Bangladesh Open University
School of Science and Technology
B.Sc in Computer Science and Engineering
First Year Second Semester Examination,
Course Code & Title: MAT1234 Linear Algebra and Differential Equation

1. (a) Define differential equation. Show that the differential equation of circle touch the x-axis at the origin is

$$(x^2 - y^2)\,dy - 2xy\,dx = 0.$$

(b) Define order and degree of a differential equation with examples. Distinguish between an ODE and a PDE.

(c) Find the differential equations from the following equation:
$y = e^x(Asin2x + Bcos2x)$.

(d) Show that, $Ax^2 + By^2 = 1$ is the solution of $x\left[y\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2\right] = y\dfrac{dy}{dx}$

(e) Find the differential equation of which $y = 2A + Blogx + C(logx)^2 + 3x^2$ is a solution

(f) Find the differential equation of which $y = ae^x + be^{-x} + c\,Cosx + d\,Sinx$ is a solution

2. Solve any three of the following equations:

(i) $dy = (y^2 - 1)\,dx$;

(ii) $\frac{dy}{dx} = 1 + e^{x-y}$;

(iii) $\frac{dy}{dx} = \sin(x+y) + \cos\,(x+y)$;

(iv) $(x^2 + y^2)\,dy = xy\,dx$.

(v) $x^2(1+y)dy + y^2(x-1)dx = 0$

(vi) $e^{x-y}dx + e^{y-x} = 0$;

(vii) $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

(viii) $(x^2 + y^2)\,dy = xy\,dx$.

(ix) $x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$.

3. (a) Prove that the differential equation $M\,dx + N\,dy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, where M and N both are functions of x, y.

(b) Verify that the differential equation $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ is exact and hence solve it.

(c) Determine whether the equation $(2x + 3y + 4)\,dx + (3x - 6y - 5)\,dy = 0$ is exact. If it is then solve it.

(d) Explain an exact differential equation and a linear differential equation with example

(e) What is integrating factor? Solve the following equations:

(i) $(12y + 4y^3 + 6x^2)dx + 3(x + xy^2)\,dy = 0$;

(ii) $y^2(ydx + 2xdy) - x^2(2ydx + xdy) = 0$.

4. Define homogeneous and linear differential equation with examples.
Solve:

Bangladesh Open University
School of Science and Technology
B.Sc in Computer Science and Engineering
First Year Second Semester Examination,
Course Code & Title: MAT1234 Linear Algebra and Differential Equation

(iii) $(x^3 + 3xy^2)\,dx + (3x^2y + y^3)\,dy = 0;$
(iv) $(1 + xy)y\,dx + (1 - xy)x\,dy = 0.$
(v) $(x^3 + 3xy^2)\,dx + (y^3 + 3x^2y)\,dy = 0$
(vi) $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$
(vii) $(x^3 + 3xy^2)\,dx + (y^3 + 3x^2y)\,dy = 0$
(viii) $\frac{dy}{dx} + 2ytanx = sinx, \quad y\left(\frac{\pi}{3}\right) = 0.$
(ix) $\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$
(x) $\frac{dy}{dx} = \frac{y-x-1}{y+x+5}$
(xi) $\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3}$
(xii) $\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$
(xiii) $(x + y + 1)dx - (2x + 2y + 1)dy = 0$

5. (a) Define Bernoulli's equation and hence solve $\frac{dy}{dx} + x\sin 2y = x^3cos^2y$

(b) Solve the equations:
$(i)\ (1 + y^2)\,dx = (\tan^{-1} y - x)\,dy;\ (ii)\ x\frac{dy}{dx} + y = y^2 \log_e x.$

(c) (i) Find the general solution of $2y'' - 7y' + 3y = 0.$
(ii) Find the particular solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ when $y(0) = 0$ and $y'(0) = 1.$

(d) It is evident that $y_p = 3x$ is a particular solution of the equation $y'' + 4y = 12x$, and that $y_c(x) = c_1cos2x + c_2sin2x$ is its complementary solution. Find a solution of this differential equation that satisfies the initial conditions $y(0) = 5, y'(0) = 7.$

(e) Find the complementary function of the equations:
i) $(D^3 - 8)y = 0;$
ii) $(D^3 + 3D^2 + 3D + 1)y = 0.$

6. (a) Solve :
(i) $(D^2 - 4D + 13)y = 0;$
(ii) $(D^2 + 4)y = e^x + x^2;$
(iii) $(D^2 + a^2)y = cosax$
(iv) $(4D^2 + 12D + 9)y = 144e^{-3x}$
(v) $(D^3 + 8)y = x^4 + 2x + 1.$
(vi) $(D^3 - 2D^2 - 19D + 20)y = 0$
(vii) $(D^2 + 1)y = sin3x$
(viii) $(D^2 + 3D + 2)y = 0$ when $y(0) = 0$ and $y'(0) = 1.$
(ix) $(D^2 - 5D + 6)y = x^3e^{2x}$
(x) $(D^2 + 4)y = 12x$ when $y(0) = 5, y'(0) = 7.$
(xi) $(D^2 - 2D + 4)y = e^xCosx;$

Bangladesh Open University

School of Science and Technology

B.Sc in Computer Science and Engineering

First Year Second Semester Examination,

Course Code & Title: MAT1234 Linear Algebra and Differential Equation