Preface

The theory of water waves has been a source of intriguing – and often difficult – mathematical problems for at least 150 years. Virtually every classical mathematical technique appears somewhere within its confines; in addition, linear problems provide a useful exemplar for simple descriptions of wave propagation, with nonlinearity adding an important level of complexity. It is, perhaps, the most readily accessible branch of applied mathematics, which is the first step beyond classical particle mechanics. It embodies the equations of fluid mechanics, the concepts of wave propagation, and the critically important rôle of boundary conditions. Furthermore, the results of a calculation provide a description that can be tested whenever an expanse of water is to hand: a river or pond, the ocean, or simply the household bath or sink. Indeed, the driving force for many workers who study water waves is to obtain information that will help to tame this most beautiful, and sometimes destructive, aspect of nature. (Perhaps 'to tame' is far too bold an ambition: at least to try to make best use of our knowledge in the design of man-made structures.) Here, though, we shall – without apology – restrict our discussion to the many and varied aspects of water-wave theory that are essentially mathematical. Such studies provide an excellent vehicle for the introduction of the modern approach to applied mathematics: complete governing equations; nondimensionalisation and scaling; rational approximation; solution; interpretation. This will be the type of systematic approach that is adopted throughout this text.

The comments that we have offered above describe the essential character of the study of water waves, particularly as it appeared during its first 120 years. However, the last 25 or 30 years have seen an altogether amazing explosion in the complexity of mathematical theories for water waves. The development of *soliton theory*, which itself started life in the context of water waves, has completely transformed many aspects of the mathematical description of nonlinear wave propagation. If it was needed, soliton theory has certainly brought the theory of water waves into the era of modern applied mathematics. This book, it is hoped, presents the material in a way that emphasises the mathematical aspects of classical water-wave theory, and also provides a description of the intrinsic relation between soliton theory and water waves.

This book is based on material which has been taught to either final-year honours mathematicians or to MSc students at the University of Newcastle upon Tyne, at various times over the last 20 years or so. The topics in classical water-wave theory (mainly in Chapter 2) are a considerable extension of those taught, in four or five different lecture courses, by the author during his time at Newcastle. The material on soliton theory is based on an introductory course given to MSc students in Applied Mathematics (and which also provided one of the bases for the book *Solitons: an introduction*, written jointly with Professor Philip Drazin). In all these courses, the aim has been to introduce mathematical ideas and techniques directly, rather than to present a formal and rigorous development. This approach, which is very much in the British tradition, enables the main principles of modern applied mathematics to be seen in a context that both has practical overtones and is mathematically exciting. It is intended that this text will provide an introduction to the theory of water waves (and associated mathematical techniques) to final-year undergraduate students in mathematics, physics, or engineering, as well as to postgraduate students in similar areas. Some of the more elementary material could be taught in the second year of some undergraduate programmes. However, it must be emphasised that there is no attempt to provide such an extensive treatment that the borders of current research are reached, although the book may allow the student to go some way in this direction. It should also be clear that *ad hoc* attempts to describe complicated phenomena are not part of our remit, important though some of these studies are. Furthermore, mainly in the interests of space, a section on numerical methods, which certainly play a rôle in the broader aspects of water-wave theory, is not included.

Chapter 1 introduces the appropriate equations of fluid mechanics, together with the relevant boundary conditions that are needed to describe water waves. In addition, the ideas of nondimensionalisation, scaling and asymptotic expansions are briefly explored, as are simple concepts in wave propagation. A student with a background in elementary fluid mechanics, and some knowledge of simple mathematical methods, could ignore this chapter and move directly to Chapter 2. (The only essential background that the student requires is in advanced calculus (for example, some familiarity with vector calculus), and in the methods of applied mathematics (for example, methods of solution of some classical ordinary and partial differential equations).) Chapter 2 looks, first, at some of the classical problems in linear water-wave theory. These include the speed of gravity and capillary waves, the effects of variable depth and the ship-wave pattern; the application of ray theory to problems where the background flow slowly varies is also developed. The second part of this chapter is devoted to nonlinear problems, but still those that are generally regarded as classical. In this area we include the Stokes wave, nonlinear long waves via the method of characteristics (and Riemann invariants), the hydraulic jump and bore, waves on a sloping beach and the solitary wave. Many additional examples and applications can be explored through the exercises at the end of the chapter.

Chapters 3 and 4 are devoted to the more modern aspects: problems that give rise to soliton-type equations. These are, first, the equations that belong to the Korteweg–de Vries family; some relevant results from soliton theory are quoted, and these are used to help in the interpretation of the various equations and solutions that arise. The applications are extended to include the effects of shear and variable depth. Then the Nonlinear Schrödinger family of equations is discussed in a similar fashion, although the rôles of an underlying shear flow or variable depth are treated less fully for this family, mainly because the calculations are very much more involved. For both families, some two-dimensional configurations of waves are also discussed.

The final chapter provides a brief introduction to the rôle and effects of viscosity, as they are relevant in a few water-wave phenomena. This is intended to add a broader view to water-wave theory; all the previous discussions here are solely for an inviscid fluid (but the flow is sometimes allowed to be rotational).

All the mathematical developments are presented in the most straightforward manner, with worked examples and simple cases carefully explained. Many other aspects, relevant calculations and additional examples are provided in the numerous exercises at the end of each chapter. Also at the end of each chapter is a section of further reading which indicates where more information can be found about some of the topics; these references include both research papers and other texts. Sections are numbered following the decimal system, and equations are numbered according to the chapter in which they appear: for example, equation (1.2) is equation 2 in Chapter 1. The exercises are numbered in a similar fashion (for example, Q2.3), as are the answers and hints at the end of the book (for example, A2.3). Also provided at the end of the book is a fairly extensive bibliography and author index, and also a collection of brief historical notes on some of the important characters who have worked on the theory of water waves. The quotations at the beginning of each chapter, and at the start of some sections, are taken from the poetical works of Alfred, Lord Tennyson.

I wish to put on record my very grateful thanks for the typing of the manuscript to Mrs Heather Bliss, Mrs Helen Bell and Miss Jackie Tait, who all played a part, but most particularly to Mrs Susan Cassidy, who carried by far the major burden. This she did with great efficiency, speed, dedication and, throughout, with the greatest good humour when faced with (a) my handwritten manuscript and (b) my changes of mind. The originals of the figures were produced on my PC, using a combination of *Mathematica* and *KeyDraw*, and printed on my Hewlett-Packard DeskJet printer (so I carry full responsibility for their clarity and accuracy). Finally, I wish to thank Cambridge University Press, and particularly Professor David Crighton, for their encouragement to write this text (and their patience when I got behind the planned schedule).

RSJ
Newcastle upon Tyne
December 1996