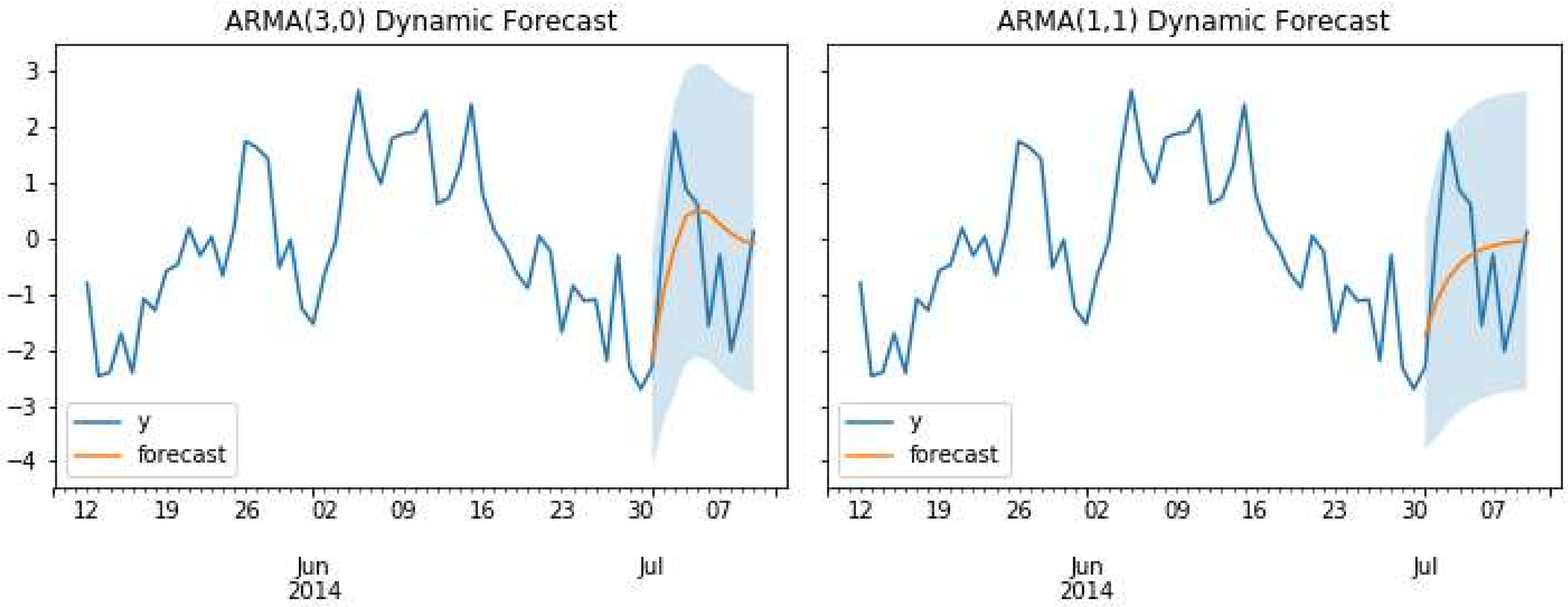
Intro to ACF and PACF

FORECASTING USING ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

Motivation


ARMA(3,0) Dynamic Forecast
ARMA(1,1) Dynamic Forecast
3
2
1
0
−1
−2
−3
−4
y
forecast
12
19
26
02
09
16
23
30
07
Jun
2014
Jul

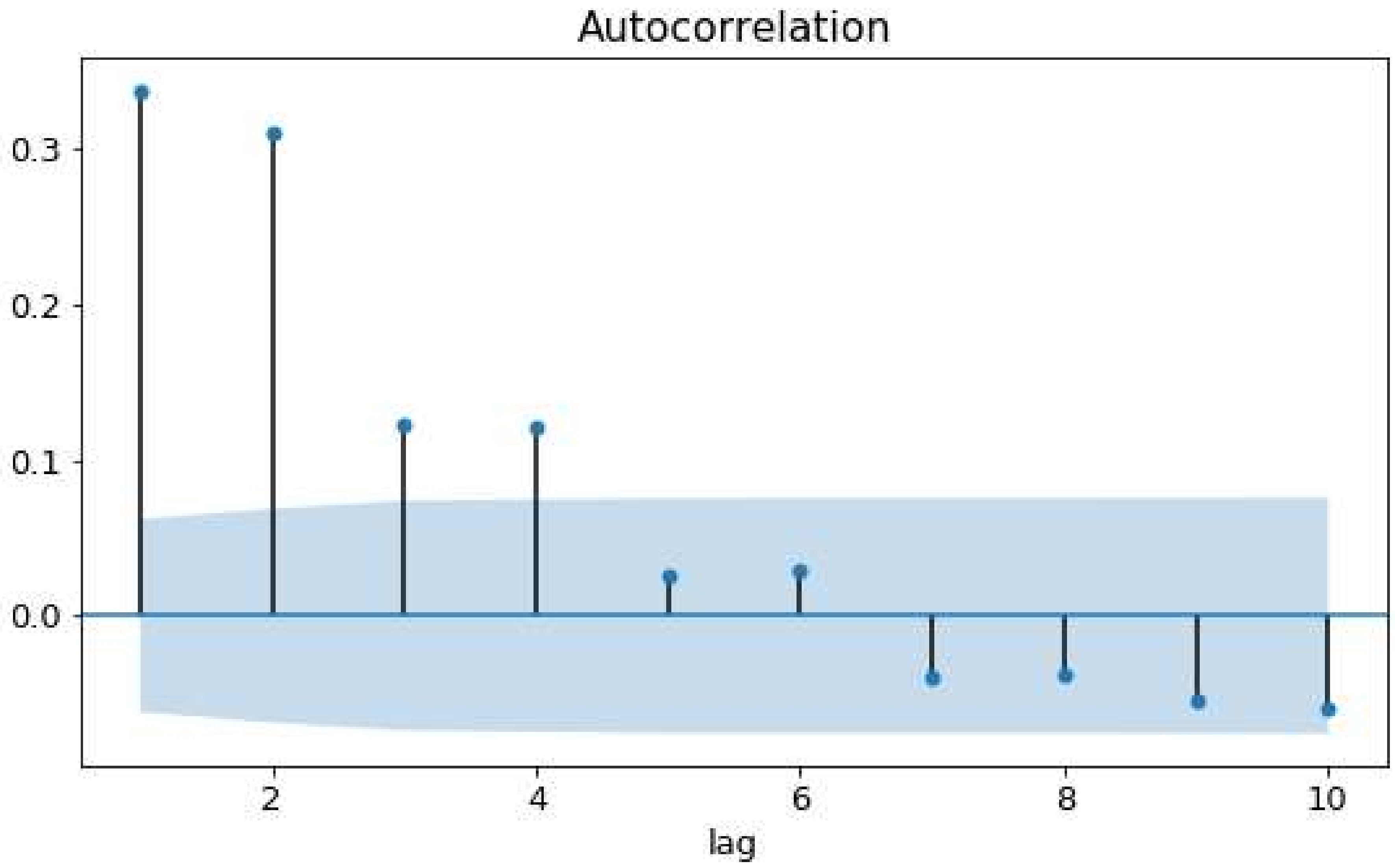
ACF and PACF

- ACF - Autocorrelation Function
- PACF - Partial autocorrelation function

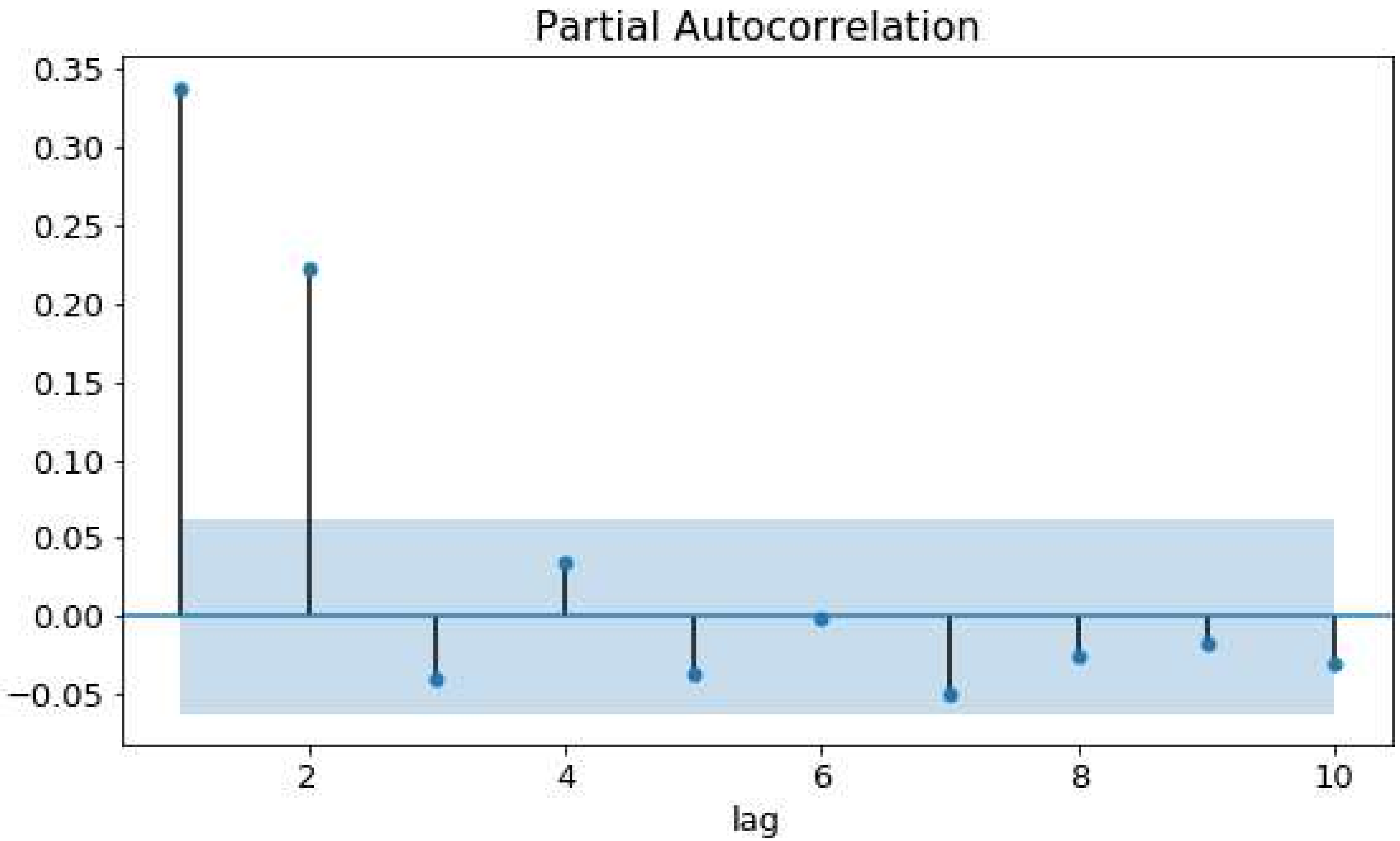
What is the ACF

- lag-1 autocorrelation $\rightarrow corr(y_t, y_{t-1})$
- lag-2 autocorrelation $\rightarrow corr(y_t, y_{t-2})$
- ...
- lag-n autocorrelation $\rightarrow corr(y_t, y_{t-n})$

What is the ACF



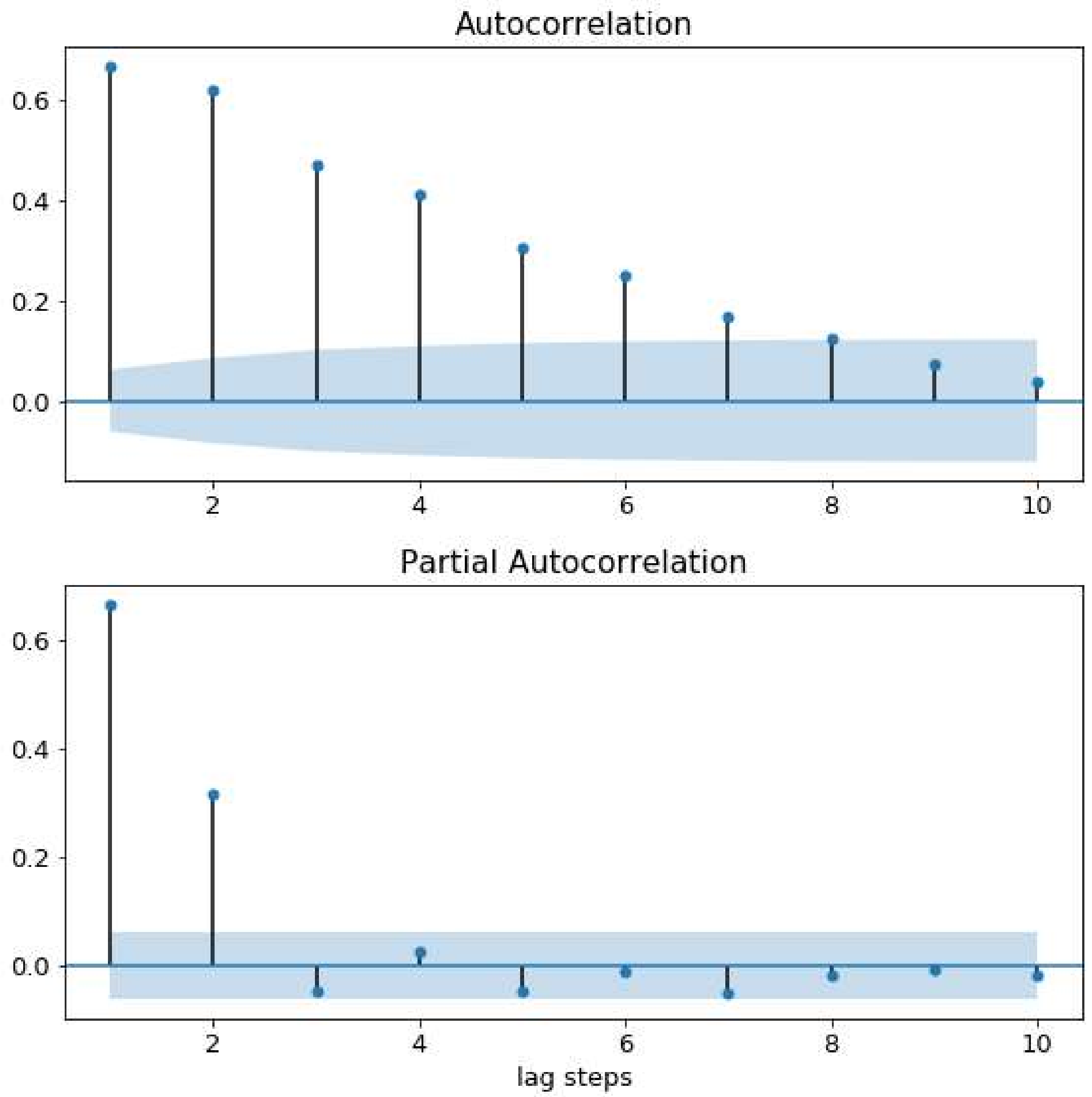
What is the PACF



Using ACF and PACF to choose model order

	AR(p)
ACF	Tails off
PACF	Cuts off after lag p

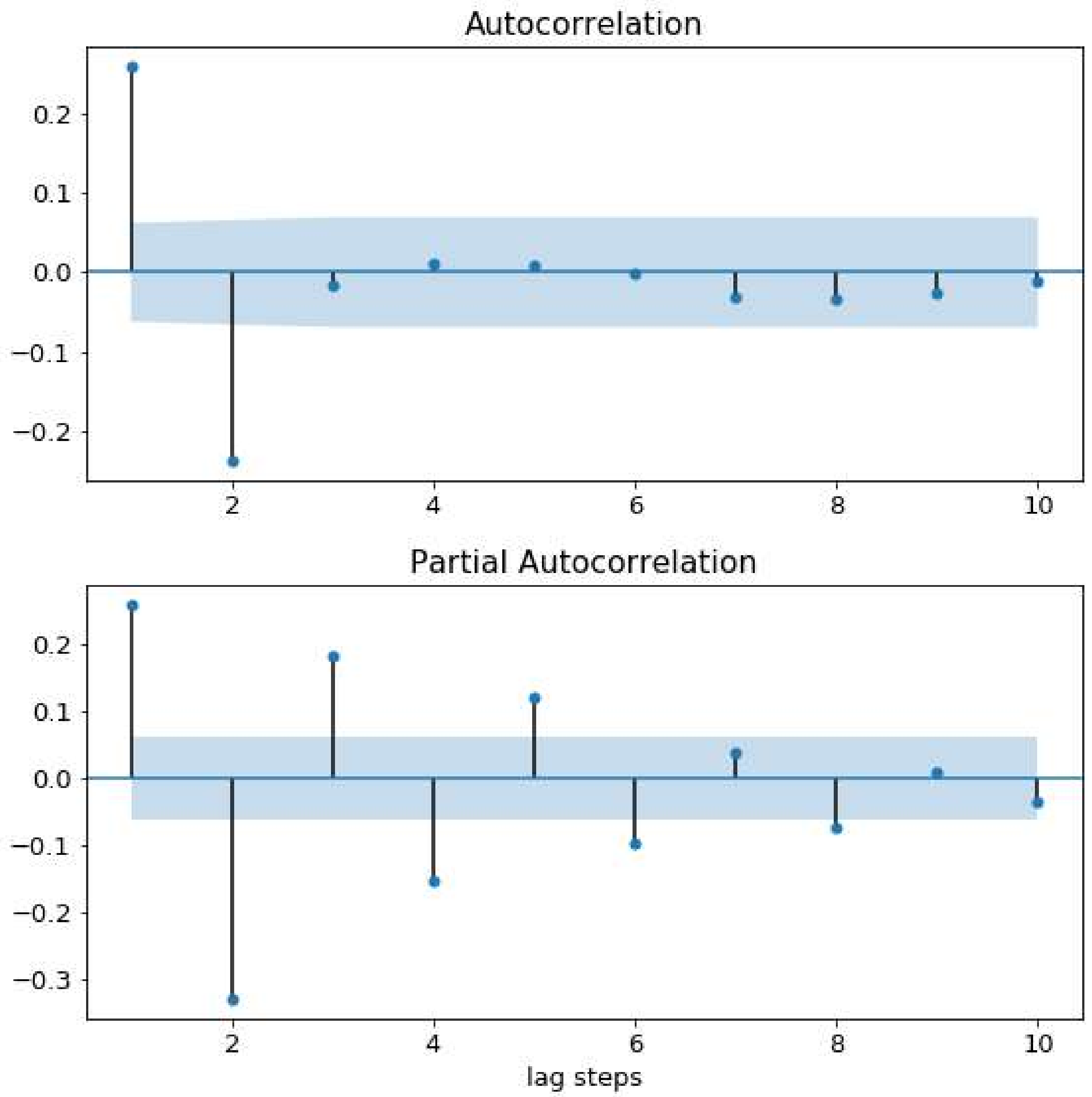
- AR(2) model →



Using ACF and PACF to choose model order

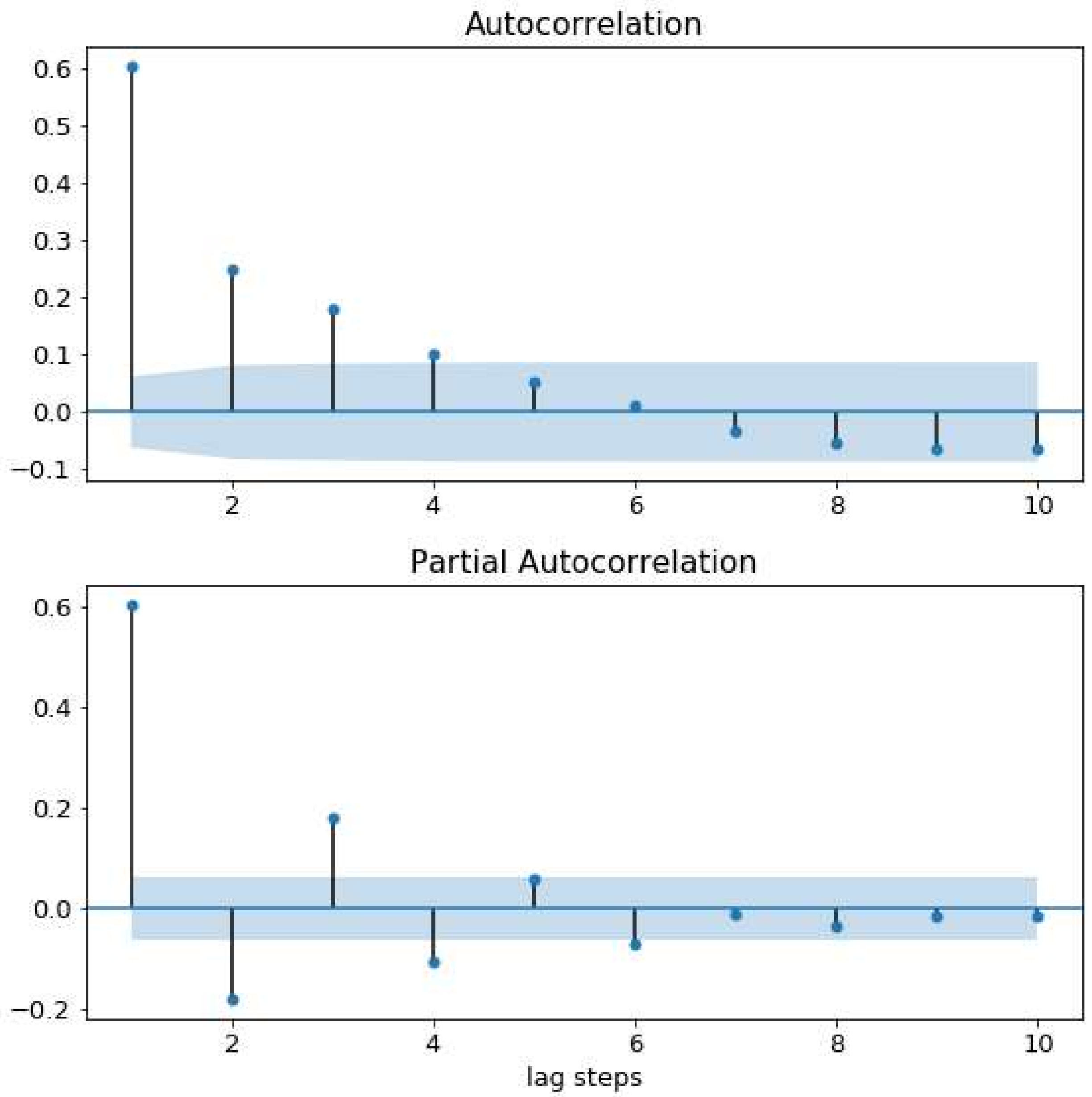
	MA(q)
ACF	Cuts off after lag q
PACF	Tails off

- MA(2) model →

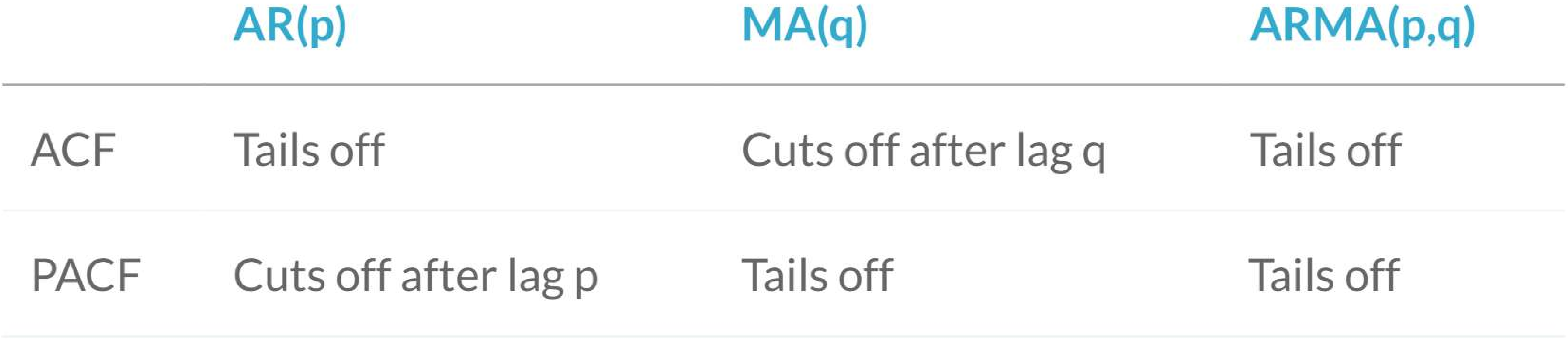


Using ACF and PACF to choose model order

	ARMA(p,q)
ACF	Tails off
PACF	Tails off



Using ACF and PACF to choose model order



	AR(p)	MA(q)	ARMA(p,q)
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

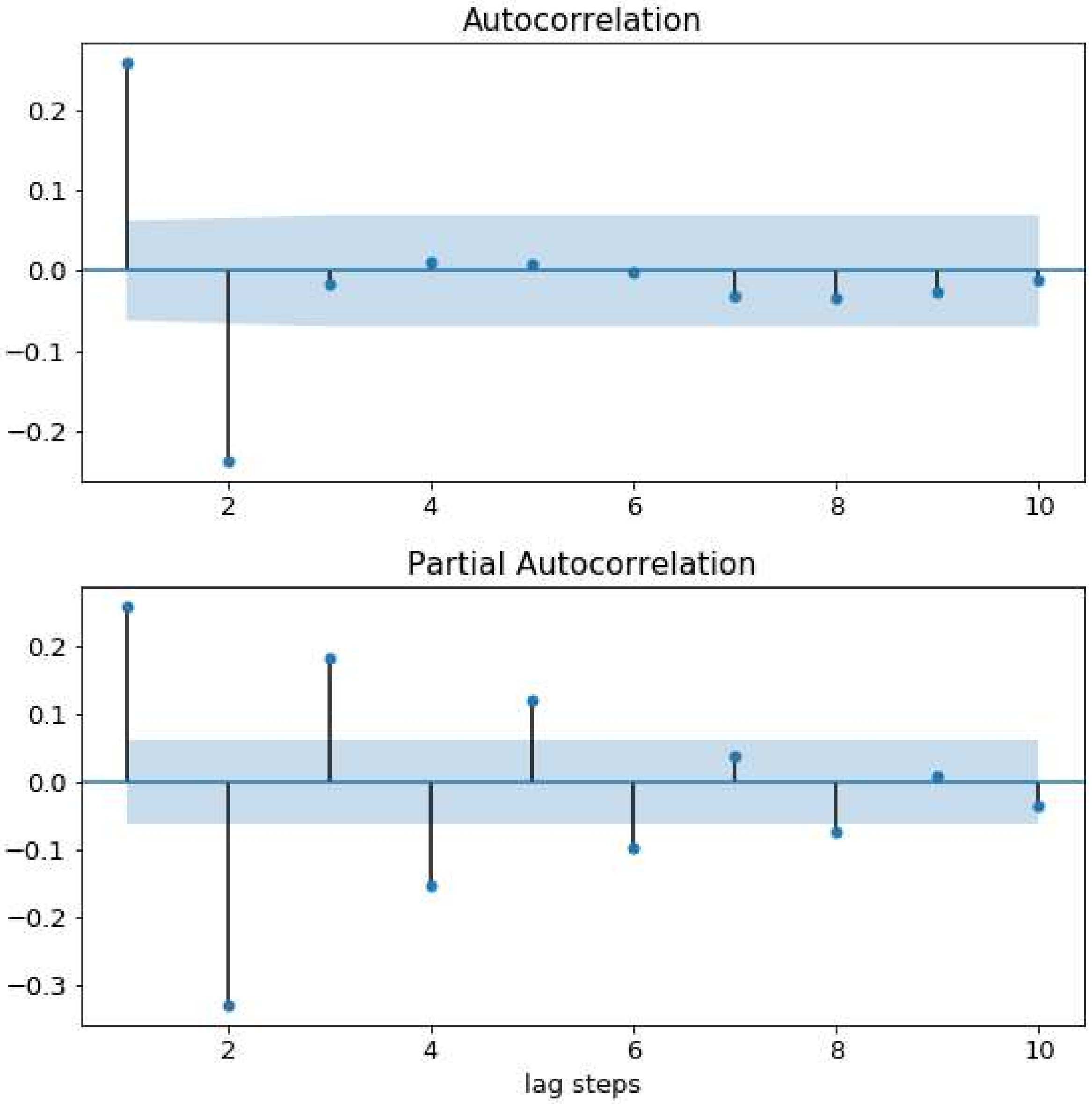
Implementation in Python

```python
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
```

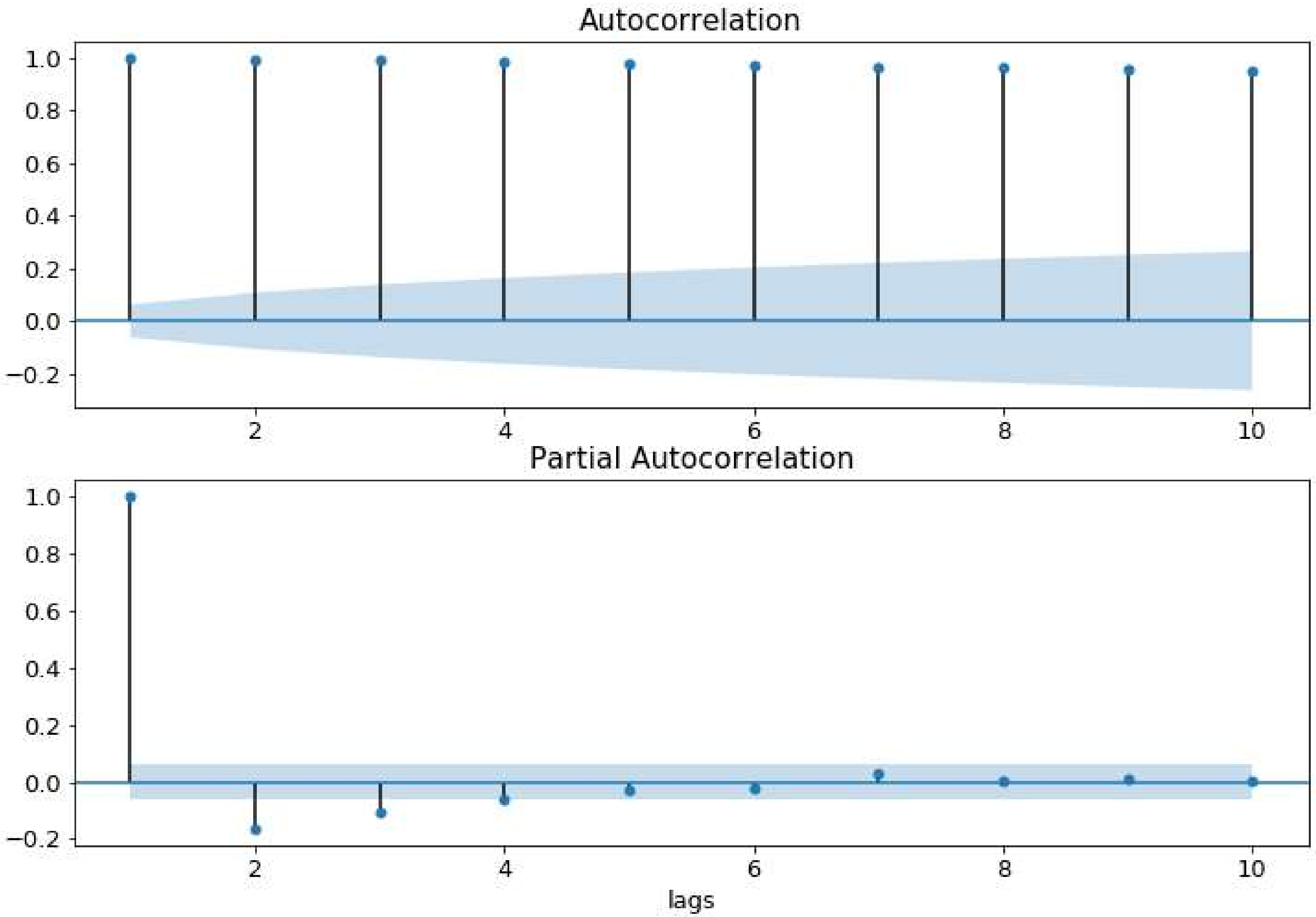
```python
# Create figure
fig, (ax1, ax2) = plt.subplots(2,1, figsize=(8,8))
# Make ACF plot
plot_acf(df, lags=10, zero=False, ax=ax1)
# Make PACF plot
plot_pacf(df, lags=10, zero=False, ax=ax2)

plt.show()
```

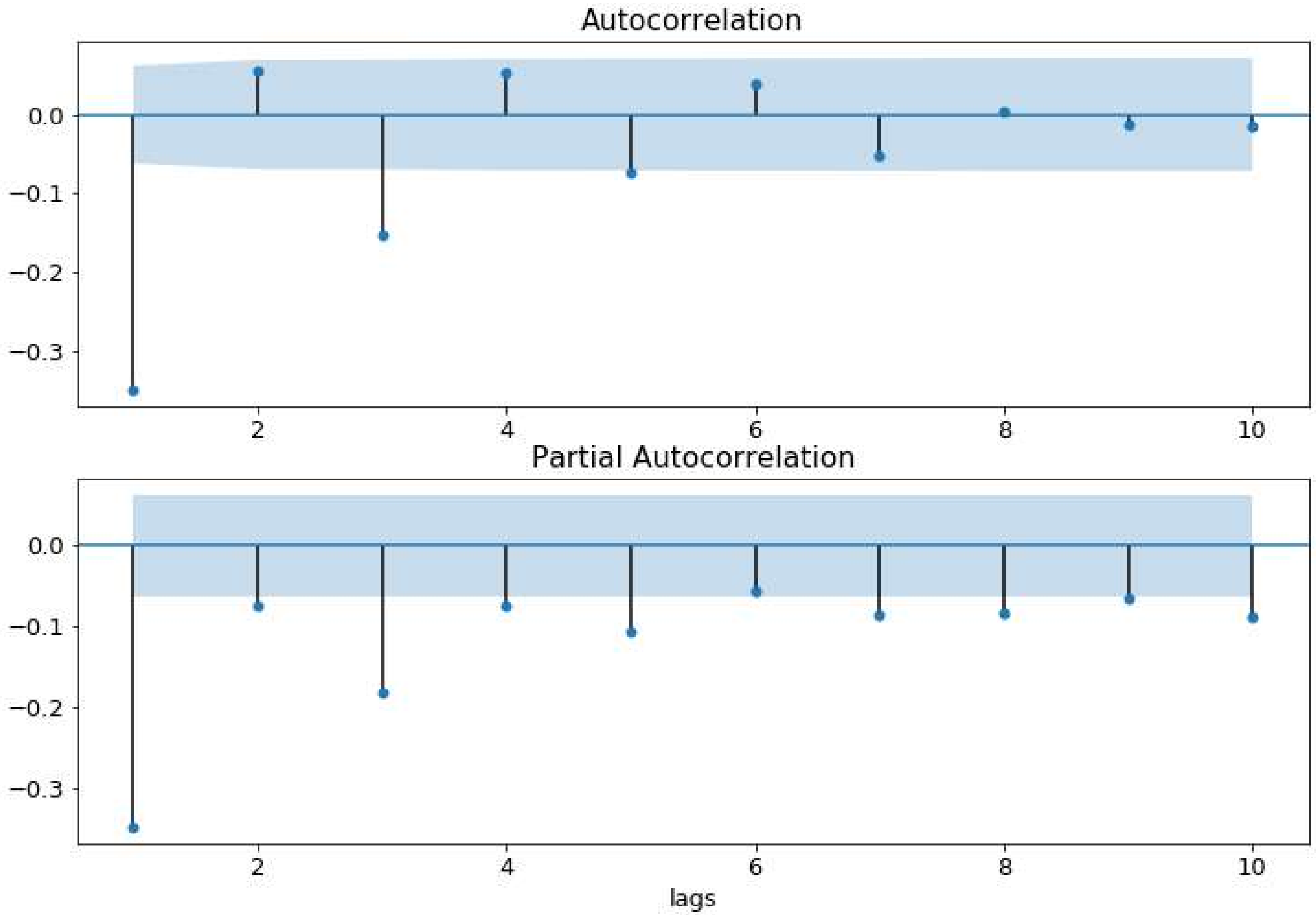
Implementation in Python



Over/under differencing and ACF and PACF



Over/under differencing and ACF and PACF



Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON

AIC and BIC

FORECASTING USING ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

AIC - Akaike information criterion

- Lower AIC indicates a better model
- AIC likes to choose simple models with lower order

BIC - Bayesian information criterion

- Very similar to AIC
- Lower BIC indicates a better model
- BIC likes to choose simple models with lower order

AIC vs BIC

- BIC favors simpler models than AIC
- AIC is better at choosing predictive models
- BIC is better at choosing good explanatory model

AIC and BIC in statsmodels

```
# Create model
model = SARIMAX(df, order=(1,0,1))
# Fit model
results = model.fit()
# Print fit summary
print(results.summary())
```

```
                            Statespace Model Results
==============================================================================
Dep. Variable                       y   No. Observations                  1000
Model                SARIMAX(2, 0, 0)   Log Likelihood               -1399.704
Date                 Fri, 10 May 2019   AIC                           2805.407
Time                         01 06 11   BIC                           2820.131
Sample                     01-01-2013   HQIC                          2811.003
                         - 09-27-2015
Covariance Type                   opg
```

AIC and BIC in statsmodels

```
# Create model
model = SARIMAX(df, order=(1,0,1))
# Fit model
results = model.fit()
# Print AIC and BIC
print('AIC ', results.aic)
print('BIC ', results.bic)
```

```
AIC  2806.36
BIC  2821.09
```

Searching over AIC and BIC

```
# Loop over AR order
for p in range(3):
    # Loop over MA order
    for q in range(3):
        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()
        # print the model order and the AIC/BIC values
        print(p, q, results.aic, results.bic)
```

```
0 0 2900.13 2905.04
0 1 2828.70 2838.52
0 2 2806.69 2821.42
1 0 2810.25 2820.06
1 1 2806.37 2821.09
1 2 2807.52 2827.15
...
```

Searching over AIC and BIC

```
order_aic_bic =[]
# Loop over AR order
for p in range(3)
    # Loop over MA order
    for q in range(3)
        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()
        # Add order and scores to list
        order_aic_bic.append((p, q, results.aic, results.bic))
```

```
# Make DataFrame of model order and AIC/BIC scores
order_df = pd.DataFrame(order_aic_bic, columns=['p','q', 'aic', 'bic'])
```

Searching over AIC and BIC

```
# Sort by AIC
print(order_df.sort_values('aic'))
```

```
   p  q      aic      bic
7  2  1  2804.54  2824.17
6  2  0  2805.41  2820.13
4  1  1  2806.37  2821.09
2  0  2  2806.69  2821.42
...
```

```
# Sort by BIC
print(order_df.sort_values('bic'))
```

```
   p  q      aic      bic
3  1  0  2810.25  2820.06
6  2  0  2805.41  2820.13
4  1  1  2806.37  2821.09
2  0  2  2806.69  2821.42
...
```

Non-stationary model orders

```
# Fit model
model = SARIMAX(df, order=(2,0,1))
results = model.fit()
```

```
ValueError  Non-stationary starting autoregressive parameters
found with `enforce_stationarity` set to True.
```

When certain orders don't work

```
# Loop over AR order
for p in range(3)
    # Loop over MA order
    for q in range(3)


        # Fit model
        model = SARIMAX(df, order=(p,0,q))
        results = model.fit()

        # Print the model order and the AIC/BIC values
        print(p, q, results.aic, results.bic)
```

When certain orders don't work

```
# Loop over AR order
for p in range(3)
    # Loop over MA order
    for q in range(3)

        try
            # Fit model
            model = SARIMAX(df, order=(p,0,q))
            results = model.fit()

            # Print the model order and the AIC/BIC values
            print(p, q, results.aic, results.bic)

        except
            # Print AIC and BIC as None when fails
            print(p, q, None, None)
```

Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON

Model diagnostics

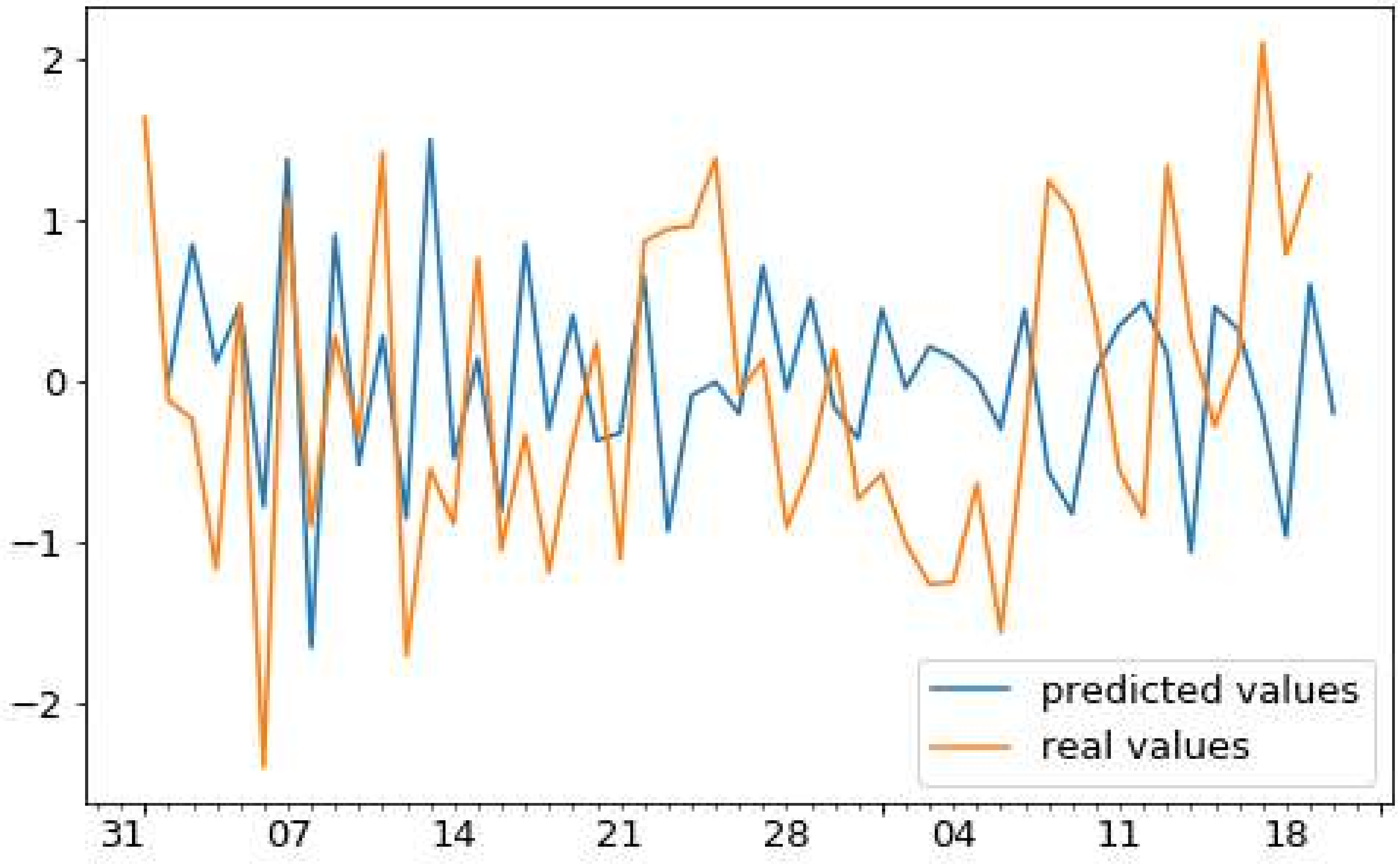
FORECASTING USING ARIMA MODELS IN PYTHON

James Fulton
Climate informatics researcher

Introduction to model diagnostics

- How good is the final model?

Residuals



Residuals

```python
# Fit model
model = SARIMAX(df, order=(p,d,q))
results = model.fit()

# Assign residuals to variable
residuals = results.resid
```

```
2013-01-23    1.013129
2013-01-24    0.114055
2013-01-25    0.430698
2013-01-26   -1.247046
2013-01-27   -0.499565
       ...         ...
```

Mean absolute error

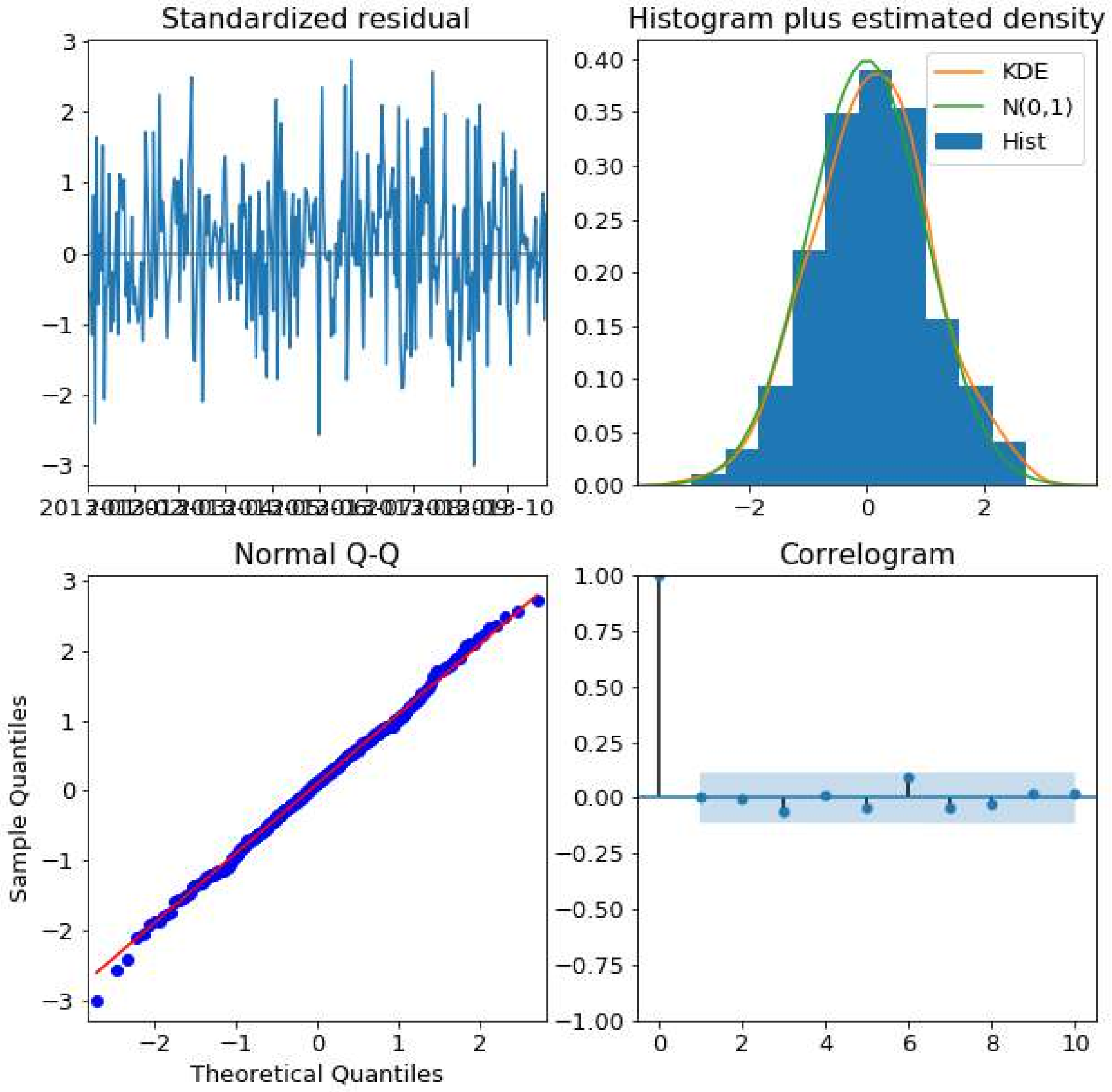
How far our the predictions from the real values?

```
mae = np.mean(np.abs(residuals))
```

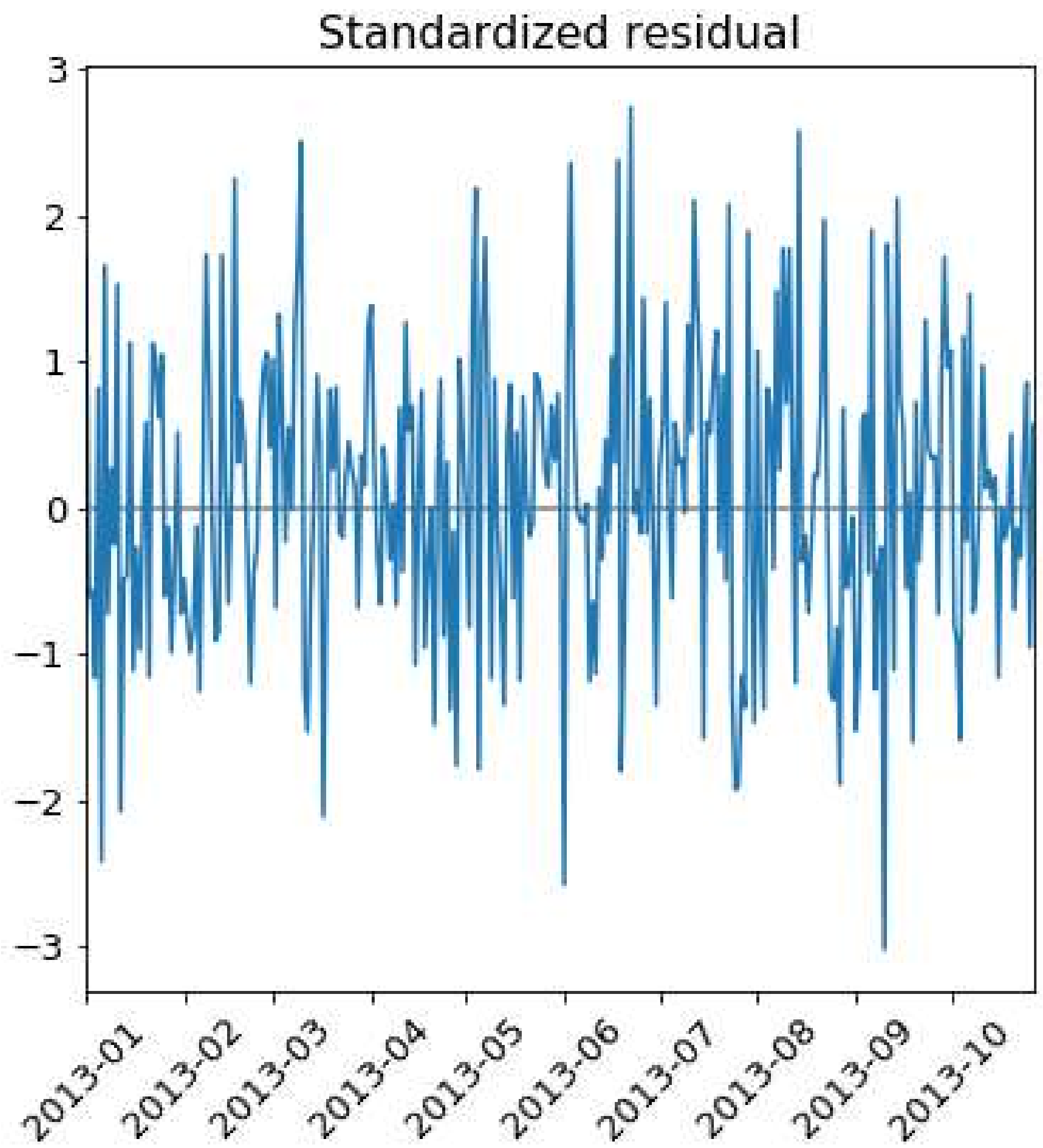
Plot diagnostics

If the model fits well the residuals will be white Gaussian noise

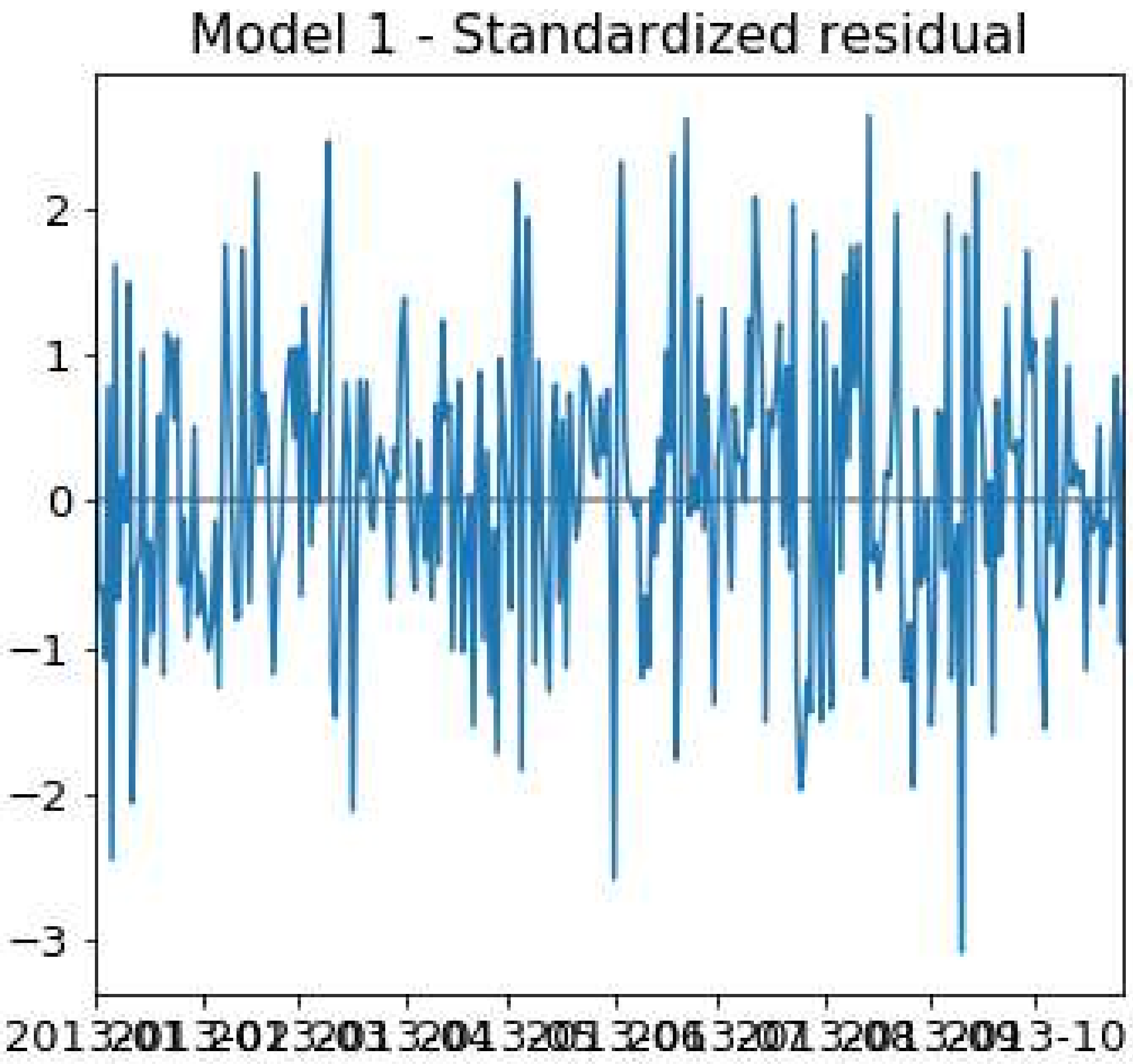
```
# Create the 4 diagostics plots
results.plot_diagnostics()
plt.show()
```

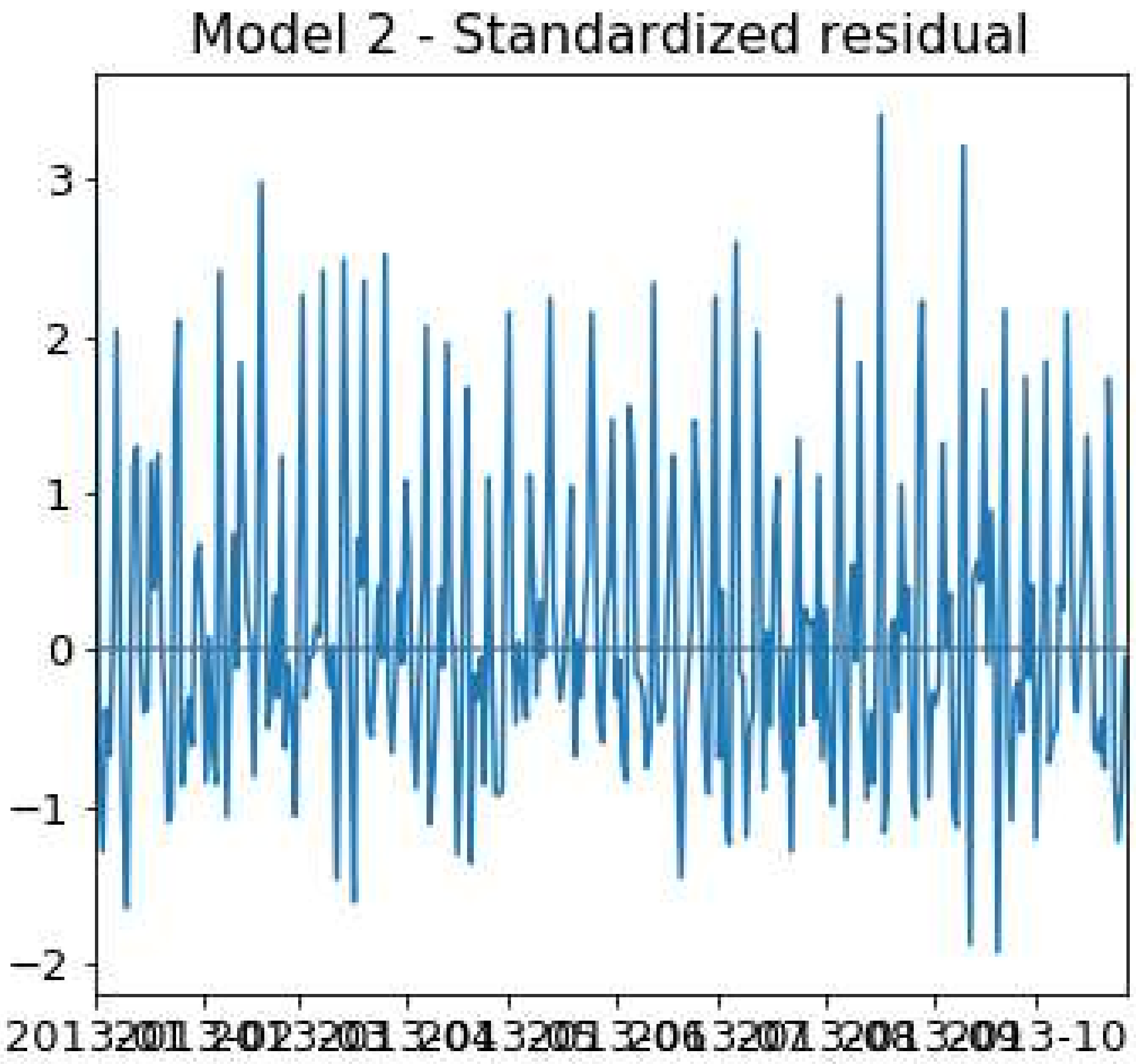


Residuals plot

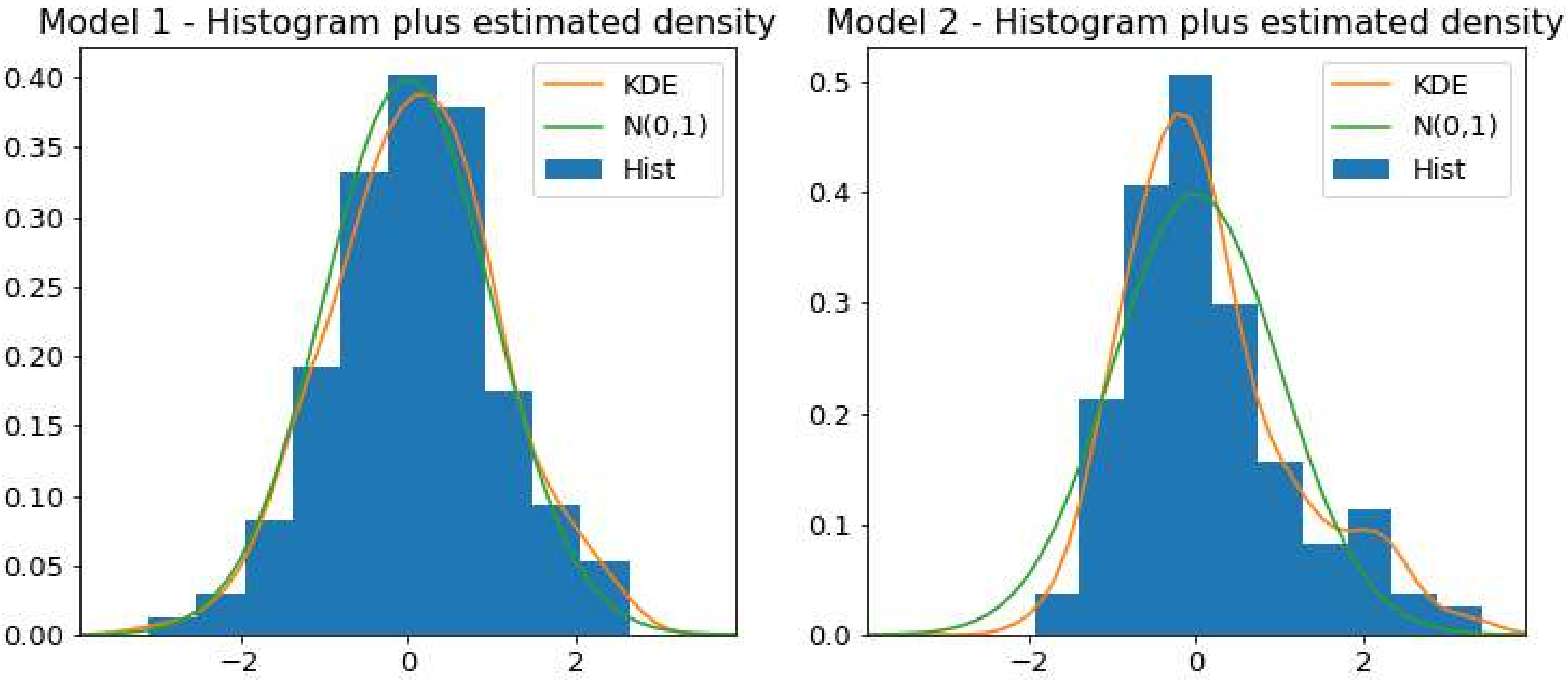


Residuals plot

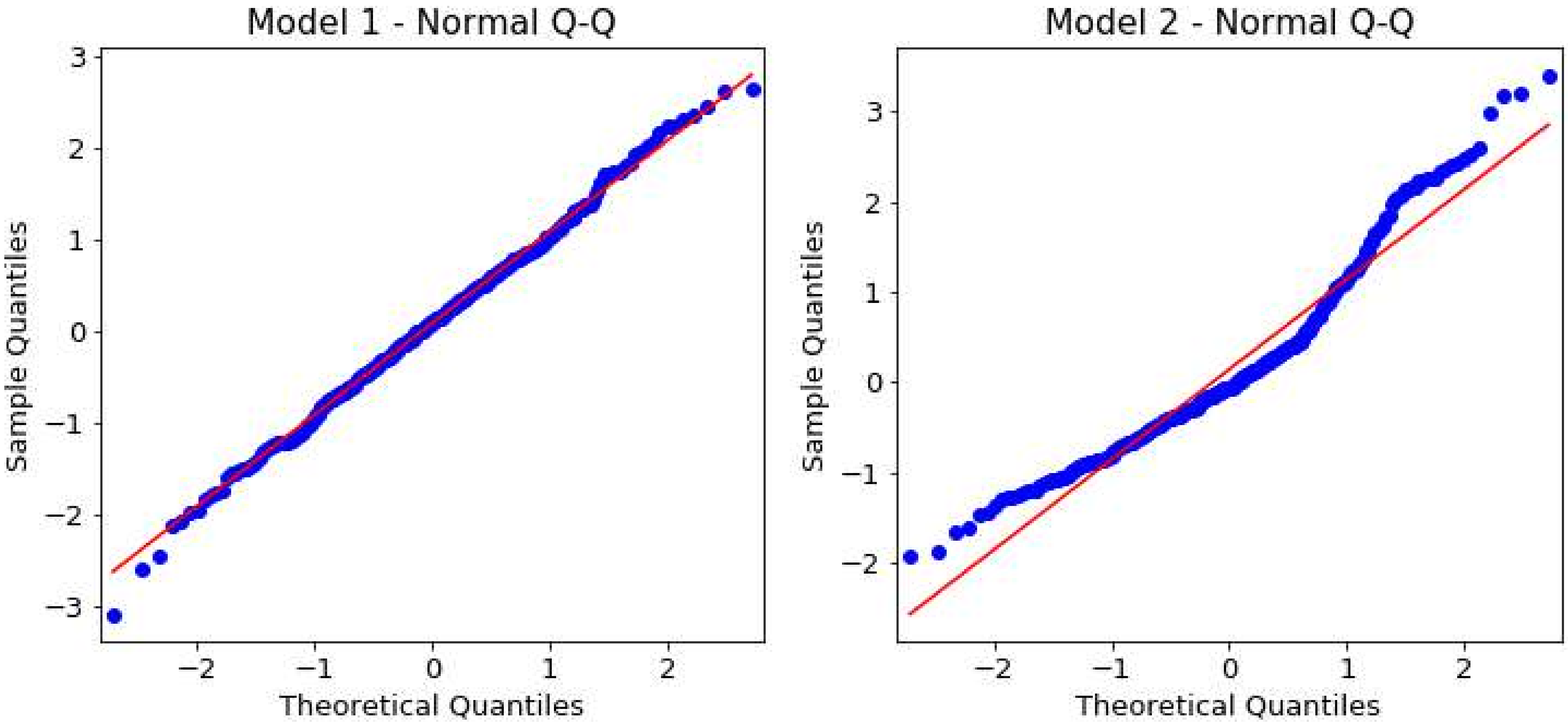

Model 1 - Standardized residual


Model 2 - Standardized residual

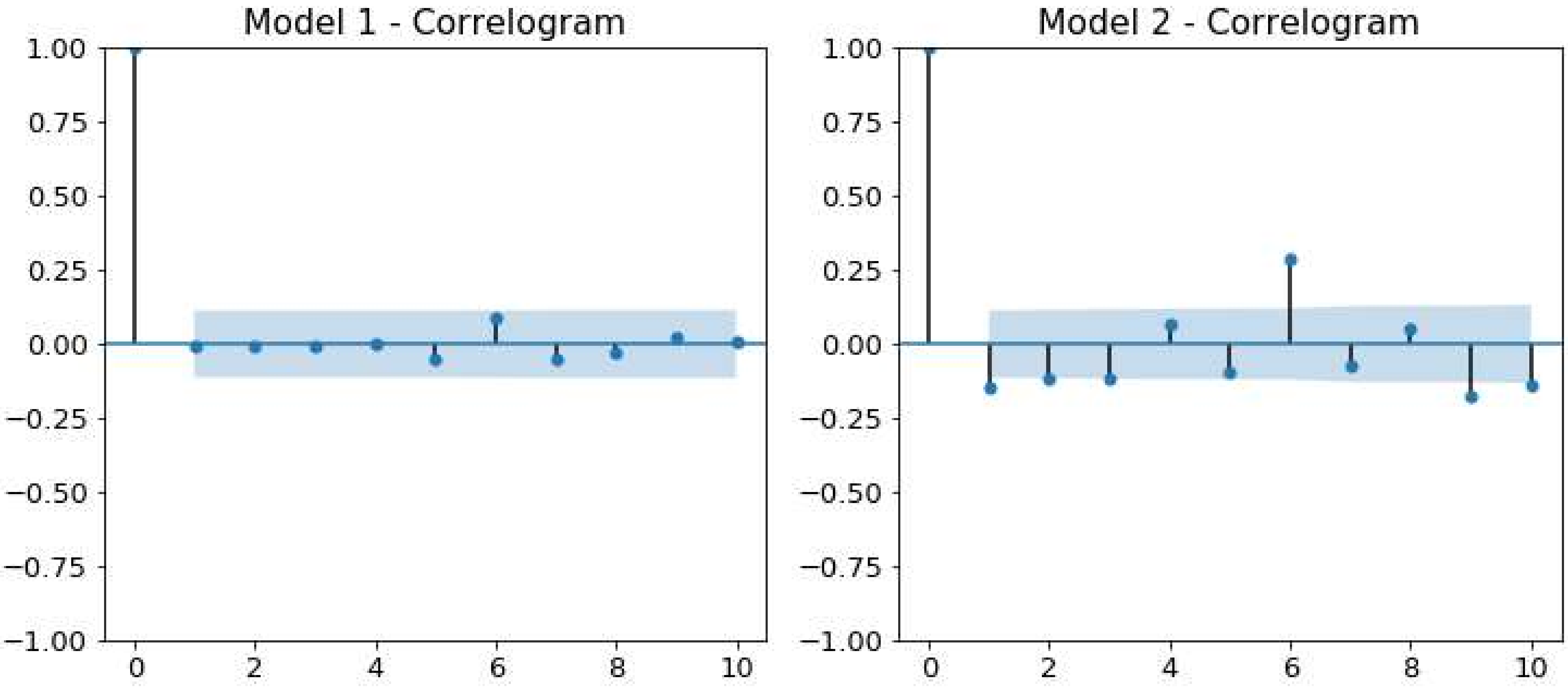
Histogram plus estimated density



Normal Q-Q


Model 1 - Normal Q-Q
Sample Quantiles
Theoretical Quantiles
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
Model 2 - Normal Q-Q
Sample Quantiles
Theoretical Quantiles
3
2
1
0
−1
−2
−2
−1
0
1
2

Correlogram



Summary statistics

```
print(results.summary())
```

```
...
====================================================================================
Ljung-Box (Q)                        32.10   Jarque-Bera (JB)                   0.02
Prob(Q)                               0.81   Prob(JB)                           0.99
Heteroskedasticity (H)                1.28   Skew                              -0.02
Prob(H) (two-sided)                   0.21   Kurtosis                           2.98
====================================================================================
```

- `Prob(Q)` - p-value for null hypothesis that residuals are uncorrelated
- `Prob(JB)` - p-value for null hypothesis that residuals are normal

Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON

Box-Jenkins method

FORECASTING USING ARIMA MODELS IN PYTHON

James Fulton

Climate informatics researcher

The Box-Jenkins method

From raw data $\rightarrow$ production model

- identification
- estimation
- model diagnostics

Identification

- Is the time series stationary?
- What differencing will make it stationary?
- What transforms will make it stationary?
- What values of p and q are most promising?

time series
identifiction

Identification tools

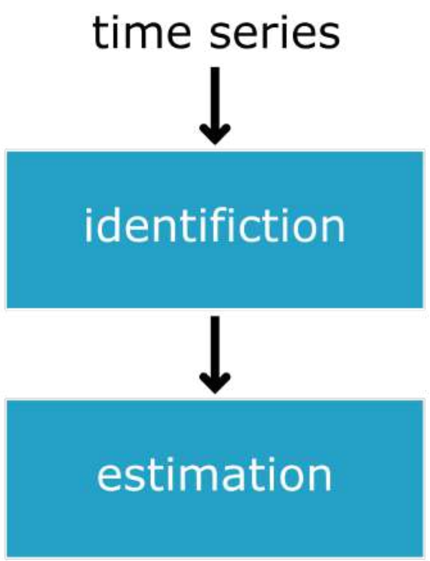
- Plot the time series
  - `df.plot()`
- Use augmented Dicky-Fuller test
  - `adfuller()`
- Use transforms and/or differencing
  - `df.diff()` , `np.log()` , `np.sqrt()`
- Plot ACF/PACF
  - `plot_acf()` , `plot_pacf()`

time series

identifiction

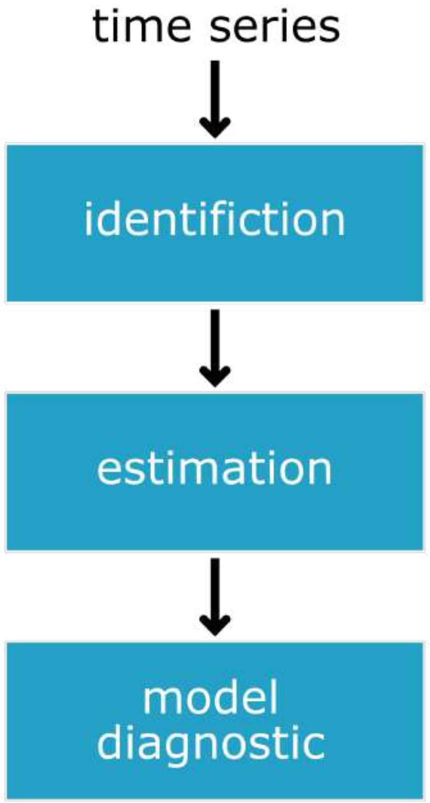
Estimation

- Use the data to train the model coefficients
- Done for us using `model.fit()`
- Choose between models using AIC and BIC
  - `results.aic` , `results.bic`

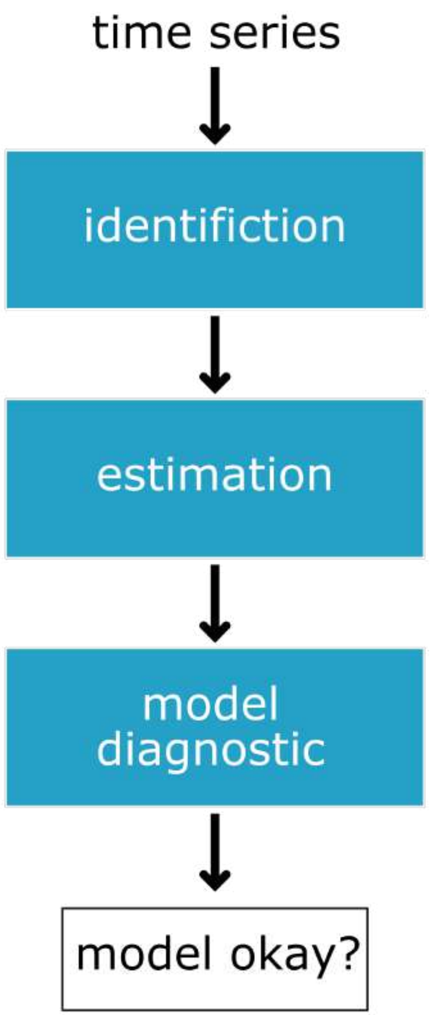


Model diagnostics

- Are the residuals uncorrelated
- Are residuals normally distributed
  - `results.plot_diagnostics()`
  - `results.summary()`



Decision



time series

identifiction

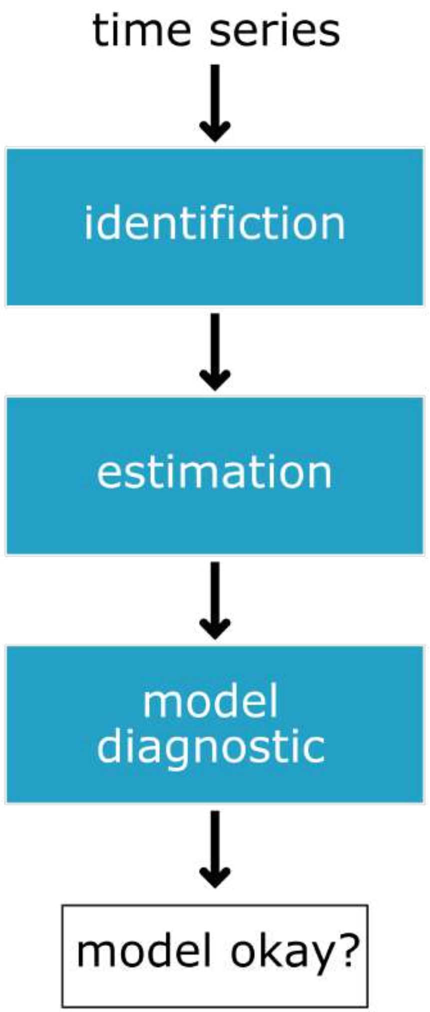
estimation

model diagnostic

model okay?

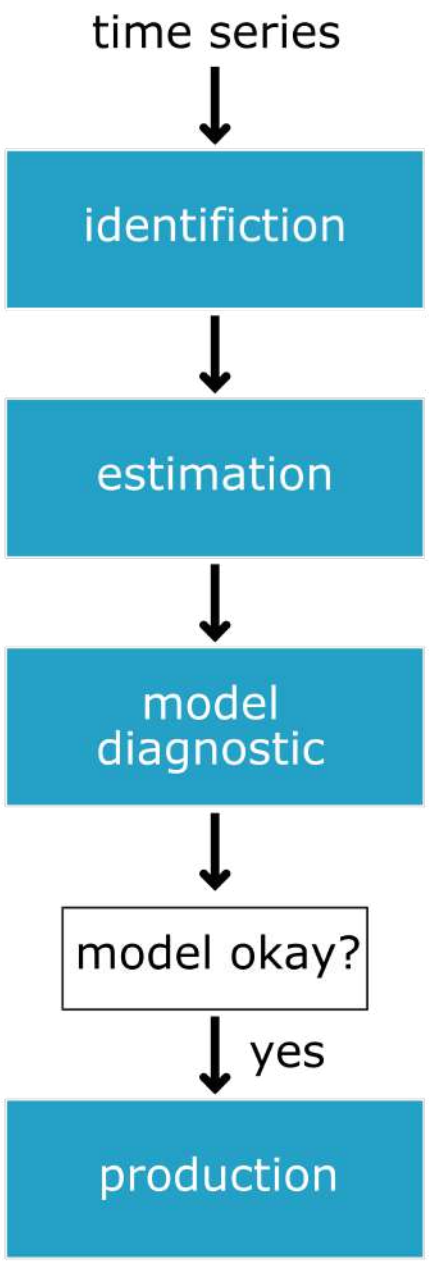
Repeat

- We go through the process again with more information
- Find a better model

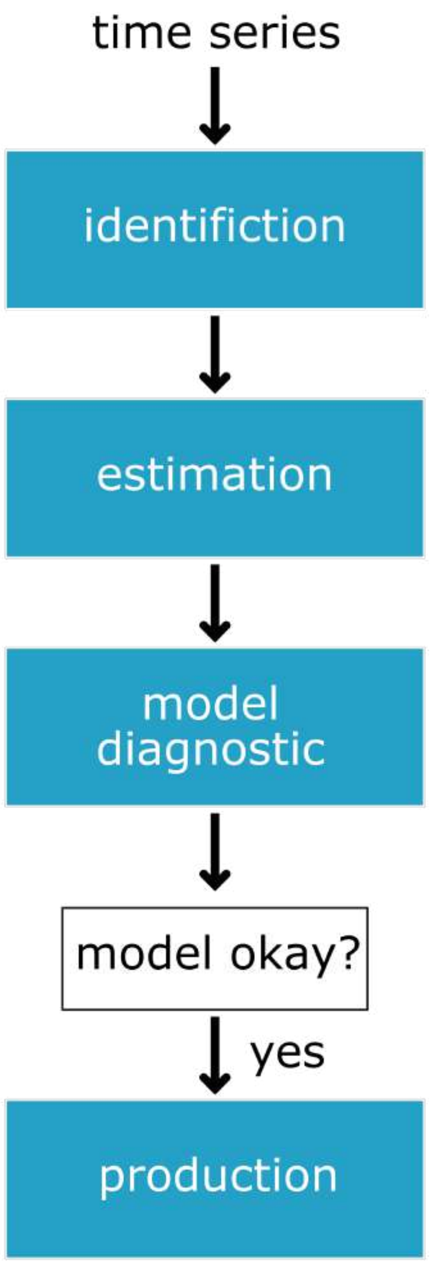


Production

- Ready to make forecasts
  - `results.get_forecast()`



Box-Jenkins


time series
identifiction
estimation
model diagnostic
model okay?
yes
production

Let's practice!

FORECASTING USING ARIMA MODELS IN PYTHON