Problem 2.1

Suppose $x(t)$ is a real-valued speech-signal whose Fourier transform is $X(j\omega)$ and it is known that $|X(j\omega)| = 0$ for $|\omega| \geq 10,000\pi$. Let $x[n] = x(nT)$ where T represents the sampling interval. Answer the following questions about $X(e^{j\omega})$, the DTFT of $x[n]$, for the specified values of T.

(a) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.0001$ secs. *Justify your answer.*

(b) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.00005$ secs. *Justify your answer.*

(c) For what values of ω is $X(e^{j\omega})$ guaranteed to be zero if $T = 0.00001$ secs. *Justify your answer.*

Problem 2.2

Sketch the magnitude of the DTFT of each of the following discrete-time signals. *Explain your reasoning in each part.*

a) $x[n] = \delta[n-3]$

b) $x[n] = \delta[n-3] + \delta[n+3]$

c) $x[n] = u[n] - u[n-4]$, where $u[n]$ is the unit step.

d) $x[n] = u[n+2] - u[n-4]$

Problem 2.3

Calculate the convolution $y[n] = x[n] * h[n]$ in each of the following cases and *show your work*:

a) $x[n] = u[n] - u[n-5]$ and $h[n] = 0.5\delta[n-3]$

b) $x[n] = n\{u[n-1] - u[n-5]\}$ and $h[n] = 2\delta[n+3]$

c) $x[n] = u[n] - u[n-5]$ and $h[n] = u[n] - u[n-5]$

d) $x[n] = u[n] - u[n-5]$ and $h[n] = u[n] - u[n-3]$

e) $x[n] = u[n] - u[n-5]$ and $h[n] = u[n]$

Problem 2.4

Sketch the phase of the DTFT of $x[n] = u[n] - u[n-4]$. *Justify your answer.*

Problem 2.5

Sketch the magnitude and the phase of the DTFT of the signal $r[n] = \sum\limits_{k=-\infty}^{\infty} x[k]x[n-k]$,

where $x[n] = u[n] - u[n-6]$. *Show your work.*

Problem 2.6

a) *Show* that if $x[n]$ has DTFT $X(e^{j\omega})$, then $x[n-n_0]$ has DTFT $e^{-j\omega_0}X(e^{j\omega})$

b) *Show* that a real and even $x[n]$ has a real and even DTFT $X(e^{j\omega})$.

c) *Show* that a real and odd $x[n]$ has an imaginary and odd DTFT $X(e^{j\omega})$.