Basic Concepts of Density-Based Clustering

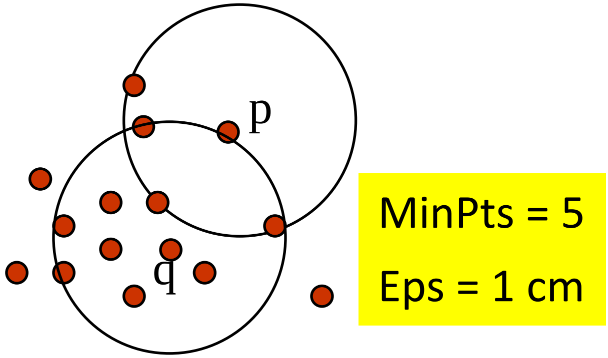
Density-Based Clustering Methods

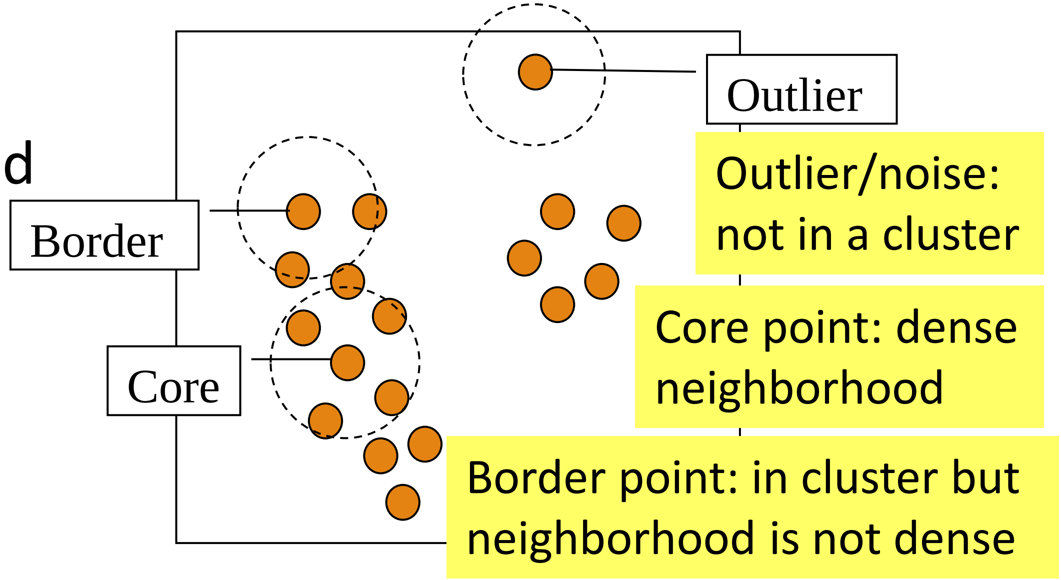
- Clustering based on density (a local cluster criterion), such as density-connected points
- Major features:
  - Discover clusters of arbitrary shape
  - Handle noise
  - One scan (only examine the local region to justify density)
  - Need density parameters as termination condition
- Several interesting studies:
  - DBSCAN: Ester, et al. (KDD'96) — To be covered in this lecture
  - OPTICS: Ankerst, et al (SIGMOD'99) — To be covered in this lecture
  - DENCLUE: Hinneburg & D. Keim (KDD'98)
  - CLIQUE: Agrawal, et al. (SIGMOD'98) (also, grid-based) — To be covered in this lecture

DBSCAN: A Density-Based Clustering Algorithm

DBSCAN: A Density-Based Spatial Clustering Algorithm

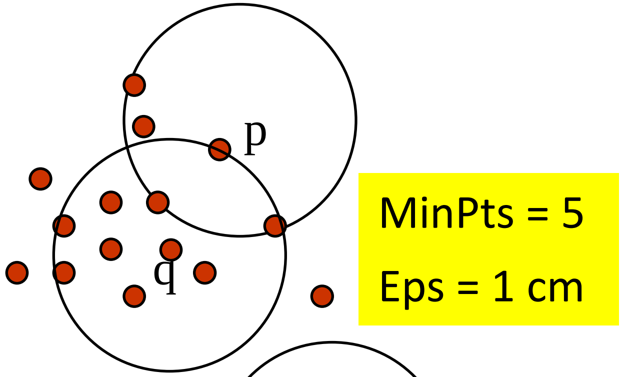
- DBSCAN (M. Ester, H.-P. Kriegel, J. Sander, and X. Xu, KDD'96)
  - Discovers clusters of arbitrary shape: <u>D</u>ensity-<u>B</u>ased <u>S</u>patial <u>C</u>lustering of <u>A</u>pplications with <u>N</u>oise
- A *density-based* notion of cluster
  - A *cluster* is defined as a maximal set of density-connected points
- Two parameters*:*
  - *Eps* (*ε*): Maximum radius of the neighborhood
  - *MinPts*: Minimum number of points in the Eps-neighborhood of a point
- The Eps(ε)-neighborhood of a point *q*:
  - $N_{Eps}(q)$: {p belongs to D | dist(p, q) ≤ Eps}

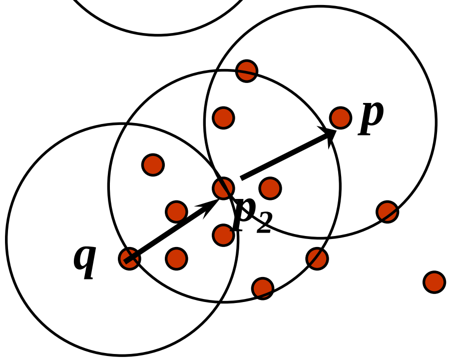


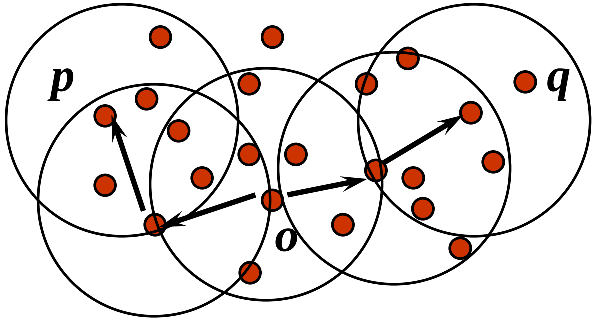


DBSCAN: Density-Reachable and Density-Connected

- **Directly density-reachable**:
  - A point p is directly density-reachable from a point q w.r.t. *Eps* (ε), *MinPts* if
    - p belongs to $N_{Eps}(q)$
    - **core point** condition: $|N_{Eps}(q)| \geq MinPts$
- **Density-reachable**:
  - A point p is density-reachable from a point q w.r.t. *Eps*, *MinPts* if there is a chain of points $p_1, \ldots, p_n$, $p_1 = q$, $p_n = p$ such that p_{i+1} is directly density-reachable from p_i
- **Density-connected:**
  - A point p is density-connected to a point q w.r.t. *Eps*, *MinPts* if there is a point o such that both p and q are density-reachable from o w.r.t. *Eps* and *MinPts*

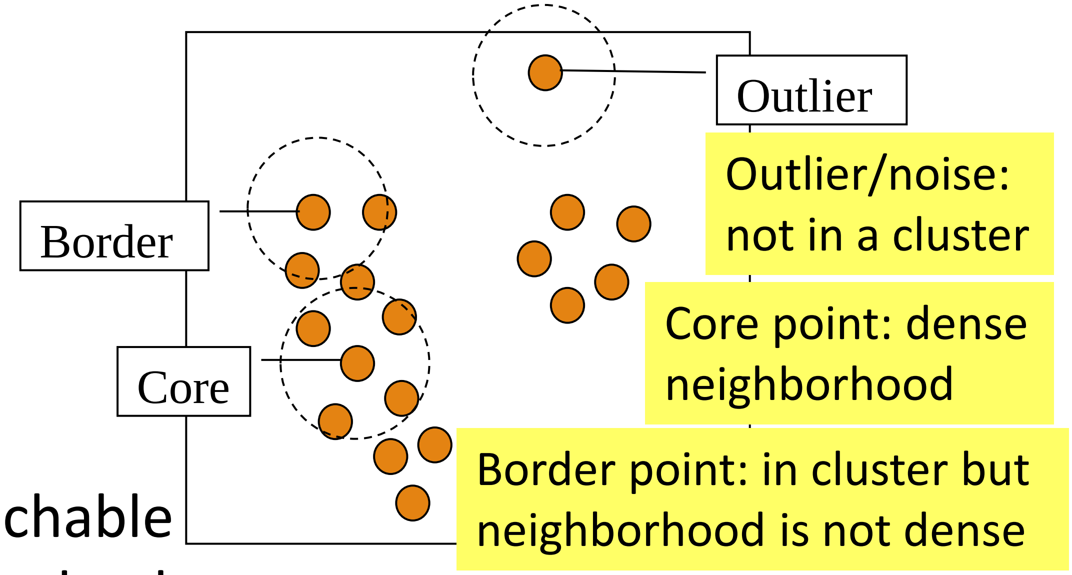






DBSCAN: The Algorithm

- **Algorithm**
  - Arbitrarily select a point *p*
  - Retrieve all points density-reachable from *p* w.r.t. *Eps* and *MinPts*
    - If *p* is a core point, a cluster is formed
    - If *p* is a border point, no points are density-reachable from *p,* and DBSCAN visits the next point of the database
  - Continue the process until all of the points have been processed



- **Computational complexity**
  - If a spatial index is used, the computational complexity of DBSCAN is O(nlogn), where n is the number of database objects
  - Otherwise, the complexity is $O(n^2)$

DBSCAN Is Sensitive to the Setting of Parameters

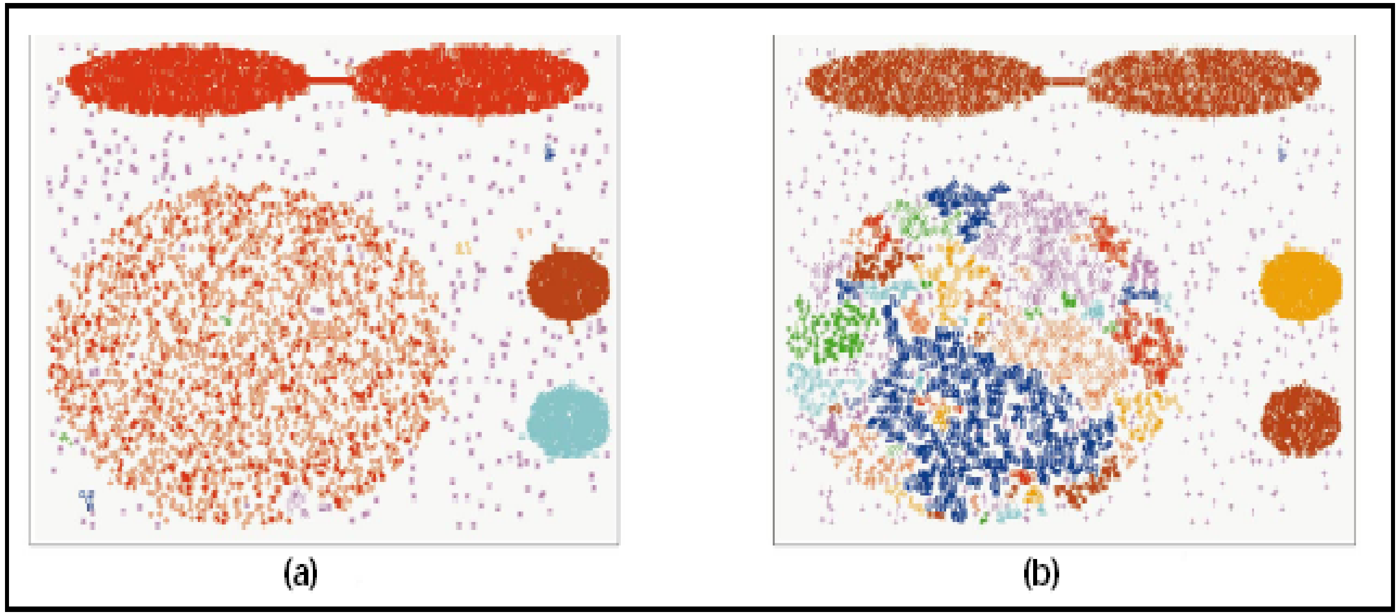


Figure 8. DBScan results for DS1 with MinPts at 4 and Eps at (a) 0.5 and (b) 0.4.

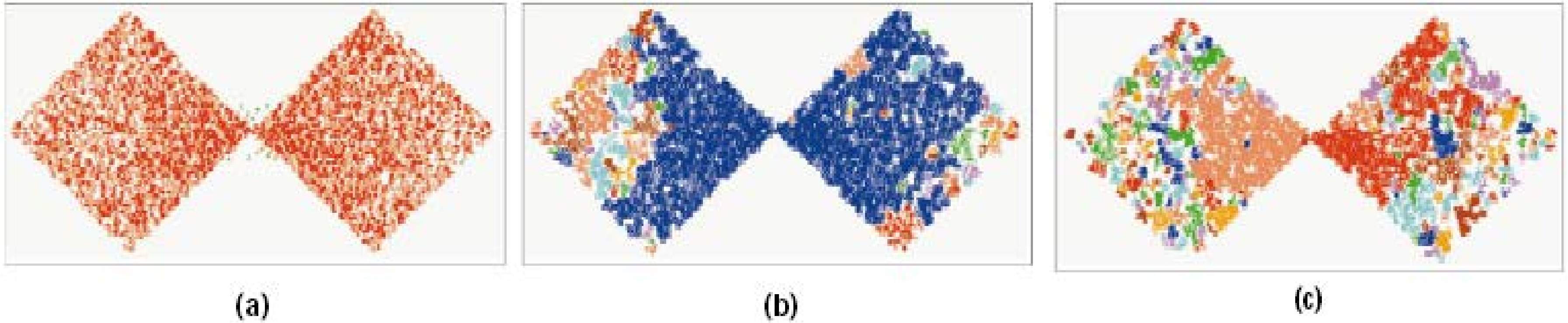


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.

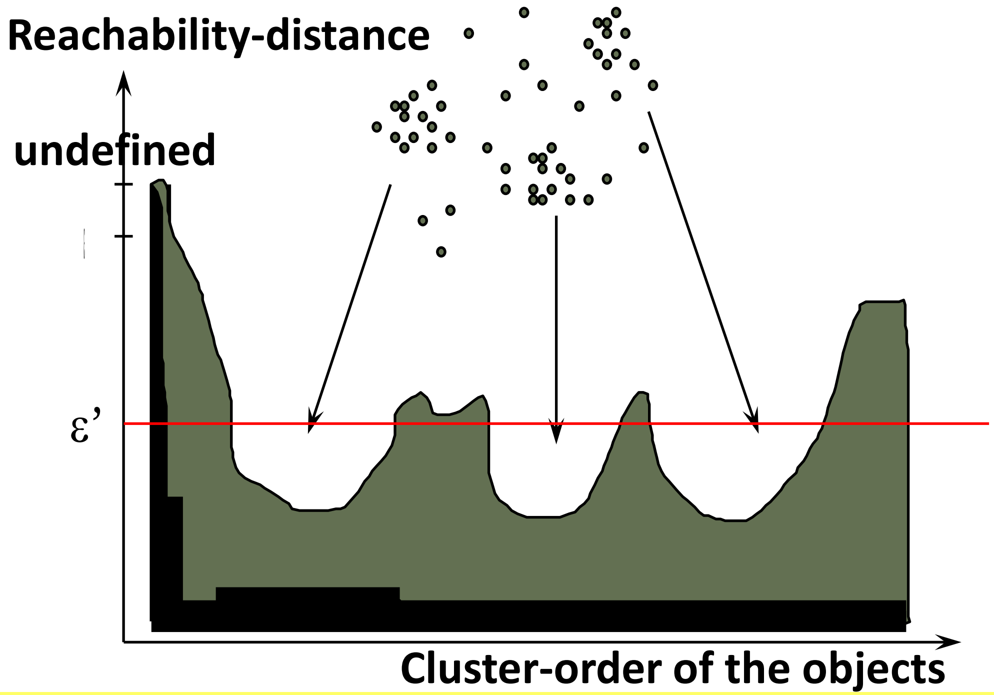
Ack. Figures from G. Karypis, E.-H. Han, and V. Kumar, *COMPUTER*, 32(8), 1999

OPTICS: Ordering Points To Identify Clustering Structure

OPTICS: Ordering Points To Identify Clustering Structure

- OPTICS (Ankerst, Breunig, Kriegel, and Sander, SIGMOD'99)
  - DBSCAN is sensitive to parameter setting
  - An extension: finding clustering structure
- Observation: Given a *MinPts*, density-based clusters w.r.t. a higher density are completely contained in clusters w.r.t. to a lower density
- Idea: Higher density points should be processed first—find high-density clusters first
- OPTICS stores such a clustering order using two pieces of information:
  - *Core distance* and *reachability distance*

Reachability plot for a dataset



- Since points belonging to a cluster have a low reachability distance to their nearest neighbor, valleys correspond to clusters
- The deeper the valley, the denser the cluster

OPTICS: An Extension from DBSCAN

- **Core distance** of an object p: The smallest value ε such that the ε-neighborhood of p has at least *MinPts* objects

  Let $N_\varepsilon(p)$: ε-neighborhood of p

  ε is a distance value

  Core-distance$_{\varepsilon,\, MinPts}(p)$ = Undefined if card($N_\varepsilon(p)$) < *MinPts*

  MinPts-distance(p), otherwise

- **Reachability distance** of object p from core object q is the min. radius value that makes p density-reachable from q

  Reachability-distance$_{\varepsilon,\, MinPts}(p, q)$ =

  Undefined, *if q is not a core object*

  max(core-distance(q), distance (q, p)), *otherwise*

- Complexity: O(N log$N$) (if index-based)

  where N: # of points

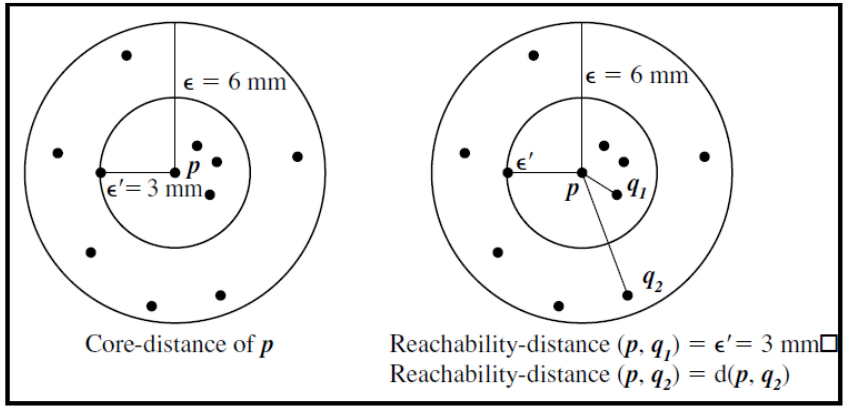
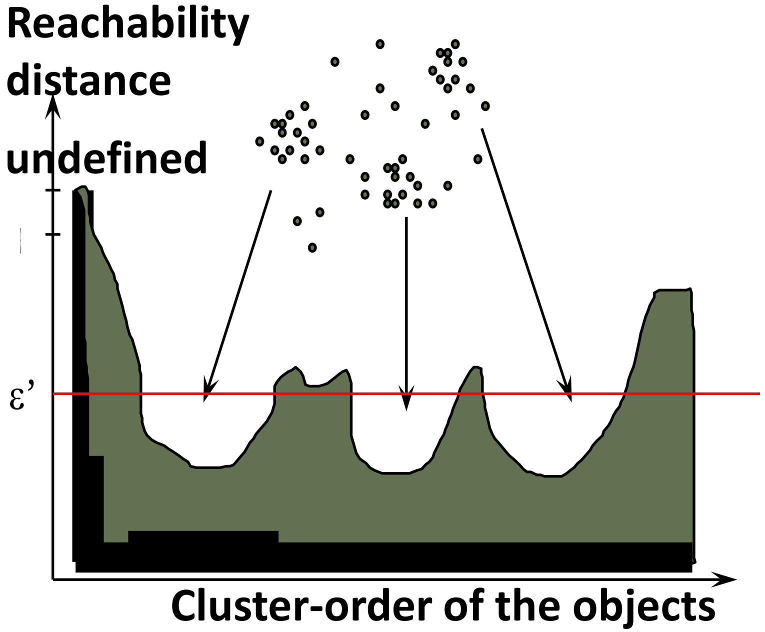
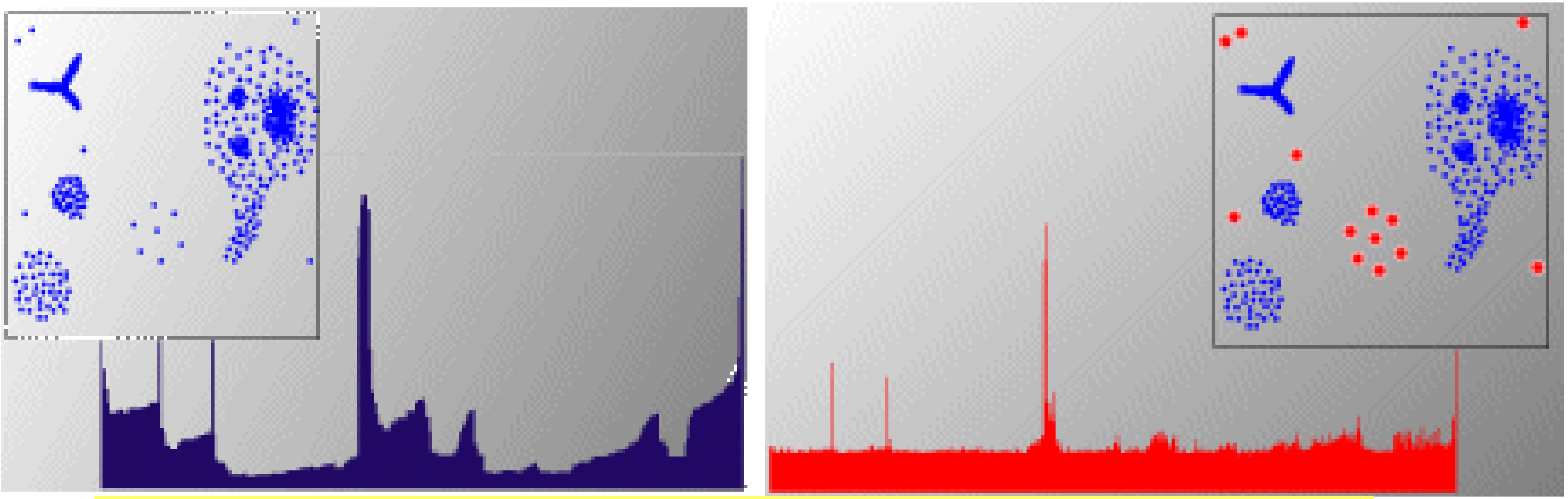


Figure 10.16: OPTICS terminology. Based on [ABKS99].



OPTICS: Finding Hierarchically Nested Clustering Structures

- OPTICS produces a special cluster-ordering of the data points with respect to its density-based clustering structure
  - The cluster-ordering contains information equivalent to the density-based clusterings corresponding to a broad range of parameter settings
  - Good for both automatic and interactive cluster analysis—finding intrinsic, even hierarchically nested clustering structures



Finding nested clustering structures with different parameter settings

Grid-Based Clustering Methods

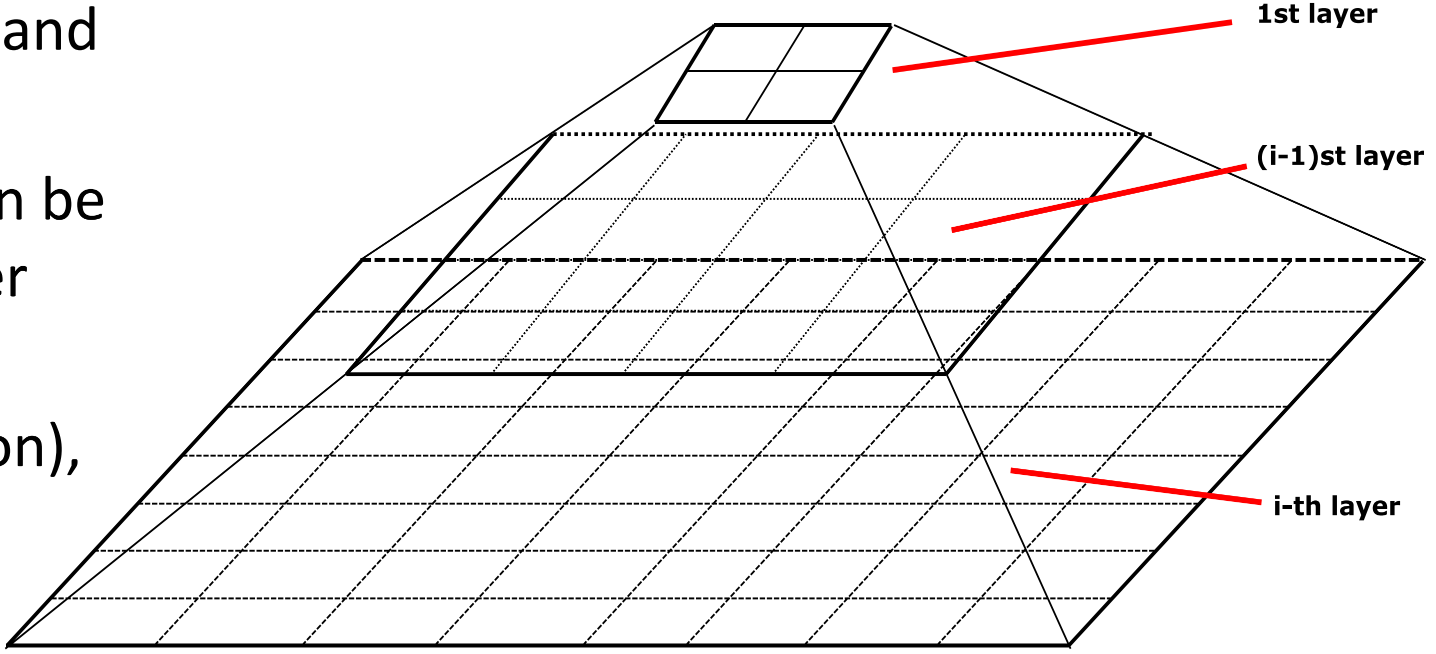
Grid-Based Clustering Methods

- Grid-Based Clustering: Explore multi-resolution grid data structure in clustering
  - Partition the data space into a finite number of cells to form a grid structure
  - Find clusters (dense regions) from the cells in the grid structure
- Features and challenges of a typical grid-based algorithm
  - Efficiency and scalability: # of cells << # of data points
  - Uniformity: Uniform, hard to handle highly irregular data distributions
  - Locality: Limited by predefined cell sizes, borders, and the density threshold
  - Curse of dimensionality: Hard to cluster high-dimensional data
- Methods to be introduced
  - **STING** (a STatistical INformation Grid approach) (Wang, Yang and Muntz, VLDB'97)
  - **CLIQUE** (Agrawal, Gehrke, Gunopulos, and Raghavan, SIGMOD'98)
    - Both grid-based and subspace clustering

STING: A Statistical Information Grid Approach

STING: A Statistical Information Grid Approach

- STING (Statistical Information Grid) (Wang, Yang and Muntz, VLDB'97)
- The spatial area is divided into rectangular cells at different levels of resolution, and these cells form a tree structure
- A cell at a high level contains a number of smaller cells of the next lower level
- Statistical information of each cell is calculated and stored beforehand and is used to answer queries
- Parameters of higher level cells can be easily calculated from that of lower level cell, including
  - *count*, *mean*, *s*(standard deviation), *min*, *max*
  - type of distribution—*normal*, *uniform*, etc.



Query Processing in STING and Its Analysis

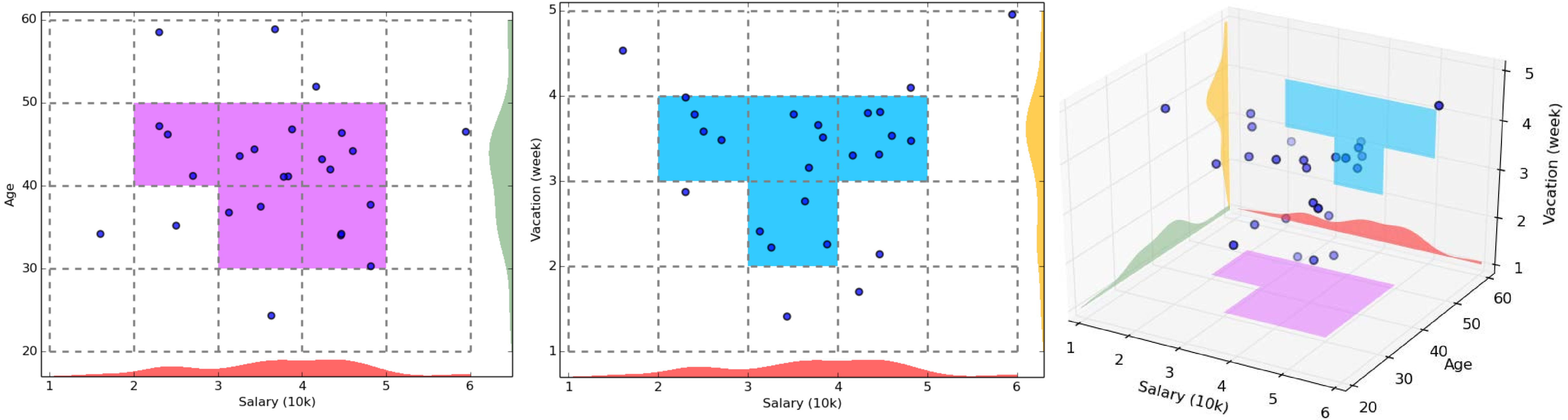
- To process a region query
  - Start at the root and proceed to the next lower level, using the STING index
  - Calculate the likelihood that a cell is relevant to the query at some confidence level using the statistical information of the cell
  - Only children of likely relevant cells are recursively explored
  - Repeat this process until the bottom layer is reached
- Advantages
  - Query-independent, easy to parallelize, incremental update
  - Efficiency: Complexity is O(K)
    - K: # of grid cells at the lowest level, and K << N (i.e., # of data points)
- Disadvantages
  - Its probabilistic nature may imply a loss of accuracy in query processing

CLIQUE: Grid-Based Subspace Clustering

CLIQUE: Grid-Based Subspace Clustering

- CLIQUE (Clustering In QUEst) (Agrawal, Gehrke, Gunopulos, Raghavan: SIGMOD'98)
- CLIQUE is a **density-based** and **grid-based** subspace clustering algorithm
  - **Grid-based**: It discretizes the data space through a grid and estimates the density by counting the number of points in a grid cell
  - **Density-based**: A cluster is a maximal set of connected dense units in a subspace
    - A unit is dense if the fraction of total data points contained in the unit exceeds the input model parameter
  - **Subspace clustering**: A subspace cluster is a set of neighboring dense cells in an arbitrary subspace. It also discovers some minimal descriptions of the clusters
- It automatically identifies subspaces of a high dimensional data space that allow better clustering than original space using the Apriori principle

CLIQUE: SubSpace Clustering with Aprori Pruning



- Start at 1-D space and discretize numerical intervals in each axis into grid
- Find dense regions (clusters) in each subspace and generate their minimal descriptions
- Use the dense regions to find promising candidates in 2-D space based on the Apriori principle
- Repeat the above in level-wise manner in higher dimensional subspaces

Major Steps of the CLIQUE Algorithm

- Identify subspaces that contain clusters
  - Partition the data space and find the number of points that lie inside each cell of the partition
  - Identify the subspaces that contain clusters using the Apriori principle
- Identify clusters
  - Determine dense units in all subspaces of interests
  - Determine connected dense units in all subspaces of interests
- Generate minimal descriptions for the clusters
  - Determine maximal regions that cover a cluster of connected dense units for each cluster
  - Determine minimal cover for each cluster

Additional Comments on *CLIQUE*

- Strengths
  - *Automatically* finds subspaces of the highest dimensionality as long as high density clusters exist in those subspaces
  - *Insensitive* to the order of records in input and does not presume some canonical data distribution
  - Scales *linearly* with the size of input and has good scalability as the number of dimensions in the data increases
- Weaknesses
  - As in all grid-based clustering approaches, the quality of the results crucially depends on the appropriate choice of the number and width of the partitions and grid cells

Recommended Readings

- M. Ester, H.-P. Kriegel, J. Sander, and X. Xu. A Density-Based Algorithm for Discovering Clusters in Large Spatial Databases. KDD'96
- W. Wang, J. Yang, R. Muntz, STING: A Statistical Information Grid Approach to Spatial Data Mining, VLDB’97
- R. Agrawal, J. Gehrke, D. Gunopulos, and P. Raghavan. Automatic Subspace Clustering of High Dimensional Data for Data Mining Applications. SIGMOD’98
- A. Hinneburg and D. A. Keim. An Efficient Approach to Clustering in Large Multimedia Databases with Noise. KDD’98
- M. Ankerst, M. M. Breunig, H.-P. Kriegel, and J. Sander. Optics: Ordering Points to Identify the Clustering Structure. SIGMOD’99
- M. Ester. Density-Based Clustering. In (Chapter 5) Aggarwal and Reddy (eds.), Data Clustering: Algorithms and Applications . CRC Press. 2014
- W. Cheng, W. Wang, and S. Batista. Grid-based Clustering. In (Chapter 6) Aggarwal and Reddy (eds.), Data Clustering: Algorithms and Applications. CRC Press. 2014