

# Requirements for Semester 1 Report

## Student: Sultan Aitzhan

### Supervisors

Katie Oliveras (Seattle University)

David Smith (Yale-NUS College)

For the Semester 1 Report, the general expencation is that Sultan will write up a detailed introduction to the water-wave problems as well as our current progress towards deriving the KdV equation on the half-line. As this project requires a significant investment into background material, it is expected that the first report will contain mostly background material, preliminary problems, and a short summary of the current state of our research. It is expected that significant progress will be made during the month of December when Sultan will visit Katie Oliveras in the US.

For a more detailed list of expectations, the following content should be included to the best of Sultan's ability. The supervisors expect that this report will be significantly extended during the December visit. The supervisors will expected an updated version of this report at the beginning of January 2020<sup>1</sup>.

## 1 The Water-Wave Problem: Description

- A description of the general water-wave problem. This includes a description of the equations describing conservation of mass and momentum within the bulk of a fluid. This section should also include a “physical intitution” for the equations of motion.
- A description of the irrotational formulation and how the equations can be reformulated via a velocity potential.

## 2 The Free-Surface Boundary Conditions

- A description of the free-surface problem.
- Derivation of the appropriate boundary conditions. This includes the boundary condtions as  $|x| \rightarrow \infty$ , the conditions at  $z = -h$ , and the conditions at  $z = \eta(x, t)$ . Physical motivations for these conditions should also be included.

## 3 The Derivation of the KdV Equation on the Whole-Line

- A description of the objective of asymptotic expansions. That is, *why should we derive the KdV equation*.
- A discussion of dispersion including the derivation of the dispersion relationship for the full water-wave problem.
- The derivation of the KdV equation using the velocity potential formulation of the water-wave problem.
- A discussion about the need for multiple time scales.

## 4 The Derivation of the KdV Equation on the Half-Line

- A description of the new boundary condition at  $x = 0$  including information about how the boundary conditions change.
- The current state of the derivation on the half-line.

## 5 Additional Content

- A summary of current challenges.
- A summary of future research directions.

---

<sup>1</sup>Though this is not an official requirement, it is expected that a significant amount of work will be completed during the December visit; it will be important for these items to be clearly documented and presented to the supervisors for early review as the Semester 2 portion begins.