$= (1 + a^2 + b^2)^2 \{1 - a^2 - b^2 + 2a^2 + 0 - 2b(0 - b)\}$

$= (1 + a^2 + b^2)^2 \{1 + a^2 - b^2 + 2b^2\}$

$= (1 + a^2 + b^2)^2 (1 + a^2 + b^2) = (1 + a^2 + b^2)^3$ [Proved]

**Example-15** Prove that [প্রমাণ কর যে,]

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

*[NUH '02; DUH '88; DUHT '86; RUH '81]*

**Solution** L.H.S. $= \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

$$= \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (c+a)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix} \quad [C_2' = C_2 - C_1 \;\&\; C_3' = C_3 - C_1]$$

$$= \begin{vmatrix} (b+c)^2 & (a+b+c)(a-b-c) & (a+b+c)(a-b-c) \\ b^2 & (c+a+b)(c+a-b) & 0 \\ c^2 & 0 & (a+b+c)(a+b-c) \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} 2bc & -2c & -2b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix} \quad [R_1' = R_1 - (R_2 + R_3)]$$

$$= 2(a+b+c)^2 \begin{vmatrix} bc & -c & -b \\ b^2 & c+a-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$

$$= \frac{2(a+b+c)^2}{bc} \begin{vmatrix} bc & -bc & -bc \\ b^2 & bc+ab-b^2 & 0 \\ c^2 & 0 & ca+bc-c^2 \end{vmatrix}$$

$$= \frac{2(a+b+c)^2}{bc}\begin{vmatrix} bc & 0 & 0 \\ b^2 & bc+ab & b^2 \\ c^2 & c^2 & ca+bc \end{vmatrix} \quad [C_2' = C_2 + C_1 \;\&\; C_3' = C_3 + C_1]$$

$$= \frac{2(a+b+c)^2 bc}{bc}\begin{vmatrix} b(c+a) & b^2 \\ c^2 & c(a+b) \end{vmatrix}$$

$$= 2bc(a+b+c)^2\begin{vmatrix} c+a & b \\ c & a+b \end{vmatrix}$$

$$= 2bc(a+b+c)^2\,(ca+bc+a^2+ab-bc)$$

$$= 2bc(a+b+c)^2\,(ca+a^2+ab) = 2abc(a+b+c)^3 = \text{R.H.S. [Proved]}$$

**Example-16** If $s = a+b+c$, then prove that [যদি $s = a+b+c$ হয়, তবে প্রমাণ কর যে]

$$\begin{vmatrix} (s-a)^2 & a^2 & a^2 \\ b^2 & (s-b)^2 & b^2 \\ c^2 & c^2 & (s-c)^2 \end{vmatrix} = 2abcs^3$$

***[DUH '73, '78; DUHT '80, '86; RUH '81; CUH '86; JUH '76]***

**Solution** Given, $s = a+b+c$

$\therefore s-a = b+c,\; s-b = c+a$ and $s-c = a+b$.

$$\text{L.H.S.} = \begin{vmatrix} (s-a)^2 & a^2 & a^2 \\ b^2 & (s-b)^2 & b^2 \\ c^2 & c^2 & (s-c)^2 \end{vmatrix}$$

$$= \begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$$= 2abc\,(a+b+c)^3 \quad \text{[According to Example-15]}$$

$$= 2abcs^3 = \text{L.H.S. [Proved]}$$

**Example-17** If $2s = a+b+c$, then prove that [যদি $2s = a+b+c$ হয়, তবে প্র
কর যে],

***[DUH '78; DUHT '***

**Example-28** Prove that [প্রমাণ কর যে,]

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (3abc-a^3-b^3-c^3)^2$$

**Solution** $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= -\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix}$$

$$= \begin{vmatrix} -a^2+bc+bc & -ab+ba+c^2 & -ac+b^2+ca \\ -ba+c^2+ab & -b^2+ca+ac & -bc+cb+a^2 \\ -ca+ac+b^2 & -cb+a^2+bc & -c^2+ab+ba \end{vmatrix}$$

$$= \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$

Again $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc-a^2) - b(b^2-ca) + c(ab-c^2)$$

$$= abc - a^3 - b^3 + abc + abc - c^3$$

$$= (3abc - a^3 - b^3 - c^3)$$

$$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = (3abc-a^3-b^3-c^3)^2$$

Hence, $$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (3abc-a^3-b^3-c^3)^2.$$

[Proved]

$$= xyzabc \begin{vmatrix} \cdots \\ -2 & -2 & -2 & c \\ x & y & z & 0 \end{vmatrix}$$

$$= xyzabc \begin{vmatrix} 0 & 0 & 0 & a-b \\ 0 & 0 & 0 & b-c \\ -2 & -2 & -2 & c \\ x & y & z & 0 \end{vmatrix} \begin{bmatrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{bmatrix}$$

$$= -xyzabc\,(a-b) \begin{vmatrix} 0 & 0 & 0 \\ -2 & -2 & -2 \\ x & y & z \end{vmatrix}$$

$= -xyzabc\,(a-b) \times 0 = 0 =$ R.H.S. [Proved]

**Example-48** Prove that [প্রমাণ কর যে,] $\begin{vmatrix} 1 & a & a^2 & 0 \\ 0 & 1 & a & a^2 \\ a^2 & 0 & 1 & a \\ a & a^2 & 0 & 1 \end{vmatrix} = 1 + a^4 + a^8$

*[NUH '01; RUH '85; CUH '79]*

**Solution** L.H.S. $= \begin{vmatrix} 1 & a & a^2 & 0 \\ 0 & 1 & a & a^2 \\ a^2 & 0 & 1 & a \\ a & a^2 & 0 & 1 \end{vmatrix}$

$$= \begin{vmatrix} 1+a+a^2 & a & a^2 & 0 \\ 1+a+a^2 & 1 & a & a^2 \\ 1+a+a^2 & 0 & 1 & a \\ 1+a+a^2 & a^2 & 0 & 1 \end{vmatrix} [C_1' = C_1 + C_2 + C_3 + C_4]$$

$$= (1+a+a^2) \begin{vmatrix} 1 & a & a^2 & 0 \\ 1 & 1 & a & a^2 \\ 1 & 0 & 1 & a \\ 1 & a^2 & 0 & 1 \end{vmatrix}$$

$$= (1+a+a^2) \begin{vmatrix} 1 & a & a^2 & 0 \\ 0 & 1-a & a-a^2 & a^2 \\ 0 & -a & 1-a^2 & a \\ 0 & a^2-a & -a^2 & 1 \end{vmatrix} \begin{bmatrix} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \\ R_4' = R_4 - R_1 \end{bmatrix}$$

$$= (1+a+a^2) \begin{vmatrix} 1-a & a-a^2 & a^2 \\ -a & 1-a^2 & a \\ a^2-a & -a^2 & 1 \end{vmatrix}$$

$$= (1+a+a^2) \begin{vmatrix} 1-a+a^2 & a-a^2 & a^2 \\ 0 & 1-a^2 & a \\ 1-a+a^2 & -a^2 & 1 \end{vmatrix} [C_1' = C_1 + C_3]$$

$$= (1+a+a^2)(1-a+a^2)\begin{vmatrix} 1 & a-a^2 & a^2 \\ 0 & 1-a^2 & a \\ 1 & -a^2 & 1 \end{vmatrix}$$

$$= (1+a+a^2)(1-a+a^2)\{(1-a^2)+a^3-(a-a^2)(-a)+a^2(-1+$$

$$= (1+a^2+a^4)(1-a^2+a^3+a^2-a^3-a^2+a^4)$$

$$= (1+a^2+a^4)(1-a^2+a^4) = 1+a^4+a^8$$

= R.H.S. [Proved]

**Example-49** Prove that [প্রমাণ কর যে,] $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+x & 1 \\ 1 & 1 & 1 & 1+x \end{vmatrix} = x^3(x+4)$

**Solution** L.H.S. $= \begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+x & 1 \\ 1 & 1 & 1 & 1+x \end{vmatrix}$

$$= \begin{vmatrix} x+4 & 1 & 1 & 1 \\ x+4 & 1+x & 1 & 1 \\ x+4 & 1 & 1+x & 1 \\ x+4 & 1 & 1 & 1+x \end{vmatrix} \quad [C_1' = C_1 + C_2 + C_3 + C_4]$$

$$= (x+4)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+x & 1 & 1 \\ 1 & 1 & 1+x & 1 \\ 1 & 1 & 1 & 1+x \end{vmatrix}$$

$$= (x+4)\begin{vmatrix} 0 & -x & 0 & 0 \\ 0 & x & -x & 0 \\ 0 & 0 & x & -x \\ 1 & 1 & 1 & 1+x \end{vmatrix} \quad \begin{bmatrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \\ R_3' = R_3 - R_4 \end{bmatrix}$$

$$= -(x+4)\begin{vmatrix} -x & 0 & 0 \\ x & -x & 0 \\ 0 & x & -x \end{vmatrix}$$

$$= x(x+4)\begin{vmatrix} -x & 0 \\ x & -x \end{vmatrix}$$

$= x^3(x+4) =$ R.H.S. [Proved]

**Example-50**

$$= a_1a_2a_3a_4\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+\frac{1}{a_4}\right) = \text{R.H.S. [Proved]}$$

**Example-57** Prove that [প্রমাণ কর যে,] $\begin{vmatrix} x & 1 & 1 & \cdots & 1 \\ 1 & x & 1 & \cdots & 1 \\ 1 & 1 & x & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & 1 & \cdots & x \end{vmatrix} = (x-1)^{n-1}(x+n$

where given determinant is of order $n$. [যেখানে প্রদত্ত নির্ণায়কটির ক্রম $n$]

*[DUHT '83; CUH*

**Solution** L.H.S. $= \begin{vmatrix} x & 1 & 1 & \cdots & 1 \\ 1 & x & 1 & \cdots & 1 \\ 1 & 1 & x & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & 1 & \cdots & x \end{vmatrix}$

$$= \begin{vmatrix} x+n-1 & 1 & 1 & \cdots & 1 \\ x+n-1 & x & 1 & \cdots & 1 \\ x+n-1 & 1 & x & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x+n-1 & 1 & 1 & \cdots & x \end{vmatrix} \quad [C_1' = C_1 + C_2 + \ldots\ldots + C_n]$$

$$= (x+n-1)\begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & x & 1 & \cdots & 1 \\ 1 & 1 & x & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & 1 & \cdots & x \end{vmatrix}$$

$$= (x+n-1)\begin{vmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & x-1 & 0 & \cdots & 0 \\ 1 & 0 & x-1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & x-1 \end{vmatrix} \quad \begin{bmatrix}\text{Subtracting 1st column} \\ \text{from each columns}\end{bmatrix}$$

$$= (x+n-1)\begin{vmatrix} x-1 & 0 & 0 & \cdots & 0 \\ 0 & x-1 & 0 & \cdots & 0 \\ 0 & 0 & x-1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & x-1 \end{vmatrix}$$

$$= (x-1)^{n-1}(x+n-1) = \text{R.H.S. [Proved]}$$