# WEIGHTED ESTIMATES FOR SOME CLASS OF QUASILINEAR OPERATORS

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Let $0 < q, p, r < \infty$ and $\frac{1}{p} + \frac{1}{p'} = 1$. Let $u = \{u_i\}_{i=1}^{\infty}$ and $v = \{v_i\}_{i=1}^{\infty}$ be weight sequences, i.e., positive sequences of real numbers. We denote by $l_{p,v}$ the space of sequences $f = \{f_i\}_{i=1}^{\infty}$ of non-negative real numbers such that

$$\|vf\|_p = \left(\sum_{i=1}^{\infty}(v_i f_i)^p\right)^{\frac{1}{p}} < \infty.$$

For any non-negative $f \in l_{p,v}$ we consider the following iterated discrete Hardy-type inequality with three weights

$$\left(\sum_{n=1}^{\infty} u_n^q (K^{\pm} f)_n^q\right)^{\frac{1}{q}} \leq C \left(\sum_{i=1}^{\infty}(v_i f_i)^p\right)^{\frac{1}{p}}, \tag{1}$$

where $C$ is a positive constant independent of $f$ and $K$ is a quasilinear operators defined as follows

$$(K^- f)_n = \left(\sum_{k=n}^{\infty} a_{k,n} \left(\sum_{i=k}^{\infty} f_i\right)^r\right)^{\frac{1}{r}},$$

$$(K^+ f)_n = \left(\sum_{k=1}^{n} a_{n,k} \left(\sum_{i=1}^{k} f_i\right)^r\right)^{\frac{1}{r}}.$$

where $a_{k,n}$ is a non-negative matrix. The aim of this paper is to characterize inequality (1) and its dual version for the case $0 < p \leq r < q < \infty$. Iterated inequalities involving matrix operators were studied in works [1] and [2] for the same parameters ratio.

**Theorem 1.** Let $0 < p \leq r < q < \infty$ and $a_{k,n}$ be a non-negative matrix. Then for any non-negative $f \in l_{p,v}$

(a) If $0 < p \leq 1$, the inequality (1) for operator $K^-$ holds if and only if

$$D = \sup_{j \geq 1} v_j^{-1} \left(\sum_{n=1}^{j} u_n^q \left(\sum_{i=n}^{j} a_{i,n}\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} < \infty$$

(b) If $1 < p < \infty$, the inequality (1) for operator $K^-$ holds if and only if

$$\mathcal{D} = \sup_{j \geq 1} \left(\sum_{i=j}^{\infty} v_i^{-p'}\right)^{\frac{1}{p'}} \left(\sum_{n=1}^{j} u_n^q \left(\sum_{i=n}^{j} a_{i,n}\right)^{\frac{q}{r}}\right)^{\frac{1}{q}} < \infty.$$

Moreover, $C \approx D$ in case (a) and $C \approx \mathcal{D}$ in case (b), where $C$ is the best constant in (1).
**Theorem 2.** Let $0 < p \leq r < q < \infty$ and $a_{n,k}$ be a non-negative matrix. Then for any non-negative $f \in l_{p,v}$
(a) If $0 < p \leq 1$, the inequality (1) for operator $K^+$ holds if and only if

$$M = \sup_{j\geq 1} v_j^{-1} \left( \sum_{n=j}^{\infty} u_n^q \left( \sum_{i=j}^{n} a_{n,i} \right)^{\frac{q}{r}} \right)^{\frac{1}{q}} < \infty$$

(b) If $1 < p < \infty$, the inequality (1) for operator $K^+$ holds if and only if

$$\mathcal{M} = \sup_{j\geq 1} \left( \sum_{i=1}^{j} v_i^{-p'} \right)^{\frac{1}{p'}} \left( \sum_{n=j}^{\infty} u_n^q \left( \sum_{i=j}^{n} a_{n,i} \right)^{\frac{q}{r}} \right)^{\frac{1}{q}} < \infty.$$

Moreover, $C \approx M$ in case (a) and $C \approx \mathcal{M}$ in case (b), where $C$ is the best constant in (1).

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## References

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