The theory of water waves has been a source of intriguing mathematical problems for at least 150 years. Virtually every classical mathematical technique appears somewhere within its confines. The aim of this book is to introduce mathematical ideas and techniques that are directly relevant to water-wave theory (although a formal development is not followed), enabling the main principles of modern applied mathematics to be seen in a context that both has practical overtones and is mathematically exciting.

Beginning with the introduction of the appropriate equations of fluid mechanics, the opening chapters go on to consider some classical problems in linear and nonlinear water-wave theory. This sets the scene for a study of more modern aspects, including problems that give rise to soliton-type equations. The book closes with an introduction to the effects of viscosity.

All the mathematical developments are presented in the most straightforward manner, with worked examples and simple cases carefully explained. Exercises, further reading, and historical notes on some of the important characters round off the book and help to make this an ideal text for either advanced undergraduate or beginning graduate courses on water wayes.

# A Modern Introduction to the Mathematical Theory of Water Waves

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To my parents Dorothy and Eric, to my sons Iain and Neil, and last but first to my wife Ros.