Fall 18

EC516 HW # 02 Solutions

2.1

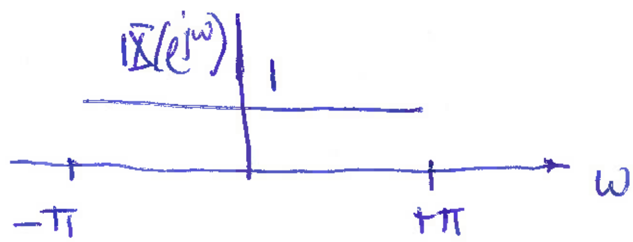
(a) For $\omega \in (-\pi, \pi]$, $X(e^{j\omega}) = 0$ when $|\omega| = \pi$

(b) For $\omega \in (-\pi, \pi]$, $X(e^{j\omega}) = 0$ when $\frac{\pi}{2} < |\omega| \leq \pi$

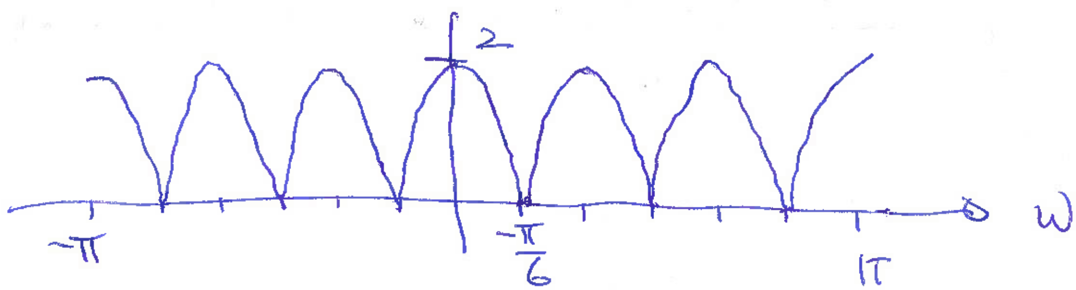
(c) For $\omega \in (-\pi, \pi]$, $X(e^{j\omega}) = 0$ when $\frac{\pi}{10} < |\omega| \leq \pi$

2.2

(a) $X(e^{j\omega}) = e^{-j3\omega}$ $\qquad |X(e^{j\omega})| = 1$

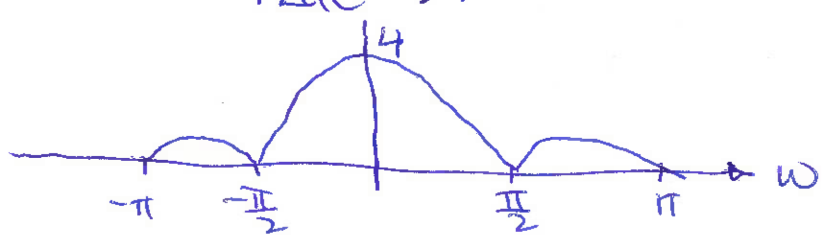


(b) $X(e^{j\omega}) = 2\cos 3\omega$



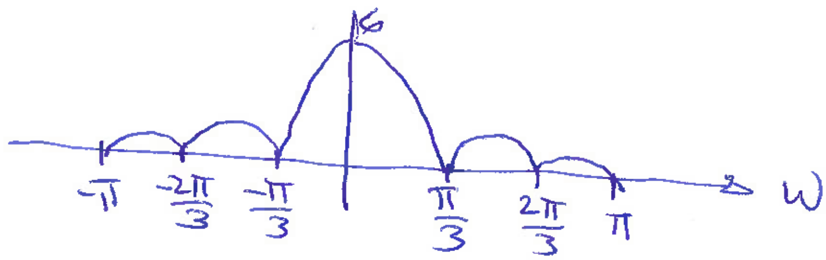
(c) $X(e^{j\omega}) = \left(\frac{\sin 2\omega}{\sin \omega/2}\right) e^{-j\frac{3}{2}\omega}$

$|X(e^{j\omega})|$



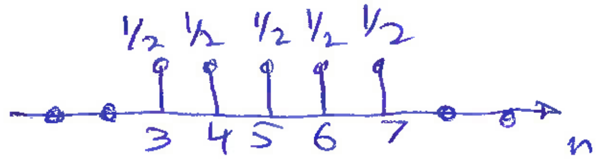
(d) $X(e^{j\omega}) = \frac{\sin 3\omega}{\sin(\omega/2)} \cdot e^{-j\frac{1}{2}\omega}$

$|X(e^{j\omega})|$

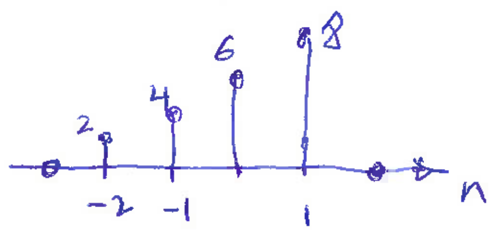


2.3

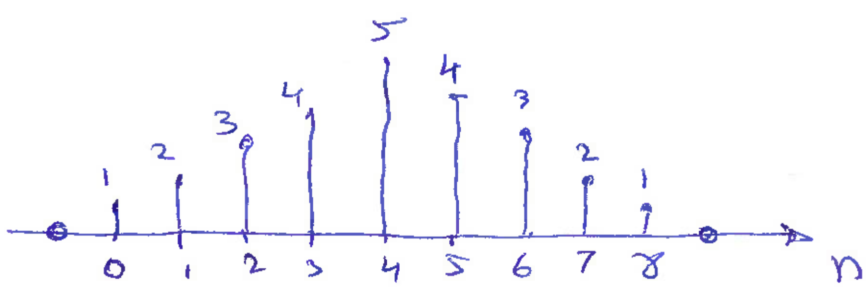
(a) $y[n] = \frac{1}{2}x[n-3] =$



(b) $y[n] = 2x[n+3] =$

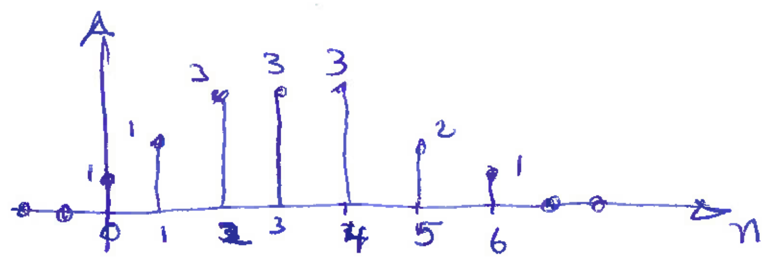


(c) $y[n] = (5\text{pt box}) * (5\text{ pt. box})$
$= \text{Triangle}$

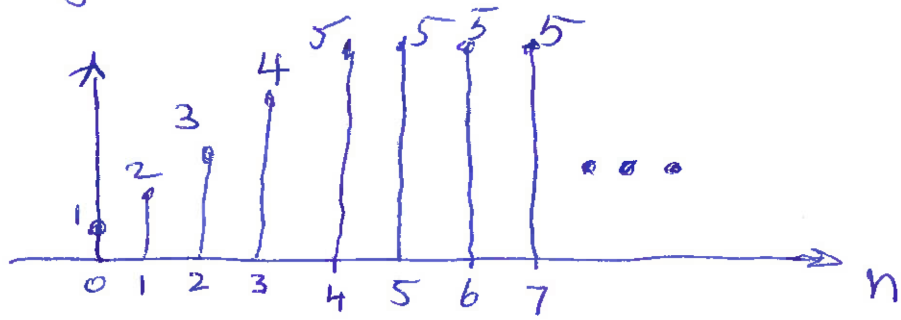


d) $y[n] = (5 \text{ pt. box}) * (3 \text{ pt. box})$
$= \text{Triangle with head chopped off}$

$$y[n] = x[n] + x[n-1] + x[n-2]$$

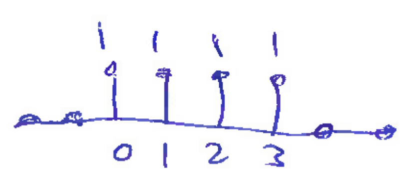


e) $y[n] = u[n] + u[n-1] + u[n-2] + u[n-3] + u[n-4]$



2.4

$$x[n] = u[n] - u[n-4] =$$

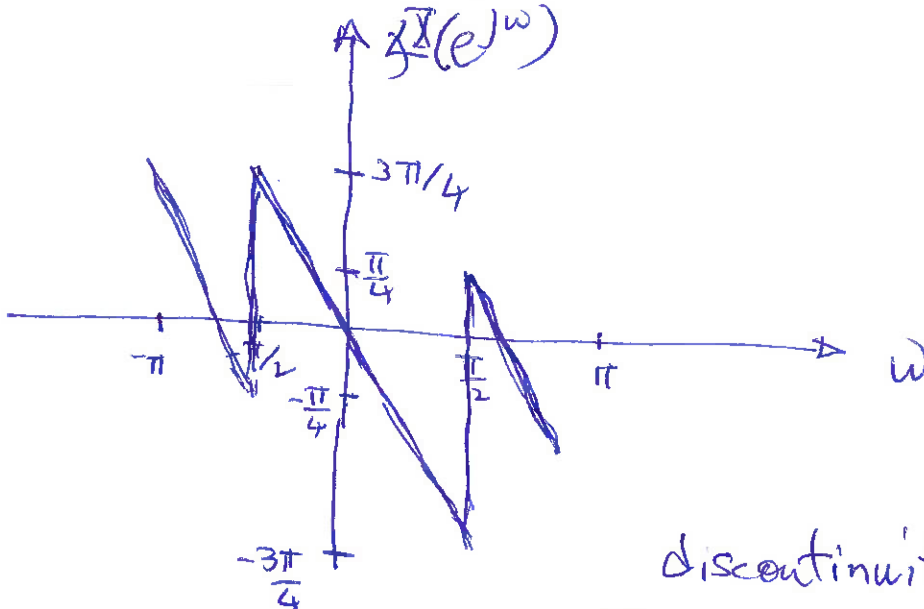


$$X(e^{j\omega}) = \frac{\sin(2\omega)}{\sin(\omega/2)} e^{-j\frac{3\omega}{2}}$$

$$\angle X(e^{j\omega}) = \underbrace{\angle\left(\frac{\sin 2\omega}{\sin(\omega/2)}\right)}_{} + \angle\left(e^{-j\frac{3\omega}{2}}\right)$$

positive sinc has phase 0
negative sinc has phase π or $-\pi$.

$$\measuredangle X(e^{j\omega}) = \measuredangle\left(\frac{\sin 2\omega}{\sin \omega/2}\right) - \frac{3\omega}{2}$$



discontinuities of π or $-\pi$ at zero crossings of $X(e^{j\omega})$.

2.5

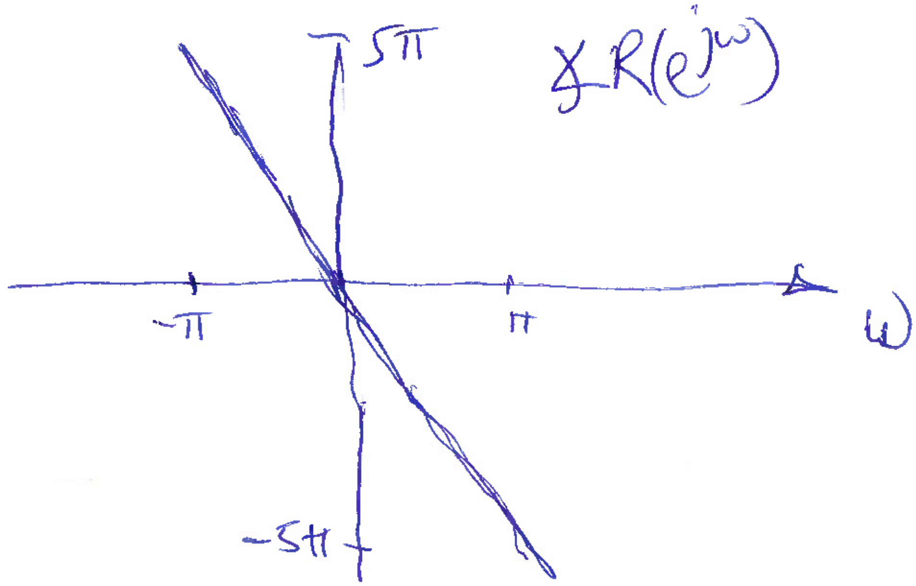
$$r[n] = x[n] * x[n]$$
$$= (6 \text{ pt. box}) * (6 \text{ pt. box})$$
$$= \text{triangle}$$
$$R(e^{j\omega}) = X(e^{j\omega}) \times X(e^{j\omega})$$
$$= \left(X(e^{j\omega})\right)^2$$

$$\therefore R(e^{j\omega}) = \left(\frac{\sin(3\omega)}{\sin \omega/2} \cdot e^{-j\frac{5}{2}\omega} \right)^2$$

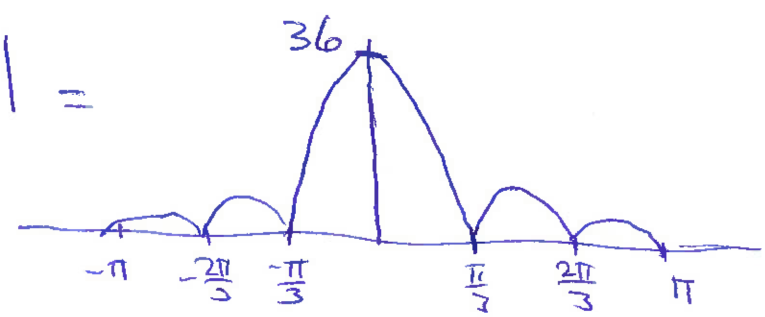
$$= \underbrace{\left(\frac{\sin(3\omega)}{\sin \omega/2} \right)^2}_{\text{never negative}} \cdot e^{-j5\omega}$$

$$\therefore \measuredangle R(e^{j\omega}) = \measuredangle \left(\frac{\sin(3\omega)}{\sin(\omega/2)} \right)^2 - 5\omega$$

$$= -5\omega$$



$|R(e^{j\omega})| =$



2.6

(a) $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$\therefore\ x[n-n_0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega(n-n_0)}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{e^{-j\omega n_0} X(e^{j\omega})}_{\text{DTFT of } x[n-n_0]} e^{j\omega n}\, d\omega$$

(b) $x[n]$ real $\Leftrightarrow$ $x[n] = x^*[n]$

$x[n]$ even $\Leftrightarrow$ $x[n] = x[-n]$

$x[n] \Leftrightarrow X(e^{j\omega})$, $x[-n] \Leftrightarrow X(e^{-j\omega})$,

$x^*[n] \Leftrightarrow X^*(e^{-j\omega})$

$x[n]$ real $\Leftrightarrow$ $X(e^{j\omega}) = X^*(e^{-j\omega})$ — (A)

$x[n]$ even $\Leftrightarrow$ $X(e^{j\omega}) = X(e^{-j\omega})$ — (B)

Substituting (B) in (A)

$X(e^{j\omega}) = X^*(e^{j\omega}) \Rightarrow X(e^{j\omega})$ real

(B) $\Rightarrow$ $X(e^{j\omega})$ is even.

(c) A Real $x[n] \Longleftrightarrow X(e^{j\omega}) = X^*(e^{-j\omega})$

B Odd $x[n] \Longleftrightarrow X(e^{j\omega}) = -X(e^{-j\omega})$

B $\Rightarrow X(e^{j\omega})$ is odd

Substituting B in A, we obtain

$$X(e^{j\omega}) = -X^*(e^{j\omega})$$

$\Rightarrow X(e^{j\omega})$ is purely imaginary.