within the signal

1.2 (a) $100\pi t = \pi k \Rightarrow t = \frac{k}{100}$

(b) $$\lim_{t \to 0} \frac{\sin(100\pi t)}{\pi t} = \lim_{t \to 0} \frac{100\pi \cos(100\pi t)}{\pi}$$

$$= 100$$

1.3 (a) $$(1-\alpha)\sum_{n=0}^{N-1}\alpha^n = 1 + \alpha + \alpha^2 + \cdots + \alpha^{N-1} - \alpha - \alpha^2 - \alpha^{N-1} - \alpha^N$$

$$= 1 - \alpha^N.$$

$$\therefore \sum_{n=0}^{N-1}\alpha^n = \frac{1-\alpha^N}{1-\alpha} \quad \text{except when } \alpha = 1$$

(b) $$\lim_{N\to\infty} \sum_{n=0}^{N-1} \alpha^n = \lim_{N\to\infty} \frac{1-\alpha^N}{1-\alpha} = \frac{1}{1-\alpha}$$

provided $|\alpha|<1$

1.4 $$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\therefore e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\therefore e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

a) $$\therefore \cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

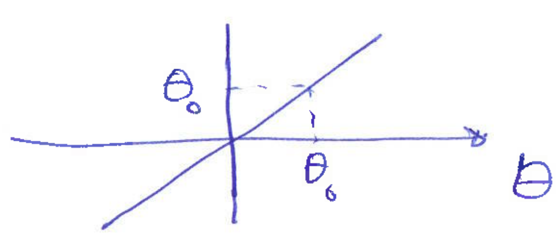
Also $e^{j\theta} - e^{-j\theta} = 2j\sin\theta$

b) $$\therefore \sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

c) $$e^{j(\theta+2\pi k)} = e^{j\theta}\cdot\underbrace{e^{j2\pi k}}_{1} = e^{j\theta}$$

d) $$|(e^{j\theta})| = +\sqrt{\cos^2\theta + \sin^2\theta} = +\sqrt{1} = 1$$

e) $\arctan(\tan\theta) = \theta$



1.5 Let ω_h be the smallest frequency for which $X(j\omega) = 0$ for $\omega > \omega_h$

Let ω_ℓ be the largest frequency for which $X(j\omega) = 0$ for $\omega < \omega_\ell$

To avoid aliasing, the sampling frequency $\frac{2\pi}{T}$ must be such that the entire bandwidth of $X(j\omega)$ does not have any overlap with its replicas, i.e.

$$\frac{2\pi}{T} > \omega_h - \omega_\ell \quad \left\{ \begin{array}{c} \text{General} \\ \text{Sampling} \\ \text{Theorem} \end{array} \right\}$$

If $x(t)$ is real, $\omega_\ell = -\omega_h$ and the Sampling Theorem becomes

$$\frac{2\pi}{T} > 2\omega_h$$