Bangladesh Open University
School of Science and Technology
B.Sc in Computer Science and Engineering
First Year Second Semester Examination,

Course Code & Title: MAT1234 Linear Algebra and Differential Equation

1. (a) Define differential equation. Show that the differential equation of circle touch the x-axis at the origin is

$$(x^2 - y^2)\, dy - 2xy\, dx = 0.$$

(b) Define order and degree of a differential equation with examples. Distinguish between an ODE and a PDE.

(c) Find the differential equations from the following equation:
$y = e^x(A sin2x + B cos2x)$.

(d) Show that, $Ax^2 + By^2 = 1$ is the solution of $x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y\frac{dy}{dx}$

(e) Find the differential equation of which $y = 2A + Blogx + C(logx)^2 + 3x^2$ is a solution

(f) Find the differential equation of which $y = ae^x + be^{-x} + c\, Cosx + d\, Sinx$ is a solution

2. Solve any three of the following equations:

(i) $dy = (y^2 - 1)\, dx$;

(ii) $\frac{dy}{dx} = 1 + e^{x-y}$;

(iii) $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$;

(iv) $(x^2 + y^2)\, dy = xy\, dx$.

(v) $x^2(1 + y)dy + y^2(x - 1)dx = 0$

(vi) $e^{x-y}dx + e^{y-x} = 0$;

(vii) $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

(viii) $(x^2 + y^2)\, dy = xy\, dx$.

(ix) $x\sqrt{1 + y^2}\, dx + y\sqrt{1 + x^2}\, dy = 0$.

3. (a) Prove that the differential equation $M\, dx + N\, dy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, where M and N both are functions of x, y.

(b) Verify that the differential equation $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$ is exact and hence solve it.

(c) Determine whether the equation $(2x + 3y + 4)\, dx + (3x - 6y - 5)\, dy = 0$ is exact. If it is then solve it.

(d) Explain an exact differential equation and a linear differential equation with example

(e) What is integrating factor? Solve the following equations:

(i) $(12y + 4y^3 + 6x^2)dx + 3(x + xy^2)\, dy = 0$;

(ii) $y^2(ydx + 2xdy) - x^2(2ydx + xdy) = 0$.

4. Define homogeneous and linear differential equation with examples.
Solve:

(iii) $(x^3 + 3xy^2)\,dx + (3x^2y + y^3)\,dy = 0;$
(iv) $(1 + xy)y\,dx + (1 - xy)x\,dy = 0.$
(v) $(x^3 + 3xy^2)\,dx + (y^3 + 3x^2y)\,dy = 0$
(vi) $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$
(vii) $(x^3 + 3xy^2)\,dx + (y^3 + 3x^2y)\,dy = 0$
(viii) $\frac{dy}{dx} + 2ytanx = sinx, \qquad y\left(\frac{\pi}{3}\right) = 0.$
(ix) $\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$
(x) $\frac{dy}{dx} = \frac{y-x-1}{y+x+5}$
(xi) $\frac{dy}{dx} = \frac{3y-7x+7}{3x-7y-3}$
(xii) $\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$
(xiii) $(x + y + 1)dx - (2x + 2y + 1)dy = 0$

5. (a) Define Bernoulli's equation and hence solve $\frac{dy}{dx} + x\sin 2y = x^3cos^2y$

(b) Solve the equations:
(i) $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$; (ii) $x\frac{dy}{dx} + y = y^2\log_e x.$

(c) (i) Find the general solution of $2y'' - 7y' + 3y = 0.$
(ii) Find the particular solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$ when $y(0) = 0$ and $y'(0) = 1.$

(d) It is evident that $y_p = 3x$ is a particular solution of the equation $y'' + 4y = 12x$, and that $y_c(x) = c_1cos2x + c_2sin2x$ is its complementary solution. Find a solution of this differential equation that satisfies the initial conditions $y(0) = 5, y'(0) = 7.$

(e) Find the complementary function of the equations:
i) $(D^3 - 8)y = 0;$
ii) $(D^3 + 3D^2 + 3D + 1)y = 0.$

6. (a) Solve :
(i) $(D^2 - 4D + 13)y = 0;$
(ii) $(D^2 + 4)y = e^x + x^2;$
(iii) $(D^2 + a^2)y = cosax$
(iv) $(4D^2 + 12D + 9)y = 144e^{-3x}$
(v) $(D^3 + 8)y = x^4 + 2x + 1.$
(vi) $(D^3 - 2D^2 - 19D + 20)y = 0$
(vii) $(D^2 + 1)y = sin3x$
(viii) $(D^2 + 3D + 2)y = 0$ when $y(0) = 0$ and $y'(0) = 1.$
(ix) $(D^2 - 5D + 6)y = x^3e^{2x}$
(x) $(D^2 + 4)y = 12x$ when $y(0) = 5, y'(0) = 7.$
(xi) $(D^2 - 2D + 4)y = e^xCosx;$

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ASSIGNMENT-01

COMPUTER SCIENCE & ENGINEERING

1ST YEAR 2ND SEMESTER

LINEAR ALGEBRA & DIFFERENTIAL EQUATION

COURSE & CODE : MAT-1231

GROUP: 01

QUESTION NO – 2 & 6

DATE OF SUBMISSION : 07-03-2020

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ASSIGNMENT- 01

ANSWER TO THE QUESTION NO. 2

(i) $\boldsymbol{dy = (y^2 - 1)dx}$

Sol^n:

$$dy = (y^2 - 1)dx$$

$$\Longrightarrow (y^2 - 1)dx - dy = 0 \quad \cdots\cdots\cdots\cdots\cdots (1)$$

Here,

$$M = y^2 - 1$$

$$\Longrightarrow \frac{\partial M}{\partial y} = 2y$$

And

$$N = -1$$

$$\Longrightarrow \frac{\partial N}{\partial x} = 0$$

Since, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ *therefore equation (1) is not exact*

Now,

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{y^2-1}(-2y)$$

$$= -\frac{2y}{y^2-1} \quad \text{which is a function of y only}$$

$$= f(y)$$

$$\therefore Integrating\ Factor \ = \ e^{\int -\frac{2y}{y^2-1}\, dy}$$

$$= e^{-\int \frac{2y}{y^2-1}\, dy}$$

$$= e^{-ln(y^2-1)}$$

$$= e^{ln(y^2-1)^{-1}}$$

$$= (y^2-1)^{-1}$$

$$= \frac{1}{y^2-1}$$

Multiplying equation (1) by $\frac{1}{y^2-1}$

$dx - \frac{1}{y^2-1}dy = 0 \cdots\cdots\cdots\cdots\cdots (2)$

Which is exact.

Now the solution is

$$\int_{y=constant} dx + \int -\frac{1}{y^2-1}dy = c'$$

$\Longrightarrow x - \int \frac{1}{y^2-1}dy = c'$

$\Longrightarrow x - \frac{1}{2}ln\left(\frac{y-1}{y+1}\right) = c'$

$\Longrightarrow 2x - ln\left(\frac{y-1}{y+1}\right) = 2c'$

$\therefore 2x - ln\left(\frac{y-1}{y+1}\right) = c \qquad [where\ 2c' = c]$

Which is the required solution.

(ii) $\boldsymbol{\frac{dy}{dx} = 1 + e^{x-y}}$

Sol^n:

$\frac{dy}{dx} = 1 + e^{x-y}$

Multiplying by e^y

$\Longrightarrow e^y\ \frac{dy}{dx} = e^y + e^x$

$\Longrightarrow e^y dy = e^y dx + e^x dx$

Integrating both side

$\Longrightarrow \int e^y dy = \int e^y dx + \int e^x dx$

$\Longrightarrow e^y = xe^y + e^x + c$

$\Longrightarrow e^y - xe^y = e^x + c$

$\Longrightarrow e^y(1-x) = e^x + c$

which is the required solution.

(iii) $\frac{dy}{dx} = \boldsymbol{sin(x+y) + cos(x+y)}$

Sol^n:

$$\frac{dy}{dx} = sin(x+y) + cos(x+y)$$

let, $x + y = z$

$\Longrightarrow 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\Longrightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$

$$\Longrightarrow \frac{dz}{dx} - 1 = sinz + cosz$$

$$\Longrightarrow \frac{dz}{dx} = 1 + sinz + cosz$$

$$\Longrightarrow \frac{dz}{dx} = 2cos^2\frac{z}{2} + 2sin\frac{z}{2} \cdot cos\frac{z}{2}$$

$$\Longrightarrow \frac{dz}{dx} = 2cos^2\frac{z}{2}(1 + \frac{\boldsymbol{sin\frac{z}{2}}}{\boldsymbol{cos\frac{z}{2}}})$$

$$\Longrightarrow \frac{dz}{dx} = 2cos^2\frac{z}{2}(1 + tan\frac{z}{2})$$

$$\Longrightarrow \frac{dz}{dx} = \frac{\boldsymbol{1 + tan\frac{z}{2}}}{\boldsymbol{\frac{1}{2}sec^2\frac{z}{2}}}$$

$$\Longrightarrow \int \frac{\boldsymbol{\frac{1}{2}sec^2\frac{z}{2}}}{\boldsymbol{1 + tan\frac{z}{2}}}\, dz = \int dx$$

$$\Longrightarrow ln\left(1 + tan\frac{z}{2}\right) = x + c$$

$$\Longrightarrow ln\left(1 + tan\frac{x+y}{2}\right) = x + c$$

Which is the required solution.

(iv) $\boldsymbol{(x^2 + y^2)dy = xy\, dx}$

Sol^n:

$$(x^2 + y^2)dy = xy\, dx$$

$$\Longrightarrow \frac{dy}{dx} = \frac{xy}{x^2+y^2} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

Let, $y = vx$

$$\Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From equation (1)

$$v + x\frac{dv}{dx} = \frac{x.\ vx}{x^2 + v^2x^2} = \frac{vx^2}{x^2(1+v^2)}$$

$\Longrightarrow x\frac{dv}{dx} = \frac{v}{1+v^2} - v$

$\Longrightarrow x\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$

$\Longrightarrow x\frac{dv}{dx} = -\frac{v^3}{1+v^2}$

$\Longrightarrow -\frac{1+v^2}{v^3}\ dv = \frac{dx}{x}$

$\Longrightarrow -\left(\frac{1}{v^3} + \frac{v^2}{v^3}\right) dv = \frac{dx}{x}$

$\Longrightarrow \int -\left(\frac{1}{v^3} + \frac{1}{v}\right) dv = \int \frac{dx}{x}$

$\Longrightarrow -\frac{1}{-2v^2} - lnv = lnx + lnc$

$\Longrightarrow \frac{1}{2v^2} = lnx + lnc + lnv$

$\Longrightarrow \frac{1}{2v^2} = ln(cvx)$

$\Longrightarrow \frac{1}{2\frac{y^2}{x^2}} = ln(c \cdot \frac{y}{x} \cdot x)$

$\Longrightarrow \frac{x^2}{2y^2} = ln(cy)$

Which is the required solution.

(v) $\boldsymbol{x^2(1+y)dy + y^2(x-1)dx = 0}$

Sol^n:

$x^2(1+y)\ dy + y^2(x-1)\ dx = 0$

Dividing by x^2y^2

$\Longrightarrow \frac{1+y}{y^2}\ dy + \frac{x-1}{x^2}\ dx = 0$

$\Longrightarrow \left(\frac{\mathbf{1}}{\mathbf{y^2}} + \frac{\mathbf{y}}{\mathbf{y^2}}\right) dy + \left(\frac{\mathbf{x}}{\mathbf{x^2}} - \frac{\mathbf{1}}{\mathbf{x^2}}\right) dx = 0$

Integrating both side

$\Longrightarrow \int \left(\frac{1}{y^2} + \frac{y}{y^2}\right) dy + \int \left(\frac{x}{x^2} - \frac{1}{x^2}\right) dx = c$

$\Longrightarrow -\frac{1}{y} + lny + lnx + \frac{1}{x} = c$

$\Longrightarrow ln(xy) + \frac{1}{x} - \frac{1}{y} = c$

$\Longrightarrow ln(xy) + \frac{y-x}{xy} = c$

$\Longrightarrow xy\ ln(xy) + y - x = cxy$

Which is the required solution.

(vi) $\boldsymbol{e^{x-y}dx + e^{y-x}dy = 0}$

Soln:

$e^{x-y}dx + e^{y-x}dy = 0$

$\Longrightarrow \frac{e^x}{e^y}dx + \frac{e^y}{e^x}dy = 0$

Multiplying by $e^x e^y$

$e^{2x}dx + e^{2y}dy = 0$

Integrating both side

$\Longrightarrow \frac{e^{2x}}{2} + \frac{e^{2y}}{2} = c'$

$\Longrightarrow e^{2x} + e^{2y} = 2c'$

$\Longrightarrow e^{2x} + e^{2y} = c$ *[where* $2c' = c$*]*

Which is the required solution.

(vii) $\boldsymbol{(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0}$

Sol^n:

$(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

$\Longrightarrow x^2(1-y)dy + y^2(1+x)dx = 0$

Dividing by x^2y^2

$\Longrightarrow \frac{1-y}{y^2}dy + \frac{1-x}{x^2}dx = 0$

$\Longrightarrow \left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \left(\frac{1}{x^2} + \frac{1}{x}\right)dx = 0$

Integrating both side

$\Longrightarrow \int\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \int\left(\frac{1}{x^2} + \frac{1}{x}\right)dx = c$

$\Longrightarrow \frac{1}{-y} - lny + \frac{1}{-x} + lnx = c$

$\Longrightarrow -\frac{1}{y} - lny - \frac{1}{x} + lnx = c$

$\Longrightarrow ln\frac{x}{y} - \left(\frac{1}{x} + \frac{1}{y}\right) = c$

$\Longrightarrow ln\frac{x}{y} - \frac{x+y}{xy} = c$

$\Longrightarrow xy\ ln\frac{x}{y} - x - y = cxy$

Which is the required solution.

(viii) $(\boldsymbol{x^2 + y^2})\boldsymbol{dy} = \boldsymbol{xy\ dx}$

Sol^n:

$(x^2 + y^2)dy = xy\ dx$

$\Longrightarrow \frac{dy}{dx} = \frac{xy}{x^2+y^2} \cdots\cdots\cdots\cdots\cdots (1)$

Let, $y = vx$

$\Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From equation (1)

$v + x\frac{dv}{dx} = \frac{x.\ vx}{x^2+v^2x^2} = \frac{vx^2}{x^2(1+v^2)}$

$\Longrightarrow x\frac{dv}{dx} = \frac{v}{1+v^2} - v$

$\Longrightarrow x\frac{dv}{dx} = \frac{v-v-v^3}{1+v^2}$

$\Longrightarrow x\frac{dv}{dx} = -\frac{v^3}{1+v^2}$

$\Longrightarrow -\frac{1+v^2}{v^3}dv = \frac{dx}{x}$

$\Longrightarrow -\left(\frac{1}{v^3} + \frac{v^2}{v^3}\right)dv = \frac{dx}{x}$

$\Longrightarrow \int -\left(\frac{1}{v^3} + \frac{1}{v}\right)dv = \int \frac{dx}{x}$

$\Longrightarrow -\frac{1}{-2v^2} - lnv = lnx + lnc$

$\Longrightarrow \frac{1}{2v^2} = lnx + lnc + lnv$

$\Longrightarrow \frac{1}{2v^2} = ln(cvx)$

$$\Rightarrow \frac{1}{2\frac{y^2}{x^2}} = ln(c \cdot \frac{y}{x} \cdot x)$$

$$\Rightarrow \frac{x^2}{2y^2} = ln(cy)$$

Which is the required solution.

(ix) $\boldsymbol{x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0}$

Sol^n:

$$x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$$

Dividing by $\sqrt{1+x^2} \cdot \sqrt{1+y^2}$

$$\Rightarrow \frac{x}{\sqrt{1+x^2}}\, dx + \frac{y}{\sqrt{1+y^2}}\, dy = 0$$

Integrating both side

$$\Rightarrow \int \frac{x}{\sqrt{1+x^2}} dx + \int \frac{y}{\sqrt{1+y^2}} dy = c'$$

$$\Rightarrow \int \frac{1}{\sqrt{z}} \cdot \frac{1}{2} dz + \int \frac{1}{\sqrt{v}} \cdot \frac{1}{2} dv = c'$$

$$\Rightarrow \frac{1}{2}\int \frac{1}{\sqrt{z}} dz + \frac{1}{2}\int \frac{1}{\sqrt{v}} dv = c'$$

$$\Rightarrow \frac{\sqrt{z}}{2} + \frac{\sqrt{v}}{2} = c'$$

$$\Rightarrow \sqrt{z} + \sqrt{v} = 2c'$$

$$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = c \qquad [\text{ where } 2c' = c]$$

Which is the required solution.

Let, $1 + x^2 = z$

$$\Rightarrow 2x = \frac{dz}{dx}$$

$$\Rightarrow x\,dx = \frac{1}{2}dz$$

Let, $1 + y^2 = v$

$$\Rightarrow 2y = \frac{dv}{dy}$$

$$\Rightarrow ydy = \frac{1}{2}dv$$

ANSWER TO THE QUESTION NO 6

(i) $(\boldsymbol{D^2 - 4D + 13})\boldsymbol{y} = \boldsymbol{0}$

Sol^n:

$(D^2 - 4D + 13)y = 0$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^2 + 4m + 13)e^{mx} = 0$

$\Longrightarrow (m^2 + 4m + 13) = 0 \qquad [\because e^{mx} \neq 0]$

$\Longrightarrow m = \frac{\boldsymbol{4 \pm \sqrt{16 - 52}}}{\boldsymbol{2}}$

$\Longrightarrow m = \frac{4 \pm 6i}{2}$

$\therefore m = 2 \pm 3i$

$The\ General\ Solution\ (G.S)\ is,$

$\therefore y = e^{2x}[A\ cas3x + B\ Sin3x]$

$Which\ is\ the\ required\ solution.$

(ii) $(\boldsymbol{D^2 + 4})\boldsymbol{y} = \boldsymbol{e^x + x^2}$

Sol^n:

$(D^2 + 4)y = e^x + x^2$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^2 + 4)e^{mx} = 0$

$\Longrightarrow (m^2 + 4) = 0 \qquad [\because e^{mx} \neq 0]$

$\therefore m = \pm 2i$

$The\ Cofficient\ Function\ (C.F)\ is,$

$\therefore y_c = [A\ cas2x + B\ Sin2x]$

$The\ Particular\ Integral\ (P.I)\ is\ ,$

$y_p = \frac{\mathbf{1}}{\boldsymbol{D^2+4}}\ (e^x + x^2)$

$= \frac{\mathbf{1}}{\boldsymbol{D^2+4}}\ .e^x + \frac{\mathbf{1}}{\boldsymbol{D^2+4}}.x^2$

$= \frac{\mathbf{1}}{\mathbf{1^2+4}}\ .e^x + \frac{1}{4}\left(1 + \frac{\boldsymbol{D^2}}{\mathbf{4}}\right)^{-1}.x^2$

$= \frac{1}{5}\ .e^x + \frac{1}{4}\left(1 - \frac{D^2}{4} + \frac{D^4}{16} - \cdots \ldots \ldots\right)\ x^2$

$= \frac{1}{5}\ .e^x + \frac{1}{4}\left(x^2 - \frac{2}{4}\right)$

$= \frac{1}{5}\ .e^x + \frac{1}{4}\left(x^2 - \frac{1}{2}\right)$

$\therefore y_p = \frac{1}{5}\ .e^x + \frac{1}{8}(2x^2 - 1)$

$The\ General\ Equation\ (G.S)\ is,$

$y = y_c + y_p$

$\therefore y = ACos2x + BSin2x + \frac{1}{5}\ .e^x + \frac{1}{8}(2x^2 - 1)$

$Which\ is\ the\ required\ solution.$

(iii) $(\boldsymbol{D^2} + \boldsymbol{a^2})\boldsymbol{y} = \boldsymbol{Cos\ ax}$

Sol^n:

$(D^2 + a^2)y = Cos\ ax$

$Let,\ y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^2 + a^2)e^{mx} = 0$$

$$\Longrightarrow (m^2 + a^2) = 0 \qquad\qquad [\because e^{mx} \neq 0]$$

$$\therefore m = \pm ai$$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$$\therefore y_c = [A\ cas\ ax + B\ Sin\ ax]$$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{\mathbf{1}}{\boldsymbol{D^2+a^2}}\ \ Cos\ ax$$

$$= x.\frac{1}{2D}\ \ Cos\ ax$$

$$= \frac{x}{2}\ \int Cos\ ax$$

$$\therefore y_p = \frac{x}{2a}\ \ Sin\ ax$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore y = ACos\ ax + BSin\ ax + \frac{x}{2a}\ \ Sin\ ax$$

$Which\ is\ the\ required\ solution.$

(iv) $\boldsymbol{(4D^2 + 12D + 9)y = 144\ e^{-3x}}$

Soln:

$$(4D^2 + 12D + 9)y = 144\ e^{-3x}$$

$$Let, y = e^{mx}$$

$$\Longrightarrow Dy = me^{mx}$$

$$\Longrightarrow D^2y = m^2e^{mx}$$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(4m^2 + 12m + l)e^{mx} = 0$$

$$\Longrightarrow (2m^2 + 3) = 0 \qquad\qquad [\because e^{mx} \neq 0]$$

$$\therefore m = -\frac{3}{2},\ -\frac{3}{2}$$

$The\ Cofficient\ Function\ (C.F)\ is,$

$$\therefore y_c = C_1 e^{-\frac{3x}{2}} + x\ C_2 e^{-\frac{3x}{2}}$$

$The\ Particular\ Integral\ (P.I)\ is$

$$y_p = \frac{\mathbf{1}}{(\mathbf{2D+3})^2}\ 144\ e^{-3x}$$

$$= 144\ \frac{\mathbf{1}}{\{\mathbf{2(-3)+3}\}^{\mathbf{2}}}\ \ e^{-3x}$$

$$= 144 . \frac{1}{9} .\ e^{-3x}$$

$$\therefore y_p = 16\ e^{-3x}$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore y = C_1 \boldsymbol{e}^{-\frac{3x}{2}} + x\ C_2 \boldsymbol{e}^{-\frac{3x}{2}} + 16\ e^{-3x}$$

$Which\ is\ the\ required\ solution.$

(v) $(\boldsymbol{D}^3 + \mathbf{8})\boldsymbol{y} = \boldsymbol{x}^4 + \mathbf{2}\boldsymbol{x} + \mathbf{1}$

Sol^n:

$(D^3 + 8)y = x^4 + 2x + 1$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$\Longrightarrow D^3y = m^3e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^3+8)e^{mx} = 0$$

$$\Longrightarrow (m^3+8) = 0 \qquad\qquad [\because e^{mx} \neq 0]$$

$$\Longrightarrow m^3 + 2m^2 - 2m^2 - 4m\ + 4m\ + 8 = 0$$

$$\Longrightarrow m^2(m+2) - 2m\ (m+2) + 4(m\ +2)\ = 0$$

$$\Longrightarrow (m+2)(m^2 - 2m\ + 4) = 0$$

$$\Longrightarrow m = -2,\ \frac{\mathbf{2 \pm \sqrt{4-16}}}{\mathbf{2}}$$

$$\Longrightarrow m = -2,\ \frac{\mathbf{2 \pm 2\sqrt{3}\ \boldsymbol{i}}}{\mathbf{2}}$$

$$\therefore m = -2,\ 1 \pm i\sqrt{3}$$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$$\therefore y_c = C_1e^{-2x} +\ e^x\ [ACos\sqrt{3}\,x + BSin\sqrt{3}\,x]$$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{1}{D^3+8}\ \ (x^4 + 2x\ + 1)$$

$$= \frac{1}{8}\left(1 + \frac{\boldsymbol{D^3}}{\mathbf{8}}\right)^{-1} (x^4 + 2x\ + 1)$$

$$= \frac{1}{8}\left(1 - \frac{D^3}{8} + \frac{D^6}{64} \ldots\ldots\ldots\right)\ (x^4 + 2x\ + 1)$$

$$= \frac{1}{8}\left(x^4 + 2x\ + 1 - \frac{24x}{8}\right)$$

$$\therefore y_p\ = \frac{1}{8}(x^4 - x + 1)$$

$The\ General\ Equation\ (G.S)\ is,$

$$y =\ y_c + y_p$$

$$\therefore y = C_1e^{-2x} +\ e^x\ [ACos\sqrt{3}\,x + BSin\sqrt{3}\,x] + \frac{1}{8}(x^4 - x + 1)$$

$Which\ is\ the\ required\ solution.$

(vi) $(\boldsymbol{D^3 - 2D^2 - 19D + 20})\boldsymbol{y = 0}$

Sol^n:

$(D^3 - 2D^2 - 19D + 20)y = 0$

$Let, \ \ y = e^{mx}$

$\Longrightarrow Dy \ = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$\Longrightarrow D^3y = m^3e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^3 - 2m^2 - 19m \ + 20)\ e^{mx} = 0$

$\Longrightarrow m^3 - 2m^2 - 19m \ + 20 = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\Longrightarrow m^3 - m^2 - m^2 + m \ - 20m \ + 20 = 0$

$\Longrightarrow m^2(m-1) - m\,(m-1) + 20(m-1) = 0$

$\Longrightarrow (m-1)\ (m^2 - m \ + 20) = 0$

$\Longrightarrow (m-1)\ (m-5)\ (m+4) = 0$

$\therefore m = 1, 5, -4$

$The\ General\ Equation\ (G.S)\ is,$

$\therefore y \ = C_1e^{x} + C_2\ e^{5x} + C_3\ e^{-4x}$

$Which\ is\ the\ required\ solution.$

(vii) $(\boldsymbol{D^2 + 1})\boldsymbol{y = Sin\ 3x}$

Sol^n:

$(D^2 + 1\,)y = Sin\ 3x$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$(m^2 + 1)e^{mx} = 0$

$\Longrightarrow (m^2 + 1) = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\therefore m = \pm\, i$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$\therefore y_c = [A\ cas\ x + B\ Sin\ x]$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{1}{D^2+1}\ \ Sin\ 3x$$

$$= \frac{1}{-3^2+1}\ \ Sin\ 3x$$

$$\therefore y_p = -\frac{1}{8}\ \ Sin\ 3x$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = y_c + y_p$$

$$\therefore y = A\ cas\ x + B\ Sin\ x\ - \frac{1}{8}\ \ Sin\ 3x$$

$Which\ is\ the\ required\ solution.$

(viii) $(\boldsymbol{D^2 + 3D + 2})\boldsymbol{y} = \boldsymbol{0}$

Sol^n:

$(D^2 + 3D + 2)y = 0$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E)is,$

$$(m^2 + 3m\ + 2)\ e^{mx} = 0$$

$\Longrightarrow m^2 + 3m\ + 2 = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\Longrightarrow (m + 2)\ (m + 1)\ = 0$

$\therefore m = -1, -2$

$The\ General\ Equation\ (G.S)\ is,$

$$y(x) = C_1\, e^{-x} + C_2\, e^{-2x} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$\Longrightarrow y'(x) = -C_1\, e^{-x} - 2C_2\, e^{-2x}$$

$Now,$

$$y(0) = C_1 + C_2$$

$$\Longrightarrow 0 = C_1 + C_2$$

$$\therefore C_1 = -C_2 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

$Again,$

$$y'(0) = -C_1 - 2C_2$$

$$\Longrightarrow 1 = -C_1 - 2C_2$$

$$\Longrightarrow -(-C_2) - 2C_2 = 1$$

$$\therefore C_2 = -1$$

$From\ equation\ (2)$

$$\Longrightarrow C_1 = -(-1)$$

$$\therefore C_1 = 1$$

$Now\ From\ equation\ (1)\ the\ \ Particular\ solution\ is,$

$$\therefore y = e^{-x} - e^{-2x}$$

$Which\ is\ the\ required\ solution.$

(ix) $\boldsymbol{(D^2 + 5D + 6)y = x^3 e^{2x}}$

Sol^n:

$$(D^2 + 5D + 6)y = x^3 e^{2x}$$

$$Let, y = e^{mx}$$

$$\Longrightarrow Dy = me^{mx}$$

$$\Longrightarrow D^2 y = m^2 e^{mx}$$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^2 - 5m\ + 6)\ e^{mx} = 0$$

$$\Longrightarrow m^2 - 3m\ - 2m + 6 = 0 \qquad [\because\ e^{mx} \neq 0]$$

$$\Longrightarrow (m-2)\ (m-3)\ = 0$$

$$\therefore\ m = 2{,}3$$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$$\therefore\ y_c = C_1\ e^{2x} + C_2\ e^{3x}$$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{\mathbf{1}}{(\boldsymbol{D}-\mathbf{2})\ (\boldsymbol{D}-\mathbf{3})}\ \ x^3 e^{2x}$$

$$= \frac{\mathbf{1}}{(\boldsymbol{D}+\mathbf{2}-\mathbf{2})\ (\boldsymbol{D}+\mathbf{2}-\mathbf{3})}\ \ x^3 e^{2x}$$

$$= e^{2x}\ \ \frac{\mathbf{1}}{\boldsymbol{D}\ (\boldsymbol{D}-\mathbf{1})}\ \ x^3$$

$$= -e^{2x}\ \ \frac{1}{D}(1-D)^{-1}\ \ x^3$$

$$= -e^{2x}\ \ \frac{1}{D}(1 + D + D^2 + D^3 + \cdots \dots \dots \dots \dots \dots \dots)\ \ x^3$$

$$= -e^{2x}\ \ \frac{1}{D}(x^3 + 3x^2 + 6x + 6\ )$$

$$= -e^{2x}\ \ \left(\frac{\boldsymbol{x^4}}{\mathbf{4}} + 3\frac{\boldsymbol{x^3}}{\mathbf{3}} + 6\frac{\boldsymbol{x^2}}{\mathbf{2}} + 6x\ \right)$$

$$\therefore\ y_p = -\ \frac{\boldsymbol{e^{2x}}}{\mathbf{4}}(x^4 + 4x^3 + 12x^2 + 24x\ )$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = \ y_c + y_p$$

$$\therefore\ \ y\ = C_1\ e^{2x} + C_2\ e^{3x}\ \ -\ \frac{\boldsymbol{e^{2x}}}{\mathbf{4}}(x^4 + 4x^3 + 12x^2 + 24x\ )$$

$Which\ is\ the\ required\ solution.$

(x) $(\boldsymbol{D^2+4})\boldsymbol{y} = \mathbf{12}\boldsymbol{x}$

Sol^n:

$(D^2+4)y = 12x$

$Let, y = e^{mx}$

$\Longrightarrow Dy = me^{mx}$

$\Longrightarrow D^2y = m^2e^{mx}$

$The\ Auxiliary\ Equation\ (A.E) is,$

$$(m^2+4)\ e^{mx} = 0$$

$\Longrightarrow (m^2+4) = 0 \qquad\qquad [\because e^{mx} \neq 0]$

$\therefore m = \pm 2i$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$$\therefore y_c = [A\ cas\ 2x + B\ Sin\ 2x]$$

$The\ Particular\ Integral\ \ (P.I)\ is$

$$y_p = \frac{1}{D^2+4}\ 12x$$

$$= 12.\frac{1}{4}\ \frac{1}{\left(1+\frac{D^2}{4}\right)}.x$$

$$= 3.\ \left(1+\frac{D^2}{4}\right)^{-1}.x$$

$$= 3.\left(1-\frac{D^2}{4}+\frac{D^4}{16}-\cdots\ldots\ldots\ldots\ldots\ldots\right).x$$

$\therefore y_p = 3x$

$The\ General\ Equation\ (G.S)\ is,$

$$y(x) = y_c + y_p$$

$\Longrightarrow y(x) = A\ Cas\ 2x + B\ Sin\ 2x + 3x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$

$\Longrightarrow y'(x) = -2A\ sin\ 2x + 2B\ cos\ 2x + 3$

$Now,$

$$y(0) = A\ Cas\ 2.0 + B\ Sin\ 2.0 + \ 3.0$$

$$\Longrightarrow 5 = A\ Cas\ 0 + B\ Sin\ 0$$

$$\Longrightarrow 5 = A\ .1 + B\ .0$$

$$\therefore A = 5$$

$Again,$

$$y'(0) = -2A\ sin\ 2.0 + 2B\ cos\ 2.0 + \ 3$$

$$\Longrightarrow 7 = -2A\ sin\ 0 + 2B\ cos\ 0 + \ 3$$

$$\Longrightarrow -2A\ .0 + 2B\ .1 = 7 - \ 3$$

$$\therefore B \ = 2$$

$From\ equation\ (1)\ the\ particular\ solution\ is\ ,$

$$\therefore y = \ 5\ Cas\ 2x + 2\ Sin\ 2x + \ 3x$$

$Which\ is\ the\ required\ solution.$

(xi) $\boldsymbol{(D^2 - 2D + 4)y = e^x Cosx}$

Sol^n:

$$(D^2 - 2D + 4)y = e^x Cosx$$

$$Let, y = e^{mx}$$

$$\Longrightarrow Dy = me^{mx}$$

$$\Longrightarrow D^2y = m^2e^{mx}$$

$The\ Auxiliary\ Equation\ (A.E)is,$

$$(m^2 - 2m + 4)\ e^{mx} = 0$$

$$\Longrightarrow m^2 - 2m + 4 = 0 \qquad [\because e^{mx} \neq 0]$$

$$\Longrightarrow m = \ \frac{2 \pm \sqrt{4-16}}{2}$$

$$\Longrightarrow m = -2,\ \frac{2 \pm 2\sqrt{3}\ i}{2}$$

$$\therefore m = \ \ 1 \pm i\sqrt{3}$$

$The\ Cofficient\ Function\ \ (C.F)\ is,$

$\therefore y_c = \ e^x\ [ACos\sqrt{3}\,x + BSin\sqrt{3}\,x]$

$The\ Particular\ Integral\ \ (P.I)\ is,$

$$y_p = \frac{\mathbf{1}}{\boldsymbol{D^2-2D+4}}\ e^x\ Cos\ x$$

$$= e^x\ \frac{\mathbf{1}}{\boldsymbol{(D+1)-2\ (D+1)+4}}\ Cos\ x$$

$$= \ e^x\ \frac{\mathbf{1}}{\boldsymbol{D^2+3}}\ \ Cos\ x$$

$$= \ e^x\ \frac{\mathbf{1}}{\boldsymbol{-1^2+3}}\ \ Cos\ x$$

$$\therefore y_p = \ \frac{1}{2}\ \ e^x\ Cos\ x$$

$The\ General\ Equation\ (G.S)\ is,$

$$y = \ y_c + y_p$$

$$\therefore y = e^x\ \left[ACos\sqrt{3}\,x + BSin\sqrt{3}\,x\right] + \ \frac{1}{2}\ \ e^x\ Cos\ x$$

$Which\ is\ the\ required\ solution.$

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BANGLADESH OPEN UNIVERSITY
(We assure education at your doorstep)

School of Science and Technology

B.Sc. in Computer Science and Engineering

Assignment-1

Assignment On: Linear Algebra and Differential Equation

Course code – MAT 1234

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3(a)

Solution: let ,$\mathrm{Mdx} + \mathrm{Ndy} = 0 - - - - - (1)$ is exact

then the differential equation function is U=U(x,y)

$\therefore \mathrm{Mdx} + \mathrm{Ndy} = \mathrm{du} - - - - - (2)$

But, $\mathrm{du} = \frac{\delta \mathrm{u}}{\delta \mathrm{x}}(\mathrm{dx}) + \frac{\delta \mathrm{u}}{\delta \mathrm{y}}(\mathrm{dy}) - - - - - (3)$

Now (2) and (3) we get,

$\frac{\delta \mathrm{u}}{\delta \mathrm{x}} = \mathrm{M} - - - - - - (4)$

$\frac{\delta \mathrm{u}}{\delta \mathrm{y}} = \mathrm{N} - - - - - - (4)$

$\therefore \frac{\partial u^2}{\delta y \partial x} = \frac{\partial u^2}{\partial x \delta y}$

$or \ \frac{\partial}{\partial y}(\frac{\partial u}{\partial x}) = \frac{\partial}{\partial x}(\frac{\partial u}{\partial y})$

or $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(proved)

3(b) Verify that the differential equation

$\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$ is exact and hence solve it.

Solution: Given that, $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$

$=> (1 + e^v)\mathrm{d}(vy) + (1 - v)e^v dy = 0$ Let, x=vy

$=> (1 + e^v)(vdy + ydv) + (1 - v)e^v dy = 0$

$=> (1 + e^v)ydv + (v + ve^v + e^v - ve^v)dy = 0$

$=> (1 + e^v)ydv + (v + e^v)dy = 0$

$=> \frac{1+e^v}{v+e^v} dv + \frac{dy}{y} = 0$

$=> ln(v + e^v) + lny = lnc$

$=> \ ln(v + e^v)y = lnc$

$=> vy + e^v y = c$

$\therefore x + e^{\frac{x}{y}} y = c$ since, V=$\frac{x}{y}$

3(c) solve it (2x+3y+4)dx +(3x-6y-5)dy =0

Solution: let, $M = 2x + 3y + 4$

$N = 3x - 6y - 5$

Now,$\frac{\delta M}{\delta y} = 3$

$\frac{\delta N}{\delta y} = 3$

$\therefore \textit{the given equation is exact}$

$\therefore \textit{the solution is } \int_{y=const.} Mdx + \int (\textit{term in } N \textit{ independent of } x)dy = c$

Or $\int (2x + 3y + 4)dx + \int (-6y - 5)dy = c$

Or $2\frac{x^2}{2} + 3xy + 4x - 6 - 5y = c$

$x^2 + 3xy + 4x - 5y - 6 -= c$

Which is the required solution

3(d)

Solution:

Exact differential equation:

The differential equation M dx+N dy=0----------(1) is called exact differential equation if L.H.S(1) is exact, that is if M dx+ N dy=du where M=M(x,y), N=N(x,y) and U=U(x,y)

Linear differential equation:

Linear equation with constant co-efficient equation of the ‘n’th order the typical form of linear differential equation with constant co-efficient of the n^{th} order is

$\therefore \frac{d^n}{dx^n} + a1\frac{d^{n-1}y}{dx^{n-1}} + a2\frac{d^{n-2}y}{dx^{n-2}} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . a_{nY=X}$

$((D^n + a1D^{n-1} + a2d^{n-2} \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . a_n)Y = X$

3(e)

(i)solve it $(12y+4y^3+6x^2)dx+3(x+xy^2)dy=0$ using integrating factor

Solution: Here, $M=12y+4y^3+6x^2$

$$\frac{\delta M}{\delta y} = 12 + 12y^2$$

$N=3x+3xy^2$

$$\frac{\delta N}{\delta x} = 3 + 3y^2$$

The given equation is not exact

Now, $\frac{1}{N}(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x})$

$= \frac{1}{3x(1+y^2)}(9 + 9y^2)$

$=\frac{3}{x}$

Which is the function is X only

$\therefore the\ I.F\ is = e^{\int f(x)dx}$

$= e^{\int \frac{3}{x}dx}$

$= e^{3lnx}$

$=x^3$

Now multiplying the given equation by I. F.= x^3

$\therefore\ x^3(12y + 4y^3 + 6x^2)dx + 3x^3(x + xy^2)dy = 0$

Which is exact

The solution is

$\int(12x^3 + 4x^3y^3 + 6x^5)dx + \int 0.dy = c \qquad (y = constant)$

$12\frac{x^4}{4}.y + 4\frac{y^3x^4}{4} + 6\frac{x^6}{6} = c$

$3x^4y + x^4y^3 + x^6 = c$ Which is the required solution.

3(e)

(ii) solve it y²(y dx+2x dy)-x²(2y dy+x dy)=0 using integrating factor

Given that, $y^2(ydx+2xdy)-x^2(2ydx+xdy)=0$

=> $y^3dx+2xy^2dy-2x^2ydx-x^3dy=0$

=> $(y^3-2x^2y)dx+(2xy^2-x^3)dy=0$ ----------------(i)

M=y^3-2x^2 => $\frac{\partial M}{\partial y}=3y^2-2x^2$ and

N = $2xy^2-x^3$ => $\frac{\partial N}{\partial x}=2y^2-3x^2$

Since $\frac{\partial M}{\partial N}\neq\frac{\partial N}{\partial x}$ So, the equation (i) is not exact.

Now, $Mx+Ny=xy^3-2x^3y+2xy^3-x^3y$

= $3xy^3-3x^3y$

= $3xy(y^2-x^2)$

IF = $\frac{1}{Mx+Ny}=\frac{1}{3xy(y^2-x^2)}$

Now multiplying both sides of equation (i) we have

$$\frac{y^3-2x^2y}{3xy(y^2-x^2)}dx+\frac{2xy^2-x^3}{3xy(y^2-x^2)}dy=0$$

$$=\frac{y\{(y^2-x^2)-x^2\}}{3x\ \ (y^2-x^2)}dx+\frac{x\{(y^2-x^2)+y^2\}}{3xy(y^2-x^2)}dy=0$$

$$=\left[\frac{1}{3x}-\frac{x}{3(y^2-x^2)}\right]dx+\left[\frac{1}{3y}-\frac{y}{3(y^2-x^2)}\right]dy=0$$ -------------------(ii)

Now the equation (ii) exact equation.

$$\int_{y=const}\left[\frac{1}{3x}-\frac{x}{3(y^2-x^2)}\right]dx+\int\frac{1}{3y}dy=0$$

$$=\frac{1}{3}\log x+\frac{1}{6}\log(y^2-x^2)+\frac{1}{3}\log y=\frac{1}{6}\log c$$

$$=\log x^2+\log(y^2-x^2)+\log y^2=\log c$$

$$\therefore\ x^2y^2(y^2-x^2)=c$$ Ans.

5.(a):

Define Bernoulli's equation and hence solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$

Bernoulli's equation: The first order differential equation of

$\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called Bernoulli's equation.

Now, $\frac{dy}{dx} + xsin2y = x^3cos^2y$

$= sec^2y\frac{dy}{dx} + 2x\ tan\ y = x^3$ Multiplying by sec^2x

$= \frac{d\ (\tan y)}{dx} + 2x\ tan\ y = x^3$ This is a first order linear differential equation of tan y

Now I.F. = $e^{\int 2x\ dx} = e^{x^2}$

So, the general solution $\tan y\ e^{x^2} = \int e^{x^2}dx$

$= \frac{1}{2}\int x^2e^{x^2}dx^2 = \frac{1}{2}\left(x^2e^{x^2} - e^{x^2}\right) + \frac{c}{2}$

$=> 2\tan y = x^2 + ce^{x^{-2}}$

5.(b): Solve

(I) $(1+y^2)dx=(\tan^{-1} y - x)dy$

Solution: $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$---------------------(1)

I.F=$e^{\int\frac{1}{1+y^2}}$ $=e^{\tan^{-1} y}$

Multiplying (1) by I.F we have

$e^{\tan^{-1} y}\ \frac{dx}{dy} + x\frac{e^{\tan^{-1} y}}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}$

Or, $\frac{d}{dx}\left(xe^{\tan^{-1} y}\right) = \frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}$

Integrating $xe^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2}.e^{\tan^{-1} y}\ dy + c$

Let, $e^{\tan^{-1} y} = z, \frac{1}{1+y^2}dy = dz$

Or, $xe^z=\int ze^zdz + c$

Or, $xe^z = e^z(z-1)+c$

Or, $xe^{\tan^{-1}y} = e^{\tan^{-1}y}\left(e^{\tan^{-1}y} - 1\right) + c$

Which is the required solution

(II)

$x\frac{dy}{dx} + y = y^2 \log x$

Let, $y^2 = z$, $\quad y^2\frac{dy}{dx} = \frac{dz}{dx}$, $\quad \frac{dy}{dx} = \frac{1}{y^2}\frac{dz}{dx}$

Or, $\frac{dy}{dx} + y\frac{1}{z} = \frac{y^2}{x}\log x$

Or, $\frac{1}{y^2}\frac{dz}{dx} = \frac{\log x}{x}$

Or, I.F $\quad = e^{\int\frac{1}{x}dx}$

$= e^{-\text{lo}} \quad = \frac{1}{x}$

5.(c):

(I) Find the general solution of $\quad 2y'' - 7y' + 3y = 0$

Let, $y = e^{mx}$

=> $y' = me^{mx}$

=> $y'' = m^2e^{mx}$

Now, $2y'' - 7y' + 3y = 0$

=> $2m^2e^{mx} - 7\,me^{mx} + 3e^{mx} = 0$

=> $e^{mx}(2m^2 - 7m + 3) = 0$

=> $2m^2 - 6m - m + 3 = 0 \qquad [e^{mx} \neq 0]$

=> $2m(m-3) - 1(m-3) = 0$

=> $(m-3)(2m-1) = 0$

So, $m = 3,\ \frac{1}{2}$

Now the general solution is $y = c_1 e^{3x} + c_2 e^{\frac{x}{2}}$

5.(c):

(II) Find the particular solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$,

when $y(0) = 0$ *and* $y'(0) = 1$

Solution: Given that, $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$

Let, $y = e^{mx}$

=> $\frac{dy}{dx} = me^{mx}$

=> $\frac{d^2y}{dx^2} = m^2e^{mx}$

So, $m^2e^{mx} + 3\,me^{mx} + 2e^{mx} = 0$

=> $e^{mx}(m^2 + 3m + 2) = 0$

=> $m^2 + 2m + m + 2$ [Hence $e^{mx} \neq 0$]

=> $m(m + 2) + 1(m + 2)$=0

So, $m = -1, -2$

Now the general solution is $y = c_1e^{-x} + c_2e^{-2x}$ --------------------(I)

$$y' = -c_1e^{-x} + c_2e^{-2x}$$

$$y(0) = c_1e^{-0} + c_2e^{-2.0} = c_1 + c_2$$

Given that $y(0) = 0$, so $c_1 + c_2 = 0$ ----------------------------------(II)

Again, $y''(0) = c_1e^{-0} + 2c_2e^{-2.0} = -c_1 + 2c_2$

And $y'(0) = 1$

=> $-c_1 + 2c_2 = 1$ ---(III)

Add equation (II) and (III) we get, $c_1 + c_2 - c_1 + 2c_2 = 1$

$$So, c_2 = \frac{1}{3}$$

Applying c_2 in equation (II)and (III) we get, $c_2 = -\frac{1}{3}$

Now, $y = -\frac{1}{3}e^{-x} + \frac{1}{3}e^{-2x}$ which is the particular solution

5(d): It is evident that $y_p = 3x$ is a particular solution of the equation $y'' + 4y = 12x$, and that $y_c(x) = c_1 cos\ 2x + c_2\ sin\ 2x$ is its complementary solution. Find a solution of this differential equation that satisfies the initial conditions $y(0) = 5, y'(0) = 7$.

Solution: Given that, $y'' + 4y = 12x$

Let, $y = e^{mx}$

=> $y' = me^{mx}$

=> $y'' = m^2 e^{mx}$

Now, $m^2 e^{mx} + 4e^{mx} = 0$

=> $e^{mx}(m^2 + 4) = 0$

=> $m^2 + 4 = 0$ $\quad [\, e^{mx} \neq 0]$

So, $m = \pm 2i$

Now the complementary solution is $y_c = A\ cos\ 2x + B\ sin\ 2x$

And the particular solution, P.I.=$\frac{1}{D^2+4} 12x$

$$=\frac{1}{4}\left(1+\frac{D^2}{4}\right)^{-1} 12x$$

$$=\frac{1}{4}\left(1-\frac{D^2}{4}\right) 12x$$

$$=\frac{1}{4} 12x$$

$$y_p = 3x$$

Now, the general solution G.S. $y = y_c + y_p = A\ cos\ 2x + B\ sin\ 2x + 3x$

Given that, $y(0) = 5$

So, $A\ cos\ 2.0 + B\ sin\ 2.0 + 3.0 = 5$

=> $A = 5$

Again, $y' = 3x - 2A\ sin\ 2x + 2B\ cos\ 2x$

=> $y'(0) = 2B$

But, $y'(0) = 7$

So, $2B = 7$

$=> B = \frac{7}{2}$

So, the solution of the differential equation is

$$y = 3x + 5\cos 2x + \frac{7}{2}\sin 2x$$

5.(e):

(i) (I) $(D^3-8)y=0$

Or, $D^3y-8y=0$

let $, y = e^{mx}, \quad \frac{dy}{dx} = me^{mx}, \frac{d^2y}{dx^2} = m^2e^{mx}, \frac{d^3y}{dx^3} = m^3e^{mx}$

or, $m^3e^{mx} - 8e^{mx} = 0$

or, $e^{mx}(m^3-8)=0$

or, $e^{mx}(m^3-2^3)=0$ $\quad [e^{mx} \neq 0]$

or$(m-2)(m^2 + 2m + 4) = 0$

0r, $m-2=0$ and $m^2 + 2m + 4 = 0$

Or, $m=2$

$$m = \frac{-2 \pm \sqrt{2^2 - 4.1.4}}{2.1}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= \frac{2(-1 \pm i\sqrt{3})}{2}$$

$$= \frac{(-1 \pm i\sqrt{3})}{}$$

The complementary function is

$y_c = c_1e^{2x} + e^{-x}(A\cos\sqrt{3}x + B\sin\sqrt{3}x)$

(II) $y(D^3+3D^2+3D+1)=0$

let,

y=e^{mx} , $\frac{d^2y}{dx^2} = m^2e^{mx}$, $\frac{d^3y}{dx^3} = m^3e^{mx}$

Or, y $(D^3+3D^2+3D+1)=0$

Or, $e^{mx}(m^3+3m^2+3m+1)=0$

Or, $e^{mx}(m^3+3m^2+3m+1)=0$

Or, $m^3+3m^2+3m+1=0$

Or, $m^2(m+1)+2m(m+1)+1(m+1)=0$

Or, $(m+1)(m^2+2m+1)=0$

Or,$(m+1)(m+1)(m+1)=0$

Or, m=-1,-1,-1

The complementary function is

$y_c = c_1e^{-x} + x\, c_2e^{-x} + x^2c_3e^{-x}$

$e^{-x}(c_1 + x\, c_2 + x^2c_3)$

Which is the required solution:

COMPUTER SCIENCE & ENGINEERING

Title: Linear Algebra and Differential Equation

COURSE & CODE: MAT1234

ASSIGNMENT: 01

GROUP: 03

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SUBMITTED TO

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Date of Submission: 07 /03/2020

Assignment-1 (Mathematics)

1(a)

Defination: An equation involving derivatives of one or more dependent variable with respect to one or more independent variable is called a different equation.

Solution:

According to the question, the circle passes through the origin at x-axis.

So, equation-

$$x^2 + y^2 + 2fy = 0 \qquad [where, c = 0, g = 0]$$

$$=> 2x + 2y.y_1 + 2f.y_1 = 0$$

$$=> 2x + 2y.y_1 + 2\frac{-x^2-y^2}{2y}.y_1 = 0$$

$$=> 2x + 2y.\frac{dy}{dx} - \frac{(x^2+y^2)}{y}.\frac{dy}{dx} = 0$$

$$=> 2xy.dy + 2y^2.dy - (x^2 + y^2)dy = 0$$ [Multiply by y.dx]

$$=> 2xy.dy + dy(2y^2 - x^2 - y^2) = 0$$

$$=> 2xy.dy + dy(y^2 - x^2) = 0$$

$$=> (x^2 - y^2)dy - 2xy.dy = 0$$

Which is the Required Solution.

1(b)

Order: The order of a different differentiates equation is the order of the highest differential of the equation.

Example:

$$2(\frac{d^3y}{dx^3})^2+3\frac{d^2y}{dx^2} + \frac{d^5y}{dx^5} = 0$$

Here, order of the differential equation is 5

Degree: The degree of a differential equation is the degree of the derivative of the highest degree in the differential equation.

Example:

$$2\left(\frac{d^3y}{dx^3}\right)^2+3\frac{d^2y}{dx^2}+\frac{d^5y}{dx^5}=0$$

Here, degree of the differential equation is 1

Distinguish between an ODE & PDE:

1. ODE's involve derivative is only one variable where as PDE's involve derivatives in multiple variables.
2. ODE has one independent variable, say x.
   Solution is y(x).
   PDE has more independent variables say $x_1, x_2 \ldots\ldots\ldots\ldots x_n$.
   Solution is $y = (x_1, x_2, \ldots \ldots \ldots \ldots \ldots .. x_n)$.

1(c)

Solution:

$$y = e^x(Asin2x + Bcos2x)$$

$$\Rightarrow \frac{dy}{dx} = e^x(Asin2x + Bcos2x) + e^x(2.Acos2x - 2.Bsin2x)$$

$$\Rightarrow \frac{dy}{dx} = y + e^x(2Acos2x - 2Bsin2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(2Acos2x - 2Bsin2x) + e^x(-4.Asin2x - 4.Bcos2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - 4e^x(Asin2x + Bcos2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y - 4y$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y=0$$

Which is the required solution.

1(d)

Solution:

$$\Rightarrow Ax^2 + By^2 = 1 \qquad \_\_\_\_\_\_\_\_(i)$$
$$\Rightarrow 2Ax + 2Byy' = 0$$
$$\Rightarrow Ax + Byy' = 0$$

Multiplying both side with "x" we find

⇨ $Ax^2 + Bxyy^{'} = 0$ ________(ii)

Now equation (ii) –(i)

⇨ $Bxyy^{'} - By^2 + Ax^2 - Ax^2 = -1$

⇨ $xyy^{'} - y^2 = -\frac{1}{B}$

Differentiating w.r.t x

⇨ $x\frac{d}{dx}(yy^{'}) + y.y^{'} - 2y.y^{'} = 0$

⇨ $x(yy^{''} + y^{'}y^{'}) - y.y^{'} = 0$

⇨ $x\left[y.\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y.\frac{dy}{dx}$

Which is the required Solution.

1(e)

Solution:

$y = (2A + Blogx + c(logx)^2 + 3x^2)$

⇨ $\frac{dy}{dx} = 0 + B.\frac{1}{x} + 2c.logx.\frac{1}{x} + 6x$

⇨ $x.\frac{dy}{dx} = B + 2clogx + 6x^2$ [Multiplying both side with x]

⇨ $x.\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 = 0 + 2c.\frac{1}{x} + 12x$

⇨ $x^2.\frac{d^2y}{dx^2} + x.\frac{dy}{dx}. = 2c + 12x^2$ [Multiplying both side with x]

⇨ $x^2.\frac{d^3y}{dx^3} + 2x.\frac{d^2y}{dx^2} + x.\frac{d^2y}{dx^2} + \frac{dy}{dx}.1 = 24x$

⇨ $x^2.\frac{d^3y}{dx^3} + 3x.\frac{d^2y}{dx^2} + \frac{dy}{dx} - 24x = 0$

Which is the required equation.

1(f)

Solution:

$y = ae^x + be^{-x} + ccosx + dsinx$

⇨ $\frac{dy}{dx} = ae^x - be^{-x} - csinx + dcosx$

$\Rightarrow \frac{d^2y}{dx^2} = ae^x + be^{-x} - ccosx - dsinx$

$\Rightarrow \frac{d^3y}{dx^3} = ae^x - be^{-x} + csinx - dcosx$

$\Rightarrow \frac{d^4y}{dx^4} = ae^x + be^{-x} + ccosx + dsinx$

$\Rightarrow \frac{d^4y}{dx^4} = y$

$\Rightarrow \frac{d^4y}{dx^4} - y = 0$

Which is the required equation.

4 (i)

Homogeneous differential equation:

When M and N of the equation Mdx + Ndy = 0 both are the same degree in x and y and homogeneous then the equation is called to be homogeneous.

Example:

F(x) = $x^2 + 2xy + y^2$

4 (ii)

Linear differential equation:

A differential equation of the form $\frac{dy}{dx} + py = Q$; where P and Q are function of x alone or constant is called a linear differential equation of 1st order.

Example:

$$\frac{dy}{dx} + 3xy = x^2$$

4 (iii)

Solution:

Given equation,

$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$

Here,

M = $x^3 + 3xy^2 \quad \therefore \frac{\partial M}{\partial y} = 6xy$

N = $3x^2y + y^3 \quad \therefore \frac{\partial N}{\partial x} = 6xy$

∴ The given differential equation is exact.

The solution is,

$\int_{y=const}(x^3 + 3xy^2)dx + \int y^3dy = \acute{c}$

$$=> \frac{x^4}{4}+3y^2\frac{x^2}{2}+\frac{y^4}{4} = \acute{c}$$

$$=> (x^4 + 6x^2y^2 + y^4) = 4\acute{c}$$

$=> x^4 + 6x^2y^2 + y^4 = \acute{c}$ [Where $4\acute{c} = c$]

Which is the required solution.

4(iv)

Solution:

Given Equation,

$$(1 + xy)y\ dx + (1 - xy)x\ dy = 0$$

Here,

M = $y + xy^2 \ \therefore \ \frac{\partial M}{\partial y} = 1 + 2xy$

N = $x - x^2y \ \therefore \ \frac{\partial N}{\partial x} = 1 - 2xy$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$; the given differential equation is not exact.

Now,

$$Mx - Ny = xy + x^2y^2 - xy + x^2y^2 = 2x^2y^2$$

$\therefore Mx - Ny \neq 0$

Therefore the integrating factor, I.F = $\frac{1}{Mx-Ny}$

= $\frac{1}{2x^2y^2}$

Now multiplying the given equation by I.F, $\frac{1}{2x^2y^2}$

$$\frac{1}{2x^2y^2}\ (1 + xy)y\ dx + \frac{1}{2x^2y^2}(1 - xy)x\ dy = 0$$

$$=> \left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \left(\frac{1}{2xy^2} - \frac{1}{2y}\right)dy = 0$$

Now the differential equation is exact.

$\therefore$ The solution is,

$$\int_{y=const}\left(\frac{1}{2x^2y} + \frac{1}{2x}\right)dx + \int\frac{1}{2y}dy = \acute{c}$$

$$=> \frac{1}{2y}\left(-\frac{1}{x}\right) + \frac{1}{2}\ lnx - \frac{1}{2}lny = \acute{c}$$

$$=> \ lnx - lny - \frac{1}{xy} = 2\acute{c}$$

$=> \ ln\frac{x}{y} - \frac{1}{xy} = c$ [Where$2\acute{c} = c$]

Which is the required solution.

4(v)

Solution:

Given equation,

$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

Here,

M = $x^3 + 3xy^2 \quad \therefore \ \frac{\partial M}{\partial y} = \ 6xy$

N = $y^3 + 3x^2y \quad \therefore \ \frac{\partial N}{\partial x} = \ 6xy$

∴ The given differential equation is exact.

The solution is,

$\int_{y=const}(x^3 + 3xy^2)dx + \int y^3 dy = \acute{c}$

$=> \ \frac{x^4}{4}+3y^2\frac{x^2}{2}+\frac{y^4}{4} = \ \acute{c}$

$=> (x^4 + 6x^2y^2 + y^4) = 4\acute{c}$

$=> x^4 + 6x^2y^2 + y^4 = \acute{c}$ [Where $4\acute{c} = c$]

Which is the required solution.

4(vi)

Solution:

Given equation,

$$\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$$

$$=> \frac{dy}{dx} = \frac{2(2x + 3y + 5)}{2x + 3y + 4} \cdots\cdots\cdots (1)$$

Let,

$2x + 3y = v$

$=> 2 + 3\ \frac{dy}{dx} = \frac{dv}{dx}$

$$=> \frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

Now eq(1) becomes,

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{2v+5}{v+4}$$

$$=> \left(\frac{dv}{dx} - 2\right) = \frac{6v+15}{v+4}$$

$$=> \frac{dv}{dx} = \frac{6v+15}{v+4} + 2$$

$$=> \frac{dv}{dx} = \frac{6v+15+2v+8}{v+4}$$

$$=> \frac{dv}{dx} = \frac{8v+23}{v+4}$$

$$=> dv\left(\frac{v+4}{8v+23}\right) = dx$$

$$=> dv\left(\frac{v+4}{8(v+4)-9}\right) = dx$$

$$=> dv\left(\frac{1}{8} - \frac{v+4}{9}\right) = dx$$

Now integrating both side,

$$\int dv\left(\frac{1}{8} - \frac{v+4}{9}\right) = \int dx$$

$$=> \frac{v}{8} - \frac{1}{9}\left(\frac{v^2}{2} + 4v\right) = x + ć$$

$$=> \frac{9v - 4v^2 + 32v}{72} = x + ć$$

$$=> 9v - 4v^2 + 32v = 72x + 72ć$$

$$=> 41v - 4v^2 = 72x + 72ć$$

$$=> 41(2x+3y) - 4(2x+3y)^2 = 72x + \mathrm{c}$$

$$=> 82x + 123y - 4(2x+3y)^2 = 72x + \mathrm{c}$$

$$=> 10x + 123y - 4(2x+3y)^2 = \mathrm{c}$$

Which is the required solution.

4(vii)

Solution:

Given equation,

$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

Here,

M = $x^3 + 3xy^2 \quad \therefore \frac{\partial M}{\partial y} = 6xy$

N = $y^3 + 3x^2y \quad \therefore \frac{\partial N}{\partial x} = 6xy$

∴ The given differential equation is exact.

The solution is,

$\int_{y=const}(x^3 + 3xy^2)dx + \int y^3 dy$ = $\acute{c}$

=> $\frac{x^4}{4}+3y^2\frac{x^2}{2}+\frac{y^4}{4} = \acute{c}$

=> $(x^4 + 6x^2y^2 + y^4) = 4\acute{c}$

=> $x^4 + 6x^2y^2 + y^4 = \acute{c}$ [Where $4\acute{c} = c$]

Which is the required solution.

4(viii)

Solution:

Given equation

$$\frac{dy}{dx} + 2ytanx = sinx;\ y\left(\frac{\pi}{3}\right) = 0$$

Differential equation of the form,

$\frac{dy}{dx} + py = Q$

$\frac{dy}{dx} + 2ytanx = sinx$ ………….(1)

Where,

P = 2tanx

Q = sinx

∴ I.F = $e^{\int 2tanx\, dx}$

= $e^{2\log secx}$

= $e^{log\, sec^2 x}$

= $sec^2 x$

Multiplying both side of (1) by $sec^2 x$

$$sec^2 x\frac{dy}{dx} + sec^2 x\, 2ytanx = sec^2 x\, sinx$$

$\frac{d}{dx}(sec^2\text{x y})$ = $sec^2\text{x sinx}$

Now integrating both side,

$$\int d\,(sec^2\text{x y}) = \int \text{tanx secx dx}$$

$$=> sec^2\text{xy} = \text{secx} + \text{c}$$

4(ix)

Solution:

Given equation,

$\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$ ....(1)

Let,

Y = vx

$=> \frac{dy}{dx} = v + x\frac{dv}{dx}$ ....(2)

From (1) and (2) we get,

$$\frac{vx(x+vx)}{x(vx-x)} = v + x\frac{dv}{dx}$$

$$=> \frac{v(1+v)}{(v-1)} = v + x\frac{dv}{dx}$$

$$=> \frac{v+v^2-v^2+v}{v-1} = x\frac{dv}{dx}$$

$$=> \frac{2v}{v-1} = x\frac{dv}{dx}$$

$$=> \frac{1}{2}\left(\frac{v-1}{2v}\right)dv = \frac{dx}{x}$$

Now integrating both side,

$$\int \frac{1}{2}\left(\frac{v-1}{2v}\right)\text{dv} = \int \frac{dx}{x}$$

$$=> \frac{1}{2}v - \frac{1}{2}lnv = lnx + \acute{c}$$

$$=> \frac{y}{x} - ln\frac{y}{x} = 2lnx + 2\,\acute{c}$$

$$=> \frac{y}{x} - ln\frac{y}{x} = 2lnx + c$$

4(x)

Solution:

Given equation,

$\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$ ....(1)

Let,

$x = x+h \quad ; y = y+k$

$=> \frac{dY}{dX} = \frac{Y+K-X-h+1}{Y+X+K+h+5}$

$=> \frac{dY}{dX} = \frac{Y-X+K-h+1}{Y+X+K+h+5}$

Let,

$K - h + 1 = 0$ ....(2)

$k + h + 5 = 0$ .....(3)

From (1) and (3) we get

$2k + 6 = 0$

$=> k = -3$

$\therefore h = -2$

Now,

$\frac{dY}{dX} = \frac{Y-X}{Y+X}$ ......(4)

Let,

$Y = vX$

$$=> \frac{dY}{dX} = v + X\frac{dv}{dX}$$

Now eq(4)becomes,

$$v + X\frac{dv}{dX} = \frac{vX - X}{vX + X}$$

$$=> X\frac{dv}{dX} = \frac{v-1}{v+1} - v$$

$$=> -X\frac{dv}{dX} = \frac{1-v^2}{v+1}$$

$$=> -\frac{dX}{X} = \left(\frac{1}{1+v^2} + \frac{2}{2+2v^2}\right)dv$$

Now integrating both side,

$$\int \left(\frac{1}{1+v^2} + \frac{2}{2+2v^2}\right) dv = \int -\frac{dX}{X}$$

$$=> \tan^{-1} v + \frac{1}{2}\ln(1+v^2) = -lnX + c$$

$$=> \tan^{-1}\frac{Y}{X} + \frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right) = -lnX + c$$

$$=> tan^{-1}\left(\frac{y+3}{x+2}\right)) + \frac{1}{2}\ln\left(1+\frac{(y+3)^2}{(x+2)^2}\right) = -lnX + c$$

$$=> tan^{-1}\left(\frac{y+3}{x+2}\right)) + \ln\sqrt{1+\frac{(y+3)^2}{(x+2)^2}} = -lnX + c$$

Which is the required equation.

4(xii)

Solution:

Given equation,

$$\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x} \text{............}(1)$$

Let,

$$Y = vx$$

$$=> \frac{dy}{dx} = v + x\frac{dv}{dx} \text{..............}(2)$$

From (1) and (3) we get,

$$v + x\frac{dv}{dx} = v + tanv$$

$$=> x\frac{dv}{dx} = tanv$$

$$=> \frac{dv}{tanv} = \frac{dx}{x}$$

Now integrating both side,

$$\int cotv\, dv = \int \frac{dx}{x}$$

$$=> \log(sinv) + c = logx$$

$=> \log(c\ sinv) = logx$
$=> c\ sinv = x$
$=> x = c \sin\frac{y}{x}$
Which is the required solution.

4(xiii)

Solution:

Given equation,

$\frac{dy}{dx} = \frac{x+y+1}{2x+2y+1}$ ....(1)

$=> \frac{d}{dx}(v-x) = \frac{v+1}{2v+1}$ [Let x+y = v]

$=> \frac{dv}{dx} - 1 = \frac{v+1}{2v+1}$

$=> \frac{dv}{dx} = \frac{3v+2}{2v+1}$

$=> \frac{2v+1}{3v+2}\, dv = dx$

Now integrating both side,

$$\int \frac{2v+1}{3v+2}\, dv = \int dx$$

$=> \frac{1}{3}\left(2v - \frac{1}{3}\ln(3v+2)\right) = x + \acute{c}$
$=> 6(x+y) - \ln(3x+3y+2) = 9x + 9\acute{c}$

$=> 6x + 6y - \ln(3x+3y+2) = 9x + c$

$=> 6y - 3x - \ln(3x+3y+2) = c$

Which is the required solution.