COMPUTER SCIENCE & ENGINEERING

Title: Linear Algebra and Differential Equation

COURSE & CODE: MAT1234

ASSIGNMENT: 01

GROUP: 03

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Assignment-1 (Mathematics)

1(a)

Defination: An equation involving derivatives of one or more dependent variable with respect to one or more independent variable is called a different equation.

Solution:

According to the question, the circle passes through the origin at x-axis.

So, equation-

$$x^2 + y^2 + 2fy = 0 \qquad [where, c = 0, g = 0]$$

$$=> 2x + 2y.y_1 + 2f.y_1 = 0$$

$$=> 2x + 2y.y_1 + 2\frac{-x^2-y^2}{2y}.y_1 = 0$$

$$=> 2x + 2y.\frac{dy}{dx} - \frac{(x^2+y^2)}{y}.\frac{dy}{dx} = 0$$

$$=> 2xy.dy + 2y^2.dy - (x^2 + y^2)dy = 0$$ [Multiply by y.dx]

$$=> 2xy.dy + dy(2y^2 - x^2 - y^2) = 0$$

$$=> 2xy.dy + dy(y^2 - x^2) = 0$$

$$=> (x^2 - y^2)dy - 2xy.dy = 0$$

Which is the Required Solution.

1(b)

Order: The order of a different differentiates equation is the order of the highest differential of the equation.

Example:

$$2(\frac{d^3y}{dx^3})^2 + 3\frac{d^2y}{dx^2} + \frac{d^5y}{dx^5} = 0$$

Here, order of the differential equation is 5

Degree: The degree of a differential equation is the degree of the derivative of the highest degree in the differential equation.

Example:

$$2\left(\frac{d^3y}{dx^3}\right)^2+3\frac{d^2y}{dx^2}+\frac{d^5y}{dx^5} = 0$$

Here, degree of the differential equation is 1

Distinguish between an ODE & PDE:

1. ODE's involve derivative is only one variable where as PDE's involve derivatives in multiple variables.
2. ODE has one independent variable, say x.
   Solution is y(x).
   PDE has more independent variables say $x_1, x_2 \ldots\ldots\ldots\ldots x_n$.
   Solution is $y = (x_1, x_2, \ldots \ldots \ldots \ldots \ldots \ldots x_n)$.

1(c)

Solution:

$$y = e^x(Asin2x + Bcos2x)$$

$$\Rightarrow \frac{dy}{dx} = e^x(Asin2x + Bcos2x) + e^x(2.Acos2x - 2.Bsin2x)$$

$$\Rightarrow \frac{dy}{dx} = y + e^x(2Acos2x - 2Bsin2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(2Acos2x - 2Bsin2x) + e^x(-4.Asin2x - 4.Bcos2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - 4e^x(Asin2x + Bcos2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - 4y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y=0$$

Which is the required solution.

1(d)

Solution:

$$\Rightarrow Ax^2 + By^2 = 1 \quad \_\_\_\_\_\_\_\_(i)$$
$$\Rightarrow 2Ax + 2Byy' = 0$$
$$\Rightarrow Ax + Byy' = 0$$

Multiplying both side with "x" we find

$\Rightarrow Ax^2 + Bxyy' = 0$ ________ (ii)

Now equation (ii) –(i)

$\Rightarrow Bxyy' - By^2 + Ax^2 - Ax^2 = -1$

$\Rightarrow xyy' - y^2 = -\frac{1}{B}$

Differentiating w.r.t x

$\Rightarrow x\frac{d}{dx}(yy') + y.y' - 2y.y' = 0$

$\Rightarrow x(yy'' + y'y') - y.y' = 0$

$\Rightarrow x\left[y.\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y.\frac{dy}{dx}$

Which is the required Solution.

1(e)

Solution:

$y = (2A + Blogx + c(logx)^2 + 3x^2)$

$\Rightarrow \frac{dy}{dx} = 0 + B.\frac{1}{x} + 2c.logx.\frac{1}{x} + 6x$

$\Rightarrow x.\frac{dy}{dx} = B + 2clogx + 6x^2$ [Multiplying both side with x]

$\Rightarrow x.\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 = 0 + 2c.\frac{1}{x} + 12x$

$\Rightarrow x^2.\frac{d^2y}{dx^2} + x.\frac{dy}{dx}. = 2c + 12x^2$ [Multiplying both side with x]

$\Rightarrow x^2.\frac{d^3y}{dx^3} + 2x.\frac{d^2y}{dx^2} + x.\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 = 24x$

$\Rightarrow x^2.\frac{d^3y}{dx^3} + 3x.\frac{d^2y}{dx^2} + \frac{dy}{dx} - 24x = 0$

Which is the required equation.

1(f)

Solution:

$y = ae^x + be^{-x} + ccosx + dsinx$

$\Rightarrow \frac{dy}{dx} = ae^x - be^{-x} - csinx + dcosx$

$\Rightarrow \frac{d^2y}{dx^2} = ae^x + be^{-x} - ccosx - dsinx$

$\Rightarrow \frac{d^3y}{dx^3} = ae^x - be^{-x} + csinx - dcosx$

$\Rightarrow \frac{d^4y}{dx^4} = ae^x + be^{-x} + ccosx + dsinx$

$\Rightarrow \frac{d^4y}{dx^4} = y$

$\Rightarrow \frac{d^4y}{dx^4} - y = 0$

Which is the required equation.

4 (i)

Homogeneous differential equation:

When M and N of the equation Mdx + Ndy = 0 both are the same degree in x and y and homogeneous then the equation is called to be homogeneous.

Example:

F(x) = $x^2 + 2xy + y^2$

4 (ii)

Linear differential equation:

A differential equation of the form $\frac{dy}{dx} + py = Q$; where P and Q are function of x alone or constant is called a linear differential equation of 1st order.

Example:

$$\frac{dy}{dx} + 3xy = x^2$$

4 (iii)

Solution:

Given equation,

$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$

Here,

M = $x^3 + 3xy^2 \quad \therefore \frac{\partial M}{\partial y} = 6xy$

N = $3x^2y + y^3 \quad \therefore \frac{\partial N}{\partial x} = 6xy$

$\therefore$ The given differential equation is exact.

The solution is,

$\int_{y=const}(x^3 + 3xy^2)dx + \int y^3 dy = \acute{c}$

$=> \frac{x^4}{4}+3y^2\frac{x^2}{2}+\frac{y^4}{4} = ć$

$=> (x^4 + 6x^2y^2 + y^4) = 4ć$

$=> x^4 + 6x^2y^2 + y^4 = ć$ [Where $4ć = c$]

Which is the required solution.

4(iv)

Solution:

Given Equation,

$(1 + xy)y\ dx + (1 - xy)x\ dy = 0$

Here,

M = $y + xy^2\ \therefore\ \frac{\partial M}{\partial y} = 1 + 2xy$

N = $x - x^2y\ \therefore\ \frac{\partial N}{\partial x} = 1 - 2xy$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$; the given differential equation is not exact.

Now,

$Mx - Ny = xy + x^2y^2 - xy + x^2y^2 = 2x^2y^2$

$\therefore Mx - Ny \neq 0$

Therefore the integrating factor, I.F = $\frac{1}{Mx-Ny}$

= $\frac{1}{2x^2y^2}$

Now multiplying the given equation by I.F, $\frac{1}{2x^2y^2}$

$$\frac{1}{2x^2y^2}\ (1 + xy)y\ dx + \frac{1}{2x^2y^2}(1 - xy)x\ dy = 0$$

$$=> \left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \left(\frac{1}{2xy^2} - \frac{1}{2y}\right)dy = 0$$

Now the differential equation is exact.

$\therefore$ The solution is,

$$\int_{y=const}\left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \int \frac{1}{2y}dy = ć$$

$$=> \frac{1}{2y}\left(-\frac{1}{x}\right) + \frac{1}{2}\ lnx - \frac{1}{2}lny = ć$$

$$=>\ lnx - lny - \frac{1}{xy} = 2ć$$

$=> ln\frac{x}{y} - \frac{1}{xy} = c$ [Where$2\acute{c} = c$]
Which is the required solution.

4(v)

Solution:

Given equation,

$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

Here,

M = $x^3 + 3xy^2 \quad \therefore \frac{\partial M}{\partial y} = 6xy$

N = $y^3 + 3x^2y \quad \therefore \frac{\partial N}{\partial x} = 6xy$

$\therefore$ The given differential equation is exact.

The solution is,

$\int_{y=const}(x^3 + 3xy^2)dx + \int y^3 dy = \acute{c}$

$=> \frac{x^4}{4} + 3y^2\frac{x^2}{2} + \frac{y^4}{4} = \acute{c}$

$=> (x^4 + 6x^2y^2 + y^4) = 4\acute{c}$

$=> x^4 + 6x^2y^2 + y^4 = \acute{c}$ [Where $4\acute{c} = c$]

Which is the required solution.

4(vi)

Solution:

Given equation,

$$\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$$

$$=> \frac{dy}{dx} = \frac{2(2x + 3y + 5)}{2x + 3y + 4} \cdots\cdots\cdots (1)$$

Let,

$2x + 3y = v$

$=> 2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$$=> \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

Now eq(1) becomes,

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{2v+5}{v+4}$$

$$=> \left(\frac{dv}{dx} - 2\right) = \frac{6v+15}{v+4}$$

$$=> \frac{dv}{dx} = \frac{6v+15}{v+4} + 2$$

$$=> \frac{dv}{dx} = \frac{6v+15+2v+8}{v+4}$$

$$=> \frac{dv}{dx} = \frac{8v+23}{v+4}$$

$$=> dv\left(\frac{v+4}{8v+23}\right) = dx$$

$$=> dv\left(\frac{v+4}{8(v+4)-9}\right) = dx$$

$$=> dv\left(\frac{1}{8} - \frac{v+4}{9}\right) = dx$$

Now integrating both side,

$$\int dv\left(\frac{1}{8} - \frac{v+4}{9}\right) = \int dx$$

$$=> \frac{v}{8} - \frac{1}{9}\left(\frac{v^2}{2} + 4v\right) = x + ć$$

$$=> \frac{9v - 4v^2 + 32v}{72} = x + ć$$

$$=> 9v - 4v^2 + 32v = 72x + 72ć$$

$$=> 41v - 4v^2 = 72x + 72ć$$

$$=> 41(2x+3y) - 4(2x+3y)^2 = 72x + \text{c}$$

$$=> 82x + 123y - 4(2x+3y)^2 = 72x + \text{c}$$

$$=> 10x + 123y - 4(2x+3y)^2 = \text{c}$$

Which is the required solution.

4(vii)

Solution:

Given equation,

$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

Here,

M = $x^3 + 3xy^2 \quad \therefore \dfrac{\partial M}{\partial y} = 6xy$

N = $y^3 + 3x^2y \quad \therefore \dfrac{\partial N}{\partial x} = 6xy$

∴ The given differential equation is exact.

The solution is,

$\int_{y=const}(x^3 + 3xy^2)dx + \int y^3 dy = \acute{c}$

$=> \frac{x^4}{4} + 3y^2\frac{x^2}{2} + \frac{y^4}{4} = \acute{c}$

$=> (x^4 + 6x^2y^2 + y^4) = 4\acute{c}$

$=> x^4 + 6x^2y^2 + y^4 = \acute{c}$ [Where $4\acute{c} = c$]

Which is the required solution.

4(viii)

Solution:

Given equation

$$\frac{dy}{dx} + 2ytanx = sinx;\ y\left(\frac{\pi}{3}\right) = 0$$

Differential equation of the form,

$\frac{dy}{dx} + py = Q$

$\frac{dy}{dx} + 2ytanx = sinx$ ………….(1)

Where,

P = 2tanx

Q = sinx

∴ I.F = $e^{\int 2tanx\, dx}$

= $e^{2\log secx}$

= $e^{log\ sec^2 x}$

= $sec^2 x$

Multiplying both side of (1) by $sec^2 x$

$$sec^2 x\frac{dy}{dx} + sec^2 x\, 2ytanx = sec^2 x\, sinx$$

$\frac{d}{dx}(sec^2\text{x y}) = sec^2\text{x sinx}$

Now integrating both side,

$$\int d\ (sec^2\text{x y}) = \int \text{tanx secx dx}$$

$$=> sec^2\text{xy} = \text{secx} + \text{c}$$

<u>4(ix)</u>

<u>Solution:</u>

Given equation,

$\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$ ....(1)

Let,

Y = vx

$=> \frac{dy}{dx} = v + x\frac{dv}{dx}$ ....(2)

From (1) and (2) we get,

$$\frac{vx(x+vx)}{x(vx-x)} = v + x\frac{dv}{dx}$$

$$=> \frac{v(1+v)}{(v-1)} = v + x\frac{dv}{dx}$$

$$=> \frac{v+v^2-v^2+v}{v-1} = x\frac{dv}{dx}$$

$$=> \frac{2v}{v-1} = x\frac{dv}{dx}$$

$$=> \frac{1}{2}\left(\frac{v-1}{2v}\right)dv = \frac{dx}{x}$$

Now integrating both side,

$$\int \frac{1}{2}\left(\frac{v-1}{2v}\right)\text{dv} = \int \frac{dx}{x}$$

$$=> \frac{1}{2}v - \frac{1}{2}lnv = lnx + \acute{c}$$

$$=> \frac{y}{x} - ln\frac{y}{x} = 2lnx + 2\,\acute{c}$$

$$=> \frac{y}{x} - ln\frac{y}{x} = 2lnx + c$$

4(x)

Solution:

Given equation,

$\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$ ....(1)

Let,

$x = x+h \quad ; y = y+k$

$=> \frac{dY}{dX} = \frac{Y+K-X-h+1}{Y+X+K+h+5}$

$=> \frac{dY}{dX} = \frac{Y-X+K-h+1}{Y+X+K+h+5}$

Let,

$K - h + 1 = 0$ ....(2)

$k + h + 5 = 0$ .....(3)

From (1) and (3) we get

$2k + 6 = 0$

$=> k = -3$

$\therefore h = -2$

Now,

$\frac{dY}{dX} = \frac{Y-X}{Y+X}$ ......(4)

Let,

Y= vX

$$=> \frac{dY}{dX} = v + X\frac{dv}{dX}$$

Now eq(4)becomes,

$$v + X\frac{dv}{dX} = \frac{vX - X}{vX + X}$$

$$=> X\frac{dv}{dX} = \frac{v-1}{v+1} - v$$

$$=> -X\frac{dv}{dX} = \frac{1-v^2}{v+1}$$

$$=> -\frac{dX}{X} = \left(\frac{1}{1+v^2} + \frac{2}{2+2v^2}\right)dv$$

Now integrating both side,

$$\int \left(\frac{1}{1+v^2} + \frac{2}{2+2v^2}\right) dv = \int -\frac{dX}{X}$$

$$=> \tan^{-1} v + \frac{1}{2}\ln(1+v^2) = -lnX + c$$

$$=> \tan^{-1}\frac{Y}{X} + \frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right) = -lnX + c$$

$$=> tan^{-1}\left(\frac{y+3}{x+2}\right)) + \frac{1}{2}\ln\left(1+\frac{(y+3)^2}{(x+2)^2}\right) = -lnX + c$$

$$=> tan^{-1}\left(\frac{y+3}{x+2}\right)) + \ln\sqrt{1+\frac{(y+3)^2}{(x+2)^2}} = -lnX + c$$

Which is the required equation.

4(xii)

Solution:

Given equation,

$\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$ ....(1)

Let,

$Y = vx$

$=> \frac{dy}{dx} = v + x\frac{dv}{dx}$ ....(2)

From (1) and (3) we get,

$$v + x\frac{dv}{dx} = v + tanv$$

$$=> x\frac{dv}{dx} = tanv$$

$$=> \frac{dv}{tanv} = \frac{dx}{x}$$

Now integrating both side,

$$\int cotv\, dv = \int \frac{dx}{x}$$

$$=> \log(sinv) + c = logx$$

$=> \log(c\ sinv) = logx$
$=> c\ sinv = x$
$=> x = c \sin\dfrac{y}{x}$
Which is the required solution.

4(xiii)

Solution:

Given equation,

$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$ …………(1)

$=> \frac{d}{dx}(v - x) = \frac{v+1}{2v+1}$ [Let x+y = v]

$$=> \frac{dv}{dx} - 1 = \frac{v+1}{2v+1}$$

$$=> \frac{dv}{dx} = \frac{3v+2}{2v+1}$$

$$=> \frac{2v+1}{3v+2}\, dv = dx$$

Now integrating both side,

$$\int \frac{2v+1}{3v+2}\, dv = \int dx$$

$$=> \frac{1}{3}\left(2v - \frac{1}{3}\ln(3v+2)\right) = x + ć$$

$$=> 6(x+y) - \ln(3x+3y+2) = 9\, x + 9\, ć$$

$$=> 6x + 6y - \ln(3x+3y+2) = 9x + c$$

$$=> 6y - 3x - \ln(3x+3y+2) = c$$

Which is the required solution.