Introduction to Time Series Analysis

Predictive Analytics

What We'll Learn Today

1. What is Time Series Data?
2. Moving Averages
3. Exponential Smoothing
4. Holt's Method (Trend)
5. Holt-Winters (Trend + Seasonality)
6. Choosing the Right Method
7. Practical Tips

What is Time Series Data?

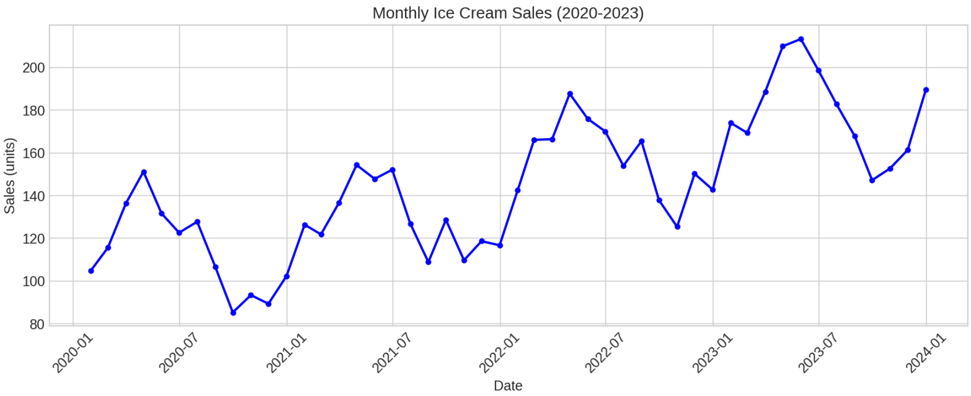
Simple Definition

A **time series** is data collected over time, in order.

Examples:

- Daily temperature readings
- Monthly sales figures
- Weekly website visitors
- Quarterly company revenue
- Yearly population counts

Example: Monthly Ice Cream Sales



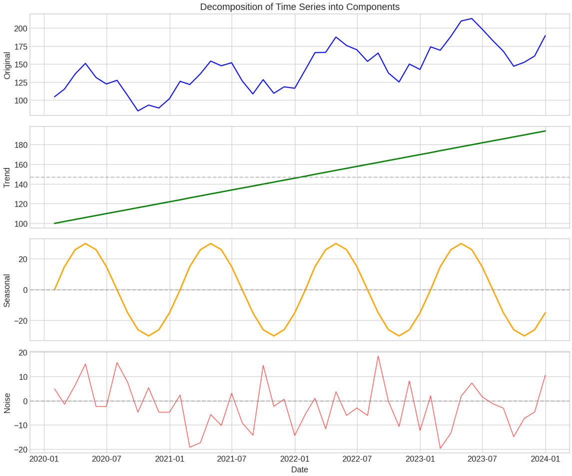
- Sales go up and down throughout the year
- There seems to be a pattern that repeats
- Overall, sales might be growing over time

The Building Blocks of Time Series

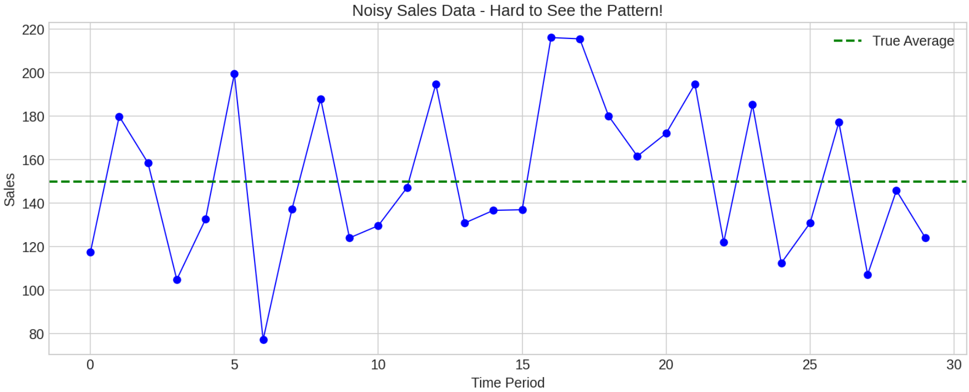
1. **Trend**
   - Long-term direction
   - Going up? Going down? Flat?
2. **Seasonality**
   - Regular, repeating patterns
   - Summer vs. winter
   - Weekday vs. weekend
3. **Noise (Random Fluctuations)**
   - Unpredictable ups and downs
   - "Static" in the data

Task:
Separate signal from noise!

Visualizing the Components



The Problem: Too Much Noise!



The Moving Average

The Idea

Instead of looking at each point alone, look at the **average of nearby points**.

3-Month Moving Average Example:

Month	Sales	Moving Average
January	100	–
February	120	$(100 + 120 + 110) \div 3 = 110$
March	110	$(120 + 110 + 130) \div 3 = 120$
April	130	$(110 + 130 + 125) \div 3 = 122$
May	125	–

Moving Average Formula

Simple Formula

$$\text{Moving Average} = \frac{\text{Sum of last } n \text{ values}}{n}$$

Example: 3-Month Moving Average

$$\text{MA}_{\text{March}} = \frac{\text{Jan} + \text{Feb} + \text{Mar}}{3} = \frac{100 + 120 + 110}{3} = 110$$

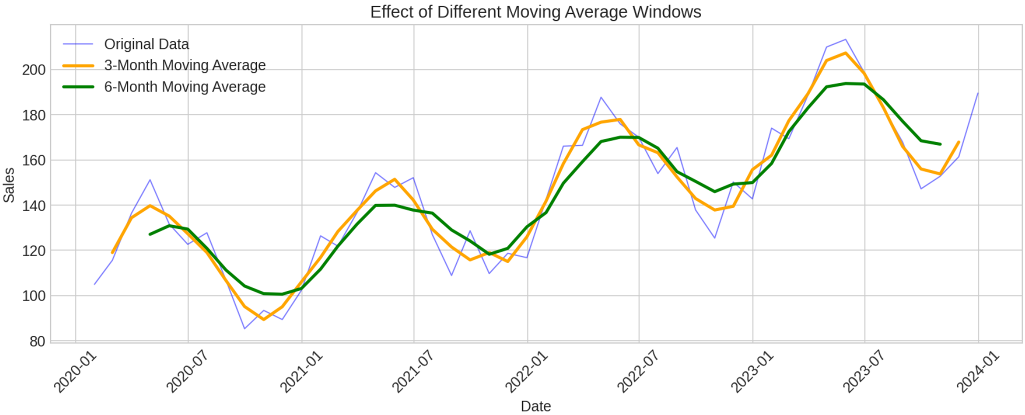
Small window (3 months):

- More responsive
- Still somewhat bumpy

Large window (12 months):

- Very smooth
- Slower to react

Moving Average in Action



Removing Seasonality with Moving Averages

The Magic of 12-Month Moving Average

When you average exactly **one full year** of monthly data, the seasonal ups and downs **cancel out**!

Why does this work?

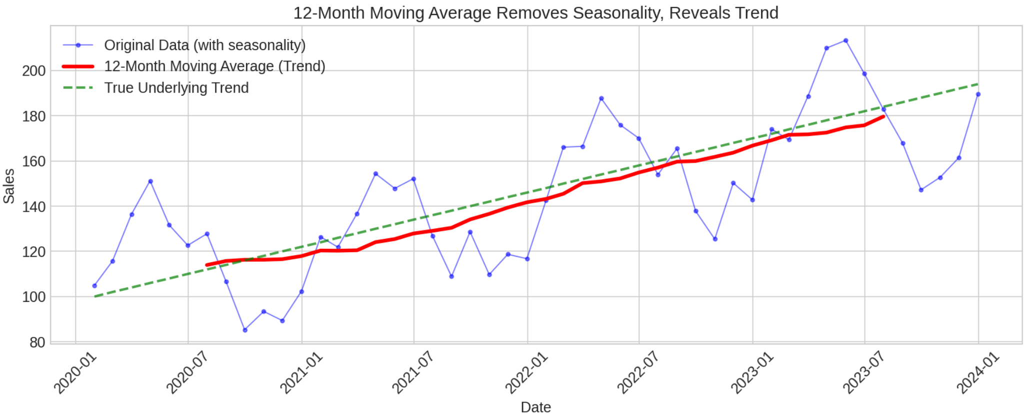
- High summer sales + Low winter sales = Average
- Every month appears exactly once in each window
- What's left is the **underlying trend**

Rule of Thumb

Use a window size equal to your seasonal period:

- Monthly data with yearly pattern → 12-month MA
- Daily data with weekly pattern → 7-day MA
- Quarterly data with yearly pattern → 4-quarter MA

12-Month Moving Average: Revealing the Trend



Notice: The 12-month MA shows the **pure trend** without seasonal bumps!

Limitation of Moving Averages

Problem with Simple MA:

- All points weighted equally
- January counts same as March
- But shouldn't **recent** data matter more?

Month	Weight
January	33%
February	33%
March	33%

Simple MA: Equal weights

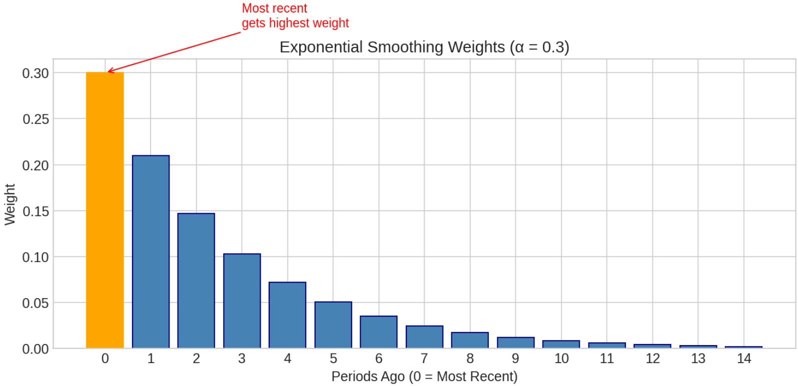
Better Idea

Give **more weight to recent** observations, **less weight to older** ones.

Exponential Smoothing: The Concept

Key Idea

Weights decrease **exponentially** as data gets older.



Recent data = High weight, Old data = Low weight

Simple Exponential Smoothing Formula

The Formula

$$\text{Forecast} = \alpha \times (\text{Actual}) + (1 - \alpha) \times (\text{Previous Forecast})$$

Where α (alpha) is between 0 and 1

High α (e.g., 0.8):

- Trust new data more
- React quickly to changes
- More "jumpy" forecasts

Low α (e.g., 0.2):

- Trust history more
- Smooth, stable forecasts
- Slow to react

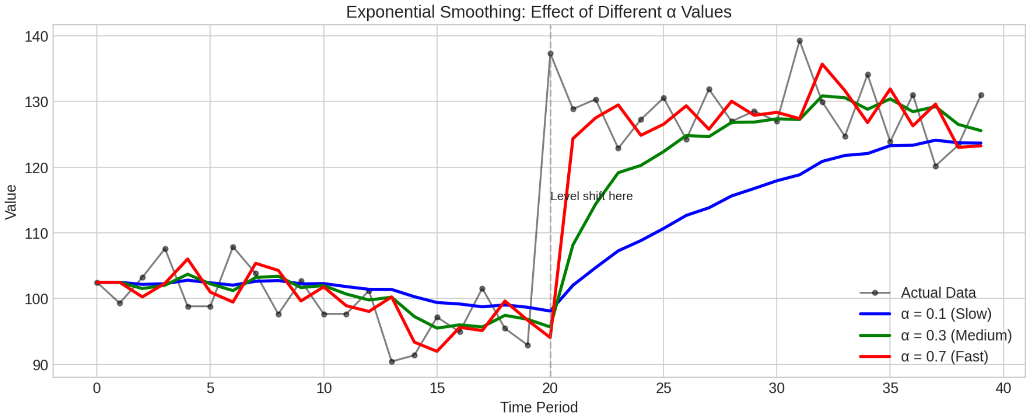
Exponential Smoothing Example

Data: Sales = 100, 120, 115, 130 (using $\alpha = 0.3$)

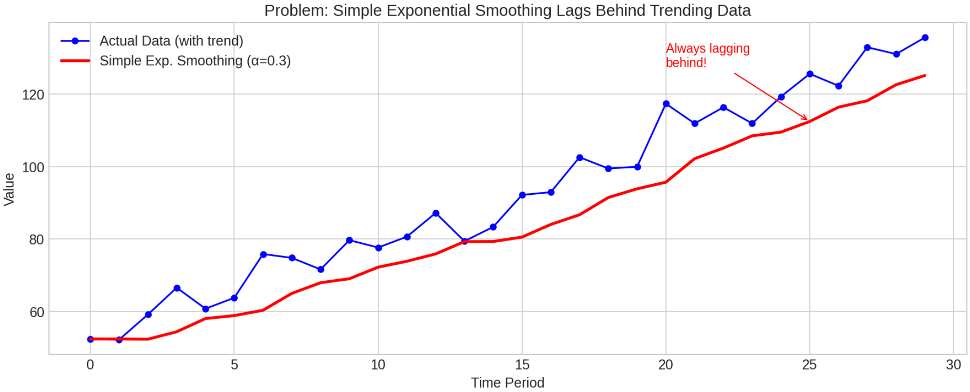
Step by step:

1. Start: $\text{Forecast}_1 = 100$ (use first value)
2. $\text{Forecast}_2 = 0.3 \times 100 + 0.7 \times 100 = 100$
3. $\text{Forecast}_3 = 0.3 \times 120 + 0.7 \times 100 = 36 + 70 = 106$
4. $\text{Forecast}_4 = 0.3 \times 115 + 0.7 \times 106 = 34.5 + 74.2 = 108.7$
5. $\text{Forecast}_5 = 0.3 \times 130 + 0.7 \times 108.7 = 39 + 76.1 = 115.1$

Comparing Different Alpha Values



The Problem: Data with a Trend

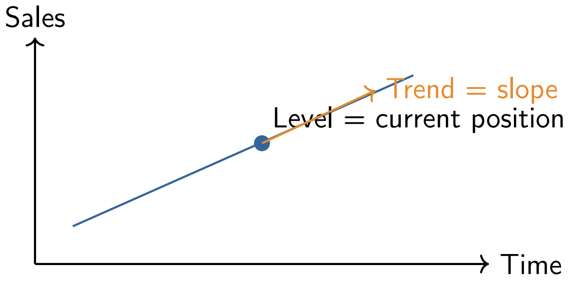


Holt's Method: Adding Trend

The Solution

Track **two things** separately:

1. The **Level** (where are we now?)
2. The **Trend** (how fast are we going up/down?)



Holt's Method: The Formulas

Two Equations

1. **Update the Level:**

$$\text{Level} = \alpha \times (\text{Actual}) + (1 - \alpha) \times (\text{Previous Level} + \text{Previous Trend})$$

2. **Update the Trend:**

$$\text{Trend} = \beta \times (\text{Level} - \text{Previous Level}) + (1 - \beta) \times (\text{Previous Trend})$$

Two smoothing parameters:

- α controls smoothing of the **level**
- β controls smoothing of the **trend**

Holt's Method: Making Forecasts

Forecast Formula

$$\text{Forecast}_{h \text{ steps ahead}} = \text{Level} + h \times \text{Trend}$$

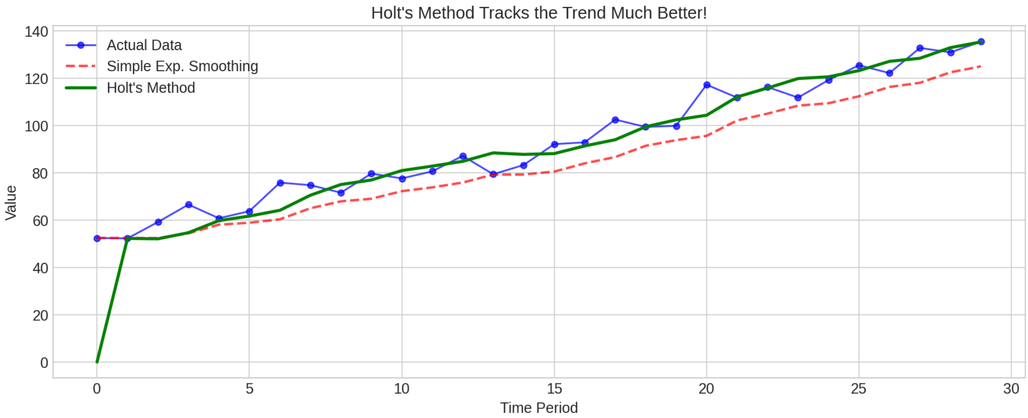
Example:

- Current Level = 500
- Current Trend = +10 per month

Forecasts:

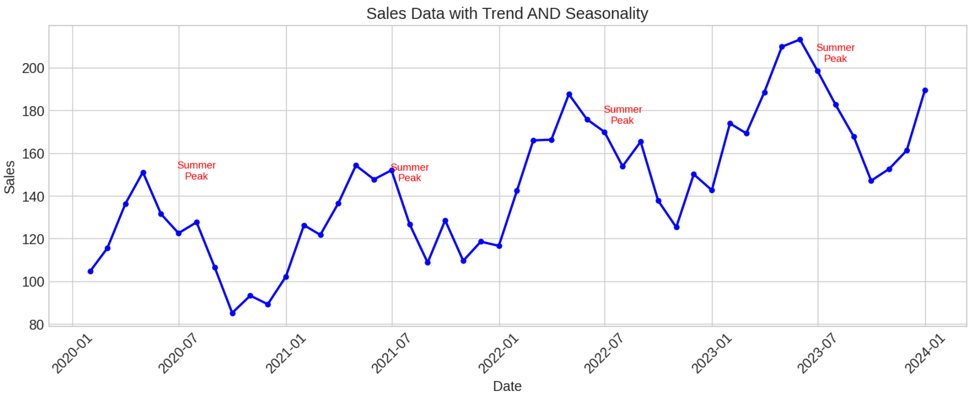
- Next month ($h = 1$): $500 + 1 \times 10 = 510$
- In 2 months ($h = 2$): $500 + 2 \times 10 = 520$
- In 6 months ($h = 6$): $500 + 6 \times 10 = 560$

Holt's Method in Action



Notice: Holt's method **follows the trend** instead of lagging behind!

The Final Challenge: Seasonality



Real business data often has:

- An overall trend (growing or shrinking)
- AND seasonal patterns (summer highs, winter lows)

Holt-Winters: The Complete Solution

Track Three Things

1. **Level** — Where are we on average?
2. **Trend** — Which direction are we heading?
3. **Seasonality** — What's the pattern within each year?

Component	**Smoothing Parameter**
Level	α (alpha)
Trend	β (beta)
Seasonality	γ (gamma)

Two Types of Seasonality

Additive Seasonality

Seasonal swings are **constant**

"Summer is always +50 units"

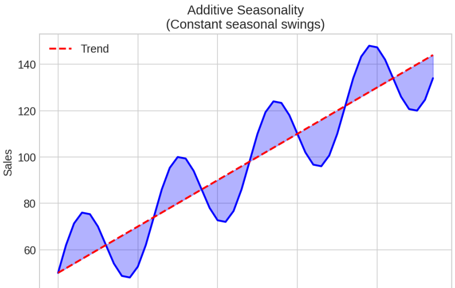
$$\text{Forecast} = \text{Level} + \text{Trend} + \text{Season}$$

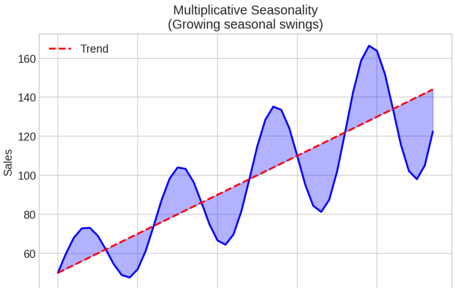
Multiplicative Seasonality

Seasonal swings **grow with level**

"Summer is always +20%"

$$\text{Forecast} = (\text{Level} + \text{Trend}) \times \text{Season}$$





Understanding Seasonal Factors

Example: Ice Cream Sales with Multiplicative Seasonality

Month	Seasonal Factor	Meaning
January	0.6	40% below average
February	0.7	30% below average
March	0.9	10% below average
April	1.0	Average
May	1.1	10% above average
June	1.3	30% above average
July	1.5	50% above average
August	1.4	40% above average
...	...	...

If Level = 1000 and it's July: Forecast $= 1000 \times 1.5 = 1500$

Holt-Winters: The Formulas (Multiplicative)

Three Update Equations

Level:

$$L_t = \alpha \times \frac{\text{Actual}_t}{S_{t-m}} + (1-\alpha) \times (L_{t-1} + T_{t-1})$$

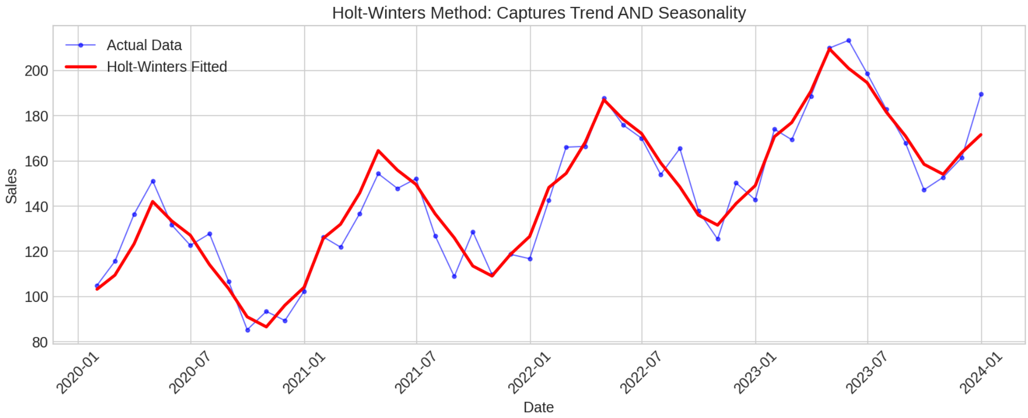
Trend:

$$T_t = \beta \times (L_t - L_{t-1}) + (1-\beta) \times T_{t-1}$$

Seasonal:

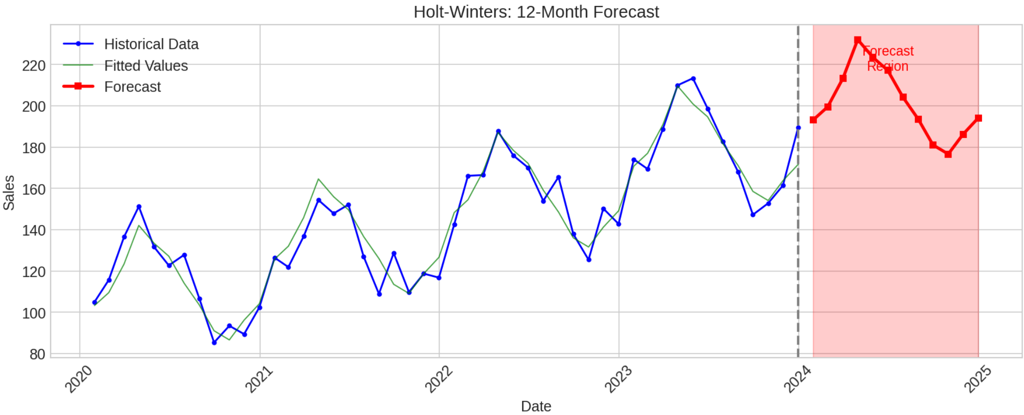
$$S_t = \gamma \times \frac{\text{Actual}_t}{L_t} + (1-\gamma) \times S_{t-m}$$

Holt-Winters in Action



Holt-Winters captures both the trend AND the seasonal pattern!

Holt-Winters Forecast Example

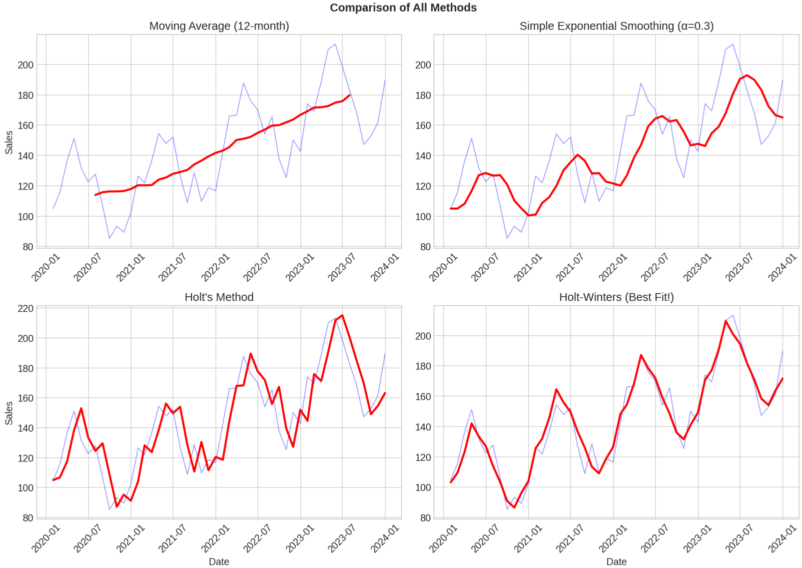


Notice: The forecast continues the trend while maintaining the seasonal pattern.

Summary: Method Comparison

Method	Trend	Seasonality	Parameters
Moving Average	×	×	Window size
Simple Exp. Smoothing	×	×	α
Holt's Method	✓	×	α, β
Holt-Winters	✓	✓	α, β, γ

All Methods Compared



Practical Tips

1. **Always plot your data first!**
   - Look for trends, seasons, outliers
2. **Start simple**
   - Try moving average first
   - Add complexity only if needed
3. **Use software to find best parameters**
   - Excel, Python, R can optimize α, β, γ
4. **Check your forecasts**
   - Compare predictions to actual values
   - Calculate forecast errors