Reduction theorems for the discrete nonlinear operator on the cones of monotone sequences

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Let $\{u_k\}, \{a_n\}$ and $\{b_n\}$ be given non-negative sequences. Let $p, q \in (0, \infty)$. We will investigate the following inequalities:

$$\left(\sum_{n=1}^{\infty}\left(\sup_{n\le i<\infty} u_i \sum_{k=1}^{i} x_k\right)^q a_n\right)^{\frac{1}{q}} \le C\left(\sum_{n=1}^{\infty} x_n^p b_n\right)^{\frac{1}{p}}, \tag{1}$$

$$\left(\sum_{n=1}^{\infty}\left(\sup_{1\le i<n} u_i \sum_{k=1}^{i} x_k\right)^q a_n\right)^{\frac{1}{q}} \le C\left(\sum_{n=1}^{\infty} x_n^p b_n\right)^{\frac{1}{p}} \tag{2}$$

for non-negative, non-increasing sequences $x = \{x_n\}$ and the constant $C > 0$ is independent of x.

Theorem 1. Let $0 < q \le \infty,\ 1 < p < \infty$. Assume that $\{a_n\}$ and $\{b_n\}$ are given non-negative weight sequences. Then the inequality (1) holds for all non-negative, non-increasing sequences $\{x_n\}$ if and only if the following inequality:

$$\left(\sum_{n=1}^{\infty}\left(\sup_{n\le i<\infty} u_i \sum_{k=1}^{i}\left(\sum_{j=k}^{\infty} y_j\right)\right)^q a_n\right)^{\frac{1}{q}} \le C\left(\sum_{n=1}^{\infty} y_n^p B_{n+1}^{p-1} B_n b_{n+1}^{1-p}\right)^{\frac{1}{p}}, \tag{3}$$

holds for all non-negative sequences $\{y_n\}$, where $B_n = \sum_{k=1}^{n} b_k, \quad n \in N$.

Theorem 2. Let $0 < q \le \infty,\ 1 < p < \infty$. Assume that $\{a_n\}$ and $\{b_n\}$ are given non-negative weight sequences.Then the inequality (2) holds for all non-negative, non-increasing sequences $\{x_n\}$ if and only if for any $\alpha > 0$ the following inequality:

$$\left(\sum_{n=1}^{\infty}\left(\sup_{1\le i<n} u_i \sum_{i=n}^{\infty}\frac{1}{B_i^{\alpha+1}}\left(\sum_{k=1}^{i} B_k^{\alpha+1} y_k\right)\right)^q a_n\right)^{\frac{1}{q}} \le C\left(\sum_{n=1}^{\infty} y_n^p B_n^p b_n^{1-p}\right)^{\frac{1}{p}}, \tag{4}$$

holds for all non-negative sequences $\{y_n\}$, where $B_n = \sum_{k=1}^{n} b_k, \quad n \in N$.

In the continuous case, similar questions were considered in [1] and [2].

Список литературы

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