C4N2

November 8, 2017

1 Exercise Sheet 3: Multilayer Perceptrons and Backpropagation Algorithm

Machine Intelligence 1
WS 2017/18, Obermayer/Augustin
due: 2017-11-08
Group: C4N2
Used Python version 3.5.2 (https://www.continuum.io/downloads, http://ipython.org/install.html)

```
In [1]: import sys
        print(sys.version)
```

```
3.5.2 (default, Sep 14 2017, 22:51:06)
[GCC 5.4.0 20160609]
```

1.1 Exercise H3.2: MLP Regression

```
In [2]: #Necessary libraries

        import numpy as np
        import matplotlib.pyplot as plt
        import matplotlib

        %matplotlib inline
```

```
In [3]: #Reading data
        regression_data = np.loadtxt('./RegressionData.txt')
        N = regression_data.shape[0]
        X = regression_data[:, 0].reshape(1, regression_data.shape[0])
        Y = regression_data[:, 1].reshape(1, regression_data.shape[0])

        #Initialization
        def init_params():
            W1 = np.random.rand(1, 3) - 0.5
            bias1 = np.random.rand(3, 1) - 0.5
            W2 = np.random.rand(3,1) - 0.5
```

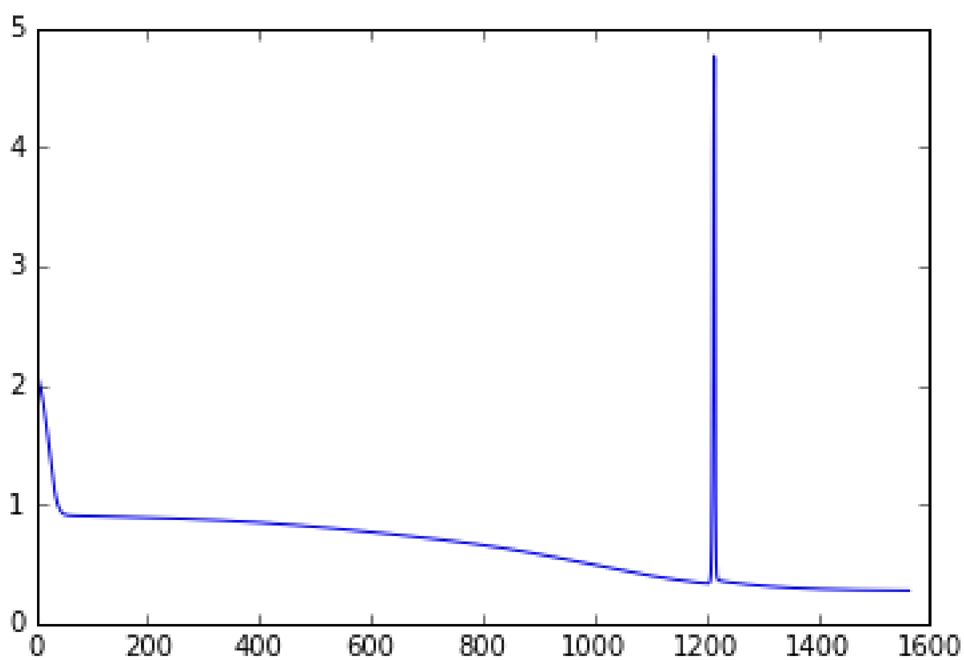
```
    bias2 = np.random.rand(1) - 0.5
    print (bias2)
    std_deviation = 0.25
    return W1, bias1, W2, bias2, std_deviation

#1. Forward propagation
def forward_propagation(W1, W2, bias1, bias2, X):
    hidden_input = W1.T.dot(X) - bias1
    hidden_output = np.tanh(hidden_input)
    output = W2.T.dot(hidden_output) - bias2
    return hidden_output, output

#2. Output error
def output_error(Y_estimate, Y):
    return 0.5*(np.sum((Y_estimate - Y)**2))

#3. Back propagation and 4. Weight Update
def bp():
    W1, bias1, W2, bias2, std_deviation = init_params()
    print (bias2)
    t = 0
    Errors = []
    Error_tmp = 0
    learning_rate = 0.5
    Y_estimate = 0
    while(t < 3000):
        hidden_output, Y_estimate = forward_propagation(W1, W2, bias1, bias2, X)
        if(len(Errors) == 0):
            Error_tmp = output_error(Y_estimate, Y)
        else:
            tmp_var = output_error(Y_estimate, Y)
            if(abs(Error_tmp - tmp_var) / tmp_var < 10e-5):
                break;
            else:
                Error_tmp = tmp_var
        Errors.append(Error_tmp)
        hidden_derivatives = - hidden_output**2 + 1
        error_output_layer = Y_estimate - Y
        error_hidden_layer = hidden_derivatives * W2.dot(error_output_layer)
        derivative_input_layer = - X.dot(error_hidden_layer.T) / (N * 1.0)
        derivative_hidden_layer = - hidden_output.dot(error_output_layer.T) / (N * 1.0)
        W1 = W1 + derivative_input_layer * learning_rate
        W2 = W2 + derivative_hidden_layer * learning_rate
        bias1 = bias1 + np.sum(error_hidden_layer, axis=1).reshape(3,1) * learning_rate/
        bias2 = bias2 + np.sum(error_output_layer)* learning_rate/(N*1.0)
        t = t + 1
    plt.plot(np.arange(0, len(Errors)), Errors)
    plt.show()
```

```
    return W1, bias1, W2, bias2

#Plotting the output of hidden units for all inputs
def plot_hidden_output():
    hidden_layer_output, y_pred = forward_propagation(W1, W2, bias1, bias2, X)
    plt.scatter(X.T, hidden_layer_output[0, :].T, color='r', label="w: " + str(W1[:, 0])
    plt.scatter(X.T, hidden_layer_output[1, :].T, color='g', label="w: " + str(W1[:, 1])
    plt.scatter(X.T, hidden_layer_output[2, :].T, color='b', label="w: " + str(W1[:, 2])
    plt.legend()
    plt.show()

#Plotting the output values over the input space
def plot_prediction():
    hidden_layer_output, y_pred = forward_propagation(W1, W2, bias1, bias2, X)
    plt.scatter(X.T, Y.T, color='r', marker="o", label="train")
    plt.scatter(X.T, y_pred.T, color='b', marker="x", alpha= 0.5, label="pred")
    plt.legend()
    plt.show()
```

1.1.1 a)

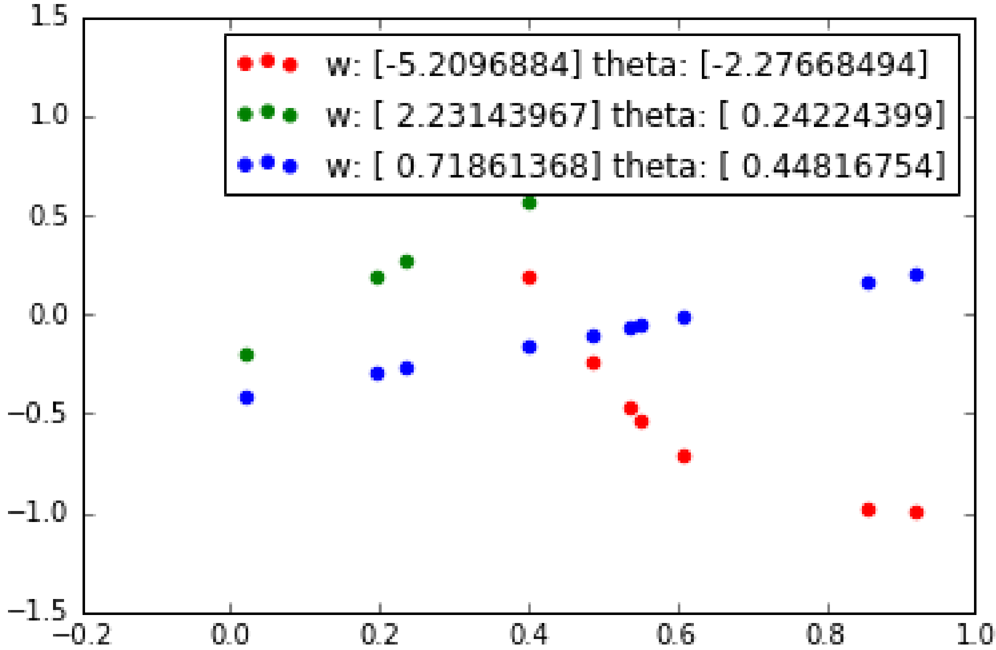
```
In [4]: W1, bias1, W2, bias2 = bp()
```

```
[-0.03715391]
[-0.03715391]
```



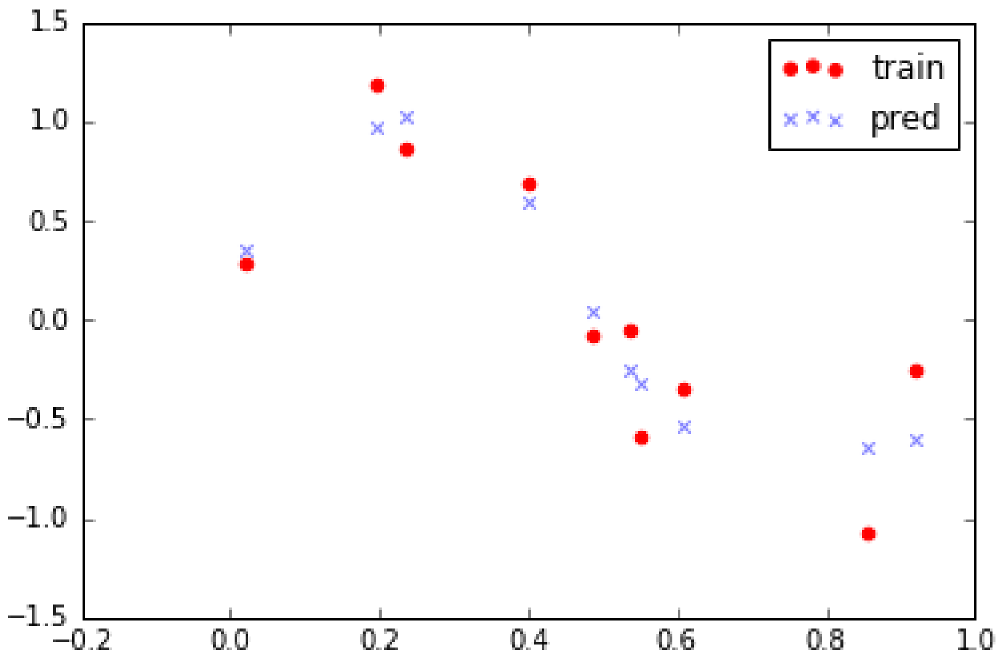
1.1.2 b)

```
In [5]: plot_hidden_output()
```



1.1.3 c)

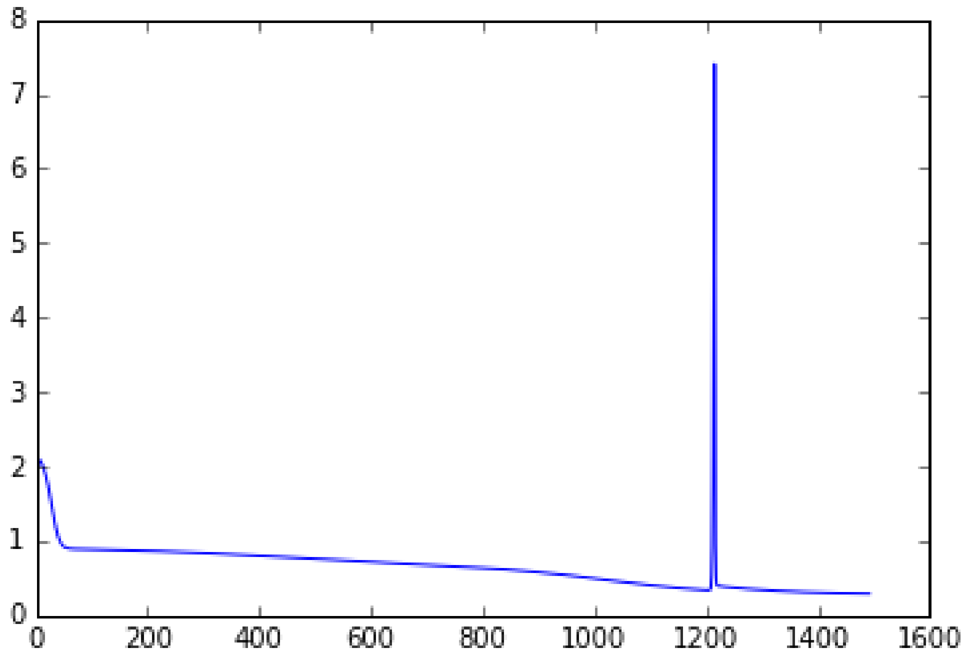
```
In [6]: plot_prediction()
```

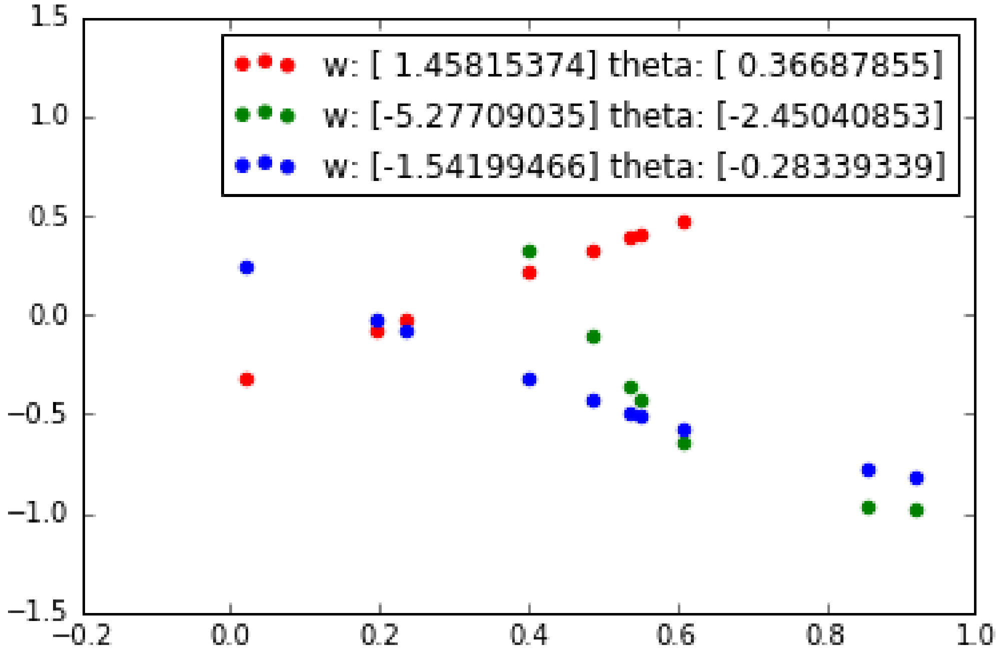


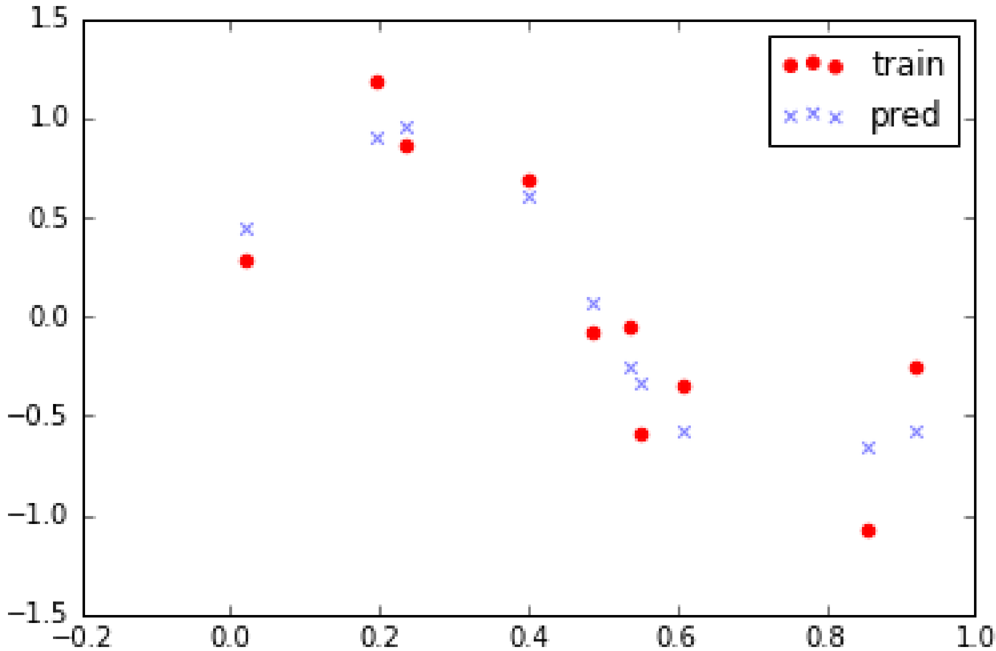
1.1.4 d)

```
In [7]: print ('First initial conditions')
        W1, bias1, W2, bias2 = bp()
        plot_hidden_output()
        plot_prediction()

        print ('Different random initial conditions')
        W1, bias1, W2, bias2 = bp()
        plot_hidden_output()
        plot_prediction()
```

```
First initial conditions
[ 0.3381478]
[ 0.3381478]
```

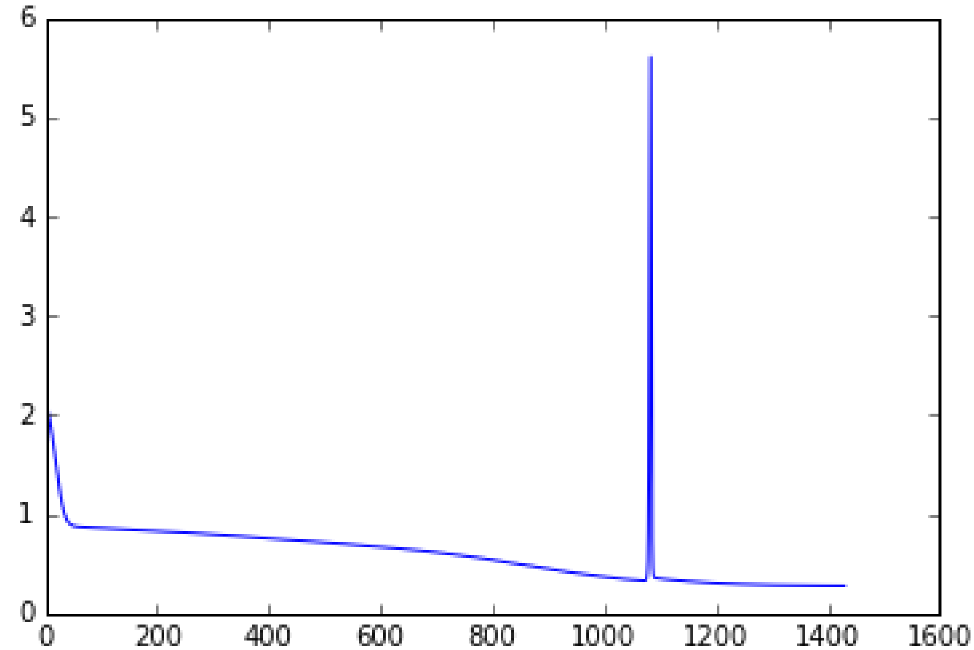


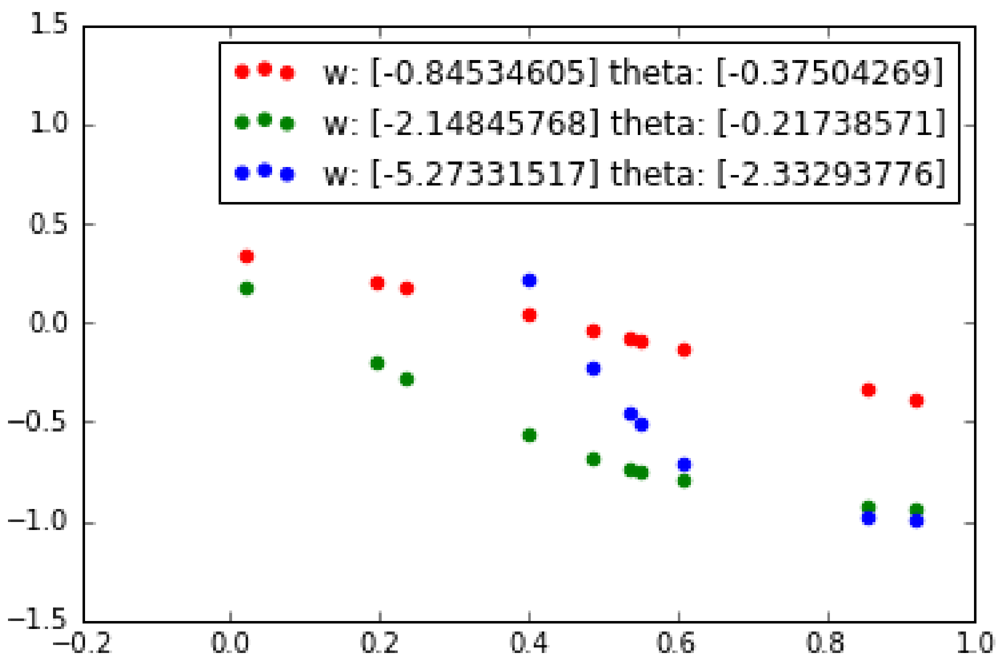


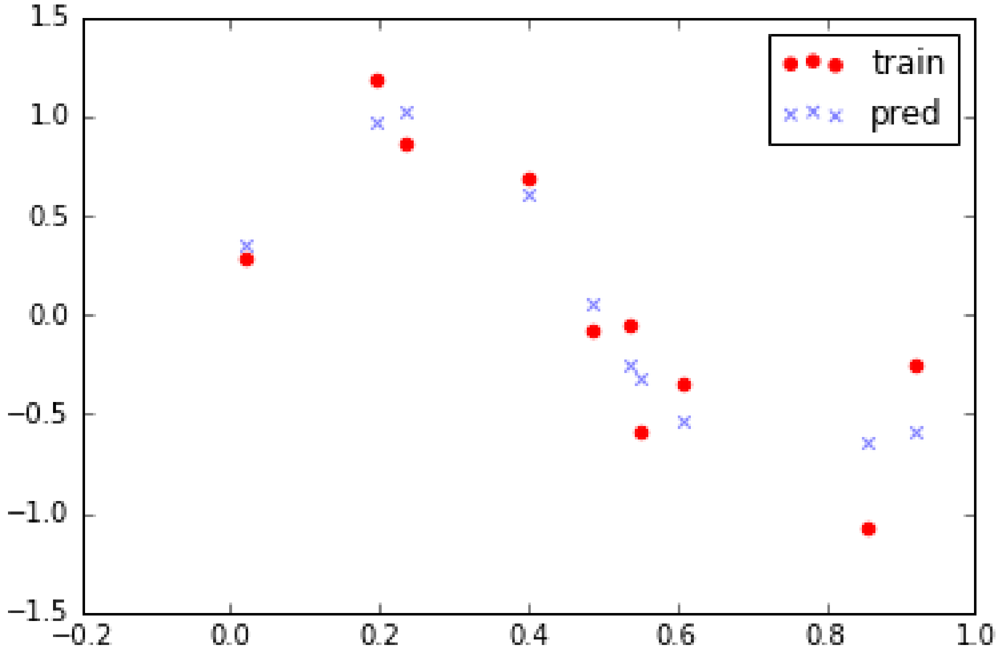



```
Different random initial conditions
[-0.37201534]
```

[-0.37201534]


6
5
4
3
2
1
0
0
200
400
600
800
1000
1200
1400
1600


w: [-0.84534605] theta: [-0.37504269]
w: [-2.14845768] theta: [-0.21738571]
w: [-5.27331517] theta: [-2.33293776]
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−0.2
0.0
0.2
0.4
0.6
0.8
1.0



Although the overall prediction of y is very similar in both MLPs, the output functions of the hidden neurons are completely different. This is caused by the random initialization of the weights: The weights are the starting point for gradient descent, which based on it finds different local optima.

1.1.5 e)

We know that the noise is Gaussian distributed, which makes big outliners very unlikely. Furthermore, the y T values vary in [1, 1] , thus quite strongly. Therefore, we want a cost functions that is strongly affected outliners.