EC516 HW04 (Fall 2018)

Due: **Wednesday, October 03** *at the beginning of lecture*

Problem 4.1

a) Show that the algebraic expression for the z-transform of $x[n] = u[n] - u[n-8]$ can be written as $X(z) = \dfrac{1 - z^{-8}}{1 - z^{-1}}$.

b) Show that $X(z)$ from the previous part can be written as a ratio of polynomials in *z*.

c) How many poles (roots of the denominator polynomial you found in part (b)) are there and where are they located? *Justify your answers.*

d) What is the region of convergence of $X(z)$? *Justify your answer*.

e) How many zeros (roots of the numerator polynomial you found in part (b)) are there and where are they located? What do the locations of these zeros tell you about the 8-point DFT of $x[n]$? *Justify your ans*wers.

Problem 4.2

Write out the W_8 matrix for the 8-point DFT and answer the following questions.

(a) Indicate the 8 distinct elements of the W_8 matrix and tabulate how many times each distinct element repeats itself.

(b) How many of the elements of W_8 are either purely real or purely imaginary?

(c) If W_8 is used to perform direct computation of the 8-point DFT of a real-valued 8-point signal, precisely how many real multiplications would have to be performed? (Assume that the 8-point signal whose DFT is being taken does not take on any trivial values like 1.0 or 0.0 within its 8-point duration. Also assume that the multiplication of a complex number (not purely real or purely imaginary) with another complex number (not purely real or purely imaginary) takes 4 real multiplications to carry out). *Show your work*.

(d) Use W_8 to perform direct computation of the 8-point DFT of $x[n] = u[n] - u[n-8]$ to show that the result is $8\delta[k]$. *Show your work*.

(e) Use W_8 to perform direct computation of the 8-point DFT of the signal $g[n] = (-1)^n \{u[n] - u[n-8]\}$. *Explain why the result makes sense*.

Problem 4.3

Let $x[n]$ be an N-point signal with z transform $X(z)$.

(a) Show that the z transform of $x[-n]$ is $X(z^{-1})$

(b) Does there exist an *N*-point signal $q[n]$ such that $Q[k]_{10} = X(e^{-j2\pi k/N})$ for $0 \le k \le N-1$. *Justify your answer*.