

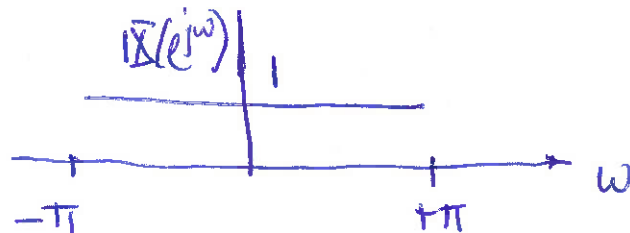
Fall 18

EC516 HW # 02 Solutions

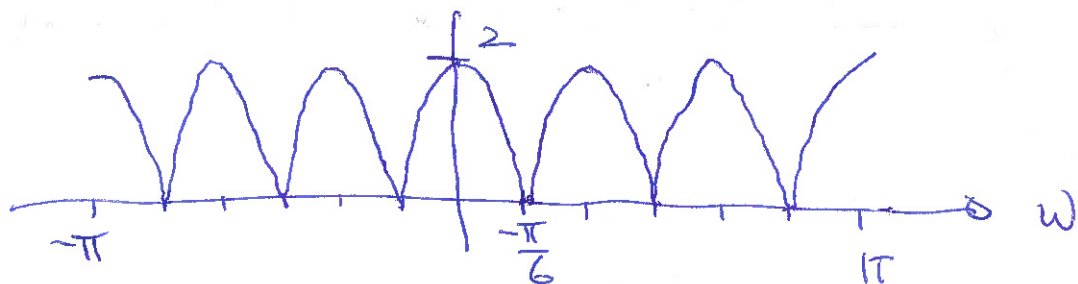
- 2.1
- (a) For $\omega \in (-\pi, \pi]$, $X(e^{j\omega}) = 0$ when $|\omega| = \pi$
- (b) For $\omega \in (-\pi, \pi]$, $X(e^{j\omega}) = 0$ when $\frac{\pi}{2} < |\omega| \leq \pi$
- (c) For $\omega \in (-\pi, \pi]$, $X(e^{j\omega}) = 0$ when $\frac{\pi}{10} < |\omega| \leq \pi$

2.2

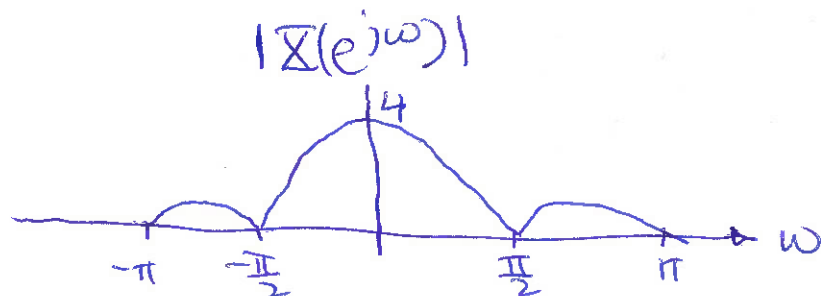
(a) $X(e^{j\omega}) = e^{-j3\omega}$ $|X(e^{j\omega})| = 1$



(b) $X(e^{j\omega}) = 2 \cos 3\omega$

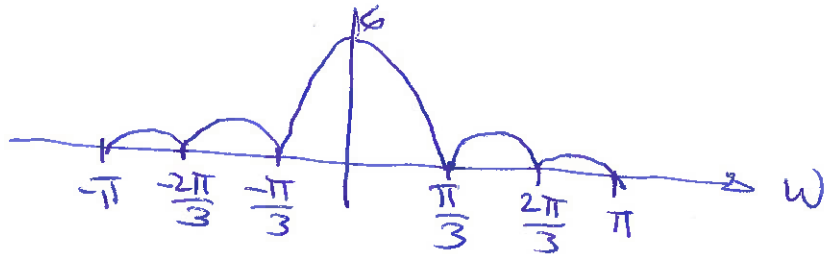


(c) $X(e^{j\omega}) = \left(\frac{\sin 2\omega}{\sin \omega/2} \right) e^{-j\frac{3}{2}\omega}$



$$(d) X(e^{j\omega}) = \frac{\sin 3\omega}{\sin(\omega/2)} \cdot e^{-j\frac{1}{2}\omega}$$

$$|X(e^{j\omega})|$$



2.3

$$(a) y[n] = \frac{1}{2} x[n-3] =$$

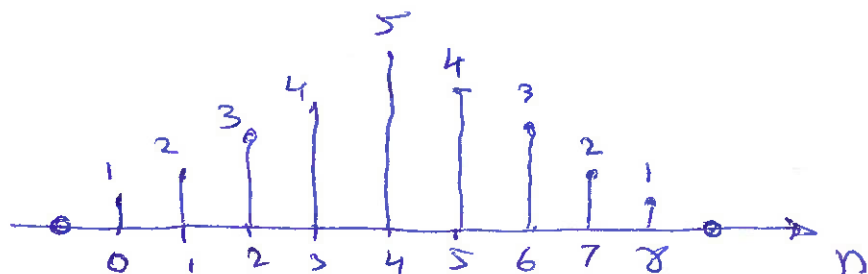
A discrete-time signal plot for $y[n]$ versus n . The signal is zero for $n < 3$ and $n > 7$. For $n = 3, 4, 5, 6, 7$, the signal values are $\frac{1}{2}$, indicated by vertical stems and labels above the stems.

$$(b) y[n] = 2 x[n+3] =$$

A discrete-time signal plot for $y[n]$ versus n . The signal is zero for $n < -3$ and $n > 1$. For $n = -2, -1, 0, 1$, the signal values are 2, 4, 6, and 8 respectively, indicated by vertical stems and labels above the stems.

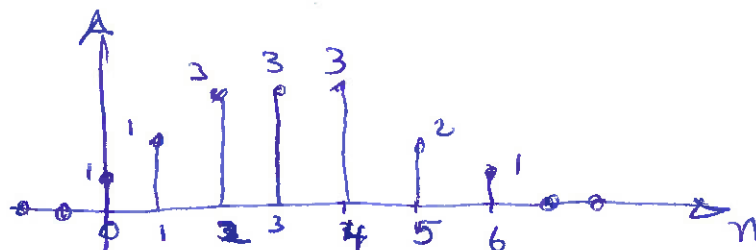
$$(c) y[n] = (5 \text{ pt box}) * (5 \text{ pt. box})$$

$$= \text{Triangle}$$

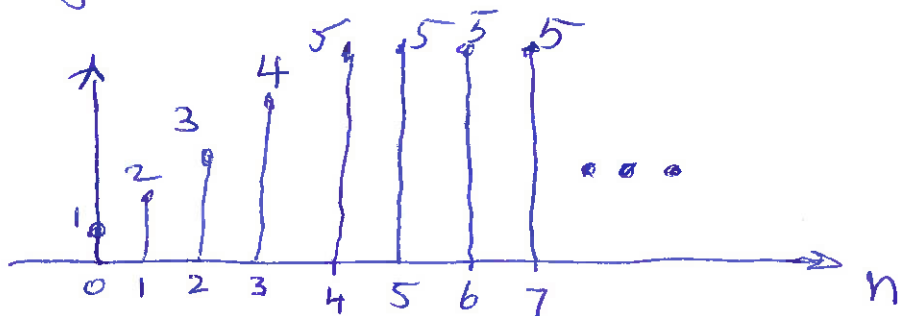


d) $y[n] = (5 \text{ pt. box}) * (3 \text{ pt. box})$
 = Triangle with head chopped off

$$y[n] = x[n] + x[n-1] + x[n-2]$$



e) $y[n] = u[n] + u[n-1] + u[n-2] + u[n-3] + u[n-4]$



2.4

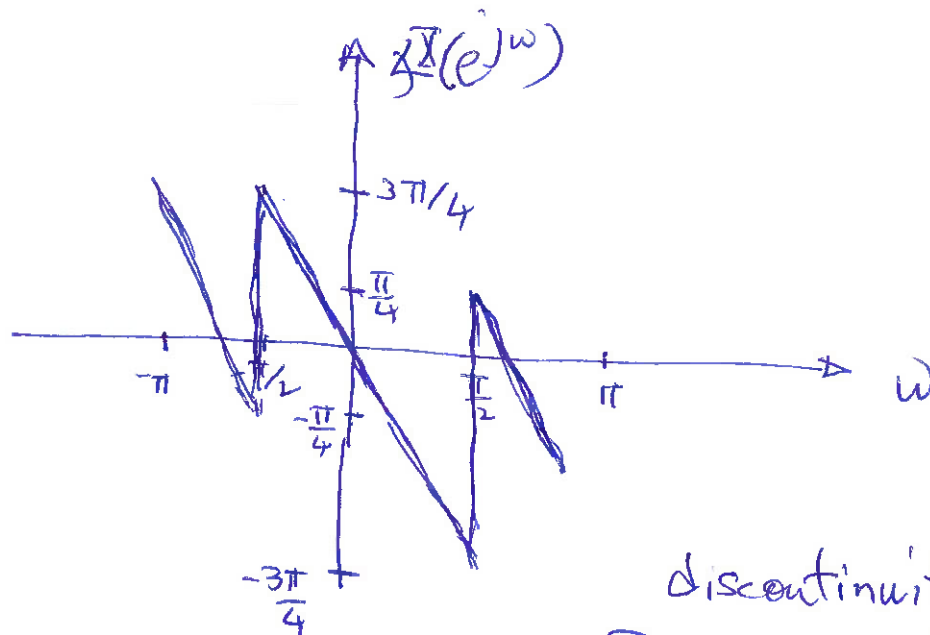
$$x[n] = u[n] - u[n-4] =$$

$$X(e^{j\omega}) = \frac{\sin(2\omega)}{\sin(\omega/2)} e^{-j\frac{3\omega}{2}}$$

$$\angle X(e^{j\omega}) = \underbrace{\angle \left(\frac{\sin 2\omega}{\sin(\omega/2)} \right)}_{\text{positive sinc has phase 0, negative sinc has phase } \pi \text{ or } -\pi} + \angle \left(e^{-j\frac{3\omega}{2}} \right)$$

positive sinc has phase 0
 negative sinc has phase π or $-\pi$.

$$\angle X(e^{j\omega}) = \angle \left(\frac{\sin 2\omega}{\sin \omega/2} \right) - \frac{3\omega}{2}$$



discontinuities of π or $-\pi$ at zero crossings of $X(e^{j\omega})$.

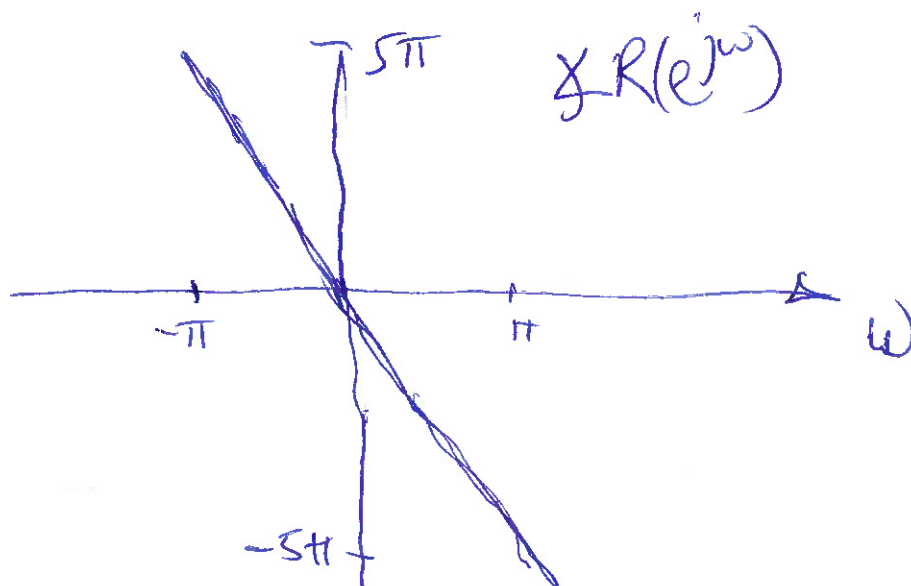
2.5

$$\begin{aligned} r[n] &= x[n] * x[n] \\ &= (6 \text{ pt. box}) * (6 \text{ pt. box}) \\ &= \text{triangle} \end{aligned}$$

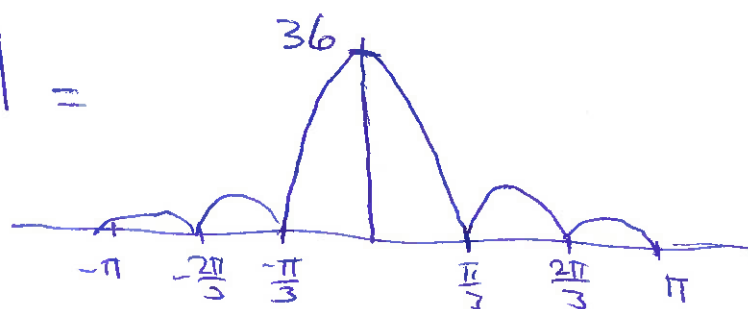
$$\begin{aligned} R(e^{j\omega}) &= X(e^{j\omega}) \times X(e^{j\omega}) \\ &= (X(e^{j\omega}))^2 \end{aligned}$$

$$\begin{aligned} \therefore R(e^{j\omega}) &= \left(\frac{\sin(3\omega)}{\sin \omega/2}, e^{-j\frac{5}{2}\omega} \right)^2 \\ &= \underbrace{\left(\frac{\sin(3\omega)}{\sin \omega/2} \right)^2}_{\text{never negative}} \cdot e^{-j5\omega} \end{aligned}$$

$$\begin{aligned} \therefore \angle R(e^{j\omega}) &= \angle \left(\frac{\sin(3\omega)}{\sin(\omega/2)} \right)^2 - 5\omega \\ &= -5\omega \end{aligned}$$



$$|R(e^{j\omega})| =$$



2.6

$$(a) \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$\therefore x[n-n_0] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega(n-n_0)} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \underbrace{e^{-j\omega n_0} X(e^{j\omega})}_{\substack{\downarrow \\ \text{DTFT of } x[n-n_0]}} e^{j\omega n} d\omega$$

DTFT of $x[n-n_0]$

$$(b) \quad x[n] \text{ real} \iff x[n] = x^*[n]$$

$$x[n] \text{ even} \iff x[n] = x[-n]$$

$$x[n] \iff X(e^{j\omega}), \quad x[-n] \iff X(e^{-j\omega}), \\ x^*[n] \iff X^*(e^{-j\omega})$$

$$x[n] \text{ real} \iff X(e^{j\omega}) = X^*(e^{-j\omega}) \quad \text{--- (A)}$$

$$x[n] \text{ even} \iff X(e^{j\omega}) = X(e^{-j\omega}) \quad \text{--- (B)}$$

Substituting (B) in (A)

$$X(e^{j\omega}) = X^*(e^{j\omega}) \Rightarrow X(e^{j\omega}) \text{ real}$$

$$(B) \Rightarrow X(e^{j\omega}) \text{ is even.}$$

$$(c) \text{ (A) Real } x[n] \Leftrightarrow X(e^{j\omega}) = X^*(e^{-j\omega})$$

$$\text{ (B) Odd } x[n] \Leftrightarrow X(e^{j\omega}) = -X(e^{-j\omega})$$

$$\text{(B)} \Rightarrow X(e^{j\omega}) \text{ is odd}$$

Substituting (B) in (A), we obtain

$$X(e^{j\omega}) = -X^*(e^{j\omega})$$

$\Rightarrow X(e^{j\omega})$ is purely imaginary.