The Downward Closure Property of Frequent Patterns

The Downward Closure Property of Frequent Patterns

- Observation: From TDB$_{1:}$ T_1: $\{a_1, \ldots, a_{50}\}$; T_2: $\{a_1, \ldots, a_{100}\}$
  - We get a frequent itemset: $\{a_1, \ldots, a_{50}\}$
  - Also, its subsets are all frequent: $\{a_1\}, \{a_2\}, \ldots, \{a_{50}\}, \{a_1, a_2\}, \ldots, \{a_1, \ldots, a_{49}\}, \ldots$
  - There must be some hidden relationships among frequent patterns!
- The downward closure (also called "Apriori") property of frequent patterns
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - Every transaction containing {beer, diaper, nuts} also contains {beer, diaper}
  - Apriori: Any subset of a frequent itemset must be frequent
- Efficient mining methodology
  - If any subset of an itemset S is infrequent, then there is no chance for S to be frequent—why do we even have to consider S!?

A sharp knife for pruning!

Apriori Pruning and Scalable Mining Methods

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not even be generated! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Scalable mining Methods: Three major approaches
  - Level-wise, join-based approach: Apriori (Agrawal & Srikant@VLDB'94)
  - Vertical data format approach: Eclat (Zaki, Parthasarathy, Ogihara, Li @KDD'97)
  - Frequent pattern projection and growth: FPgrowth (Han, Pei, Yin @SIGMOD'00)

The Apriori Algorithm

Apriori: A Candidate Generation & Test Approach

- Outline of Apriori (level-wise, candidate generation and test)
  - Initially, scan DB once to get frequent 1-itemset
  - Repeat
    - Generate length-(k+1) candidate itemsets from length-k frequent itemsets
    - Test the candidates against DB to find frequent (k+1)-itemsets
    - Set k := k +1
  - Until no frequent or candidate set can be generated
  - Return all the frequent itemsets derived

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k

F_k : Frequent itemset of size k

K := 1;

F_k := {frequent items}; // frequent 1-itemset

While (F_k != ∅) **do {** // when F_k is non-empty

C_{k+1} := candidates generated from F_k; // candidate generation

Derive F_{k+1} by counting candidates in C_{k+1} with respect to *TDB* at minsup;

k := k + 1

}

return $\cup_k F_k$ // return F_k generated at each level

The Apriori Algorithm—An Example

minsup = 2

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1^{st} scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

F_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2^{nd} scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

F_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

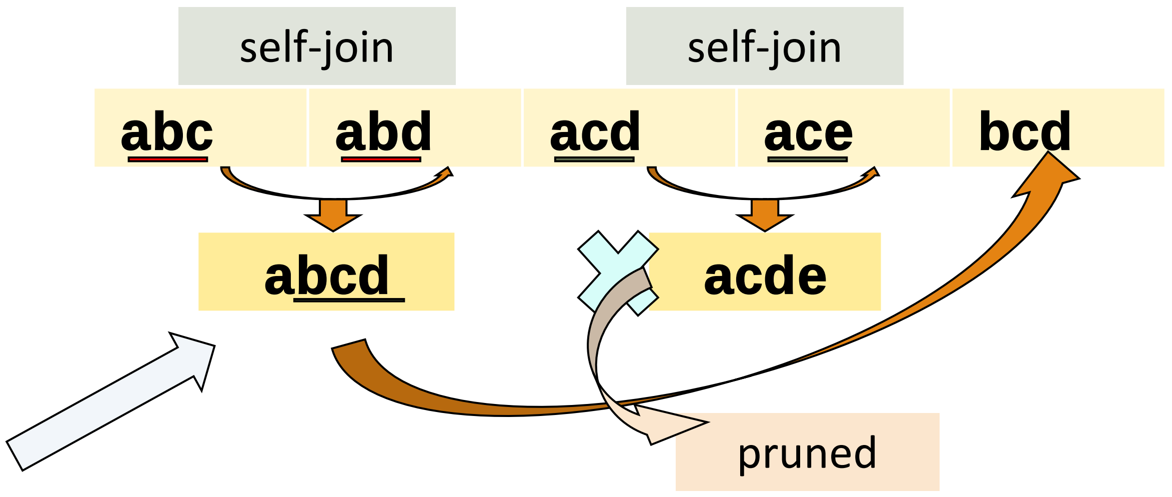
3^{rd} scan

F_3

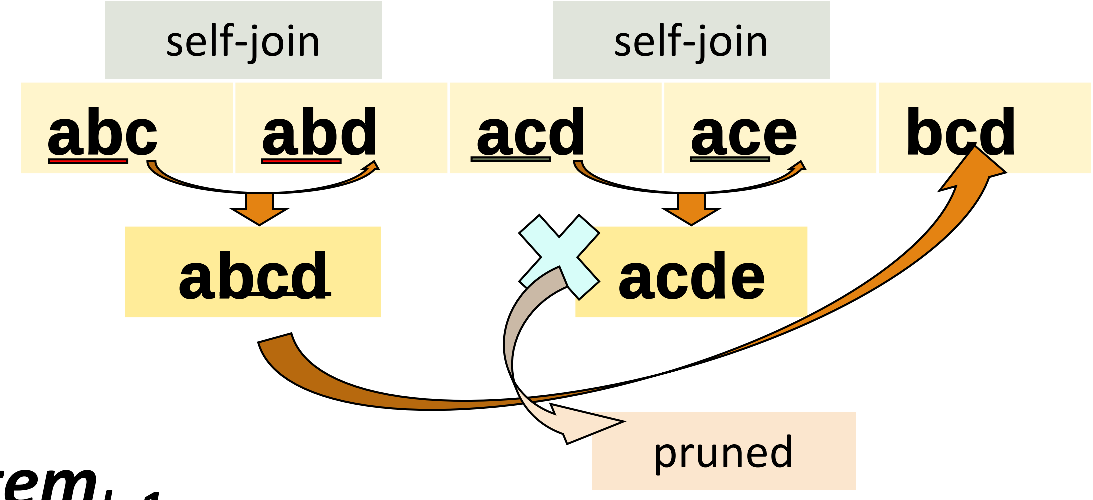
Itemset	sup
{B, C, E}	2

Apriori: Implementation Tricks

- How to generate candidates?
  - Step 1: self-joining F_k
  - Step 2: pruning
- Example of candidate-generation
  - $F_3 = \{abc, abd, acd, ace, bcd\}$
  - Self-joining: $F_3 * F_3$
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in F_3
  - $C_4 = \{abcd\}$



Candidate Generation: An SQL Implementation



- Suppose the items in F_{k-1} are listed in an order
- Step 1: self-joining F_{k-1}

  insert into $\boldsymbol{C_k}$

  select $\boldsymbol{p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}}$

  from $\boldsymbol{F_{k-1}}$ ***as p***, $\boldsymbol{F_{k-1}}$ ***as q***

  where $\boldsymbol{p.item_1 = q.item_1, \ldots, p.item_{k-2} = q.item_{k-2}, p.item_{k-1} < q.item_{k-1}}$
- Step 2: pruning

  for all ***itemsets c in*** $\boldsymbol{C_k}$ do

  for all ***(k-1)-subsets s of c*** do

  if *(s is not in* F_{k-1}*)* **then delete** *c* **from** C_k

Extensions or Improvements of Apriori

Apriori: Improvements and Alternatives

- Reduce passes of transaction database scans
  - Partitioning (e.g., Savasere, et al., 1995)
  - Dynamic itemset counting (Brin, et al., 1997)

To be discussed in subsequent slides

- Shrink the number of candidates
  - Hashing (e.g., DHP: Park, et al., 1995)
  - Pruning by support lower bounding (e.g., Bayardo 1998)
  - Sampling (e.g., Toivonen, 1996)

To be discussed in subsequent slides

- Exploring special data structures
  - Tree projection (Agarwal, et al., 2001)
  - H-miner (Pei, et al., 2001)
  - Hypecube decomposition (e.g., LCM: Uno, et al., 2004)

Partitioning: Scan Database Only Twice

- Theorem: *Any itemset that is potentially frequent in TDB must be frequent in at least one of the partitions of TDB*

Here is the proof!

TDB_1	+	TDB_2	+	. . .	+	TDB_k	=	TDB
$sup_1(X) < \sigma\|TDB_1\|$		$sup_2(X) < \sigma\|TDB_2\|$		. . .		$sup_k(X) < \sigma\|TDB_k\|$		$sup(X) < \sigma\|TDB\|$

- Method: (A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*)
  - Scan 1: Partition database (how?) and find local frequent patterns
  - Scan 2: Consolidate global frequent patterns (how to?)
- Why does this method guarantee to scan TDB only twice?

Direct Hashing and Pruning (DHP)

- DHP (Direct Hashing and Pruning): Reduce the number of candidates (J. Park, M. Chen, and P. Yu, SIGMOD'95)
- Observation: A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - Hash entries
    - {ab, ad, ae}
    - {bd, be, de}
    - …
  - Frequent 1-itemset: a, b, d, e
  - ab is not a candidate 2-itemset if the sum of count of {ab, ad, ae} is below support threshold

Itemsets	Count
{ab, ad, ae}	35
{bd, be, de}	298
……	…
{yz, qs, wt}	58

Hash Table

Mining Frequent Patterns by Exploring Vertical Data Format

Exploring Vertical Data Format: ECLAT

- ECLAT (Equivalence Class Transformation): A depth-first search algorithm using set intersection [Zaki et al. @KDD'97]
- Tid-List: List of transaction-ids containing an itemset
- Vertical format: $t(e) = \{T_{10}, T_{20}, T_{30}\}$; $t(a) = \{T_{10}, T_{20}\}$; $t(ae) = \{T_{10}, T_{20}\}$
- Properties of Tid-Lists
  - $t(X) = t(Y)$: X and Y always happen together (e.g., t(ac} = t(d})
  - $t(X) \subset t(Y)$: transaction having X always has Y (e.g., $t(ac) \subset t(ce)$)
- Deriving frequent patterns based on vertical intersections
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - $t(e) = \{T_{10}, T_{20}, T_{30}\}$, $t(ce) = \{T_{10}, T_{30}\}$ → Diffset $(ce, e) = \{T_{20}\}$

A transaction DB in Horizontal Data Format

Tid	Itemset
10	a, c, d, e
20	a, b, e
30	b, c, e

The transaction DB in Vertical Data Format

Item	TidList
a	10, 20
b	20, 30
c	10, 30
d	10
e	10, 20, 30

FPGrowth: A Pattern Growth Approach

FPGrowth: Mining Frequent Patterns by Pattern Growth

- Idea: Frequent pattern growth (FPGrowth)
  - Find frequent single items and partition the database based on each such item
  - Recursively grow frequent patterns by doing the above for each partitioned database (also called *conditional database*)
  - To facilitate efficient processing, an efficient data structure, FP-tree, can be constructed
- Mining becomes
  - Recursively construct and mine (conditional) FP-trees
  - Until the resulting FP-tree is empty, or until it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Example: Construct FP-tree from a Transational DB

TID	Items in the Transaction	Ordered, frequent items
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

1. Scan DB once, find single item frequent pattern: **Let min_support = 3**

   f:4, a:3, c:4, b:3, m:3, p:3

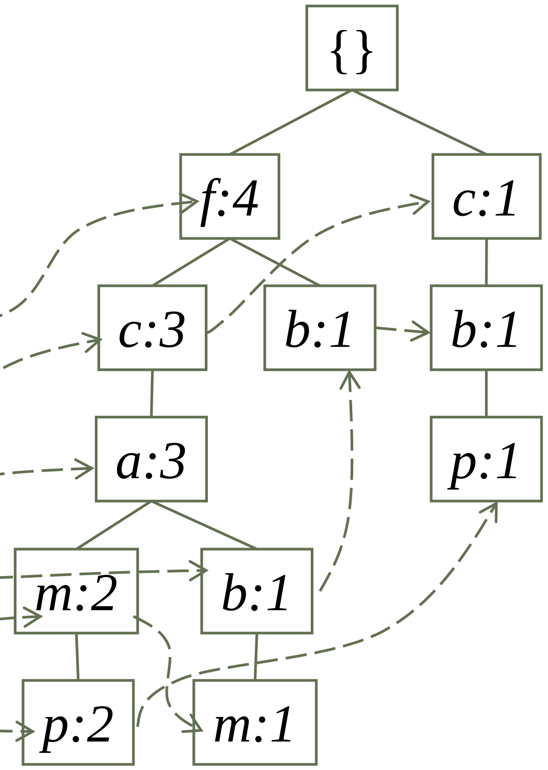
2. Sort frequent items in frequency descending order, f-list

   F-list = f-c-a-b-m-p

3. Scan DB again, construct FP-tree

Header Table

Item	Frequency	header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	

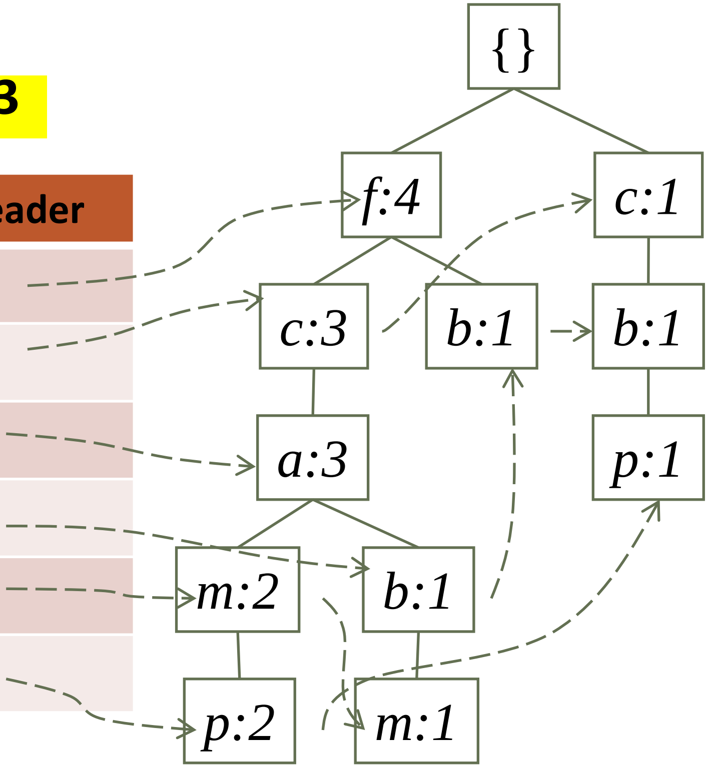


Divide and Conquer Based on Patterns and Data

- Pattern mining can be partitioned according to current patterns
  - Patterns containing p: p's conditional database: *fcam:2, cb:1*
  - Patterns having m but no p: m's conditional database: *fca:2, fcab:1*
  - …… ……
- *p*'s conditional pattern base: *transformed prefix paths* of item *p*

min_support = 3

Item	Frequency	Header
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***Conditional* pattern bases**

Item	*Conditional pattern base*
c	***f:3***
a	***fc:3***
b	***fca:1, f:1, c:1***
m	***fca:2, fcab:1***
p	***fcam:2, cb:1***

Mine Each Conditional Pattern-Base Recursively

***Conditional* pattern bases**

item ***cond. pattern base***

min_support = 3

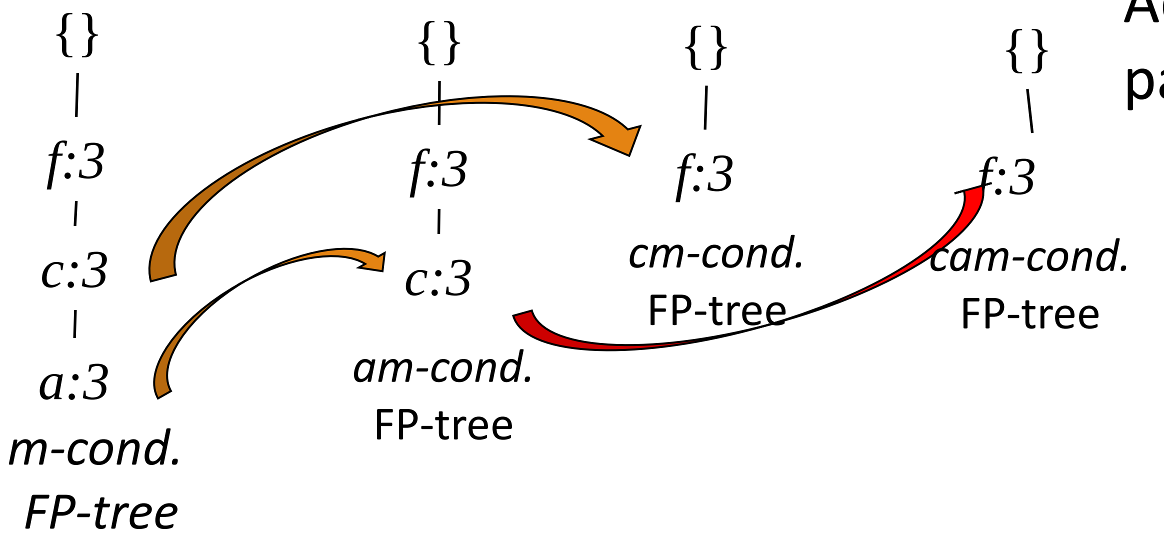
item	cond. pattern base
c	*f:3*
a	*fc:3*
b	*fca:1, f:1, c:1*
m	*fca:2, fcab:1*
p	*fcam:2, cb:1*

- For each conditional pattern-base
  - Mine single-item patterns
  - Construct its FP-tree & mine it

p-conditional PB: ***fcam:2, cb:1 → c: 3***

m-conditional PB: ***fca:2, fcab:1 → fca: 3***

b-conditional PB: ***fca:1, f:1, c:1 → ϕ***



Actually, for single branch FP-tree, all frequent patterns can be generated in one shot

m: 3

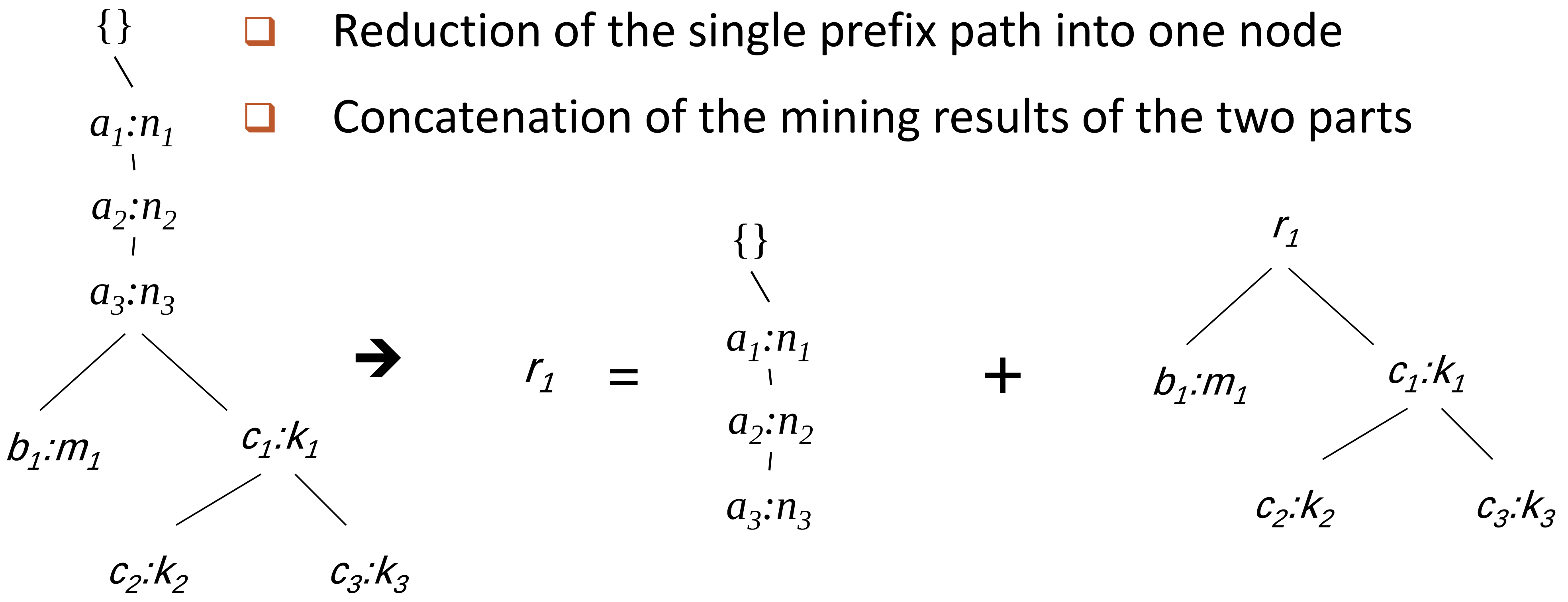
fm: 3, cm: 3, am: 3

fcm: 3, fam:3, cam: 3

fcam: 3

A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts



Scaling FP-growth by Database Projection

- What if FP-tree cannot fit in memory? — DB projection
  - Project the DB based on patterns
  - Construct & mine FP-tree for each projected DB
- Parallel projection vs. partition projection
  - Parallel projection: Project the DB on each frequent item
    - Space costly, all partitions can be processed in parallel
  - Partition projection: Partition the DB in order
    - Passing the unprocessed parts to subsequent partitions

Trans. DB
f_2 f_3 f_4 g h
f_3 f_4 i j
f_2 f_4 k
f_1 f_3 h
…

Assume only f's are freq. & the freq. item ordering is: f_1-f_2-f_3-f_4

Parallel projection

f_4-proj. DB	f_3-proj. DB
f_2 f_3	f_2
f_3	f_1
f_2	…
…	

Partition projection

f_4-proj. DB	f_3-proj. DB
f_2 f_3	f_1
f_3	…
f_2	
…	

f_2 will be projected to f_3-proj. DB only when processing f_4-proj. DB

Mining Closed Patterns

CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Efficient, *direct* mining of closed itemsets
- Ex. Itemset merging: If Y appears in every occurrence of X, then Y is merged with X
  - d-proj. db: {acef, acf} → acfd-proj. db: {e}, thus we get: acfd:2
- Many other tricks (but not detailed here), such as
  - Hybrid tree projection
    - Bottom-up physical tree-projection
    - Top-down pseudo tree-projection
  - Sub-itemset pruning
  - Item skipping
  - Efficient subset checking
- For details, see J. Wang, et al., "CLOSET+: ……", KDD'03

TID	Items
1	acdef
2	abe
3	cefg
4	acdf

Let minsupport = 2

a:3, c:3, d:2, e:3, f:3

F-List: a-c-e-f-d

Recommended Readings

- R. Agrawal and R. Srikant, "Fast algorithms for mining association rules", VLDB'94
- A. Savasere, E. Omiecinski, and S. Navathe, "An efficient algorithm for mining association rules in large databases", VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu, "An effective hash-based algorithm for mining association rules", SIGMOD'95
- S. Sarawagi, S. Thomas, and R. Agrawal, "Integrating association rule mining with relational database systems: Alternatives and implications", SIGMOD'98
- M. J. Zaki, S. Parthasarathy, M. Ogihara, and W. Li, "Parallel algorithm for discovery of association rules", Data Mining and Knowledge Discovery, 1997
- J. Han, J. Pei, and Y. Yin, "Mining frequent patterns without candidate generation", SIGMOD'00
- M. J. Zaki and Hsiao, "CHARM: An Efficient Algorithm for Closed Itemset Mining", SDM'02
- J. Wang, J. Han, and J. Pei, "CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets", KDD'03
- C. C. Aggarwal, M.A., Bhuiyan, M. A. Hasan, "Frequent Pattern Mining Algorithms: A Survey", in Aggarwal and Han (eds.): Frequent Pattern Mining, Springer, 2014