sheet07_C4M2

December 12, 2017

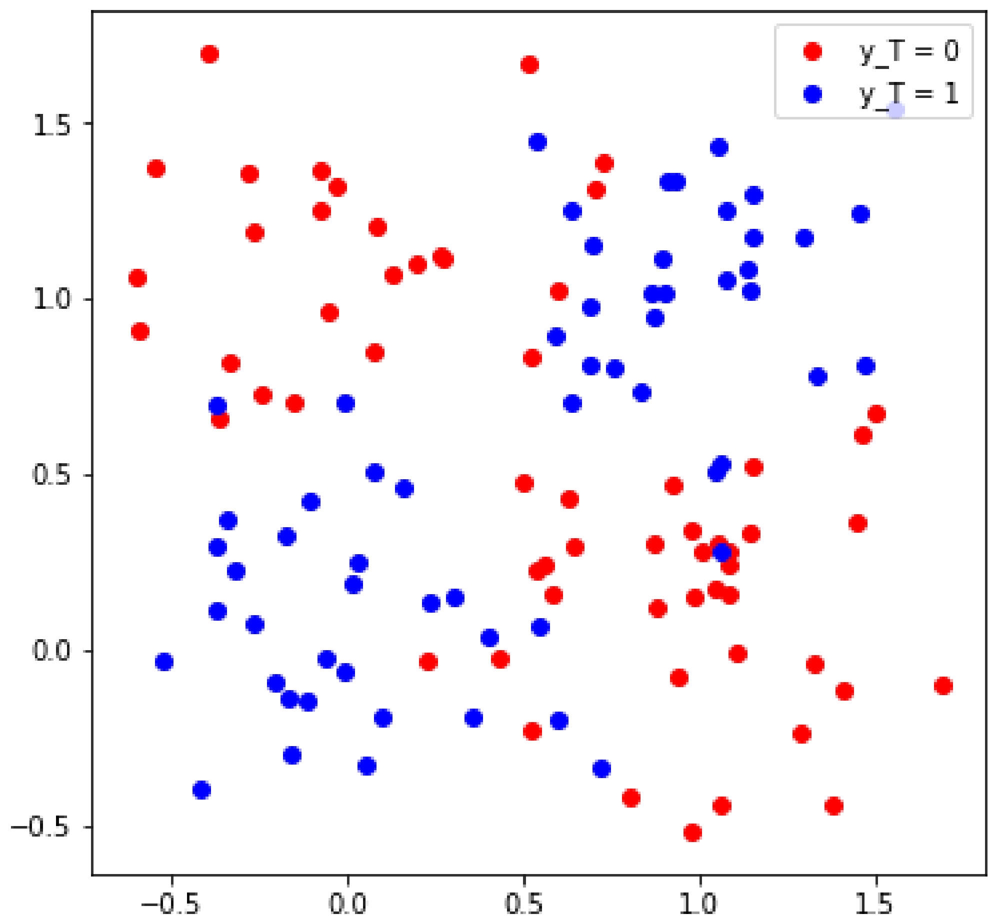
```
In [29]: import numpy as np
         import scipy.cluster
         import matplotlib
         import matplotlib.pyplot as plt
         %matplotlib inline
```

1 Exercise H7.1: Training data

1.1 (a) Input Samples

```
In [276]: def SampleMixtureNormal2D(m1, s1, m2, s2): # m1, m2 are the means, s1, s2 the variance
              if (np.random.choice([0,1])):
                  return np.random.multivariate_normal(m1, [[s1, 0], [0, s1]])
              else:
                  return np.random.multivariate_normal(m2, [[s2, 0], [0, s2]])


          def getSamples(m1, m2, m3, m4, s, N = 0, M = 0):

              samples = np.zeros((N + M, 2))
              classes = np.zeros((N + M, 2))

              for i in range(N):
                  samples[i] = SampleMixtureNormal2D(m1, s, m2, s)
                  classes[i] = np.array([1,0])
              for i in range(M):
                  samples[i + N] = SampleMixtureNormal2D(m3, s, m4, s)
                  classes[i + N] = np.array([0,1])

              return samples, classes

          samples, classes = getSamples([0, 1], [1, 0], [0, 0], [1, 1], 0.1, 60, 60)

          def getClassColor(i):
              colors = [(1,0,0),(0,0,1),(0,1,0),(1,1,0),(1,0,1),(0,1,1)]
              return colors[i%6]
```

```
def plotSamples(ax, samples, classes):

    K = classes.shape[1]

    handles = [None] * K

    for k in range(K):
        c = samples[np.where(classes[:,k] == 1)]
        handles[k] = ax.scatter(c[:,0],c[:,1], c = getClassColor(k), label='y_T = '  +

    ax.set_aspect('equal')
    ax.legend(handles = [y_0, y_1])

fig = plt.figure(figsize=(6,6))
ax = fig.add_subplot(111)
plotSamples(ax, samples, classes)
```



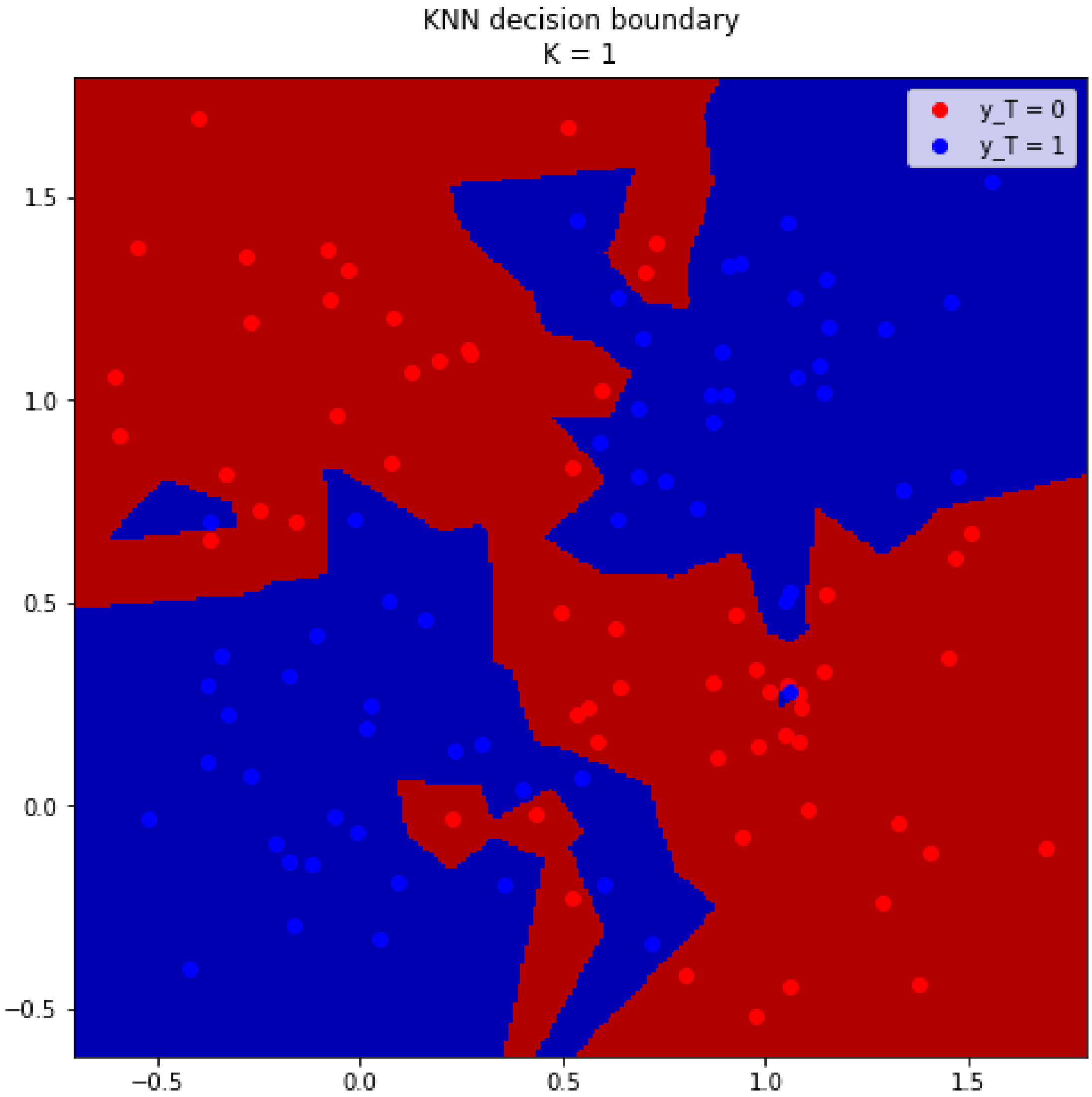
2 Exercise H7.2: k nearest neighbors (kNN)

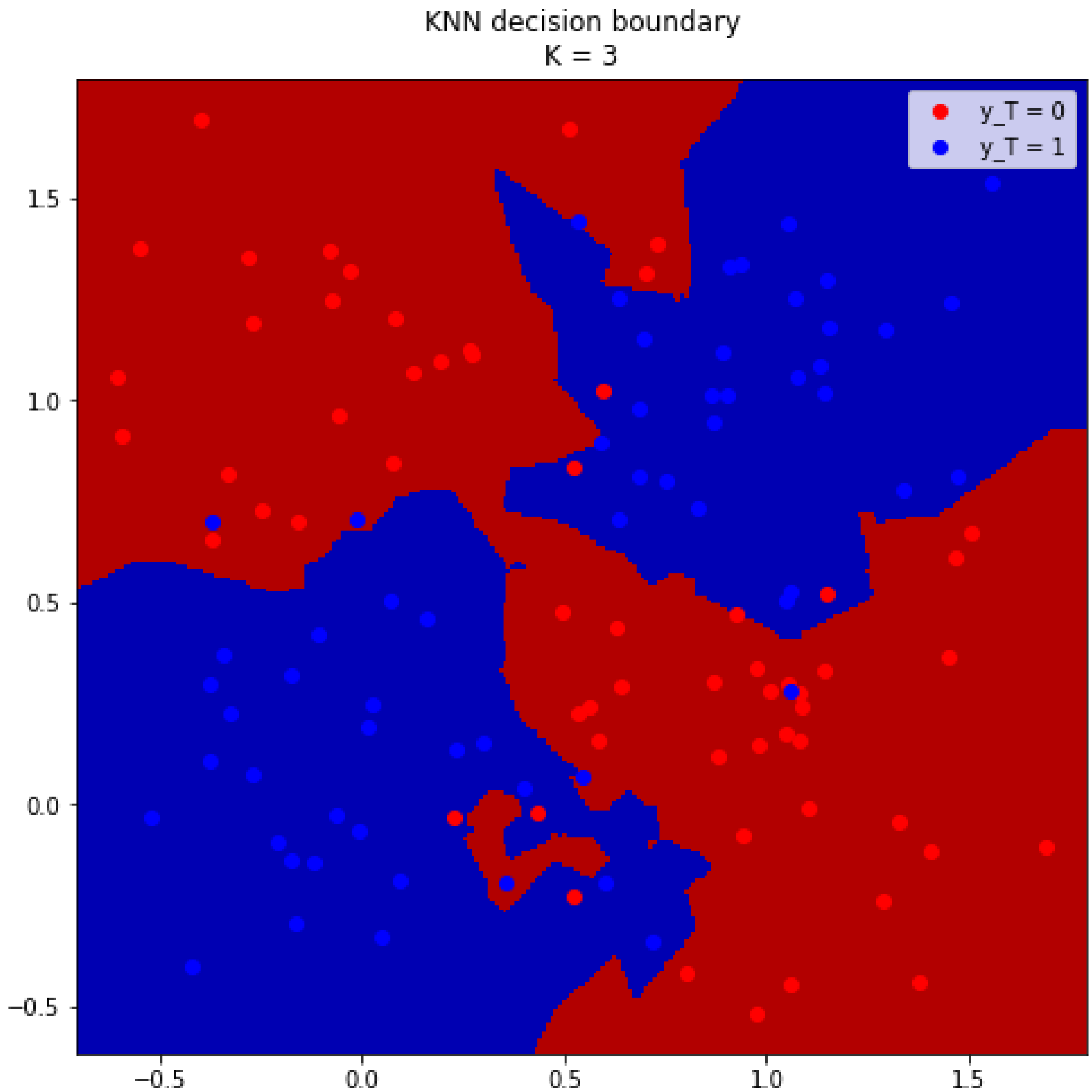
2.1 (a) Patterns and decision boundary

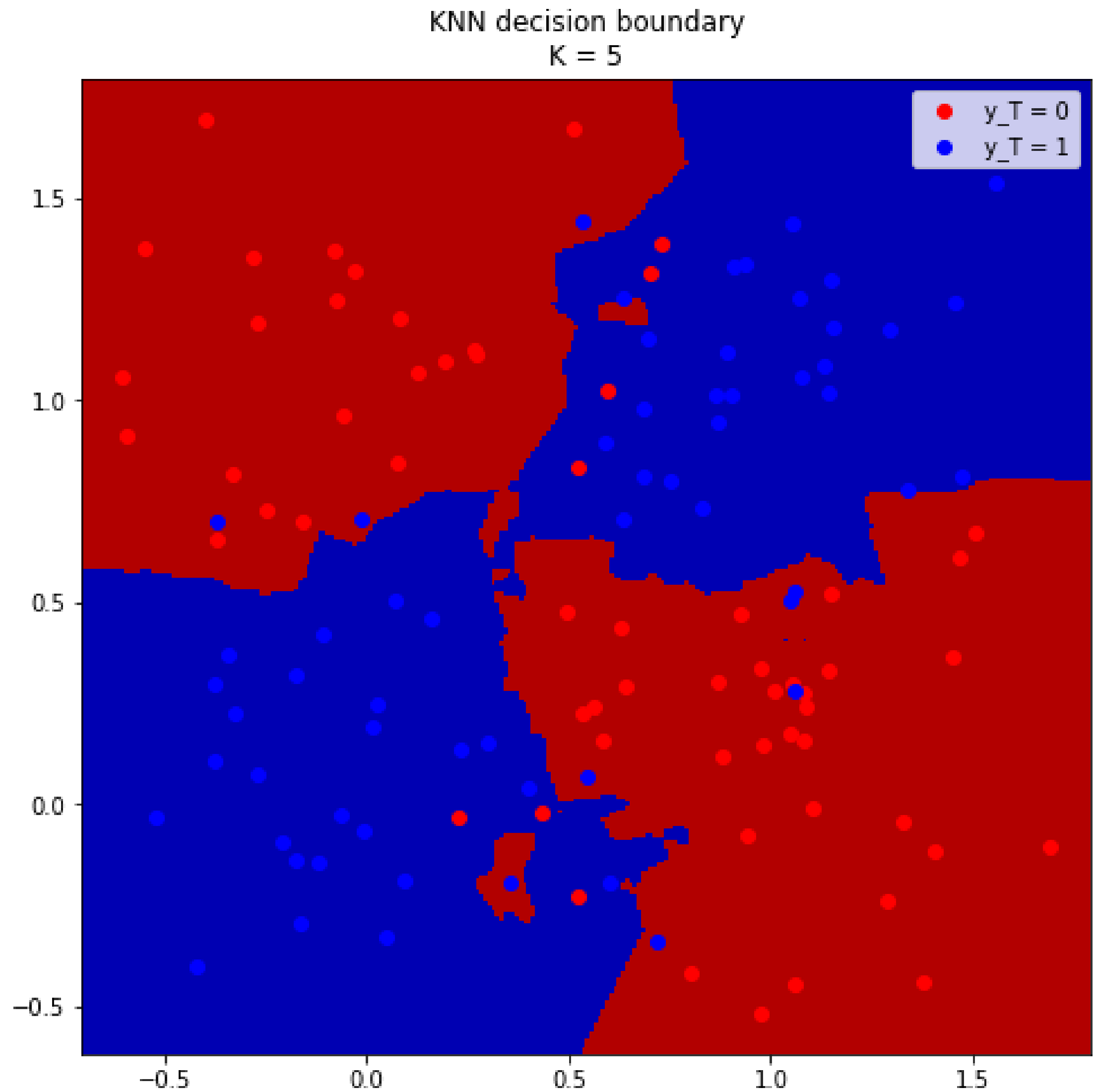
In [303]:
```
def plotClassifier(x, y, probabilities, samples, classes, title):

    K = classes.shape[1]

    fig = plt.figure(figsize=(8,8))
    ax = fig.add_subplot(111)

    plotSamples(ax, samples, classes)

    # plot decision boundary
    n = probabilities.shape[0]
    m = probabilities.shape[1]

    min_x, max_x = samples[:,0].min() - 0.1, samples[:,0].max() + 0.1
    min_y, max_y = samples[:,1].min() - 0.1, samples[:,1].max() + 0.1

    image = np.zeros((n, m, 3))
    for i in range(n):
        for j in range(m):
            c = getClassColor(np.argmax(probabilities[j,n-1-i]))
            image[i,j, :] = np.array([c[0]*0.7, c[1]*0.7, c[2]*0.7])
            #image[i,j, :] = np.array([j/m, 0, 0])
    ax.imshow(image, extent = (min_x,max_x,min_y,max_y))
    ax.set_title(title)

    return ax
```

In [304]:
```
def getKNNClassProb(x, k, samples, classes):
    m = np.subtract(samples, x)
    d = np.sum(np.multiply(m,m), axis = 1)
    return np.average(classes[d.argsort()[:k]], axis = 0)

def getKNNClassProbabilities(x, y, k, samples, classes):
    n = x.size
    m = y.size
    probs = np.zeros((n,m,classes.shape[1]))
    for i in range(n):
        for j in range(m):
            probs[i,j,:] = getKNNClassProb([x[i], y[j]], k, samples, classes)
    return probs

def getGrid(res, samples):
    min_x, max_x = samples[:,0].min() - 0.1, samples[:,0].max() + 0.1
    min_y, max_y = samples[:,1].min() - 0.1, samples[:,1].max() + 0.1
    return np.linspace(min_x, max_x, res), np.linspace(min_y, max_y, res)
```

```
x, y = getGrid(256, samples)

for K in [1,3,5]:
    probs = getKNNClassProbabilities(x, y, K, samples, classes)
    plotClassifier(x, y, probs, samples, classes, 'KNN decision boundary\nK = ' + str(
```




KNN decision boundary
K = 3
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5



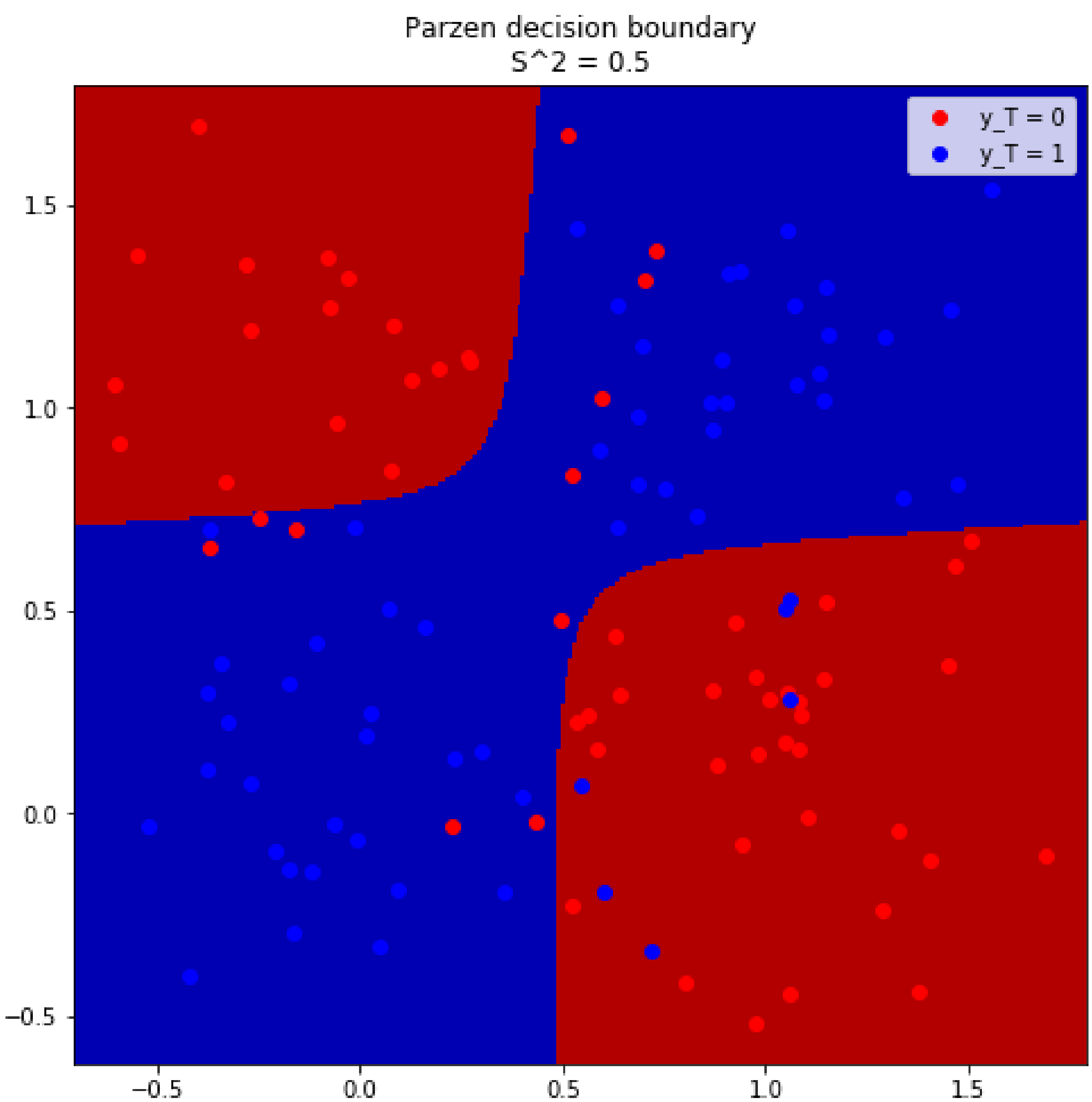
With increasing K, the model becomes more robust against outliers but sometimes we can observer small artifacts which are probably not desirable. Also with increasing K, the decision boundary becomes more of an average and does not extend to points which are in the neighbourhood of points of a different class. This sometimes does not seem reasonable.

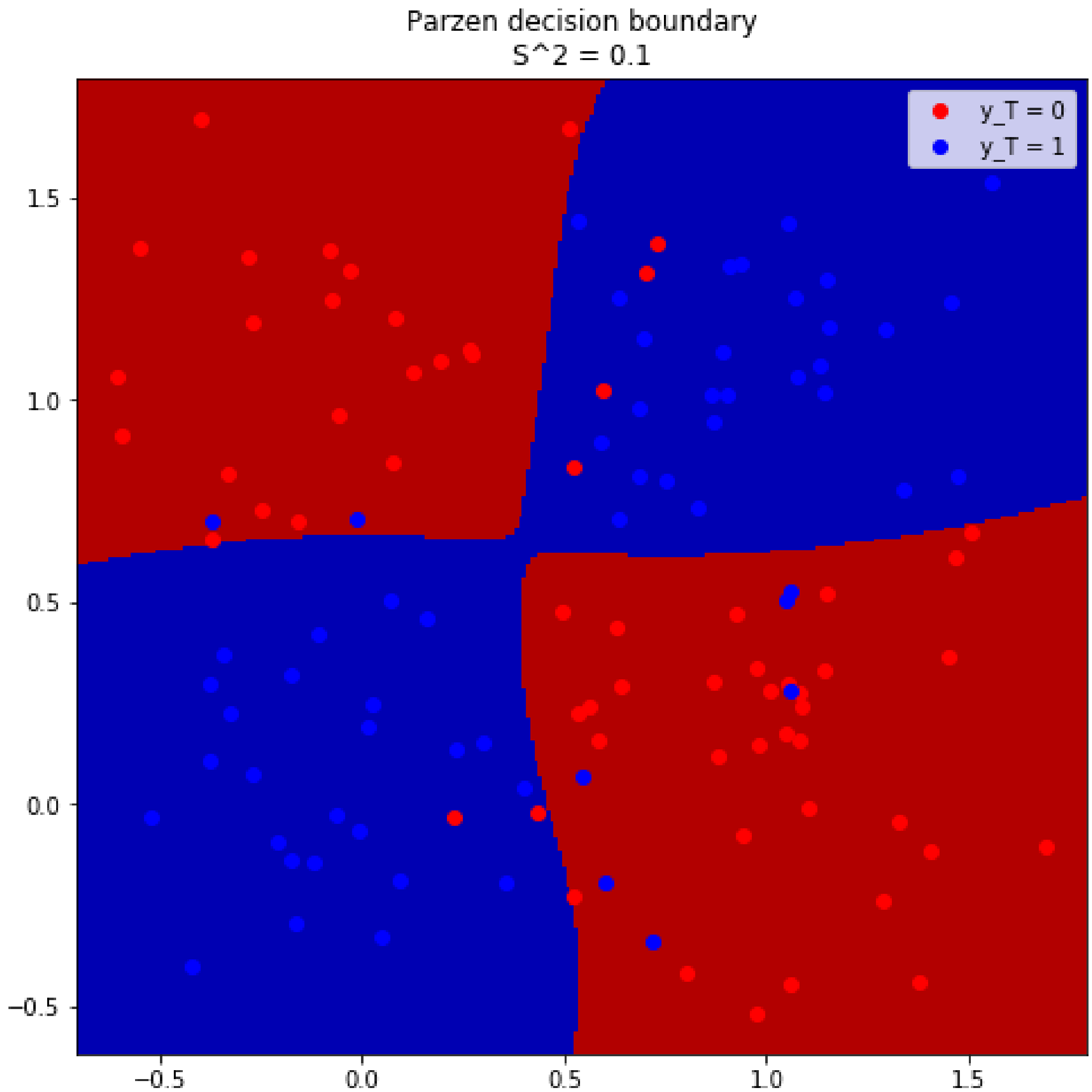
3 Exercise H7.3: "Parzen window" classifier

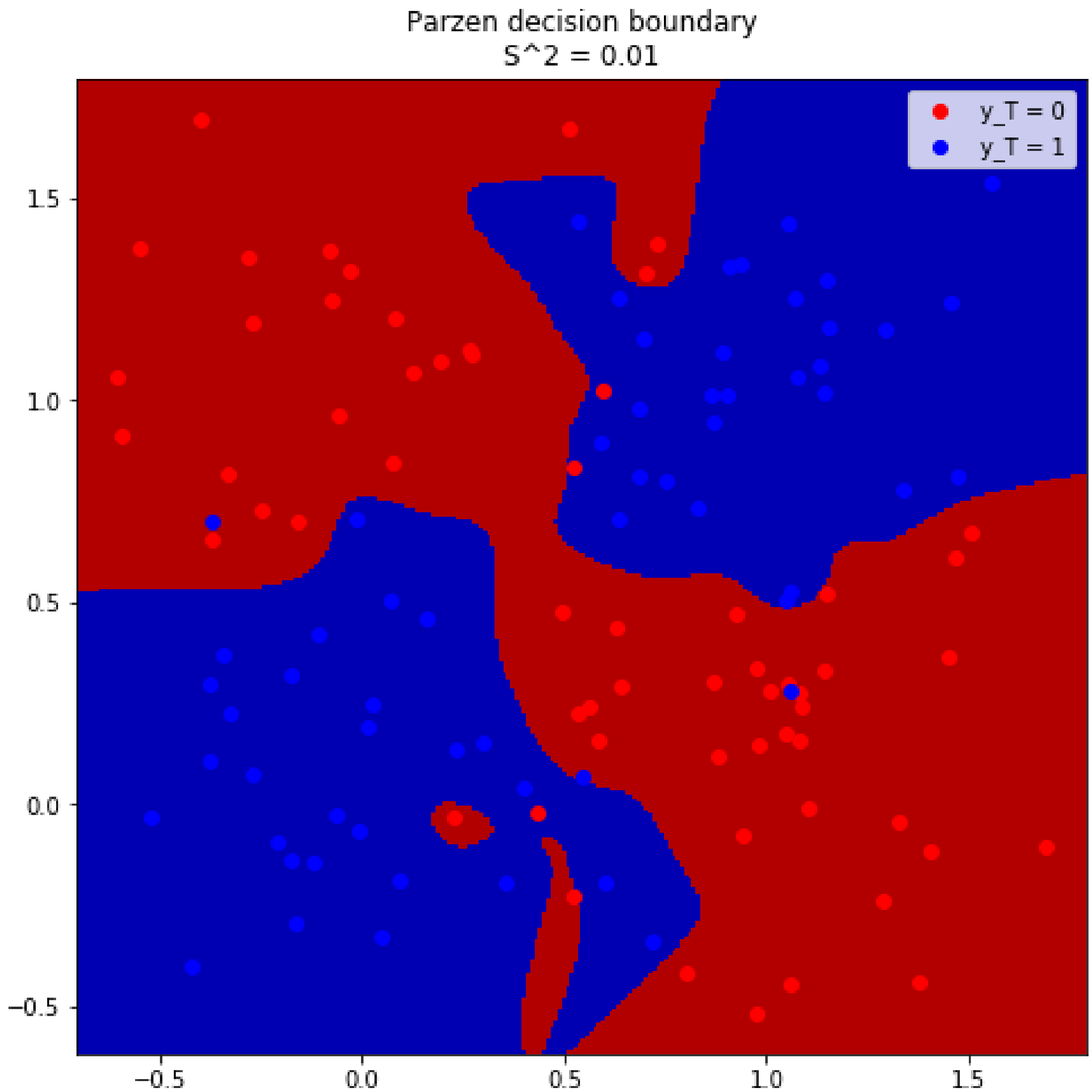
3.1 (a) Training patterns and the decision boundary

```
In [298]: def getParzenClassProb(x, s, samples, classes):
              m = np.subtract(samples[:,0:2],x) # help array
              d = np.sum(np.multiply(m,m), axis = 1) # distance between x and every point in sam
              w = np.exp(np.multiply(- 1 / (2*s), d)) # weight for every point in samples
              return np.divide(np.sum(np.multiply(classes, w[:,None]), axis = 0), np.sum(w))
```

```
def getParzenClassProbablities(x, y, s, samples, classes):
    n = x.size
    m = y.size
    probs = np.zeros((n,m,classes.shape[1]))
    for i in range(n):
        for j in range(m):
            probs[i,j,:] = getParzenClassProb([x[i], y[j]], s, samples, classes)
    return probs

x, y = getGrid(256, samples)

for S in [0.5, 0.1, 0.01]:
    probs = getParzenClassProbablities(x, y, S, samples, classes)
    plotClassifier(x, y, probs, samples, classes, 'Parzen decision boundary\nS^2 = ' +
```



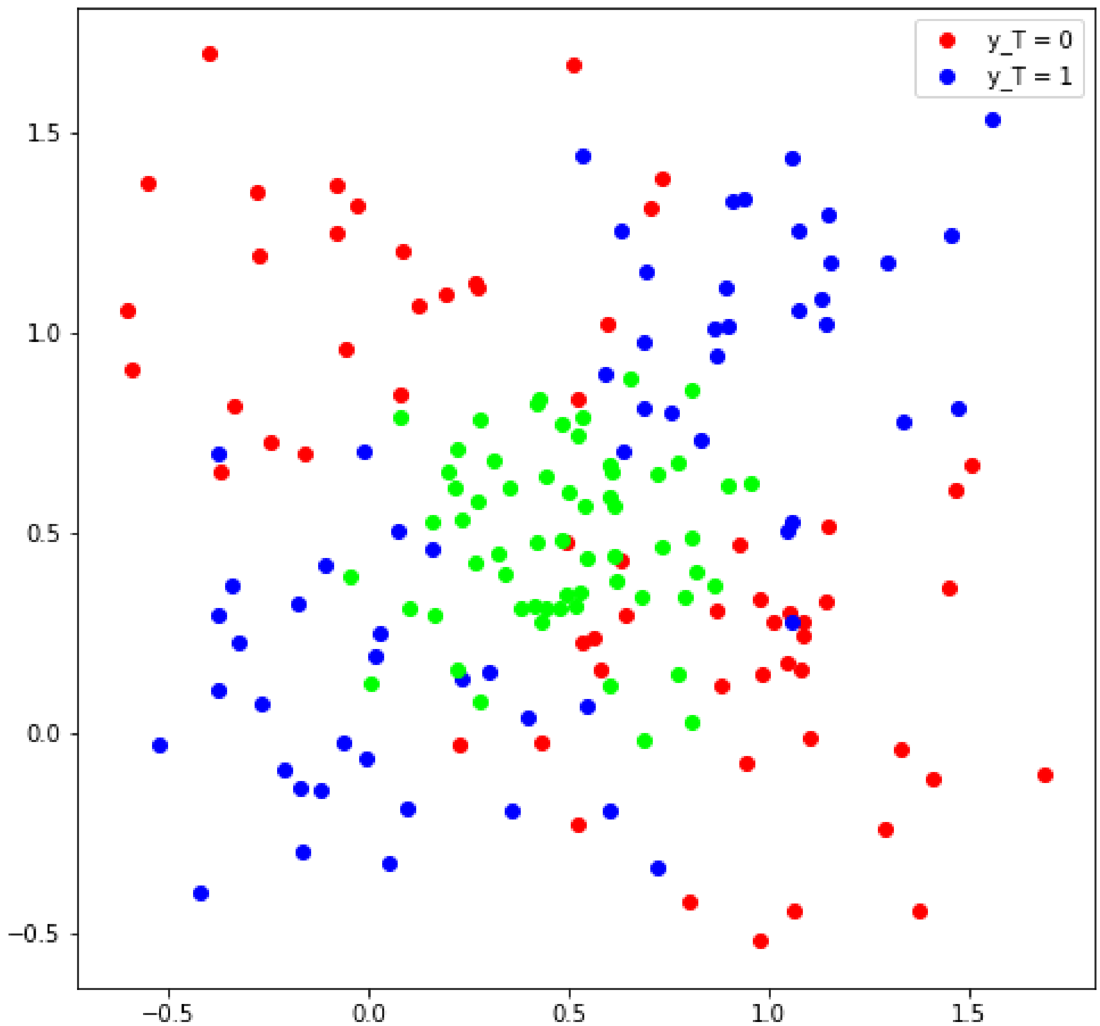

Parzen decision boundary
S^2 = 0.1
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5

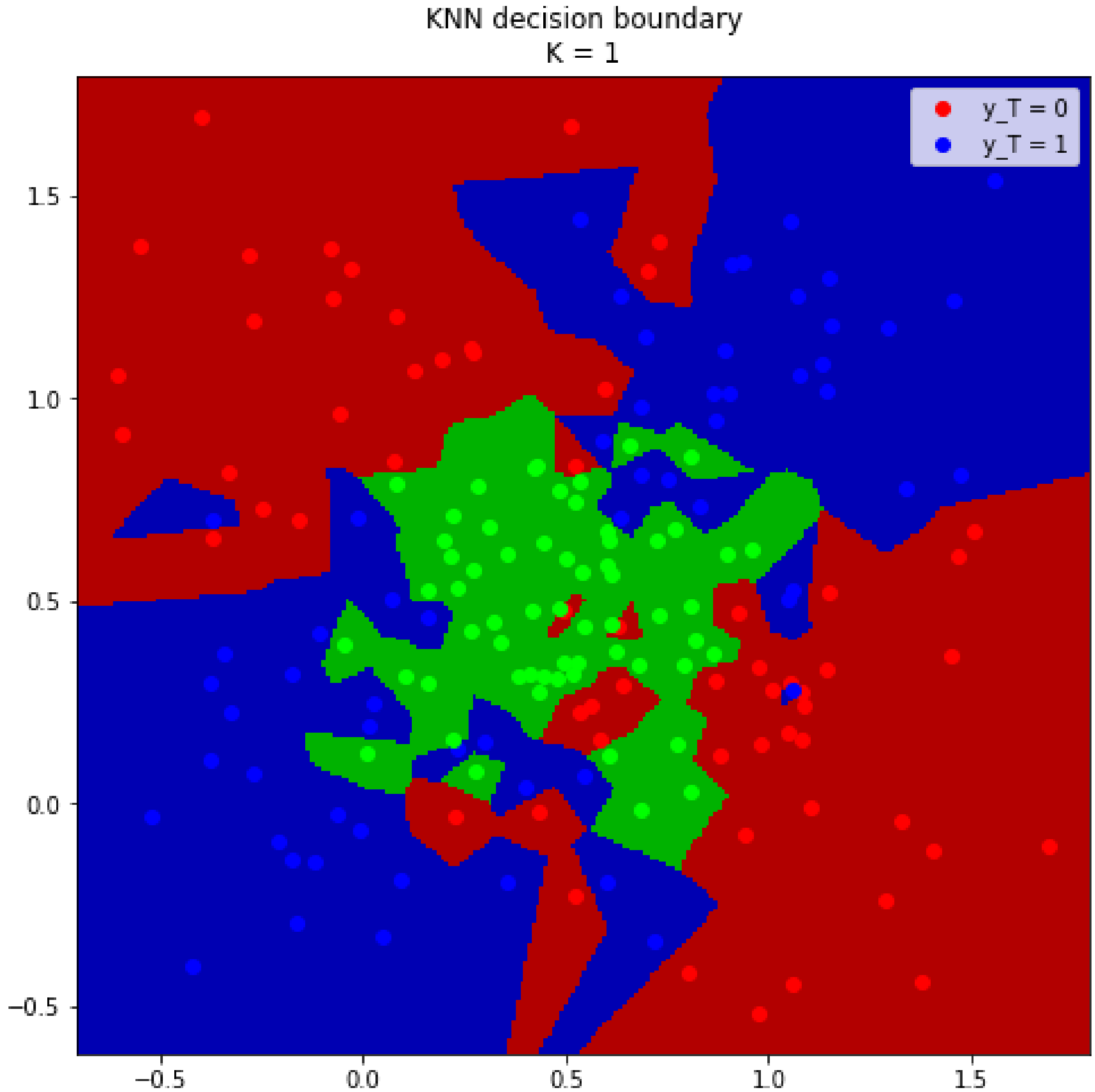


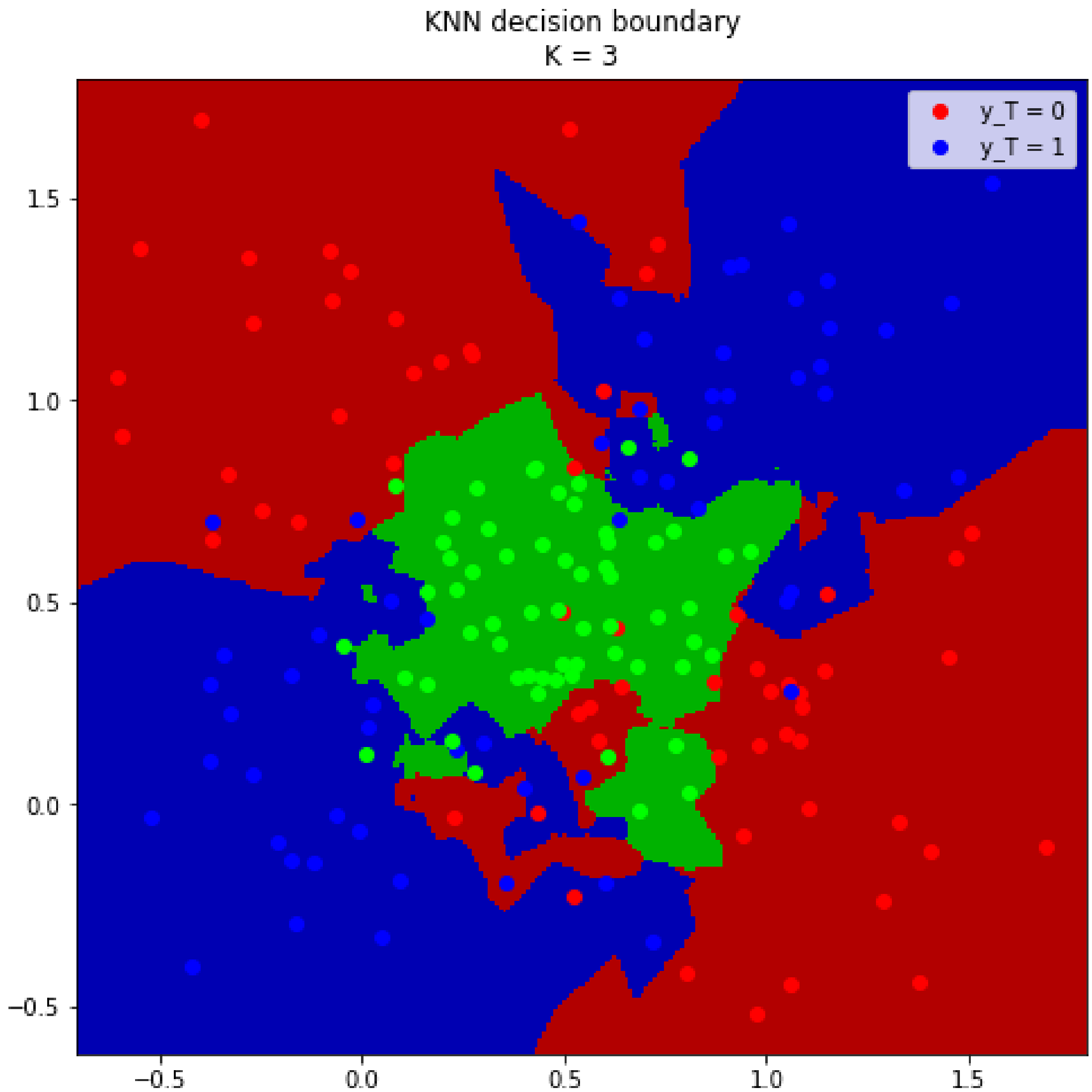
3.2 (b) 60 more data points

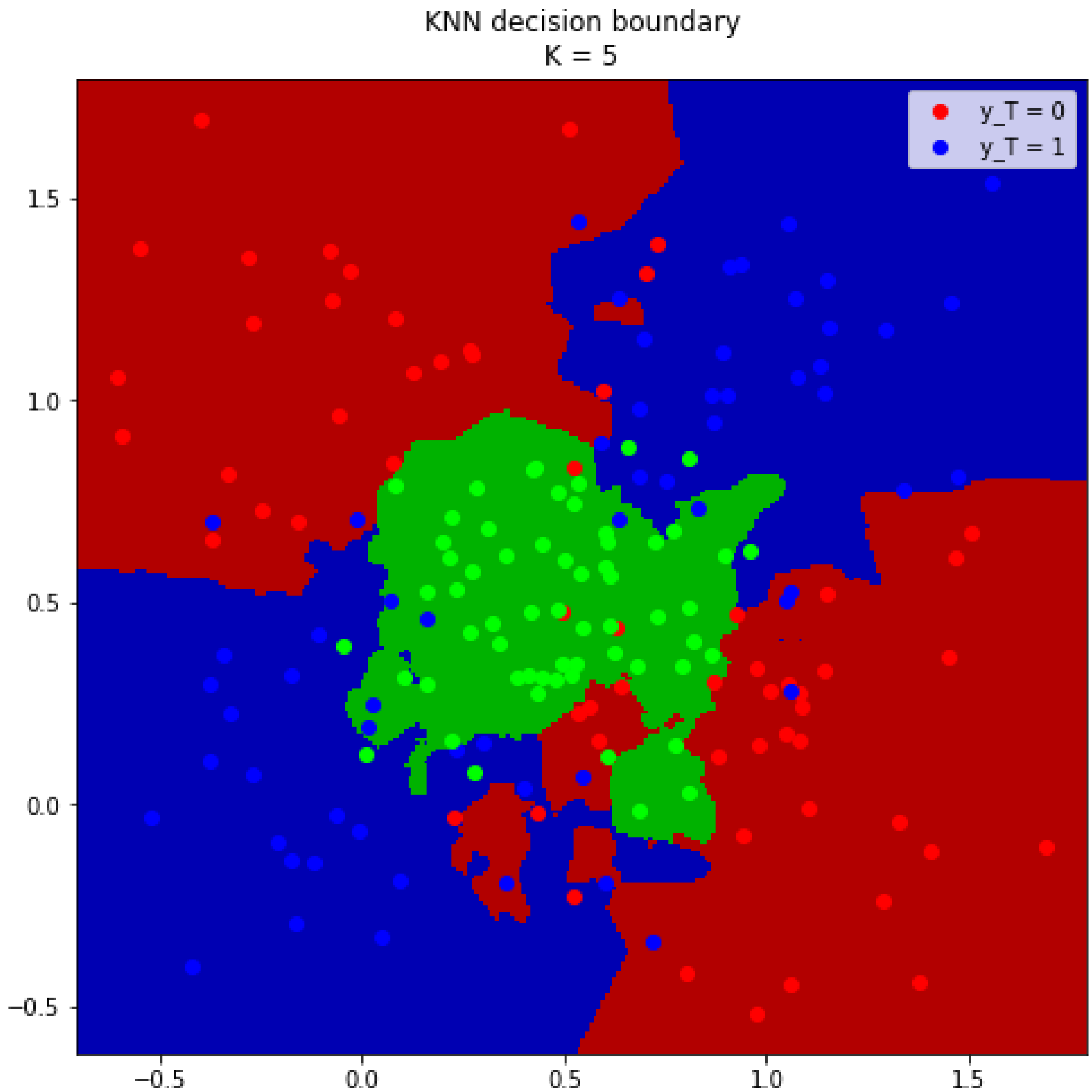
In [299]:
```
# generating points for 3rd class:
samples_3 = np.zeros((60,2))
classes_3 = np.zeros((60,3))

for i in range(60):
    samples_3[i] = np.random.multivariate_normal([0.5, 0.5], [[0.05, 0], [0, 0.05]])
    classes_3[i] = np.array([0,0,1])

# add new samples
ex_samples = np.append(samples, samples_3, axis = 0)
```

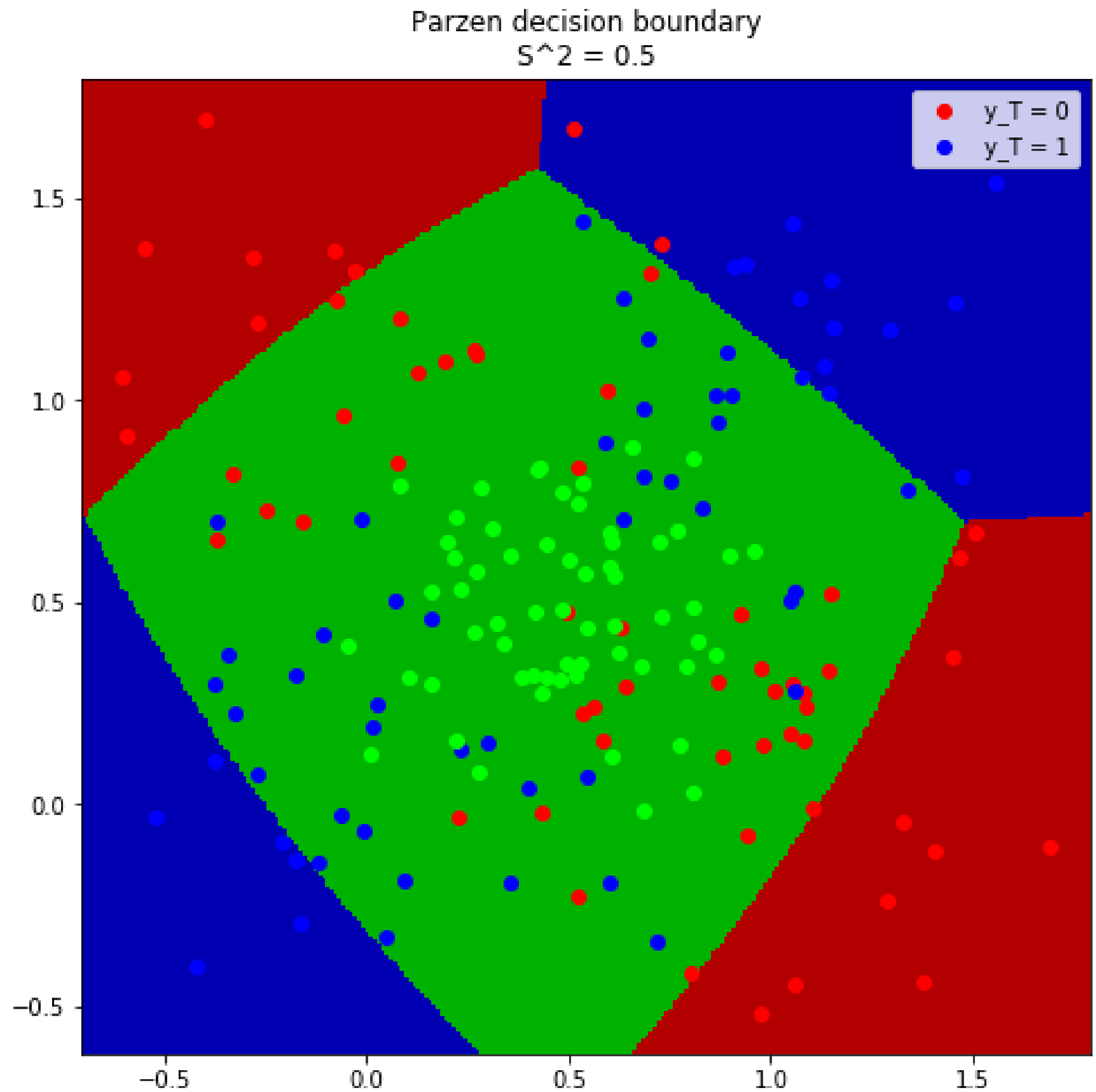
```
# add new classes
ex_classes = np.append(classes, np.zeros((classes.shape[0],1)), axis = 1) # add new cl
ex_classes = np.append(ex_classes, classes_3, axis = 0)

fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111)
plotSamples(ax, ex_samples, ex_classes)

x, y = getGrid(256, samples)

for K in [1,3,5]:
    probs = getKNNClassProbabilities(x, y, K, ex_samples, ex_classes)
    plotClassifier(x, y, probs, ex_samples, ex_classes, 'KNN decision boundary\nK = '
for S in [0.5, 0.1, 0.01]:
    probs = getParzenClassProbablities(x, y, S, ex_samples, ex_classes)
    plotClassifier(x, y, probs, ex_samples, ex_classes, 'Parzen decision boundary\nS^2
```

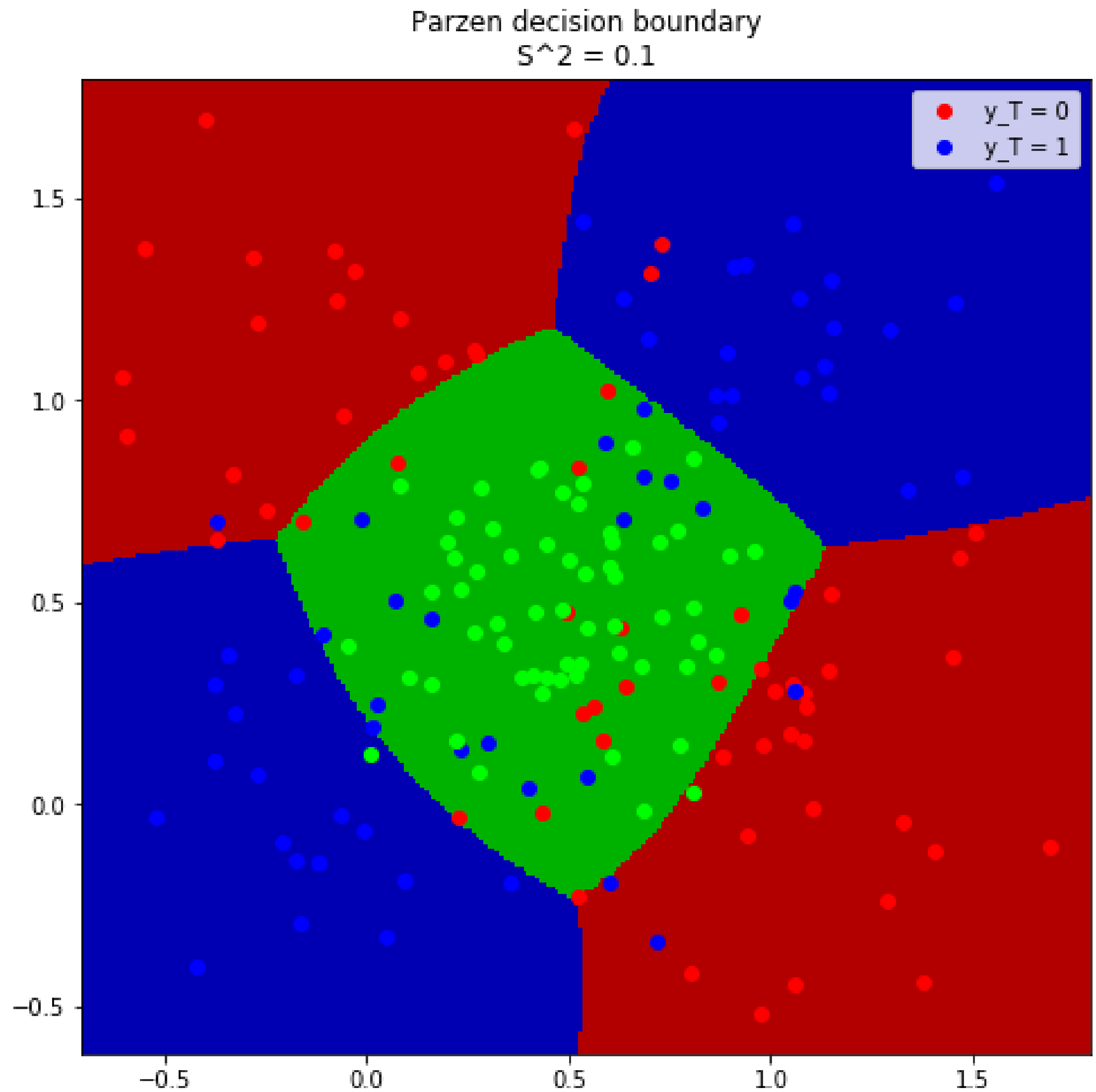



KNN decision boundary
K = 1
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5

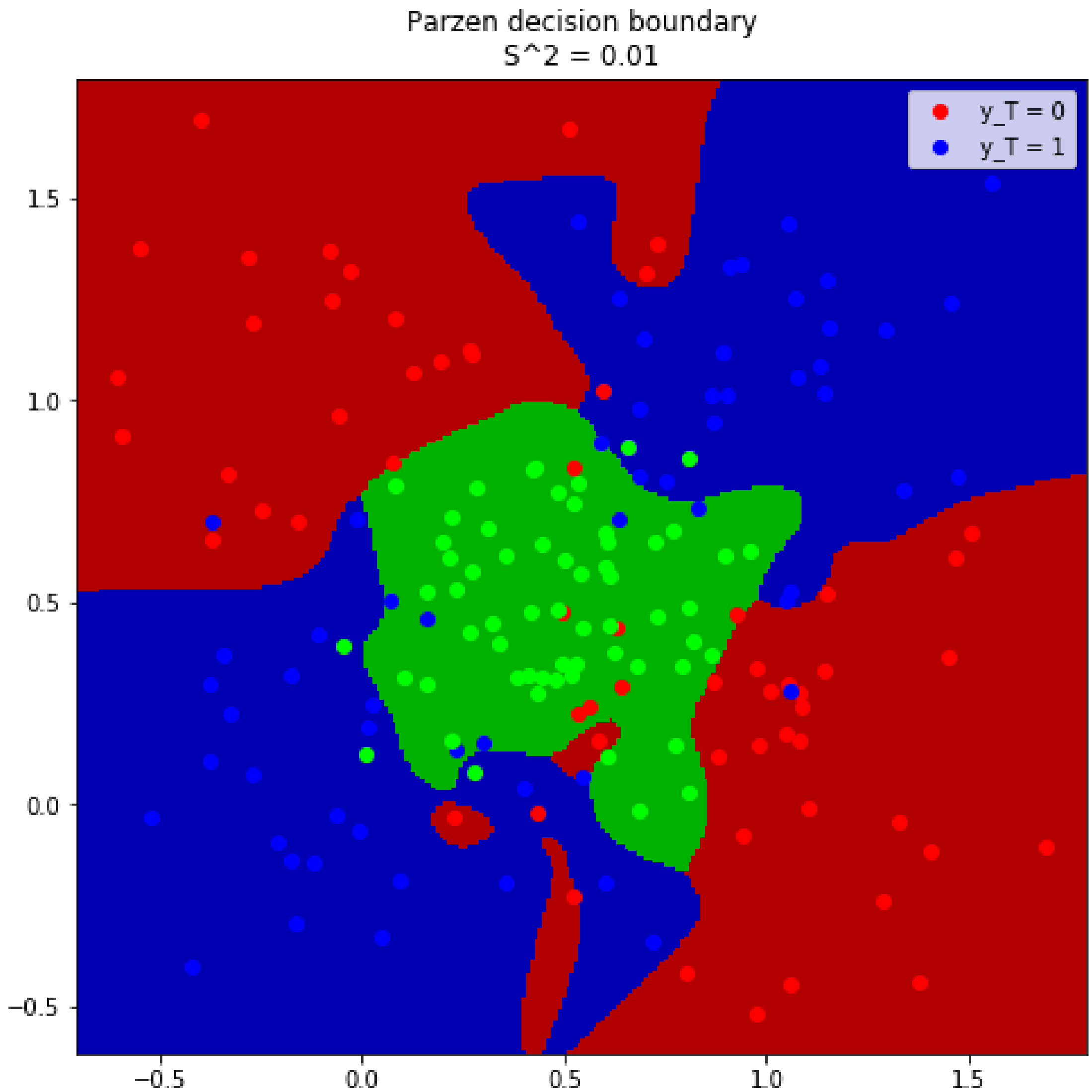

KNN decision boundary
K = 3
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5


KNN decision boundary
K = 5
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5


Parzen decision boundary
S^2 = 0.5
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5



Parzen decision boundary
S^2 = 0.1
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5



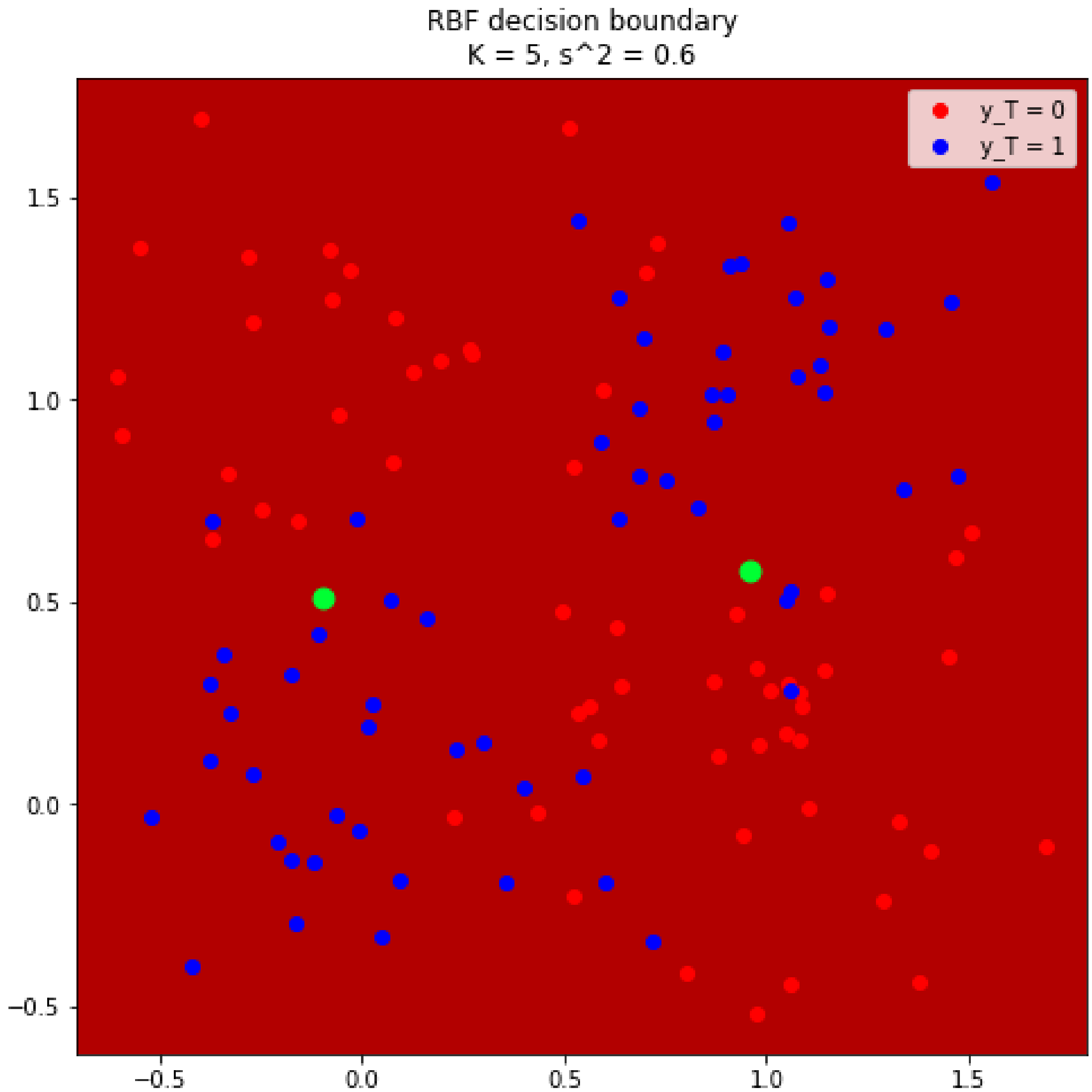
4 Exercise H7.4: RBF networks

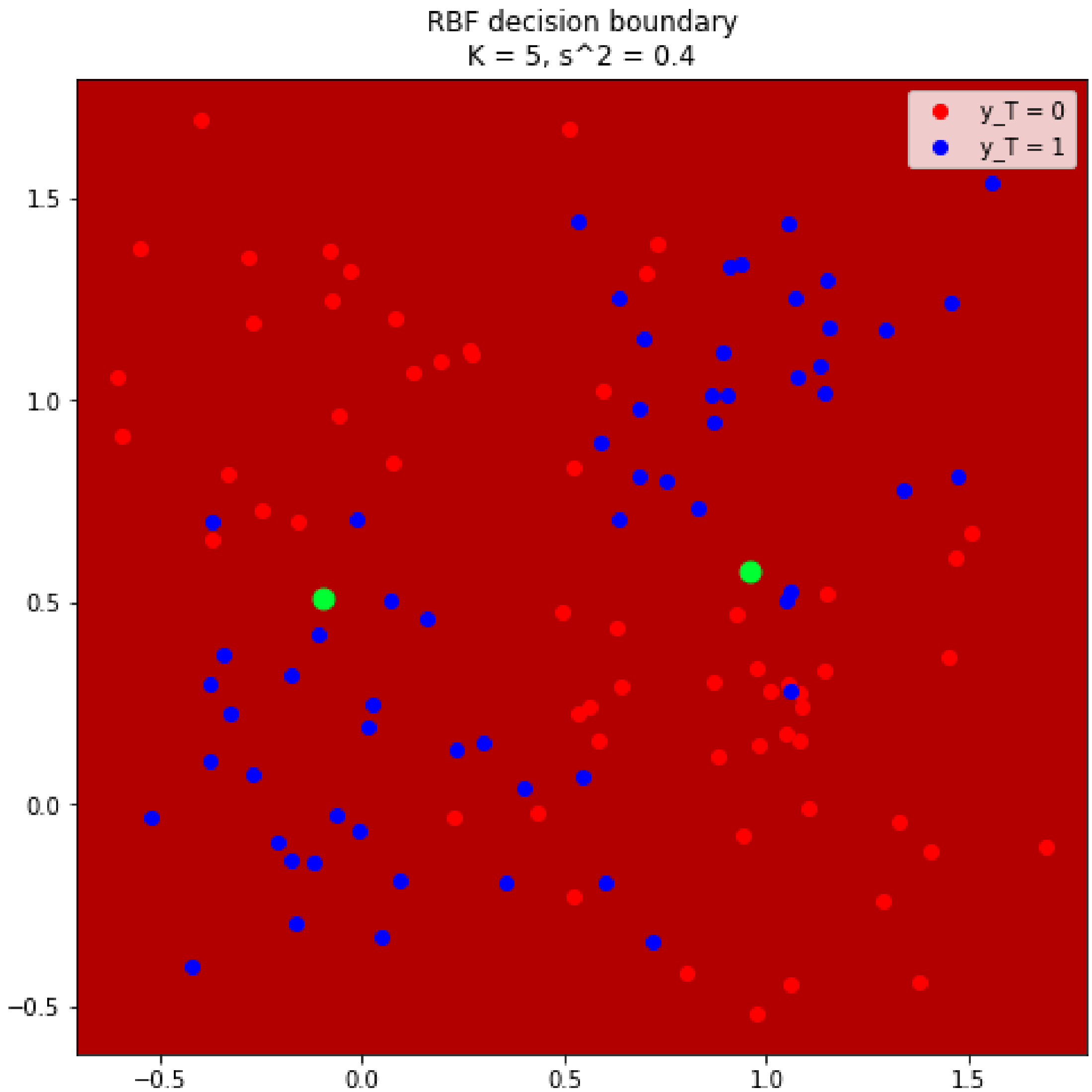
4.1 (a) Decision boundaries and training patterns

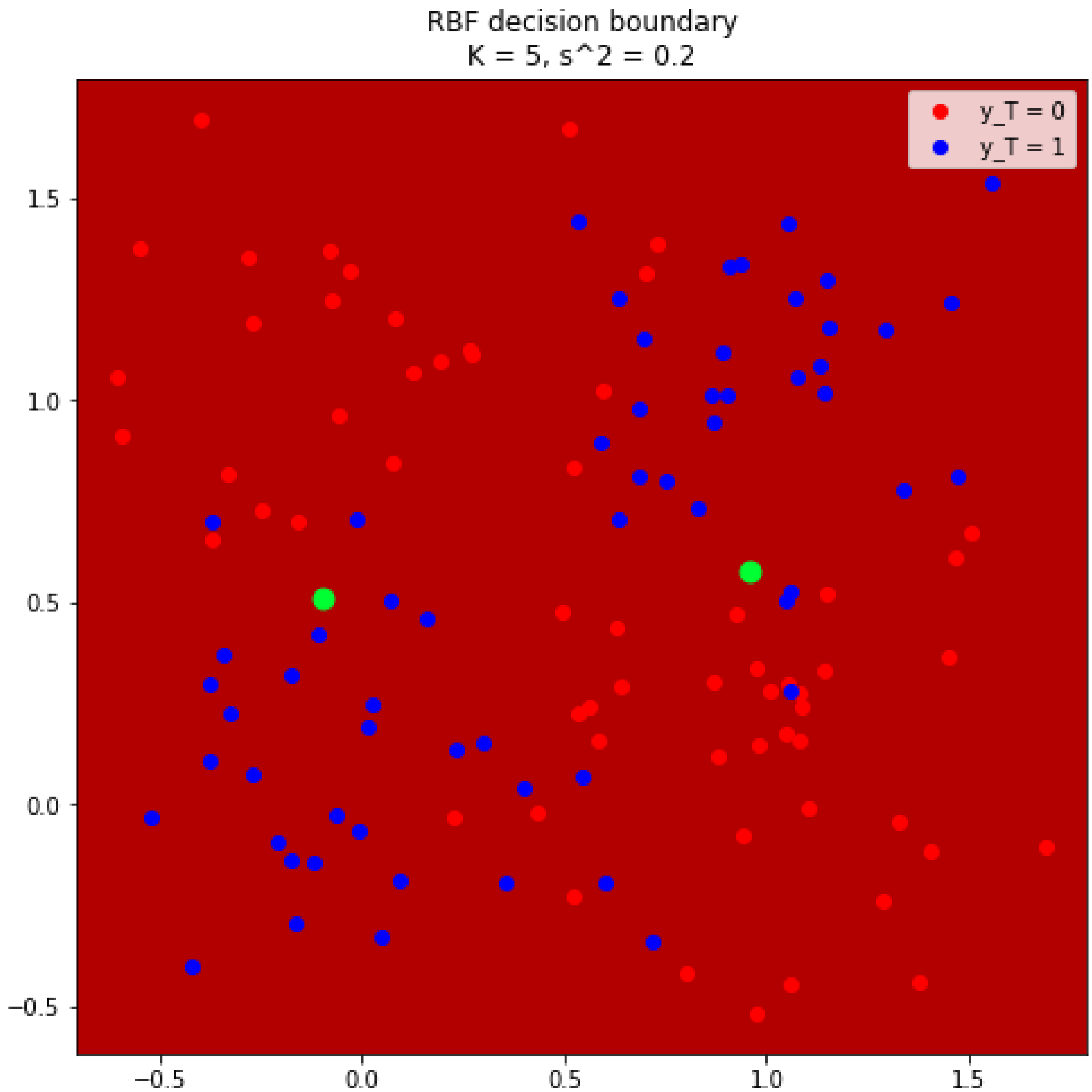
```
In [317]: def getKMeans(k, samples):
              return scipy.cluster.vq.kmeans(samples, k)[0]

          def sign(h):
              if (h>= 0):
                  return 1
              else:
                  return 0
              #return h    # <-- this works quite well
```

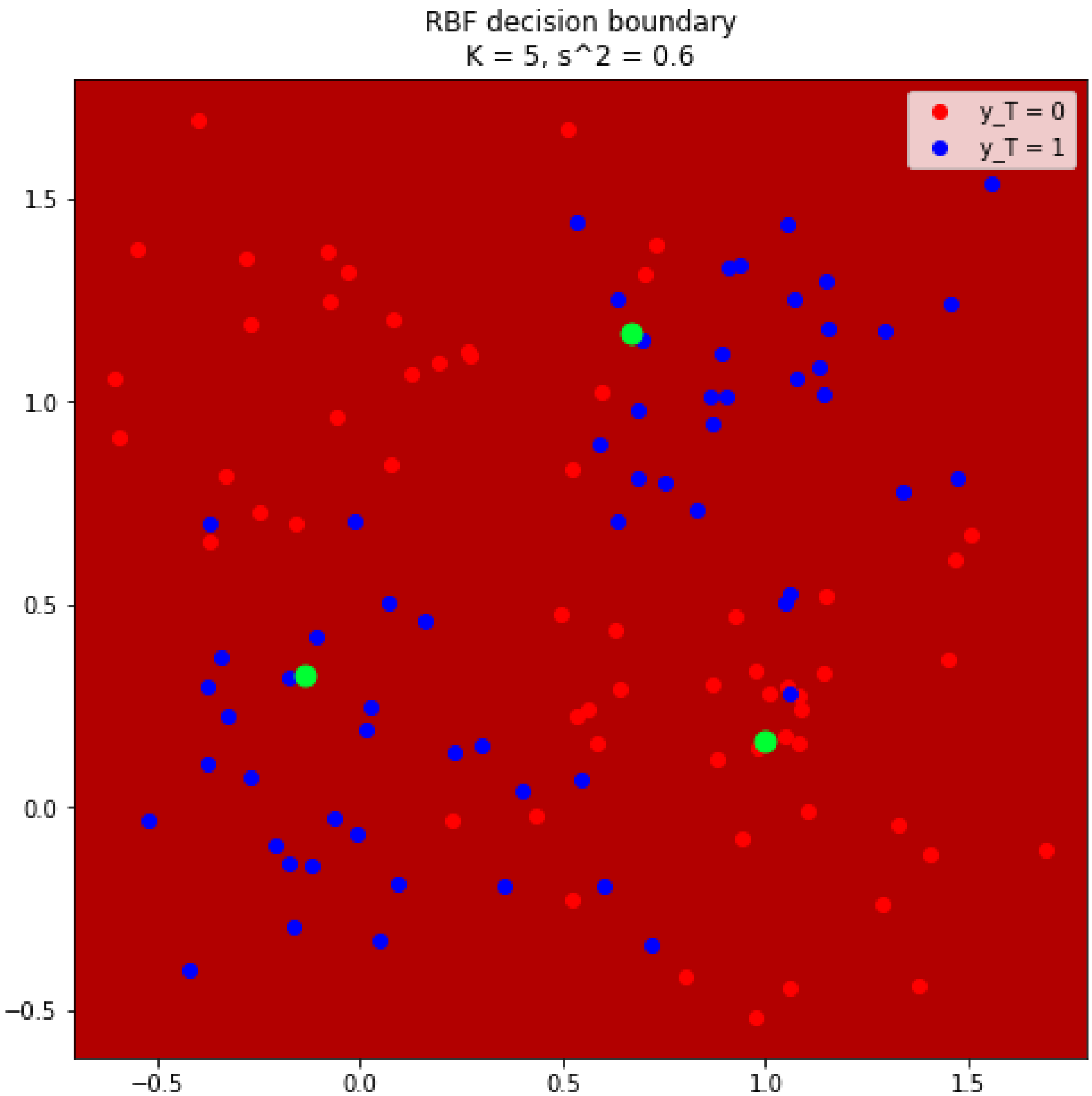
```python
def getWeight(T, s, samples, classes): # T - representives, s - RBF variance

    yT = classes[:,0]
    X = samples

    # calculate the RBF function values
    A = T[:,None,:] - X[None,:,:]
    A = np.sum(A * A, axis = 2)
    A = np.multiply(- 1 / (2 * s), A)
    A = np.exp(A)

    # add bias
    Phi = np.lib.pad(A, ((1,0),(0,0)), 'constant', constant_values=1)

    return np.dot(np.dot(np.linalg.inv(np.dot(Phi, Phi.T)), Phi), yT)

def getphi(T, s, x):

    A = T - x
    A = np.sum(A * A, axis = 1)
    A = np.multiply(- 1 / (2 * s), A)
    A = np.exp(A)

    return np.append([1], A)

def RBFPrediction(T, s, w, x):
    return sign(np.dot(w.T, getphi(T, s, x)))
    #return np.dot(w.T, getphi(T, s, x))

def getRBFClassProbabilities(x, y, T, s, w):
    n = x.size
    m = y.size
    probs = np.zeros((n,m,2))
    for i in range(m):
        for j in range(n):
            probs[i,j,0] = RBFPrediction(T, s, w, [x[j], y[i]])
            probs[i,j,1] = 1 - probs[i,j,0]

    return probs

x, y = getGrid(256, samples)

for k in [2,3,4]:
    T = getKMeans(k, samples)
    for s in [0.6,0.4, 0.2]:
        w = getWeight(T, s, samples, classes)
        probs = getRBFClassProbabilities(x, y, T, 0.4, w)
```

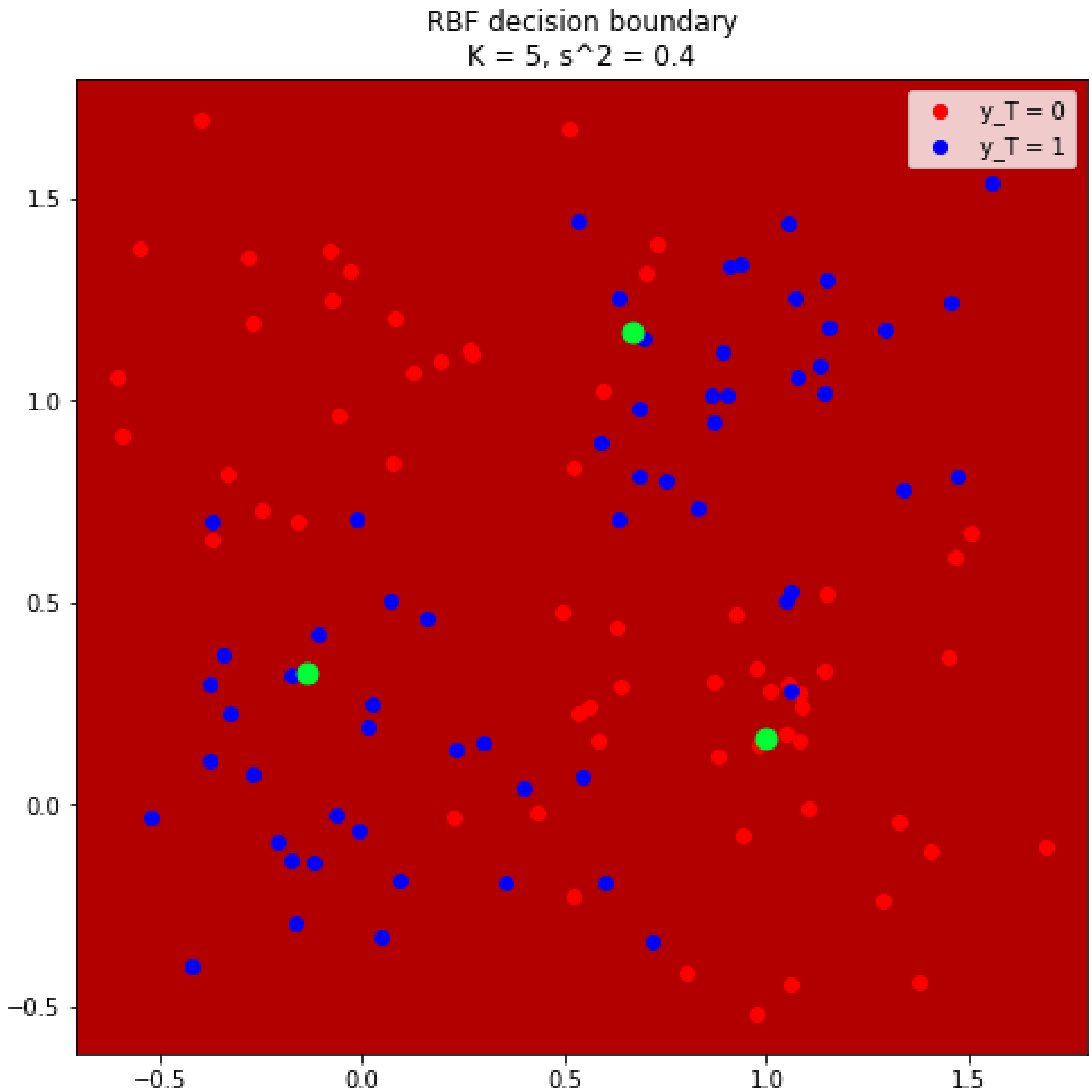
```
ax = plotClassifier(x, y, probs, samples, classes, 'RBF decision boundary\nK =
ax.scatter(T[:,0],T[:,1],c='#00ff33', s=80)
```

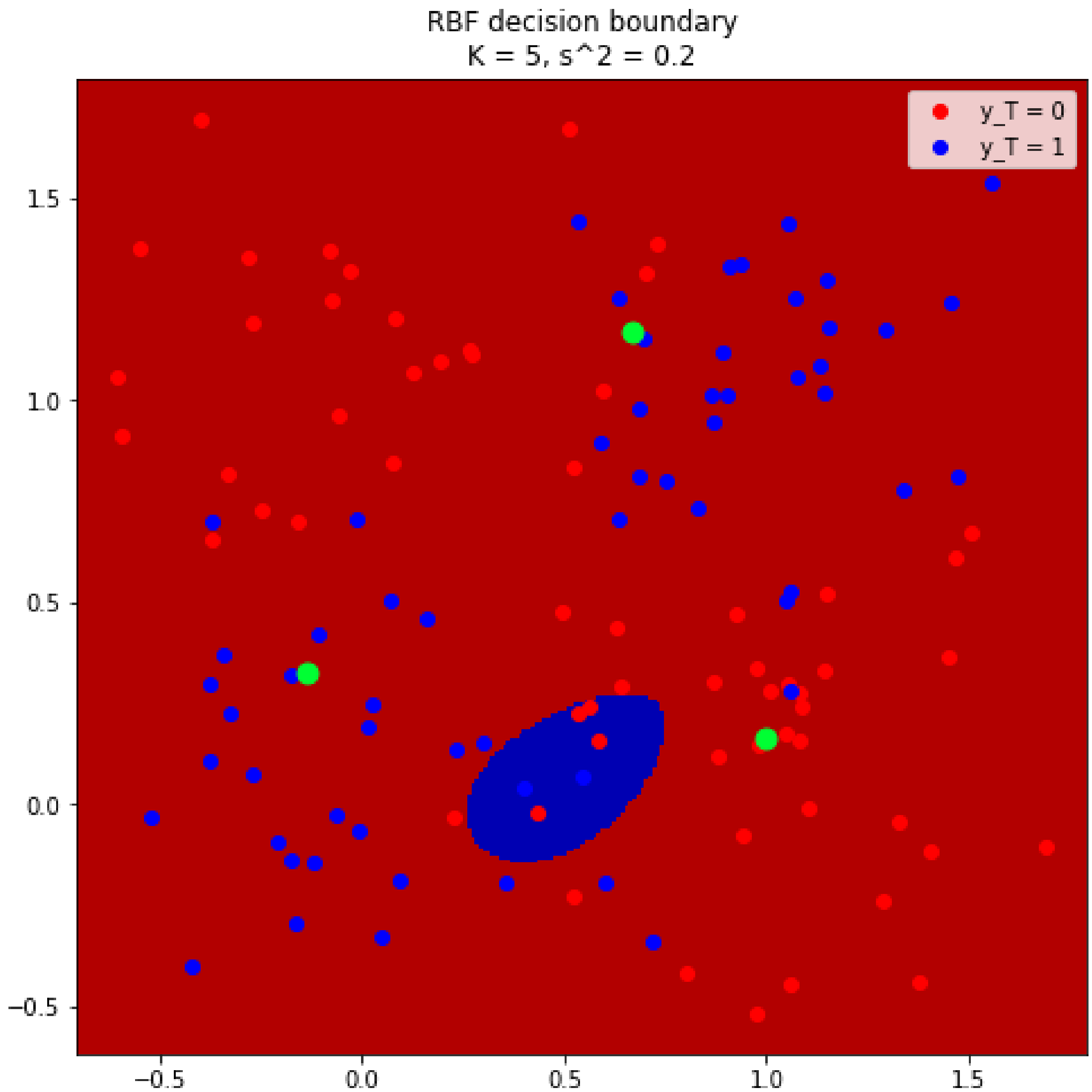


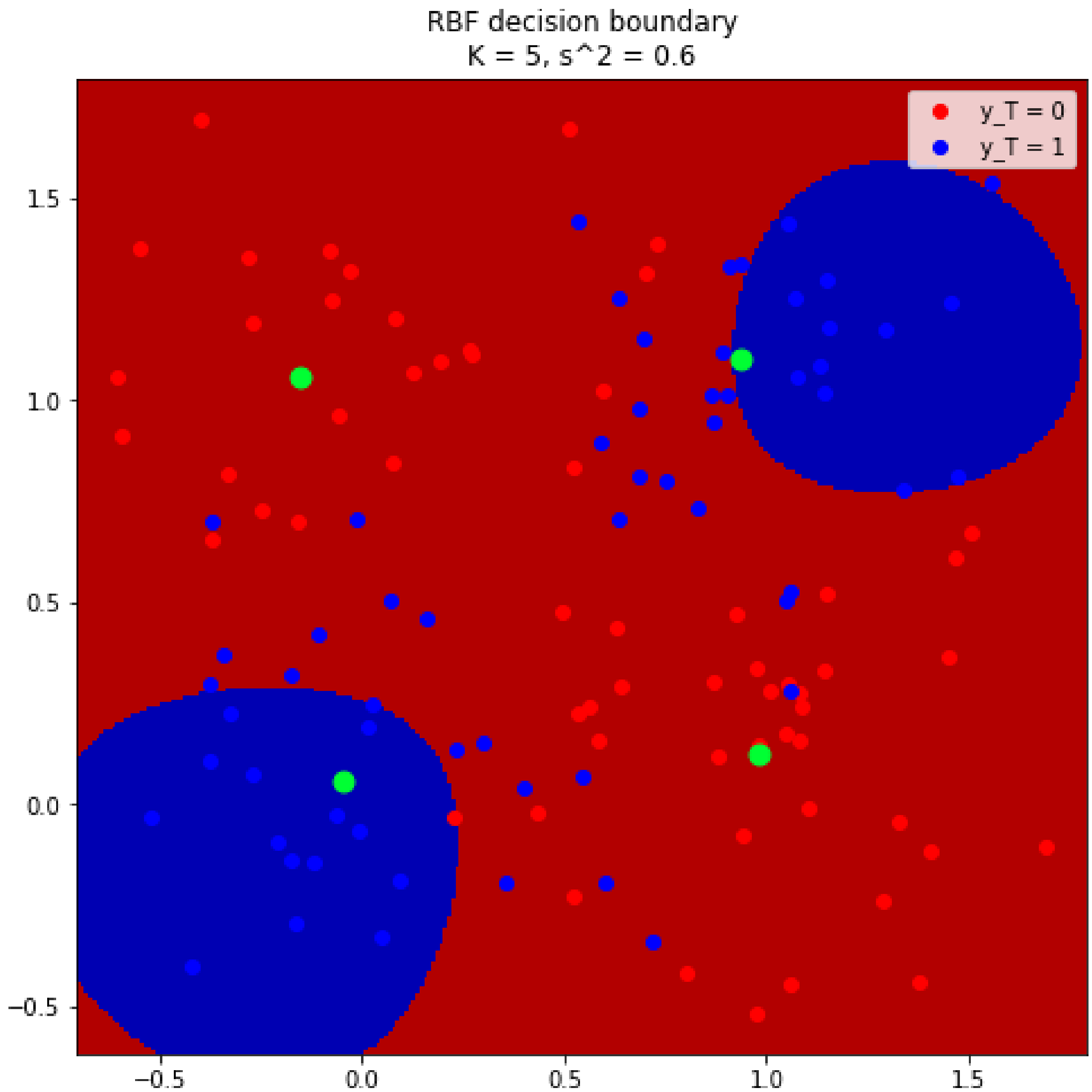

RBF decision boundary
K = 5, s^2 = 0.4
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5

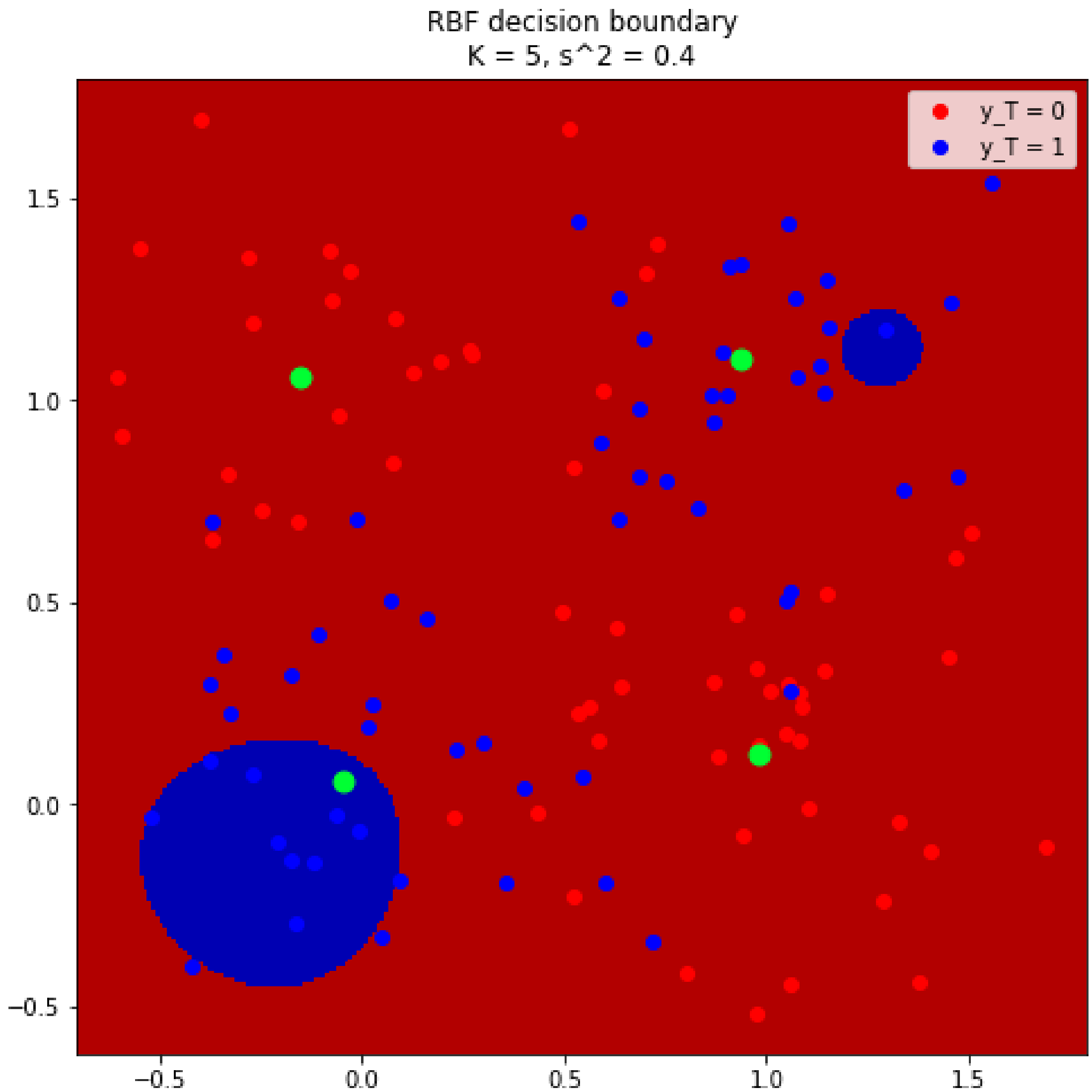

RBF decision boundary
K = 5, s^2 = 0.2
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5

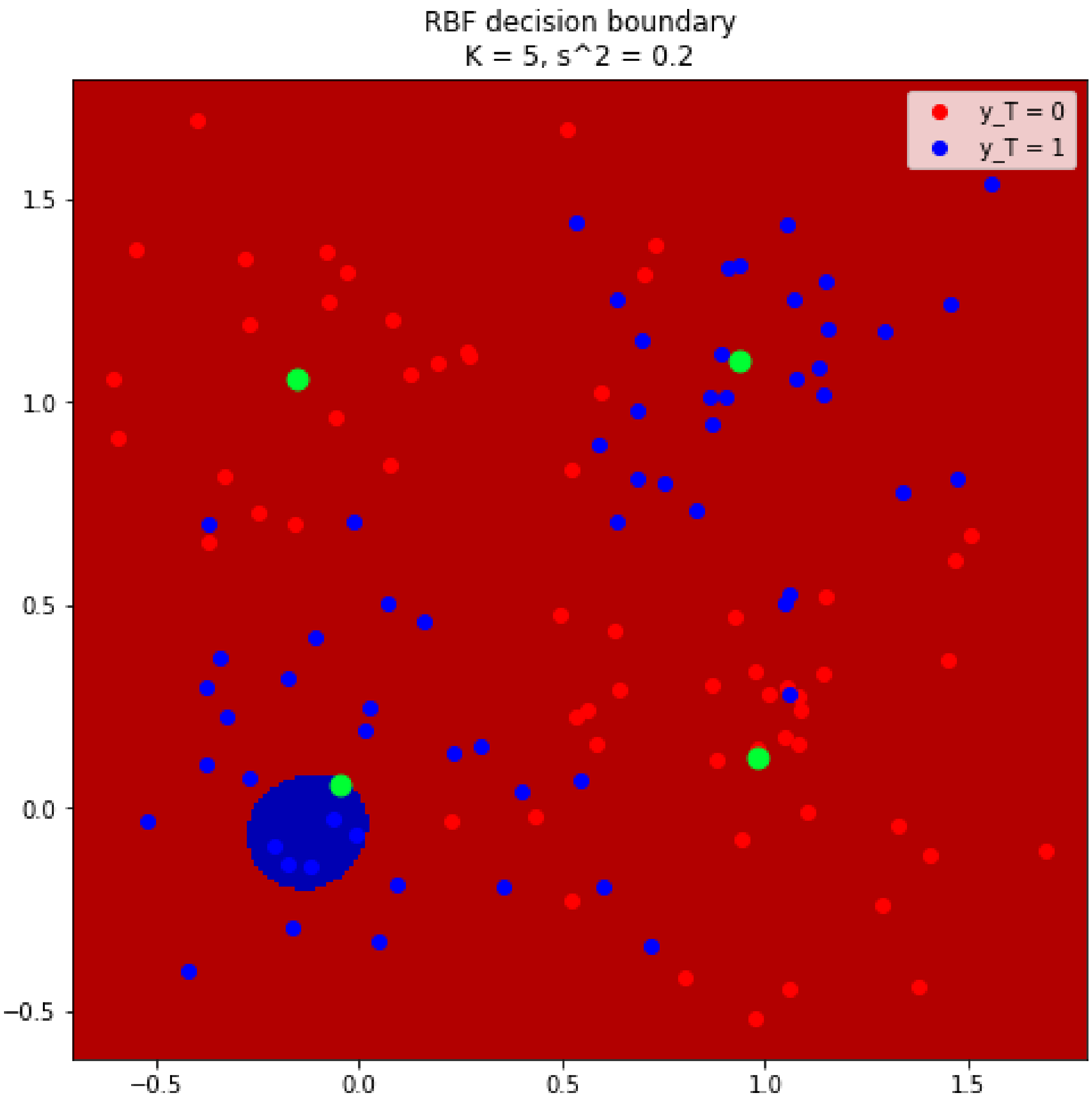

RBF decision boundary
K = 5, s^2 = 0.6
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5


RBF decision boundary
K = 5, s^2 = 0.4
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5


RBF decision boundary
K = 5, s^2 = 0.2
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5


RBF decision boundary
K = 5, s^2 = 0.6
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5


RBF decision boundary
K = 5, s^2 = 0.4
y_T = 0
y_T = 1
1.5
1.0
0.5
0.0
−0.5
−0.5
0.0
0.5
1.0
1.5



4.2 (b) RBF-network with 2 RBFs