$$\phi_{zz} + \varepsilon \phi_{xx} = 0 \qquad z \in (-1, \varepsilon\eta)$$

$$\phi_z = 0 \qquad z = -1$$

$$\left\{ \begin{array}{ll} \varepsilon \eta_t + \varepsilon^2 \phi_x \eta_x = \phi_z & z = \varepsilon\eta \\ \phi_t + \eta + \frac{1}{2}\phi_z^2 + \frac{\varepsilon}{2}\phi_x^2 = 0 & z = \varepsilon\eta \end{array} \right\}$$

$\eta(x;t)$

$\phi(x,z;t)$

ε

$\phi_z = 0$ at $z=-1$ $\Rightarrow$ $\phi(x,z,t) = \sum_{n=0}^{\infty} (z+1)^n \phi_n(x,t)$ (Assume)

$$\phi_{zz} + \varepsilon\phi_{xx} \Rightarrow \varepsilon \sum_{n=0}^{\infty} (z+1)^n (\phi_n)_{xx} + \sum_{n=0}^{\infty} (z+1)^n (n+2)(n+1)(\phi_{n+2}) = 0$$

$$\Rightarrow \phi_{n+2} = -\frac{\varepsilon(\phi_n)_{xx}}{(n+1)(n+2)} \qquad n = 0, 1, 2, \ldots$$

$\phi_z = 0$ at $z=-1$ $\Rightarrow$ $\phi_1 = 0$.

$$\Rightarrow \phi_3 = \phi_5 = \ldots = 0$$

$$\Rightarrow \phi(x,z,t) = \phi_0 + \varepsilon \frac{(z+1)^2}{2!}(\phi_0)_{xx} + \varepsilon^2 \frac{(z+1)^4}{4!}(\phi_0)_{xxxx} + \ldots$$

Rafael Aceraga

$$\phi_z(x,z,t) = -\varepsilon(z+1)(\phi_0)_{xx} + \frac{\varepsilon^2(z+1)^3}{3!}(\phi_0)_{xxxx} + \dots$$

$$\phi_z(x,\varepsilon\eta,t) = -\varepsilon(\varepsilon\eta+1)(\phi_0)_{xx} + \frac{\varepsilon^2(\varepsilon\eta+1)^3}{3!}(\phi_0)_{xxxx} + \dots$$

$$= -\varepsilon\phi_{0xx} + O(\varepsilon^2)$$

$$\varepsilon\eta_t + \varepsilon^2\phi_x\eta_x = \phi_z = -\varepsilon(\phi_0)_{xx} + O(\varepsilon^2)$$

$$\varepsilon\eta_t = -\varepsilon(\phi_0)_{xx} + O(\varepsilon^2)$$

$$\eta_t = -(\phi_0)_{xx} + O(\varepsilon)$$

velocity

$$\Rightarrow \quad \eta_t + \partial_x\left[\boxed{\phi_{0x}}\right] = O(\varepsilon) \quad (1)$$

$$(\phi_{0x})_t + \partial_x\eta = O(\varepsilon) \quad (2)$$

<u>why θ over σ ?</u>

<u>Multiple Scales Analysis</u>

$$f(t) = \tilde{f}(t,\varepsilon t)$$
$$= \tilde{f}(T_0, T_1, \dots)$$

why ~~pow~~ integer powers of epsilon ?

$$\phi_{0x} = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots$$
$$\eta = \eta_0 + \varepsilon\eta_1 + \varepsilon^2\eta_2 + \dots$$

Introduce slow time scales:

$$\cancel{t = t + \varepsilon t + \varepsilon^2 t} + \varepsilon^3 t$$
$$T_0 \quad T_1 \quad T_2 \quad T_3$$

$t \approx T_0$ since ε small

☆ (don't expand in power T, just try without & see why you get stuck)

?

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial T_0} + \varepsilon\frac{\partial}{\partial T_1} + \varepsilon^2\frac{\partial}{\partial T_2} + \dots$$

$\phi_{0x} = u_0 \qquad \eta = \eta_0$ ~~$+\varepsilon\eta_0$~~

$$\begin{cases} \dfrac{\partial \eta_0}{\partial \tau_0} + \dfrac{\partial u_0}{\partial x} = 0 & \Rightarrow \partial_x(\ldots) = 0 \\ \dfrac{\partial u_0}{\partial \tau_0} + \dfrac{\partial \eta}{\partial x} = 0 & \Rightarrow \partial_{\tau_0}(\ldots) = 0 \end{cases}$$

[all epsilon terms lump tgt]

Assume we can interchange x, τ_0 in derivatives : compatibility.

Clairaut Theorem.

$$\Rightarrow \begin{cases} \dfrac{\partial^2 \eta_0}{\partial \tau_0^2} - \dfrac{\partial^2 \eta_0}{\partial x^2} = 0 \\ \dfrac{\partial^2 u_0}{\partial \tau_0^2} - \dfrac{\partial^2 u_0}{\partial x^2} = 0 \end{cases}$$

☆ horizontal BC & see how they occur in the asymptotics

$$\Rightarrow \quad \eta_0 = f(x - \tau_0, \tau_1, \ldots) + g(x + \tau_0, \tau_1, \ldots)$$
$$u_0 = f(x - \tau_0, \tau_1, \ldots) + g(x + \tau_0, \tau_1, \ldots)$$

☆ Animation of propagation (related to characteristics)

☆ reflection at the half-line condition

☆ ~~some~~ notes on wave equation (on a half line).

KdV:

$$\phi_z(x, \varepsilon\eta, t) = -$$
$$= -\varepsilon(1 + \varepsilon\eta)(\phi_0)_{xx} + \varepsilon^2 \frac{(1 + \varepsilon\eta)^3}{6}(\phi_0)_{xxxx} + O(\varepsilon^3)$$

$$= -\varepsilon(1 + \varepsilon\eta_0)\underbrace{(\phi_0)_{xx}}_{u_{0x} + \varepsilon u_{1x}} + \varepsilon^2 \frac{1}{6}(\phi_0)_{xxxx} + O(\varepsilon^3)$$

$$= -\varepsilon u_{0x} + \varepsilon^2\left(-\eta_0 u_{0x} - u_{1x} + \frac{1}{6} u_{0xxx}\right) + O(\varepsilon^3)$$

$$\phi_x(x, \varepsilon\eta, t) = u_0 + \varepsilon\left(u_1 - \frac{1}{2}u_{0xx}\right) + O(\varepsilon^2)$$

$$\phi_{xt}(x, \varepsilon\eta, t) = (u_0)_{\tau_0} + \varepsilon\left(u_{0\tau_1} + u_{1\tau_0} - \frac{1}{2}(u_0)_{xx\tau_0}\right) + O(\varepsilon^2)$$

$$\eta_t = (\eta_0)_{\tau_0} + \varepsilon\left((\eta_1)_{\tau_0} + (\eta_0)_{\tau_1}\right) + O(\varepsilon^2)$$

$$\phi_y(x, \varepsilon\eta, t) = -\varepsilon\eta_{0x} + \varepsilon^2\left(-\eta_0 u_{0x} - u_{1x} + \frac{1}{6}u_{0xxx}\right) + O(\varepsilon^3)$$

KC: $$\begin{cases} \varepsilon : & -u_{0x} = \eta_{0\tau_0} \\ \varepsilon^2 : & -\eta_0(u_0)_x - (u_1)_x + \frac{1}{6}u_{0xxx} = \eta_{1\tau_0} + \eta_{0\tau_1} + u_0\eta_{0x} \end{cases}$$

BC: $$\begin{cases} \varepsilon^0 : & u_{0\tau_0} + \eta_{0x} = 0 \\ \varepsilon^1 : & u_{0\tau_1} + u_{1\tau_0} - \frac{1}{2}(u_0)_{xx\tau_0} + \eta_{1x} + u_0 u_{0x} = 0 \end{cases}$$

(1) realise wave eq on half line
(2) characteristic variables
(3) KdV references for the para.

" also follow the dom. behaviour of waves"!!

" write as a free system & diagonalise : notes

" right & left eigenvectors"

" characteristic".

" rigorous KdV stuff" (assumption etc.)

" Ablowitz, @ Segur, → book

↓

Dispersive Waves

" horizontal BC" : $\phi_t \to 0$ $\phi_t \Rightarrow 0$

$\phi_x \to 0$

$\phi_{xx} \to 0$

Perko

Strogatz

Bernard

Monday 9a.m.

Second week of Oct till 15, October.