```
> with(PDEtools): declare(eta(x,y,u), xi__1(x,y,u), xi__2(x,y,u))
```

$$\eta(x, y, u) \text{ will now be displayed as } \eta$$

$$\xi_1(x, y, u) \text{ will now be displayed as } \xi_1$$

$$\xi_2(x, y, u) \text{ will now be displayed as } \xi_2 \tag{1}$$

```
> eq:=u[x]*u[x,x]+u[y,y];
```

$$eq := u_x u_{x,x} + u_{y,y} \tag{2}$$

```
> zeta__1 * diff(eq, u[x]) + zeta__2 * diff(eq, u[y]) + zeta__11 *
  diff(eq, u[x,x]) + zeta__12 * diff(eq, u[x,y]) + zeta__22 * diff
  (eq, u[y,y]);
```

$$\zeta_1 u_{x,x} + \zeta_{11} u_x + \zeta_{22} \tag{3}$$

```
> zeta__1 := D_Dx(eta(x,y,u), x, u(x,y))
           - u[x] * D_Dx(xi__1(x,y,u), x, u(x,y))
           - u[y] * D_Dx(xi__2(x,y,u), x, u(x,y));
```

$$\zeta_1 := \eta_x + \left(\eta_u\right) u_x - u_x \left(\xi_{1_x} + \left(\xi_{1_u}\right) u_x\right) - u_y \left(\xi_{2_x} + \left(\xi_{2_u}\right) u_x\right) \tag{4}$$

```
> zeta__2 := D_Dx(eta(x,y,u), y, u(x,y))
           - u[x] * D_Dx(xi__1(x,y,u), y, u(x,y))
           - u[y] * D_Dx(xi__2(x,y,u), y, u(x,y));
```

$$\zeta_2 := \eta_y + \left(\eta_u\right) u_y - u_x \left(\xi_{1_y} + \left(\xi_{1_u}\right) u_y\right) - u_y \left(\xi_{2_y} + \left(\xi_{2_u}\right) u_y\right) \tag{5}$$

```
> zeta__11 := D_Dx(zeta__1, x, u(x,y))
            - u[x,x] * D_Dx(xi__1(x,y,u), x, u(x,y))
            - u[x,y] * D_Dx(xi__2(x,y,u), x, u(x,y));
```

$$\zeta_{11} := \eta_{x,x} + \left(\eta_{u,x}\right) u_x + \left(\eta_{u,x} + \left(\eta_{u,u}\right) u_x\right) u_x + \left(\eta_u\right) u_{x,x} - 2\, u_{x,x} \left(\xi_{1_x} + \left(\xi_{1_u}\right) u_x\right) - u_x \left(\xi_{1_{x,x}} \right. \tag{6}$$

$$\left. + \left(\xi_{1_{u,x}}\right) u_x + \left(\xi_{1_{u,x}} + \left(\xi_{1_{u,u}}\right) u_x\right) u_x + \left(\xi_{1_u}\right) u_{x,x}\right) - 2\, u_{x,y} \left(\xi_{2_x} + \left(\xi_{2_u}\right) u_x\right) - u_y \left(\xi_{2_{x,x}} \right.$$

$$\left. + \left(\xi_{2_{u,x}}\right) u_x + \left(\xi_{2_{u,x}} + \left(\xi_{2_{u,u}}\right) u_x\right) u_x + \left(\xi_{2_u}\right) u_{x,x}\right)$$

```
> zeta__22 := D_Dx(zeta__2, y, u(x,y))
            - u[x,y] * D_Dx(xi__1(x,y,u), y, u(x,y))
            - u[y,y] * D_Dx(xi__2(x,y,u), y, u(x,y));
```

$$\zeta_{22} := \eta_{y,y} + \left(\eta_{u,y}\right) u_y + \left(\eta_{u,y} + \left(\eta_{u,u}\right) u_y\right) u_y + \left(\eta_u\right) u_{y,y} - 2\, u_{x,y} \left(\xi_{1_y} + \left(\xi_{1_u}\right) u_y\right) - u_x \left(\xi_{1_{y,y}} \right. \tag{7}$$

$$\left. + \left(\xi_{1_{u,y}}\right) u_y + \left(\xi_{1_{u,y}} + \left(\xi_{1_{u,u}}\right) u_y\right) u_y + \left(\xi_{1_u}\right) u_{y,y}\right) - 2\, u_{y,y} \left(\xi_{2_y} + \left(\xi_{2_u}\right) u_y\right) - u_y \left(\xi_{2_{y,y}} \right.$$

$$\left. + \left(\xi_{2_{u,y}}\right) u_y + \left(\xi_{2_{u,y}} + \left(\xi_{2_{u,u}}\right) u_y\right) u_y + \left(\xi_{2_u}\right) u_{y,y}\right)$$

```
> defeq:=zeta__1 * diff(eq, u[x]) + zeta__2 * diff(eq, u[y]) +
  zeta__11 * diff(eq, u[x,x]) + zeta__12 * diff(eq, u[x,y]) +
  zeta__22 * diff(eq, u[y,y]);
```

$$defeq := \left(\eta_x + \left(\eta_u\right) u_x - u_x \left(\xi_{1_x} + \left(\xi_{1_u}\right) u_x\right) - u_y \left(\xi_{2_x} + \left(\xi_{2_u}\right) u_x\right)\right) u_{x,x} + \left(\eta_{x,x} + \left(\eta_{u,x}\right) u_x \right. \tag{8}$$

$$\left. + \left(\eta_{u,x} + \left(\eta_{u,u}\right) u_x\right) u_x + \left(\eta_u\right) u_{x,x} - 2\, u_{x,x} \left(\xi_{1_x} + \left(\xi_{1_u}\right) u_x\right) - u_x \left(\xi_{1_{x,x}} + \left(\xi_{1_{u,x}}\right) u_x \right.\right.$$

$$\left.\left. + \left(\xi_{1_{u,x}} + \left(\xi_{1_{u,u}}\right) u_x\right) u_x + \left(\xi_{1_u}\right) u_{x,x}\right) - 2\, u_{x,y} \left(\xi_{2_x} + \left(\xi_{2_u}\right) u_x\right) - u_y \left(\xi_{2_{x,x}} + \left(\xi_{2_{u,x}}\right) u_x\right.$$

$$+\left(\xi_{2_{u,x}}+\left(\xi_{2_{u,u}}\right)u_x\right)u_x+\left(\xi_{2_u}\right)u_{x,x}\Big)\Big)u_x+\eta_{y,y}+\left(\eta_{u,y}\right)u_y+\left(\eta_{u,y}+\left(\eta_{u,u}\right)u_y\right)u_y$$
$$+\left(\eta_u\right)u_{y,y}-2\,u_{x,y}\left(\xi_{1_y}+\left(\xi_{1_u}\right)u_y\right)-u_x\left(\xi_{1_{y,y}}+\left(\xi_{1_{u,y}}\right)u_y+\left(\xi_{1_{u,y}}+\left(\xi_{1_{u,u}}\right)u_y\right)u_y\right.$$
$$+\left(\xi_{1_u}\right)u_{y,y}\Big)-2\,u_{y,y}\left(\xi_{2_y}+\left(\xi_{2_u}\right)u_y\right)-u_y\left(\xi_{2_{y,y}}+\left(\xi_{2_{u,y}}\right)u_y+\left(\xi_{2_{u,y}}+\left(\xi_{2_{u,u}}\right)u_y\right)u_y\right.$$
$$+\left(\xi_{2_u}\right)u_{y,y}\Big)$$

```
> defeq:=subs(u[y,y] = - u[x]*u[x,x], defeq);
```

$$defeq := \left(\eta_x+\left(\eta_u\right)u_x-u_x\left(\xi_{1_x}+\left(\xi_{1_u}\right)u_x\right)-u_y\left(\xi_{2_x}+\left(\xi_{2_u}\right)u_x\right)\right)u_{x,x}+\left(\eta_{x,x}+\left(\eta_{u,x}\right)u_x\right. \tag{9}$$
$$+\left(\eta_{u,x}+\left(\eta_{u,u}\right)u_x\right)u_x+\left(\eta_u\right)u_{x,x}-2\,u_{x,x}\left(\xi_{1_x}+\left(\xi_{1_u}\right)u_x\right)-u_x\left(\xi_{1_{x,x}}+\left(\xi_{1_{u,x}}\right)u_x\right.$$
$$+\left(\xi_{1_{u,x}}+\left(\xi_{1_{u,u}}\right)u_x\right)u_x+\left(\xi_{1_u}\right)u_{x,x}\Big)-2\,u_{x,y}\left(\xi_{2_x}+\left(\xi_{2_u}\right)u_x\right)-u_y\left(\xi_{2_{x,x}}+\left(\xi_{2_{u,x}}\right)u_x\right.$$
$$+\left(\xi_{2_{u,x}}+\left(\xi_{2_{u,u}}\right)u_x\right)u_x+\left(\xi_{2_u}\right)u_{x,x}\Big)\Big)u_x+\eta_{y,y}+\left(\eta_{u,y}\right)u_y+\left(\eta_{u,y}+\left(\eta_{u,u}\right)u_y\right)u_y$$
$$-\left(\eta_u\right)u_xu_{x,x}-2\,u_{x,y}\left(\xi_{1_y}+\left(\xi_{1_u}\right)u_y\right)-u_x\left(\xi_{1_{y,y}}+\left(\xi_{1_{u,y}}\right)u_y+\left(\xi_{1_{u,y}}+\left(\xi_{1_{u,u}}\right)u_y\right)u_y\right.$$
$$-\left(\xi_{1_u}\right)u_xu_{x,x}\Big)+2\,u_xu_{x,x}\left(\xi_{2_y}+\left(\xi_{2_u}\right)u_y\right)-u_y\left(\xi_{2_{y,y}}+\left(\xi_{2_{u,y}}\right)u_y+\left(\xi_{2_{u,y}}\right.\right.$$
$$+\left(\xi_{2_{u,u}}\right)u_y\Big)u_y-\left(\xi_{2_u}\right)u_xu_{x,x}\Big)$$

```
> allds := [u[x], u[y], u[x,x], u[x, y]];
```

$$allds := \left[u_x, u_y, u_{x,x}, u_{x,y}\right] \tag{10}$$

```
> collect(defeq, allds);
```

$$-\left(\xi_{1_{u,u}}\right)u_x^4+\left(-\left(\xi_{2_{u,u}}\right)u_y+\eta_{u,u}-2\,\xi_{1_{u,x}}\right)u_x^3+\left(-3\left(\xi_{1_u}\right)u_{x,x}-2\,u_{x,y}\left(\xi_{2_u}\right)\right. \tag{11}$$
$$-2\,u_y\left(\xi_{2_{u,x}}\right)+2\,\eta_{u,x}-\xi_{1_{x,x}}\Big)u_x^2+\left(-\left(\xi_{1_{u,u}}\right)u_y^2+\left(\left(\xi_{2_u}\right)u_{x,x}-\xi_{2_{x,x}}-2\,\xi_{1_{u,y}}\right)u_y\right.$$
$$+\left(\eta_u-3\,\xi_{1_x}+2\,\xi_{2_y}\right)u_{x,x}-2\,u_{x,y}\left(\xi_{2_x}\right)+\eta_{x,x}-\xi_{1_{y,y}}\Big)u_x-\left(\xi_{2_{u,u}}\right)u_y^3+\left(\eta_{u,u}\right.$$
$$-2\,\xi_{2_{u,y}}\Big)u_y^2+\left(-2\,u_{x,y}\left(\xi_{1_u}\right)-\left(\xi_{2_x}\right)u_{x,x}+2\,\eta_{u,y}-\xi_{2_{y,y}}\right)u_y+\left(\eta_x\right)u_{x,x}-2\,u_{x,y}\left(\xi_{1_y}\right)$$
$$+\eta_{y,y}$$

```
> defeq := expand(defeq);
```

$$defeq := -3\,u_{x,x}u_x\left(\xi_{1_x}\right)-3\,u_{x,x}\left(\xi_{1_u}\right)u_x^2-u_{x,x}u_y\left(\xi_{2_x}\right)-2\,u_xu_{x,y}\left(\xi_{2_x}\right)-2\,u_{x,y}\left(\xi_{2_u}\right)u_x^2 \tag{12}$$
$$-u_xu_y\left(\xi_{2_{x,x}}\right)-2\,u_y\left(\xi_{2_{u,x}}\right)u_x^2-u_y\left(\xi_{2_{u,u}}\right)u_x^3-2\,u_{x,y}\left(\xi_{1_u}\right)u_y+\eta_{y,y}-2\,u_x\left(\xi_{1_{u,y}}\right)u_y$$
$$-u_x\left(\xi_{1_{u,u}}\right)u_y^2+2\left(\eta_{u,y}\right)u_y-\left(\xi_{2_{u,u}}\right)u_y^3+\left(\eta_x\right)u_{x,x}-2\,u_{x,y}\left(\xi_{1_y}\right)+u_x\left(\eta_{x,x}\right)$$
$$+\left(\eta_u\right)u_xu_{x,x}-\left(\xi_{1_{u,u}}\right)u_x^4+2\left(\eta_{u,x}\right)u_x^2+\left(\eta_{u,u}\right)u_x^3-u_x^2\left(\xi_{1_{x,x}}\right)-2\left(\xi_{1_{u,x}}\right)u_x^3$$
$$+\left(\eta_{u,u}\right)u_y^2-u_x\left(\xi_{1_{y,y}}\right)-u_y\left(\xi_{2_{y,y}}\right)-2\left(\xi_{2_{u,y}}\right)u_y^2+u_{x,x}u_y\left(\xi_{2_u}\right)u_x+2\,u_xu_{x,x}\left(\xi_{2_y}\right)$$

```
> sys_pde:=[coeffs(defeq, allds)];
```

$$sys\_pde := \left[\eta_{y,y},\ \eta_u - 3\,\xi_{1_x} + 2\,\xi_{2_y},\ -3\,\xi_{1_u},\ -\xi_{2_x},\ -2\,\xi_{2_x},\ -2\,\xi_{2_u},\ -\xi_{2_{x,x}} - 2\,\xi_{1_{u,y}},\ -2\,\xi_{2_{u,x}},\ -\xi_{2_{u,u}},\right.$$
$$-2\,\xi_{1_u},\ -\xi_{1_{u,u}},\ \xi_{2_u},\ -\xi_{2_{u,u}},\ -\xi_{1_{u,u}},\ \eta_{u,u} - 2\,\xi_{1_{u,x}},\ 2\,\eta_{u,x} - \xi_{1_{x,x}},\ \eta_{u,u} - 2\,\xi_{2_{u,y}},\ 2\,\eta_{u,y} - \xi_{2_{y,y}},$$
$$\left.-2\,\xi_{1_y},\ \eta_{x,x} - \xi_{1_{y,y}},\ \eta_x\right] \tag{13}$$

```
> pdsolve(sys_pde);
```

$$\left\{\eta = (3\,\_C3 - 2\,\_C1)\,u + \_C5\,y + \_C6,\ \xi_1 = \_C3\,x + \_C4,\ \xi_2 = \_C1\,y + \_C2\right\} \tag{14}$$