Rendleman Bartter Model

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1. The Rendleman-Bartter model (1980) in finance is a short rate model that describes the change of interest rate. It does so by focusing on interest rate movements as driven by only one source of market risk; thus making it a "one factor model". It is a stochastic asset model (which will be our concentration in this paper). The Rendleman Bartter model more specifically is a one-facto short-rate model. The stochastic factor, which is the short rate, determines the future development of all the interest rates.

<u>Assumptions:</u> Usually a perfect and efficient market is assumed; there are no transaction cost, taxes or restrictions on short sales involved with the purchase or sale of any security. Information is not priced and the market is assumed to efficient in that it equates the expected returns of all securities of equivalent risks.

- The stochastic representation of the Rendleman Bartter model:

$$dr_t = \theta r_t \, dt + \sigma r_t \, dW_t$$

Where W_t is the Wiener process modeling the random mark risk factor. Our drift parameter θ is a constant expected instantaneous rate of change within the interest rate. Lastly σ is the standard deviation, determining the volatility of the interest rate.

The Rendleman Bartter model (and also the Ho-Lee model) do not capture the mean reversion of interest rates. Similar to the Cox-Ingersoll-Ross model and the Vasicek model the model we will be testing has only a finite number of free parameters, which makes it not possible to specify these parameters so that they calibrate with observed market prices. A simple solution to this to allow the parameters to vary deterministically with time (this is implemented during the input).

Rendleman Bartter Model follows the properties of geometric Brownian motion. With the following expected value and variance (in our model S_t is r_t and S_0 is r_0):

- <u>Theoretical Expected Value (mean)</u>:

$$\mathbb{E}(S_t) = S_0 e^{\mu t},$$

- <u>Theoretical Variance</u>:

$$\mathrm{Var}(S_t) = S_0^2 e^{2\mu t} \left(e^{\sigma^2 t} - 1 \right),$$

We assume that the annual rate of interest for pure discounting bonds (in this test we will work with stocks) maturing one period into the future can take on only one of two values; but in our analysis we make a linear vector that sets the path for the prices based on a calculated probability of success.

<u>NOTE:</u> The primary shortcoming of a one-factor model is that it cannot capture complicated yield curve behavior; they tend to produce parallel shifts in the yield curve but not changes in its slope or curvature.

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2) In this report I will be testing and verifying the theoretical mean and variance against the numerical (calculated via MATLAB) calculations of the mean and variance. To take a step further we will be testing the effects of the Rendleman Bartter model on European option, American option and Asian option. Where the American option the contract holder can exercise the option any time until a set date (T); European option the contract holder can exercise only the date at which the contract will end, T; last the Asian option take the average price for the period [0,T].

The payoff for each option:

<u>American Option:</u> ***any t<=T***	**<u>European Option:</u>** ***only t=T***	**<u>Asian Option:</u>** ***interval [0,T]***
Call: $\max\{(S-K),0\}$	Call: $\max\{(S-K),0\}$	Call: $P(T)=\max(A(0,T)-K,0)$,
Put: $\max\{(K-S),0\}$	Put: $\max\{(K-S),0\}$	Put: $P(T)=\max(K-A(0,T),0)$.

The test that will be undergone are:

1. The expected value and variance of the American and European options
2. Mean of the Asian option (refer to section 5 for the variance)
3. Test the rates (per option) evaluated by making a price range (20-2000) controlled stock market; this is to compared the put/call option.

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3) The code for the Rendleman Bartter Model (lines 1-25):

Parts 1. & 2. of the experiment

```
1   clear all;
2   close all;
3
4   theta = input('What is theta?');
5   sigma = input('What is sigma?');
6   r0 = input('What is the intial interest value?');
7   T = input('What is the value of T?');
8   N = input('What is the value of N?');
9   u = 5;
10  d = 1/u;
11  strike = 500;
12
13  for k = 1:10000;
14      for l = 1:N;
15          deltaW(l) = (T/N)^(1/2)*randn(1,1);
16      end;
17      r(1) = r0 + theta*r0*(T/N) + sigma*r0*deltaW(1);
18      for j = 2:N;
19          r(j) = r(j-1) + theta*r(j-1)*(T/N) + sigma*r(j-1)*deltaW(j);
20      end;
21
22      endvalueAmer(k) = max(r);
23      endvalueEuro(k) = r(N);
24      endvalueAsian(k) = mean(r);
25  end;
```

Things to note about the code; Lines 9-11 will be used to calculate the stock process that we will be using in our evaluation. Lines 14-16 is the Wiener process. Using j as subscript we get the stochastic representation of the Rendleman Bartter model:

$$r_j = r_{j-1} + ø*(T/N) + σ*deltaW(j) \quad \text{(as shown on line 19)}$$

Lines 22-24 are the rates that get selected at the end of each iteration (ie after the first iteration of k one rate is selected from r(n) based on the filter of each option). For the endvalueAmer (for the American option) we select the maximum rate from that iteration; the endvalueEuro (for

European option) we select the last (r(N)) to be inserted into the list (or array); and for the endvalueAsian (for the Asian option) we at the mean of r and insert it.

Part 3 of the experiment

```
    %risk neutral probability
for n = 1:length(endvalueEuro);
%for n = 1:length(endvalueAmer);
%for n = 1:length(endvalueAsian);

    %p(n) = ( 1 + endvalueEuro(n) - d)/( u - d);
    p(n) = ( 1 + endvalueAmer(n) - d)/( u - d);
    %p(n) = ( 1 + endvalueAsian(n) - d)/( u - d);
end;
```

This part of the code calculates the risk neutral probability and stores each in p(n) with result to each rate in the select option being observed. Each option will be evaluated and analyzed.
(***con't part 3)***

```
37    stockarray = [];
38    stockarray(1) = 500; %sake of testing limit values from around 25-1000
39                         %may vary due to the checks below
40
41  for i = 1:length(p); %run a test per calculated probability
42      m = 1;
43      while m < length(p);
44          x = binornd(1,p(i), 1, 1); %the chances of success with prob p
45          if x == 1;
46              if stockarray(m)>=1000;
47                  stockarray(m+1) = stockarray(m)*d;
48              else
49                  stockarray(m+1) = stockarray(m)*u; % if heads then mulitple previous price by u
50              end;
51          else
52              if stockarray(m) <= 25;
53                  stockarray(m+1) = stockarray(m)*u; % otherwise it went down, so multiple by d
54              else
55                  stockarray(m+1) = stockarray(m)*d;
56              end;
57          end;
58          m = m+1;
59
60          %European Option
61          %varrayEuroPut(i) = max( strike - stockarray(end) , 0 ); %put option
62          %varrayEuroCall(i) = max( stockarray(end) - strike , 0 ); %call option
63
64          %American Option
65          varrayAmerPut(i) = max( strike - max(stockarray) , 0 ); %put option
66          varrayAmerCall(i) = max( max(stockarray) - strike , 0 ); %call option
67
68          %Asian Option
69          %varrayAsianPut(i) = max( strike - mean(stockarray), 0 ); %put option
70          %varrayAsianCall(i) = max( mean(stockarray) - strike , 0 ); %call option
71      end;
72  end;
```

(***end of code)***
Quick run-down of the code above: Lines 37-38 created an array of to hold stock prices with initial price being 500. Lines 41-58 is the path made for the stocks with probability p(i) and are bound between 20-2000. Lastly lines 60-70 are to evaluate when put or call option is exercised for the respected option.

4) **Results:**

Results of Parts 1. & 2.

European option:

Test Number	θ	σ	r_0	T	N	K	Calculated mean	Theoretical mean	Calculated Variance	Theoretical Variance
1	.3	.4	.03	2	1000	10^5	0.0546	0.0547	0.0011	0.0011
2	.4	.5	.06	1	500	10^5	0.0895	0.0895	0.0023	0.0023
3	.6	.2	.2	1	10^4	10^5	0.3634	0.3644	0.0054	0.0054

American option:

Test Number	θ	σ	r_0	T	N	K	Calculated mean	Theoretical mean	Calculated Variance	Theoretical Variance
1	.3	.4	.03	2	1000	10^5	0.0348	0.0269	0.0002	0.0002
2	.4	.5	.06	1	1000	10^5	0.03238	0.02443	0.00016	0.00017
3	.6	.2	.2	1	10^4	$\mathbf{10^7}$	0.054158	0.054156	0.001139	0.001137

(Last test I let run over night to get a closer approximation)

Asian option:

Test Number	θ	σ	r_0	T	N	K	Calculated mean	Theoretical mean	Calculated Variance	Theoretical Variance
1	.3	.4	.03	2	1000	10^5	0.0234	0.0269	-----	-----
2	.4	.5	.06	1	1000	10^5	0.02407	0.02443	-----	-----
3	.6	.2	.2	1	10^4	10^5	0.05408	0.05466	----	-----

Results of part 3:

Option	θ	σ	r_0	Strike price	T	N	K	Mean of the probability array ***p***	Mean of Call option payoff	Mean of Put option payoff	Pull-Call Parity
American	.3	.4	.03	500	2	1000	10^5	0.83589	2000	480	1550
European	.4	.5	.06	500	1	1000	10^5	0.45147	101.08	379.78	-278.70
Asian	.6	.2	.2	500	1	1000	10^5	0.14531	0	348.36	348.36

(***Reference 1 &2 for the put-call parities for American and Asian option)***

5) **Commentary and future inquires:**

Once you compare the numbers it is evident that the mean and variance are very close to the theoretical values, specific cases for the American option were the code ran over night to get a closer approximation. If the experiment were to be ran infinitely many times, both the theoretical and computational values would be exactly the same. However since we ran it only a finite time there are slight differences, which would only decrease as N increases. As for why I didn't take into consideration the Asian option variance is because there isn't a solid enough closed formula to compare the results too.

The results for the last part of the test was expected once the mean of the probability (p) was taken. If the mean was relatively high then the chances of the stock will go higher, and if so then the call option holder would exercise it in order to get an expensive stock for the lower strike price agreed upon when making the deal. Likewise when the mean of p is low then it is expected that the put option would be used to sell less worthy stocks, but will suffer a loss compared to the amount spent on the call option in total. Example with Asian option we see that the probability for success in rise of stock is extremely small so in order to avoid a loss we do not exercise the option.

Looking forward…

- I would like to investigate a better method for the American option since it isn't as straight forward, in terms of computation, as the European option (even though in the actual market American option is valued more).
- Investigate the Asian option methods of calculating the variance (either using variance reduction or Monte Carlo method).
- And last to re-due this experiment with actual stock prices from an historical stock databases (ie yahoo finance, google finance etc)

Reference:

1. http://efinance.org.cn/cn/FEshuo/150102%20%20%20%20%20The%20Pricing%20of%20Options%20on%20Debt%20Securities,%20pp.%2011-24.pdf
2. http://www.math.nyu.edu/~cai/Courses/Derivatives/lecture8.pdf
3. http://michaelcarteronline.com/MCM/ExoticOptions/AsianOptions.pdf