বাংলাদেশ উন্মুক্ত বিশ্ববিদ্যালয়

BANGLADESH OPEN UNIVERSITY

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School of Science and Technology

B.Sc. in Computer Science and Engineering

Assignment-1

Assignment On: Linear Algebra and Differential Equation

Course code – MAT 1234

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Session: 2018-2019

1st Year, 2nd Semester

Date of Submission: 07/03/2020

3(a)

Solution: let ,$\mathrm{Mdx} + \mathrm{Ndy} = 0 - - - - - (1)$ is exact

then the differential equation function is U=U(x,y)

$\therefore \mathrm{Mdx} + \mathrm{Ndy} = \mathrm{du} - - - - - (2)$

But, $\mathrm{du} = \frac{\delta \mathrm{u}}{\delta \mathrm{x}}(\mathrm{dx}) + \frac{\delta \mathrm{u}}{\delta \mathrm{y}}(\mathrm{dy}) - - - - - (3)$

Now (2) and (3) we get,

$\frac{\delta \mathrm{u}}{\delta \mathrm{x}} = \mathrm{M} - - - - - - (4)$

$\frac{\delta \mathrm{u}}{\delta \mathrm{y}} = \mathrm{N} - - - - - - (4)$

$\therefore \ \frac{\partial u^2}{\delta y \partial x} = \frac{\partial u^2}{\partial x \delta y}$

$or \ \frac{\partial}{\partial y}(\frac{\partial u}{\partial x}) = \frac{\partial}{\partial x}(\frac{\partial u}{\partial y})$

or $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(proved)

3(b) Verify that the differential equation

$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$ is exact and hence solve it.

Solution: Given that, $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$

$=> (1 + e^v)\mathrm{d}(vy) + (1 - v)e^v dy = 0$ Let, x=vy

$=> (1 + e^v)(vdy + ydv) + (1 - v)e^v dy = 0$

$=> (1 + e^v)ydv + (v + ve^v + e^v - ve^v)dy = 0$

$=> (1 + e^v)ydv + (v + e^v)dy = 0$

$=> \frac{1+e^v}{v+e^v}dv + \frac{dy}{y} = 0$

$=> ln(v + e^v) + lny = lnc$

$$=> \ ln(v + e^v)y = lnc$$

$$=> vy + e^v y = c$$

$$\therefore x + e^{\frac{x}{y}} y = c \qquad \text{since, } V = \frac{x}{y}$$

3(c) solve it (2x+3y+4)dx +(3x-6y-5)dy =0

Solution: let, $M = 2x + 3y + 4$

$N = 3x - 6y - 5$

Now,$\frac{\delta M}{\delta y} = 3$

$\frac{\delta N}{\delta y} = 3$

$\therefore \textit{the given equation is exact}$

$\therefore \textit{the solution is } \int_{y=const.} Mdx + \int(\textit{term in N independent of x})dy = c$

Or $\int(2x + 3y + 4)dx + \int(-6y - 5)dy = c$

Or $2\frac{x^2}{2} + 3xy + 4x - 6 - 5y = c$

$x^2 + 3xy + 4x - 5y - 6 -= c$

Which is the required solution

3(d)

Solution:

Exact differential equation:

The differential equation M dx+N dy=0----------(1) is called exact differential equation if L.H.S(1) is exact, that is if M dx+ N dy=du where M=M(x,y), N=N(x,y) and U=U(x,y)

Linear differential equation:

Linear equation with constant co-efficient equation of the 'n'th order the typical form of linear differential equation with constant co-efficient of the n^{th} order is

$\therefore \frac{d^n}{dx^n} + a1\frac{d^{n-1}y}{dx^{n-1}} + a2\frac{d^{n-2}y}{dx^{n-2}} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . . a_{nY=X}$

$((D^n + a1D^{n-1} + a2d^{n-2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots . . a_n)Y = X$

3(e)

(i)solve it $(12y+4y^3+6x^2)dx + 3(x+xy^2)dy=0$ using integrating factor

Solution: Here, $M=12y+4y^3+6x^2$

$$\frac{\delta M}{\delta y} = 12 + 12y^2$$

$$N=3x+3xy^2$$

$$\frac{\delta N}{\delta x} = 3 + 3y^2$$

The given equation is not exact

Now, $\frac{1}{N}(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x})$

$= \frac{1}{3x(1+y^2)}(9 + 9y^2)$

$=\frac{3}{x}$

Which is the function is X only

$\therefore$ *the I.F is* $= e^{\int f(x)dx}$

$$= e^{\int \frac{3}{x}dx}$$

$$= e^{3lnx}$$

$$=x^3$$

Now multiplying the given equation by I. F.= x^3

$\therefore\ x^3(12y + 4y^3 + 6x^2)dx + 3x^3(x + xy^2)dy = 0$

Which is exact

The solution is

$\int(12x^3 + 4x^3y^3 + 6x^5)dx + \int 0.dy = c \qquad (y = constant)$

$12\frac{x^4}{4}.y + 4\frac{y^3x^4}{4} + 6\frac{x^6}{6} = c$

$3x^4y + x^4y^3 + x^6 = c$ Which is the required solution.

3(e)

(ii) solve it y^2(y dx+2x dy)-x^2(2y dy+x dy)=0 using integrating factor

Given that, $y^2(ydx + 2xdy) - x^2(2ydx + xdy) = 0$

$$=> y^3dx + 2xy^2dy - 2x^2ydx - x^3dy = 0$$

$$=> (y^3 - 2x^2y)dx + (2xy^2 - x^3)dy = 0 \qquad \text{-----------------(i)}$$

M=$y^3 - 2x^2$ $\quad => \frac{\partial M}{\partial y} = 3y^2 - 2x^2$ and

N = $2xy^2 - x^3$ $\quad => \frac{\partial N}{\partial x} = 2y^2 - 3x^2$

Since $\frac{\partial M}{\partial N} \neq \frac{\partial N}{\partial x}$ So, the equation (i) is not exact.

Now, $Mx + Ny = xy^3 - 2x^3y + 2xy^3 - x^3y$

= $3xy^3 - 3x^3y$

= $3xy(y^2 - x^2)$

IF = $\frac{1}{Mx+Ny} = \frac{1}{3xy(y^2-x^2)}$

Now multiplying both sides of equation (i) we have

$$\frac{y^3-2x^2y}{3xy(y^2-x^2)}dx + \frac{2xy^2-x^3}{3xy(y^2-x^2)}dy = 0$$

$$= \frac{y\{(y^2-x^2)-x^2\}}{3x \ (y^2-x^2)}dx + \frac{x\{(y^2-x^2)+y^2\}}{3xy(y^2-x^2)}dy = 0$$

$$= \left[\frac{1}{3x} - \frac{x}{3(y^2-x^2)}\right]dx + \left[\frac{1}{3y} - \frac{y}{3(y^2-x^2)}\right]dy = 0 \text{ --------------------(ii)}$$

Now the equation (ii) exact equation.

$$\int_{y=const} \left[\frac{1}{3x} - \frac{x}{3(y^2 - x^2)}\right]dx + \int \frac{1}{3y}dy = 0$$

$$= \frac{1}{3}\log x + \frac{1}{6}\log(y^2 - x^2) + \frac{1}{3}\log y = \frac{1}{6}\log c$$

$$= \log x^2 + \log(y^2 - x^2) + \log y^2 = \log c$$

$$\therefore \ x^2y^2(y^2 - x^2) = c \qquad \text{Ans.}$$

5.(a):

Define Bernoulli's equation and hence solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$

Bernoulli's equation: The first order differential equation of

$\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called Bernoulli's equation.

Now, $\frac{dy}{dx} + xsin2y = x^3cos^2y$

$= sec^2y\frac{dy}{dx} + 2x\ tan\ y = x^3$ Multiplying by sec^2x

$= \frac{d\ (\tan y)}{dx} + 2x\ tan\ y = x^3$ This is a first order linear differential equation of tan y

Now I.F. = $e^{\int 2x\ dx} = e^{x^2}$

So, the general solution $\tan y\ e^{x^2} = \int e^{x^2}dx$

$= \frac{1}{2}\int x^2e^{x^2}dx^2 = \frac{1}{2}\left(x^2e^{x^2} - e^{x^2}\right) + \frac{c}{2}$

$=> 2\tan y = x^2 + ce^{x^{-2}}$

5.(b): Solve

(I) $(1+y^2)dx=(\tan^{-1} y - x)dy$

Solution: $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$---------------------(1)

I.F=$e^{\int\frac{1}{1+y^2}}$ $=e^{\tan^{-1} y}$

Multiplying (1) by I.F we have

$e^{\tan^{-1} y}\ \frac{dx}{dy} + x\frac{e^{\tan^{-1} y}}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}$

Or, $\frac{d}{dx}\left(xe^{\tan^{-1} y}\right) = \frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}$

Integrating $xe^{\tan^{-1} y} = \int\frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}\ dy + c$

Let, $e^{\tan^{-1} y} = z, \frac{1}{1+y^2}dy = dz$

Or, $xe^z=\int ze^zdz + c$

Or, $xe^z = e^z(z-1)+c$

Or, $xe^{\tan^{-1}y} = e^{\tan^{-1}y}\left(e^{\tan^{-1}y} - 1\right) + c$

Which is the required solution

(II)

$x\frac{dy}{dx} + y = y^2 \log x$

Let, $y^2 = z$, $\quad y^2\frac{dy}{dx} = \frac{dz}{dx}$, $\quad \frac{dy}{dx} = \frac{1}{y^2}\frac{dz}{dx}$

Or, $\frac{dy}{dx} + y\frac{1}{z} = \frac{y^2}{x}\log x$

Or, $\frac{1}{y^2}\frac{dz}{dx} = \frac{\log x}{x}$

Or, I.F $\quad = e^{\int \frac{1}{x}dx}$

$= e^{-\text{lo}} \quad = \frac{1}{x}$

5.(c):

(I) Find the general solution of $\quad 2y'' - 7y' + 3y = 0$

Let, $y = e^{mx}$

=> $y' = me^{mx}$

=> $y'' = m^2e^{mx}$

Now, $2y'' - 7y' + 3y = 0$

=> $2m^2e^{mx} - 7\,me^{mx} + 3e^{mx} = 0$

=> $e^{mx}(2m^2 - 7m + 3) = 0$

=> $2m^2 - 6m - m + 3 = 0 \qquad [e^{mx} \neq 0]$

=> $2m(m-3) - 1(m-3) = 0$

=> $(m-3)(2m-1) = 0$

So, $m = 3,\ \frac{1}{2}$

Now the general solution is $y = c_1 e^{3x} + c_2 e^{\frac{x}{2}}$

5.(c):

(II) Find the particular solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$,

when $y(0) = 0$ *and* $y'(0) = 1$

Solution: Given that, $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

Let, $y = e^{mx}$

=> $\frac{dy}{dx} = me^{mx}$

=> $\frac{d^2y}{dx^2} = m^2 e^{mx}$

So, $m^2 e^{mx} + 3\, me^{mx} + 2e^{mx} = 0$

=> $e^{mx}(m^2 + 3m + 2) = 0$

=> $m^2 + 2m + m + 2$ [Hence $e^{mx} \neq 0$]

=> $m(m + 2) + 1(m + 2)$=0

So, $m = -1, -2$

Now the general solution is $y = c_1 e^{-x} + c_2 e^{-2x}$ --------------------(I)

$$y' = -c_1 e^{-x} + c_2 e^{-2x}$$

$$y(0) = c_1 e^{-0} + c_2 e^{-2.0} = c_1 + c_2$$

Given that $y(0) = 0$, so $c_1 + c_2 = 0$ ----------------------------------(II)

Again, $y''(0) = c_1 e^{-0} + 2c_2 e^{-2.0} = -c_1 + 2c_2$

And $y'(0) = 1$

=> $-c_1 + 2c_2 = 1$ --(III)

Add equation (II) and (III) we get, $c_1 + c_2 - c_1 + 2c_2 = 1$

$$So, c_2 = \frac{1}{3}$$

Applying c_2 in equation (II)and (III) we get, $c_2 = -\frac{1}{3}$

Now, $y = -\frac{1}{3}e^{-x} + \frac{1}{3}e^{-2x}$ which is the particular solution

5(d): It is evident that $y_p = 3x$ is a particular solution of the equation $y'' + 4y = 12x,$ and that $y_c(x) = c_1 cos\ 2x + c_2\ sin\ 2x$ is its complementary solution. Find a solution of this differential equation that satisfies the initial conditions $y(0) = 5, y'(0) = 7$.

Solution: Given that, $y'' + 4y = 12x$

Let, $y = e^{mx}$

=> $y' = me^{mx}$

=> $y'' = m^2e^{mx}$

Now, $m^2e^{mx} + 4e^{mx} = 0$

$=> \ e^{mx}(m^2 + 4) = 0$

=> $m^2 + 4 = 0$ $[\ e^{mx} \neq 0]$

So, $m = \pm 2i$

Now the complementary solution is $y_c = A\ cos\ 2x + B\ sin\ 2x$

And the particular solution, P.I.=$\frac{1}{D^2+4}12x$

$$=\frac{1}{4}\left(1+\frac{D^2}{4}\right)^{-1} 12x$$

$$=\frac{1}{4}\left(1-\frac{D^2}{4}\right) 12x$$

$$=\frac{1}{4}12x$$

$$y_p = 3x$$

Now, the general solution G.S. $y = y_c + y_p = A\ cos\ 2x + B\ sin\ 2x + 3x$

Given that, $y(0) = 5$

So, $A\ cos\ 2.0 + B\ sin\ 2.0 + 3.0 = 5$

=> $A = 5$

Again, $y' = 3x - 2A\ sin\ 2x + 2B\ cos\ 2x$

=> $y'(0) = 2B$

But, $y'(0) = 7$

So, $2B = 7$

$=> B = \frac{7}{2}$

So, the solution of the differential equation is

$$y = 3x + 5\,cos\,2x + \frac{7}{2}\,sin\,2x$$

5.(e):

(i) (I) $(D^3-8)y=0$

Or, $D^3y-8y=0$

let $, y = e^{mx}$, $\frac{dy}{dx} = me^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}$ $, \frac{d^3y}{dx^3} = m^3e^{mx}$

or, $m^3e^{mx} - 8e^{mx} = 0$

or, $e^{mx}(m^3-8)=0$

or, $e^{mx}(m^3-2^3)=0$ $[e^{mx} \neq 0]$

or$(m-2)(m^2 + 2m + 4) = 0$

0r,m-2=0 and $m^2 + 2m + 4 = 0$

Or, m=2 $m = \frac{-2 \pm \sqrt{2^2-4.1.4}}{2.1}$

$= \frac{-2 \pm \sqrt{-12}}{2}$

$= \frac{-2 \pm 2i\sqrt{3}}{2}$

$= \frac{2(-1 \pm i\sqrt{3})}{2}$

$= \frac{(-1 \pm i\sqrt{3})}{}$

The complementary function is

$y_c = c_1e^{2x} + e^{-x}(Acos\sqrt{3}x + Bsin\sqrt{3}x)$

(II) y $(D^3+3D^2+3D+1)=0$

let,

$y=e^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}$, $\frac{d^3y}{dx^3} = m^3e^{mx}$

Or, $y\,(D^3+3D^2+3D+1)=0$

Or, $e^{mx}(m^3+3m^2+3m+1)=0$

Or, $e^{mx}(m^3+3m^2+3m+1)=0$

Or, $m^3+3m^2+3m+1=0$

Or, $m^2(m+1)+2m(m+1)+1(m+1)=0$

Or, $(m+1)(m^2+2m+1)=0$

Or, $(m+1)(m+1)(m+1)=0$

Or, $m=-1,-1,-1$

The complementary function is

$y_c = c_1e^{-x} + x\,c_2e^{-x} + x^2c_3e^{-x}$

$e^{-x}(c_1 + x\,c_2 + x^2c_3)$

Which is the required solution: