Homework 2 *Digital Signal Processing* EC516

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Problem 1: Real Valued Signal

Suppose $x(t)$ is a real-valued speech-signal whose Fourier transform is $X(j\omega)$ and it is known that $|X(j\omega)| = 0$ for $|\omega| \geq 10,000\pi|$. Let $x[n] = x(nT)$ where T represents the sampling interval. Answer the following questions about $X(e^{j\omega}))$, the DTFT of x[n], for the specified values of T.

First, since the signal is real-valued, I belive the FT should be symmetric about zero (no information in negatives). Though this does not matter for the problem.

The following questions all essentially ask where the frequency content is definitely zero, given the band-limiting conditions above. Note that, when sampled, the DFT repeats every $2\pi/T$. The response then becomes "compressed" in frequency.

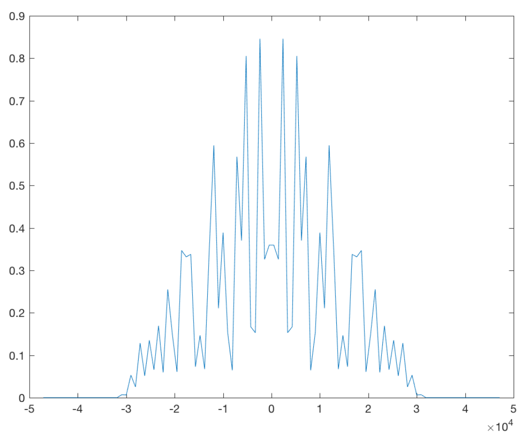

Figure 1: Basic Signal

a. Zeros

a..1 T = 0.0001 seconds

With this sample rate, we have replication occur at $2\pi/1E-4$. Setting this symmetric about zero, we have a signal repeating on the interval $[-\pi E4, \pi E4]$. This is exactly equal to our band-limiting condition. Thus the only guaranteed zeros occur at $\pi E4 * (2k-1), k \in \mathbb{Z}$.

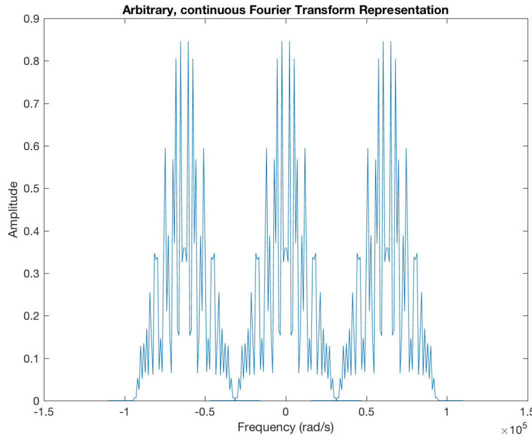

Figure 2: Repetition

a..2 T = T1/2

Similar to problem 1, but our band now becomes limited at $2\pi/5E-5$, or four times the band limiting rate. Effectively doubling the bandwidth from the last question. The bandwidth is zero at the specified value, 10,000π. The function folds at $2\pi E4*(2k-1)$. Thus we have zeros in all regions, [10,000πk, $2\pi E4*(2k-1)+10,000\pi$.

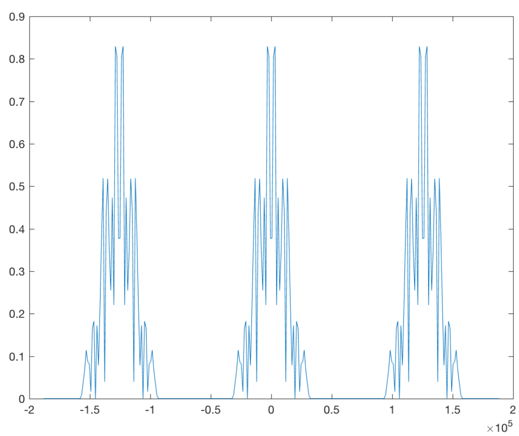

Figure 3: Repetition

a..3 T1/10

Using the same process as above, we have zeros between [10,000πk, $2\pi E4*(2k-1)+10,000\pi$.

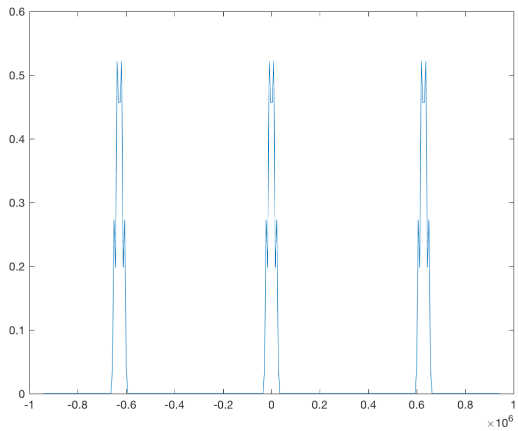

Figure 4: Repetition

b. Magnitude of Fourier Transform

$$\delta[n-3] \to |e^{3j\omega}| = 1.$$
$$\delta[n-3] + \delta[n+3] \to |e^{3j\omega}e^{-3j\omega}| = 2\cos(3\omega).$$
$$u[n] - u[n-4](\text{five sample box}) \to \frac{\sin(4\omega/2)}{\sin(\omega/2))}e^{-j\omega\cdot 3/2}$$

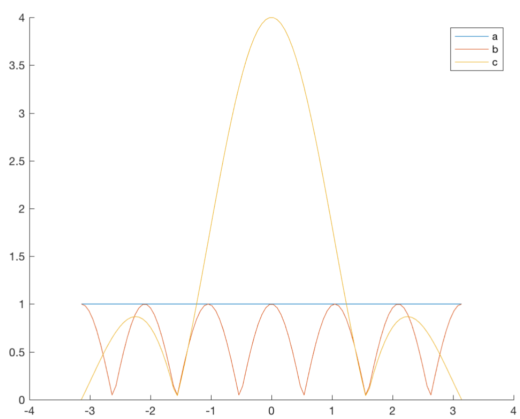

Figure 5: DFT Magnitudes

c. Phase of DTFT

First, construct the full signal. Note the convolution of a box with a box should be a triangle (potentially with its head cut off). $\Sigma conv(x[n], e^{j\omega}) = \Sigma((u[n] - u[n-4])e^{-j\omega})$