ASSIGNMENT-01

COMPUTER SCIENCE & ENGINEERING

1ST YEAR 2ND SEMESTER

LINEAR ALGEBRA & DIFFERENTIAL EQUATION

COURSE & CODE : MAT-1231

GROUP: 01

QUESTION NO – 2 & 6

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ASSIGNMENT- 01

ANSWER TO THE QUESTION NO. 2

(i) $\boldsymbol{dy = (y^2 - 1)dx}$

Sol^n:

$$dy = (y^2 - 1)dx$$

$$\Longrightarrow (y^2 - 1)dx - dy = 0 \quad \cdots\cdots\cdots\cdots (1)$$

Here,

$$M = y^2 - 1$$

$$\Longrightarrow \frac{\partial M}{\partial y} = 2y$$

And

$$N = -1$$

$$\Longrightarrow \frac{\partial N}{\partial x} = 0$$

Since, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ *therefore equation (1) is not exact*

Now,

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y^2-1}(-2y)$$

$$= -\frac{2y}{y^2-1}$$ *which is a function of y only*

$$= f(y)$$

$$\therefore Integrating\ Factor\ = \ e^{\int -\frac{2y}{y^2-1}\, dy}$$

$$= e^{-\int \frac{2y}{y^2-1}\, dy}$$

$$= e^{-ln(y^2-1)}$$

$$= e^{ln(y^2-1)^{-1}}$$

$$= (y^2-1)^{-1}$$

$$= \frac{\mathbf{1}}{\mathbf{y^2-1}}$$

Multiplying equation (1) by $\frac{\mathbf{1}}{\mathbf{y^2-1}}$

$dx - \frac{1}{y^2-1}dy = 0 \cdots\cdots\cdots\cdots\cdots (2)$

Which is exact.

Now the solution is

$$\int_{y=constant} dx + \int -\frac{1}{y^2-1}dy = c'$$

$\Longrightarrow x - \int \frac{1}{y^2-1}dy = c'$

$\Longrightarrow x - \frac{1}{2}ln\left(\frac{y-1}{y+1}\right) = c'$

$\Longrightarrow 2x - ln\left(\frac{y-1}{y+1}\right) = 2c'$

$\therefore 2x - ln\left(\frac{y-1}{y+1}\right) = c \qquad [where\ 2c' = c]$

Which is the required solution.

(ii) $\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \mathbf{1} + \boldsymbol{e^{x-y}}$

Sol^n:

$\frac{dy}{dx} = 1 + e^{x-y}$

Multiplying by e^y

$\Longrightarrow e^y\,\frac{dy}{dx} = e^y + e^x$

$\Longrightarrow e^y dy = e^y dx + e^x dx$

Integrating both side

$\Longrightarrow \int e^y dy = \int e^y dx + \int e^x dx$

$\Longrightarrow e^y = xe^y + e^x + c$

$\Longrightarrow e^y - xe^y = e^x + c$

$\Longrightarrow e^y(1-x) = e^x + c$

which is the required solution.

(iii) $\boldsymbol{\frac{dy}{dx} = sin(x+y) + cos(x+y)}$

Sol^n:

$\frac{dy}{dx} = sin(x+y) + cos(x+y)$

$let,\ x + y = z$

$\Longrightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\Longrightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$

$\Longrightarrow \frac{dz}{dx} - 1 = sinz + cosz$

$\Longrightarrow \frac{dz}{dx} = 1 + sinz + cosz$

$\Longrightarrow \frac{dz}{dx} = 2cos^2\frac{z}{2} + 2sin\frac{z}{2} \cdot cos\frac{z}{2}$

$\Longrightarrow \frac{dz}{dx} = 2cos^2\frac{z}{2}(1 + \frac{sin\frac{z}{2}}{cos\frac{z}{2}})$

$\Longrightarrow \frac{dz}{dx} = 2cos^2\frac{z}{2}(1 + tan\frac{z}{2})$

$\Longrightarrow \frac{dz}{dx} = \frac{1 + tan\frac{z}{2}}{\frac{1}{2}sec^2\frac{z}{2}}$

$\Longrightarrow \int \frac{\frac{1}{2}sec^2\frac{z}{2}}{1 + tan\frac{z}{2}}\, dz = \int dx$

$\Longrightarrow ln\left(1 + tan\frac{z}{2}\right) = x + c$

$\Longrightarrow ln\left(1 + tan\frac{x+y}{2}\right) = x + c$

Which is the required solution.

(iv) $\boldsymbol{(x^2 + y^2)dy = xy\, dx}$

Sol^n:

$(x^2 + y^2)dy = xy\, dx$

$\Longrightarrow \frac{dy}{dx} = \frac{xy}{x^2+y^2} \quad \cdots\cdots\cdots\cdots\cdots (1)$

Let, $y = vx$

$\Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From equation (1)

$v + x\frac{dv}{dx} = \frac{x.\ vx}{x^2 + v^2x^2} = \frac{vx^2}{x^2(1+v^2)}$

$$\Longrightarrow x\frac{dv}{dx} = \frac{v}{1+v^2} - v$$

$$\Longrightarrow x\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$$

$$\Longrightarrow x\frac{dv}{dx} = -\frac{v^3}{1+v^2}$$

$$\Longrightarrow -\frac{1+v^2}{v^3}\,dv = \frac{dx}{x}$$

$$\Longrightarrow -\left(\frac{1}{v^3} + \frac{v^2}{v^3}\right) dv = \frac{dx}{x}$$

$$\Longrightarrow \int -\left(\frac{1}{v^3} + \frac{1}{v}\right) dv = \int \frac{dx}{x}$$

$$\Longrightarrow -\frac{1}{-2v^2} - lnv = lnx + lnc$$

$$\Longrightarrow \frac{1}{2v^2} = lnx + lnc + lnv$$

$$\Longrightarrow \frac{1}{2v^2} = ln(cvx)$$

$$\Longrightarrow \frac{1}{2\frac{y^2}{x^2}} = ln(c\cdot\frac{y}{x}\cdot x)$$

$$\Longrightarrow \frac{x^2}{2y^2} = ln(cy)$$

Which is the required solution.

(v) $\boldsymbol{x^2(1+y)dy + y^2(x-1)dx = 0}$

Sol^n:

$$x^2(1+y)\,dy + y^2(x-1)\,dx = 0$$

Dividing by x^2y^2

$$\Longrightarrow \frac{1+y}{y^2}\,dy + \frac{x-1}{x^2}\,dx = 0$$

$$\Longrightarrow \left(\frac{1}{y^2} + \frac{y}{y^2}\right) dy + \left(\frac{x}{x^2} - \frac{1}{x^2}\right) dx = 0$$

Integrating both side

$$\Longrightarrow \int \left(\frac{1}{y^2} + \frac{y}{y^2}\right) dy + \int \left(\frac{x}{x^2} - \frac{1}{x^2}\right) dx = c$$

$$\Longrightarrow -\frac{1}{y} + lny + lnx + \frac{1}{x} = c$$

$$\Longrightarrow ln(xy) + \frac{1}{x} - \frac{1}{y} = c$$

$$\Longrightarrow ln(xy) + \frac{y-x}{xy} = c$$

$\Longrightarrow xy\ ln(xy) + y - x = cxy$

Which is the required solution.

(vi) $\boldsymbol{e^{x-y}dx + e^{y-x}dy = 0}$

Sol

$$e^{x-y}dx + e^{y-x}dy = 0$$

$\Longrightarrow \frac{e^x}{e^y}dx + \frac{e^y}{e^x}dy = 0$

Multiplying by $e^x e^y$

$$e^{2x}dx + e^{2y}dy = 0$$

Integrating both side

$\Longrightarrow \frac{e^{2x}}{2} + \frac{e^{2y}}{2} = c'$

$\Longrightarrow e^{2x} + e^{2y} = 2c'$

$\Longrightarrow e^{2x} + e^{2y} = c$ *[where* $2c' = c$*]*

Which is the required solution.

(vii) $\boldsymbol{(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0}$

Sol^n:

$$(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$$

$\Longrightarrow x^2(1-y)dy + y^2(1+x)dx = 0$

Dividing by x^2y^2

$\Longrightarrow \frac{1-y}{y^2}dy + \frac{1-x}{x^2}dx = 0$

$\Longrightarrow \left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \left(\frac{1}{x^2} + \frac{1}{x}\right)dx = 0$

Integrating both side

$\Longrightarrow \int \left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int \left(\frac{1}{x^2} + \frac{1}{x}\right)dx = c$

$\Longrightarrow \frac{1}{-y} - lny + \frac{1}{-x} + lnx = c$

$\Longrightarrow -\frac{1}{y} - lny - \frac{1}{x} + lnx = c$

$\Longrightarrow ln\frac{x}{y} - \left(\frac{1}{x} + \frac{1}{y}\right) = c$

$\Longrightarrow ln\frac{x}{y} - \frac{x+y}{xy} = c$

$\Longrightarrow xy\ ln\frac{x}{y} - x - y = cxy$

Which is the required solution.

(viii) $(\boldsymbol{x^2 + y^2})\boldsymbol{dy} = \boldsymbol{xy\ dx}$

Sol^n:

$(x^2 + y^2)dy = xy\ dx$

$\Longrightarrow \frac{dy}{dx} = \frac{xy}{x^2+y^2} \cdots\cdots\cdots\cdots\cdots (1)$

Let, $y = vx$

$\Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From equation (1)

$v + x\frac{dv}{dx} = \frac{x.\ vx}{x^2+v^2x^2} = \frac{vx^2}{x^2(1+v^2)}$

$\Longrightarrow x\frac{dv}{dx} = \frac{v}{1+v^2} - v$

$\Longrightarrow x\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$

$\Longrightarrow x\frac{dv}{dx} = -\frac{v^3}{1+v^2}$

$\Longrightarrow -\frac{1+v^2}{v^3}dv = \frac{dx}{x}$

$\Longrightarrow -\left(\frac{1}{v^3} + \frac{v^2}{v^3}\right)dv = \frac{dx}{x}$

$\Longrightarrow \int -\left(\frac{1}{v^3} + \frac{1}{v}\right)dv = \int\frac{dx}{x}$

$\Longrightarrow -\frac{1}{-2v^2} - lnv = lnx + lnc$

$\Longrightarrow \frac{1}{2v^2} = lnx + lnc + lnv$

$\Longrightarrow \frac{1}{2v^2} = ln(cvx)$

$\Longrightarrow \frac{1}{\mathbf{2}\frac{y^2}{x^2}} = ln(c \cdot \frac{y}{x} \cdot x)$

$\Longrightarrow \frac{x^2}{2y^2} = ln(cy)$

Which is the required solution.

(ix) $\boldsymbol{x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0}$

Sol^n:

$$x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$$

Dividing by $\sqrt{1+x^2} \cdot \sqrt{1+y^2}$

$\Longrightarrow \frac{x}{\sqrt{1+x^2}}\, dx + \frac{y}{\sqrt{1+y^2}}\, dy = 0$

Integrating both side

$\Longrightarrow \int \frac{x}{\sqrt{\mathbf{1+x^2}}} dx + \int \frac{y}{\sqrt{\mathbf{1+y^2}}} dy = c'$

$\Longrightarrow \int \frac{1}{\sqrt{z}} \cdot \frac{1}{2} dz + \int \frac{1}{\sqrt{v}} \cdot \frac{1}{2} dv = c'$

$\Longrightarrow \frac{1}{2}\int \frac{1}{\sqrt{z}} dz + \frac{1}{2}\int \frac{1}{\sqrt{v}} dv = c'$

$\Longrightarrow \frac{\sqrt{z}}{2} + \frac{\sqrt{v}}{2} = c'$

$\Longrightarrow \sqrt{z} + \sqrt{v} = 2c'$

$\Longrightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = c$ *[where* $2c' = c$*]*

Which is the required solution.

Let, $1 + x^2 = z$

$\Longrightarrow 2x = \frac{dz}{dx}$

$\Longrightarrow x\, dx = \frac{1}{2} dz$

Let, $1 + y^2 = v$

$\Longrightarrow 2y = \frac{dv}{dy}$

$\Longrightarrow ydy = \frac{1}{2} dv$

ANSWER TO THE QUESTION NO 6

(i) $(\boldsymbol{D^2 - 4D + 13)y = 0}$

Sol^n:

$(D^2 - 4D + 13)y = 0$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E)is,$

$(m^2 + 4m + 13)e^{mx} = 0$

$\Longrightarrow (m^2 + 4m + 13) = 0 \qquad [\because e^{mx} \neq 0]$

$\Longrightarrow m = \frac{\boldsymbol{4 \pm \sqrt{16 - 52}}}{\boldsymbol{2}}$

$\Longrightarrow m = \frac{4 \pm 6i}{2}$

$\therefore m = 2 \pm 3i$

$The\ General\ Solution\ (G.S)\ is,$

$\therefore y = e^{2x}[A\ cas3x + B\ Sin3x]$

$Which\ is\ the\ required\ solution.$

(ii) $(\boldsymbol{D^2 + 4)y = e^x + x^2}$

Sol^n:

$(D^2 + 4)y = e^x + x^2$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E)is,$

$(m^2 + 4)e^{mx} = 0$

$\Longrightarrow (m^2 + 4) = 0 \qquad [\because e^{mx} \neq 0]$

$\therefore m = \pm 2i$

$The\ Cofficient\ Function\ (C.F)\ is,$

$\therefore y_c = [A\ cas2x + B\ Sin2x]$

$The\ Particular\ Integral\ (P.I)\ is\ ,$

$$y_p = \frac{\mathbf{1}}{\boldsymbol{D^2+4}}\ (e^x + x^2)$$

$$= \frac{\mathbf{1}}{\boldsymbol{D^2+4}}\ .e^x + \frac{\mathbf{1}}{\boldsymbol{D^2+4}}.x^2$$

$$= \frac{\mathbf{1}}{\mathbf{1^2+4}}\ .e^x + \frac{1}{4}\left(1 + \frac{\boldsymbol{D^2}}{\mathbf{4}}\right)^{-1}.x^2$$

$$= \frac{1}{5}\ .e^x + \frac{1}{4}\left(1 - \frac{D^2}{4} + \frac{D^4}{16} - \cdots \ldots \ldots\right)\ x^2$$

$$= \frac{1}{5}\ .e^x + \frac{1}{4}\left(x^2 - \frac{2}{4}\right)$$

$$= \frac{1}{5}\ .e^x + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)$$

$$\therefore y_p = \frac{1}{5}\ .e^x + \frac{1}{8}(2x^2 - 1)$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore y = ACos2x + BSin2x + \frac{1}{5}\ .e^x + \frac{1}{8}(2x^2 - 1)$$

$Which\ is\ the\ required\ solution.$

(iii) $(\boldsymbol{D^2 + a^2})\boldsymbol{y} = \boldsymbol{Cos\ ax}$

Sol^n:

$(D^2 + a^2)y = Cos\ ax$

$Let,\ y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^2 + a^2)e^{mx} = 0$$

$$\Longrightarrow (m^2 + a^2) = 0 \qquad\qquad [\because e^{mx} \neq 0]$$

$$\therefore m = \pm ai$$

$The\ Cofficient\ Function\ (C.F)\ is,$

$$\therefore y_c = [A\ cas\ ax + B\ Sin\ ax]$$

$The\ Particular\ Integral\ (P.I)\ is$

$$y_p = \frac{\mathbf{1}}{\boldsymbol{D^2+a^2}}\ Cos\ ax$$

$$= x.\frac{1}{2D}\ Cos\ ax$$

$$= \frac{x}{2}\ \int Cos\ ax$$

$$\therefore y_p = \frac{x}{2a}\ Sin\ ax$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore y = ACos\ ax + BSin\ ax + \frac{x}{2a}\ Sin\ ax$$

$Which\ is\ the\ required\ solution.$

(iv) $\quad \boldsymbol{(4D^2 + 12D + 9)y = 144\ e^{-3x}}$

Sol^n:

$$(4D^2 + 12D + 9)y = 144\ e^{-3x}$$

$$Let, y = e^{mx}$$

$$\Longrightarrow Dy = me^{mx}$$

$$\Longrightarrow D^2y = m^2e^{mx}$$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(4m^2 + 12m + l)e^{mx} = 0$$

$\Longrightarrow (2m^2 + 3) = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\therefore m = -\frac{3}{2}, \ -\frac{3}{2}$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$$\therefore y_c = C_1 e^{-\frac{3x}{2}} + x\, C_2 e^{-\frac{3x}{2}}$$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{\mathbf{1}}{(\mathbf{2D+3})^{\mathbf{2}}}\ 144\, e^{-3x}$$

$$= 144\ \frac{\mathbf{1}}{\{\mathbf{2(-3)+3}\}^{\mathbf{2}}}\ \ e^{-3x}$$

$$= \ 144 \,.\frac{1}{9}.\ e^{-3x}$$

$$\therefore y_p = \ 16\, e^{-3x}$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = \ y_c + y_p$$

$$\therefore y = C_1 \boldsymbol{e}^{-\frac{3x}{2}} + x\, C_2 \boldsymbol{e}^{-\frac{3x}{2}} + \ 16\, e^{-3x}$$

$Which\ is\ the\ required\ solution.$

(v) $(\boldsymbol{D^3} + \mathbf{8})\boldsymbol{y} = \boldsymbol{x^4} + \mathbf{2}\boldsymbol{x} + \mathbf{1}$

Sol^n:

$(D^3 + 8)y = x^4 + 2x + 1$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$\Longrightarrow D^3y = m^3e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^3+8)e^{mx}=0$$

$$\Longrightarrow (m^3+8)=0 \qquad\qquad [\because e^{mx}\neq 0]$$

$$\Longrightarrow m^3+2m^2-2m^2-4m\ +4m\ +8=0$$

$$\Longrightarrow m^2(m+2)-2m\,(m+2)+4(m\ +2)\ =0$$

$$\Longrightarrow (m+2)(m^2-2m\ +4)=0$$

$$\Longrightarrow m=-2,\ \frac{\mathbf{2\pm\sqrt{4-16}}}{\mathbf{2}}$$

$$\Longrightarrow m=-2,\ \frac{\mathbf{2\pm 2\sqrt{3}\ \mathit{i}}}{\mathbf{2}}$$

$$\therefore m=-2,\ 1\pm i\sqrt{3}$$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$$\therefore y_c=C_1e^{-2x}+\ e^{x}\,[ACos\sqrt{3}\,x+BSin\sqrt{3}\,x]$$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p=\frac{1}{D^3+8}\ (x^4+2x\ +1)$$

$$=\ \frac{1}{8}\left(1+\frac{\boldsymbol{D^3}}{\mathbf{8}}\right)^{-1}(x^4+2x\ +1)$$

$$=\ \frac{1}{8}\left(1-\frac{D^3}{8}+\frac{D^6}{64}\ldots\ldots\ldots\right)(x^4+2x\ +1)$$

$$=\ \frac{1}{8}\left(x^4+2x\ +1-\frac{24x}{8}\right)$$

$$\therefore y_p\ =\ \frac{1}{8}(x^4-x+1)$$

$The\ General\ Equation\ (G.S)\ is,$

$$y=\ y_c+y_p$$

$$\therefore y=C_1e^{-2x}+\ e^{x}\,[ACos\sqrt{3}\,x+BSin\sqrt{3}\,x]+\frac{1}{8}(x^4-x+1)$$

$Which\ is\ the\ required\ solution.$

(vi) $(\boldsymbol{D^3 - 2D^2 - 19D + 20})\boldsymbol{y = 0}$

Sol^n:

$(D^3 - 2D^2 - 19D + 20)y = 0$

$Let, \;\; y = e^{mx}$

$\Longrightarrow Dy \; = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$\Longrightarrow D^3y = m^3e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^3 - 2m^2 - 19m \; + 20) \; e^{mx} = 0$

$\Longrightarrow m^3 - 2m^2 - 19m \; + 20 = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\Longrightarrow m^3 - m^2 - m^2 + m \; - 20m \; + 20 = 0$

$\Longrightarrow m^2(m-1) - m\,(m-1) + 20(m-1) = 0$

$\Longrightarrow (m-1)\;(m^2 - m \; + 20) = 0$

$\Longrightarrow (m-1)\;(m-5)\;(m+4) = 0$

$\therefore m = 1, 5, -4$

$The\ General\ Equation\ (G.S)\ is,$

$\therefore y \; = C_1e^x + C_2\,e^{5x} + C_3\,e^{-4x}$

$Which\ is\ the\ required\ solution.$

(vii) $(\boldsymbol{D^2 + 1}\,)\boldsymbol{y = Sin\ 3x}$

Sol^n:

$(D^2 + 1\,)y = Sin\ 3x$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^2 + 1)e^{mx} = 0$

$\Longrightarrow (m^2 + 1) = 0 \qquad [\because e^{mx} \neq 0]$

$\therefore m = \pm\, i$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$\therefore y_c = [A\ cas\ x + B\ Sin\ x]$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{1}{D^2+1}\ \ Sin\ 3x$$

$$= \frac{1}{-3^2+1}\ \ Sin\ 3x$$

$$\therefore y_p = -\frac{1}{8}\ \ Sin\ 3x$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore y = A\ cas\ x + B\ Sin\ x\ -\frac{1}{8}\ \ Sin\ 3x$$

$Which\ is\ the\ required\ solution.$

(viii) $(\boldsymbol{D^2} + \mathbf{3}\boldsymbol{D} + \mathbf{2})\boldsymbol{y} = \mathbf{0}$

Sol^n:

$(D^2 + 3D + 2)y = 0$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E)is,$

$$(m^2 + 3m\ + 2)\ e^{mx} = 0$$

$\Longrightarrow m^2 + 3m\ + 2 = 0 \qquad [\because e^{mx} \neq 0]$

$\Longrightarrow (m + 2)\ (m + 1)\ = 0$

$\therefore m = -1, -2$

$The\ General\ Equation\ (G.S)\ is,$

$$y(x) = C_1 e^{-x} + C_2 e^{-2x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\Longrightarrow y'(x) = -C_1 e^{-x} - 2C_2 e^{-2x}$$

$Now,$

$$y(0) = C_1 + C_2$$

$$\Longrightarrow 0 = C_1 + C_2$$

$$\therefore C_1 = -C_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$Again,$

$$y'(0) = -C_1 - 2C_2$$

$$\Longrightarrow 1 = -C_1 - 2C_2$$

$$\Longrightarrow -(-C_2) - 2C_2 = 1$$

$$\therefore C_2 = -1$$

$From\ equation\ (2)$

$$\Longrightarrow C_1 = -(-1)$$

$$\therefore C_1 = 1$$

$Now\ From\ equation\ (1)\ the\ Particular\ solution\ is,$

$$\therefore y = e^{-x} - e^{-2x}$$

$Which\ is\ the\ required\ solution.$

(ix) $\boldsymbol{(D^2 + 5D + 6)y = x^3 e^{2x}}$

Sol^n:

$$(D^2 + 5D + 6)y = x^3 e^{2x}$$

$$Let, y = e^{mx}$$

$$\Longrightarrow Dy = me^{mx}$$

$$\Longrightarrow D^2 y = m^2 e^{mx}$$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^2 - 5m + 6)\ e^{mx} = 0$$

$$\Longrightarrow m^2 - 3m - 2m + 6 = 0 \qquad [\because e^{mx} \neq 0]$$

$$\Longrightarrow (m-2)\ (m-3) = 0$$

$$\therefore m = 2,3$$

$The\ Cofficient\ Function\ (C.F)\ is,$

$$\therefore y_c = C_1\ e^{2x} + C_2\ e^{3x}$$

$The\ Particular\ Integral\ (P.I)\ is$

$$y_p = \frac{\mathbf{1}}{(\boldsymbol{D}-\mathbf{2})\ (\boldsymbol{D}-\mathbf{3})}\ x^3 e^{2x}$$

$$= \frac{\mathbf{1}}{(\boldsymbol{D}+\mathbf{2}-\mathbf{2})\ (\boldsymbol{D}+\mathbf{2}-\mathbf{3})}\ x^3 e^{2x}$$

$$= e^{2x}\ \frac{\mathbf{1}}{\boldsymbol{D}\ (\boldsymbol{D}-\mathbf{1})}\ x^3$$

$$= -e^{2x}\ \frac{1}{D}(1-D)^{-1}\ x^3$$

$$= -e^{2x}\ \frac{1}{D}(1 + D + D^2 + D^3 + \cdots \ldots \ldots \ldots \ldots \ldots \ldots)\ \ x^3$$

$$= -e^{2x}\ \frac{1}{D}(x^3 + 3x^2 + 6x + 6)$$

$$= -e^{2x}\ \left(\frac{\boldsymbol{x^4}}{\mathbf{4}} + 3\frac{\boldsymbol{x^3}}{\mathbf{3}} + 6\frac{\boldsymbol{x^2}}{\mathbf{2}} + 6x\right)$$

$$\therefore y_p = -\ \frac{\boldsymbol{e^{2x}}}{\mathbf{4}}(x^4 + 4x^3 + 12x^2 + 24x)$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore\ y = C_1\ e^{2x} + C_2\ e^{3x} - \frac{\boldsymbol{e^{2x}}}{\mathbf{4}}(x^4 + 4x^3 + 12x^2 + 24x)$$

$Which\ is\ the\ required\ solution.$

(x) $(\boldsymbol{D^2+4})\boldsymbol{y} = \mathbf{12}\boldsymbol{x}$

Sol^n:

$(D^2+4)y = 12x$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^2+4)\ e^{mx} = 0$

$\Longrightarrow (m^2+4) = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\therefore m = \pm 2i$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$\therefore y_c = [A\ cas\ 2x + B\ Sin\ 2x]$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{1}{D^2+4}\ 12x$$

$$= 12.\frac{1}{4}\ \frac{1}{\left(1+\frac{D^2}{4}\right)}.x$$

$$= 3.\ \left(1+\frac{D^2}{4}\right)^{-1}.x$$

$$= 3.\left(1-\frac{D^2}{4}+\frac{D^4}{16}-\cdots \ldots\ldots\ldots\ldots\ldots\right).x$$

$\therefore y_p = \ 3x$

$The\ General\ Equation\ (G.S)\ is,$

$y(x) = \ y_c + y_p$

$\Longrightarrow y(x) = A\ Cas\ 2x + B\ Sin\ 2x + \ 3x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$

$\Longrightarrow y'(x) = -2A\ sin\ 2x + 2B\ cos\ 2x + \ 3$

$Now,$

$$y(0) = A\ Cas\ 2.0 + B\ Sin\ 2.0 + \ 3.0$$

$$\Longrightarrow 5 = A\ Cas\ 0 + B\ Sin\ 0$$

$$\Longrightarrow 5 = A\ .1 + B\ .0$$

$$\therefore A = 5$$

$Again,$

$$y'(0) = -2A\ sin\ 2.0 + 2B\ cos\ 2.0 + \ 3$$

$$\Longrightarrow 7 = -2A\ sin\ 0 + 2B\ cos\ 0 + \ 3$$

$$\Longrightarrow -2A\ .0 + 2B\ .1 = 7 - \ 3$$

$$\therefore B \ = 2$$

$From\ equation\ (1)\ the\ particular\ solution\ is\ ,$

$$\therefore y = \ 5\ Cas\ 2x + 2\ Sin\ 2x + \ 3x$$

$Which\ is\ the\ required\ solution.$

(xi) $\boldsymbol{(D^2 - 2D + 4)y = e^x Cosx}$

Sol^n:

$(D^2 - 2D + 4)y = e^x Cosx$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E)is,$

$$(m^2 - 2m + 4)\ e^{mx} = 0$$

$$\Longrightarrow m^2 - 2m + 4 = 0 \qquad [\because e^{mx} \neq 0]$$

$$\Longrightarrow m = \ \frac{\mathbf{2 \pm \sqrt{4-16}}}{\mathbf{2}}$$

$$\Longrightarrow m = -2,\ \frac{\mathbf{2 \pm 2\sqrt{3}\ \boldsymbol{i}}}{\mathbf{2}}$$

$$\therefore m = \ \ 1 \pm i\sqrt{3}$$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$\therefore y_c = \ e^x\ [ACos\sqrt{3}\,x + BSin\sqrt{3}\,x]$

$The\ Particular\ Integral\ \ (P.I)\ is,$

$$y_p = \frac{\mathbf{1}}{\boldsymbol{D^2 - 2D + 4}}\ e^x\ Cos\ x$$

$$= e^x\ \ \frac{\mathbf{1}}{\boldsymbol{(D+1) - 2\ (D+1) + 4}}\ Cos\ x$$

$$= \ e^x\ \frac{\mathbf{1}}{\boldsymbol{D^2 + 3}}\ \ Cos\ x$$

$$= \ e^x\ \frac{\mathbf{1}}{\boldsymbol{-1^2 + 3}}\ \ Cos\ x$$

$$\therefore y_p = \ \frac{1}{2}\ \ e^x\ Cos\ x$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = \ y_c + y_p$$

$$\therefore y = e^x\left[ACos\sqrt{3}\,x + BSin\sqrt{3}\,x\right] + \ \frac{1}{2}\ \ e^x\ Cos\ x$$

$Which\ is\ the\ required\ solution.$