sheet4_zli

November 14, 2017

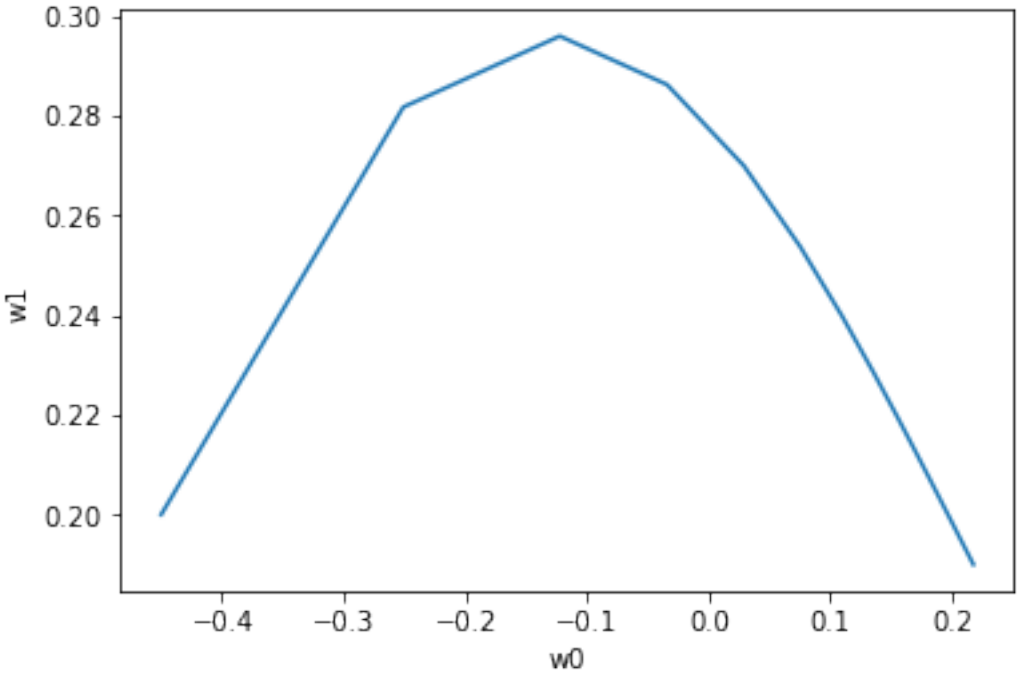
1 Exercise H4.2: Comparison of gradient descent methods

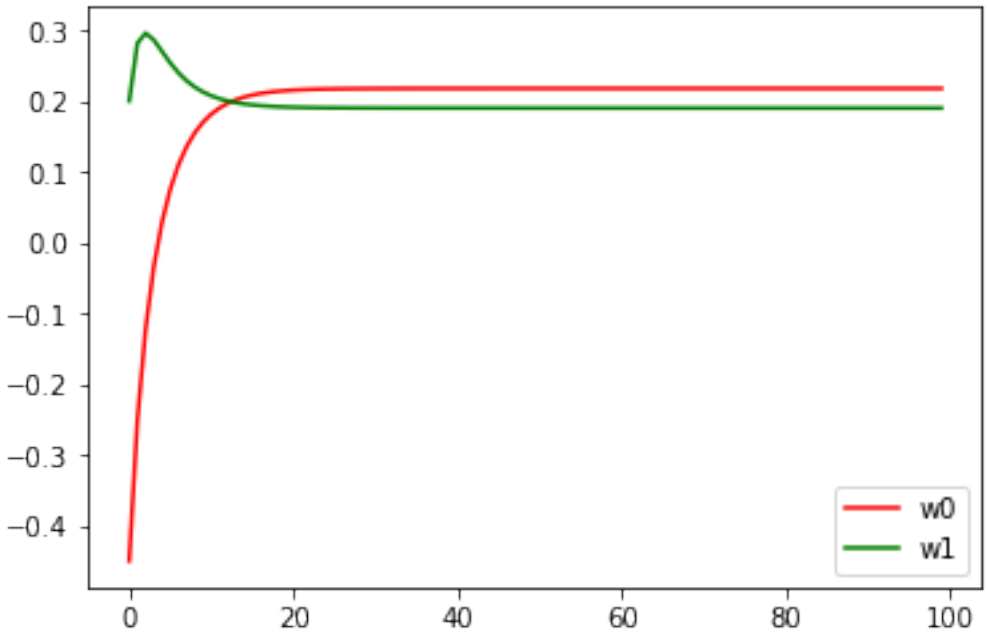
```
In [1]: import numpy as np
        import matplotlib
        import matplotlib.pyplot as plt
        %matplotlib inline

        data = np.array([[-1, -0.1], [0.3, 0.5], [2, 0.5]])
        X = np.array(zip(np.ones(data.shape[0]), data[:, 0])).T
        Y = data[:, 1].reshape(1, 3)
        W1 = np.array([-0.45, 0.2]).reshape(2,1)


        def func_g(w, x, y):
            # x: D*N
            # y: 1*N
            # w: D*1
            H = X.dot(X.T)
            b = -X.dot(Y.T)
            return H.dot(w) + b
```

1.1 (a) Gradient Descent

```
In [2]: learning_rate = 0.1
        w_gradient_descent_array = []
        # w_gradient_descent_array.append(W1)
        w_tmp = W1
        for i in range(0, 100):
            w_gradient_descent_array.append(w_tmp)
            w_tmp = w_tmp - learning_rate * func_g(w_tmp, X, Y)
        W = np.array(w_gradient_descent_array).reshape(100, 2)

        plt.plot(W[:, 0], W[:, 1])
        plt.xlabel('w0')
        plt.ylabel("w1")
        plt.show()
```

```
plt.plot(range(0, W.shape[0]), W[:, 0], color='r', label='w0')
plt.plot(range(0, W.shape[0]), W[:, 1], color='g', label='w1')
plt.legend()
plt.show()
```

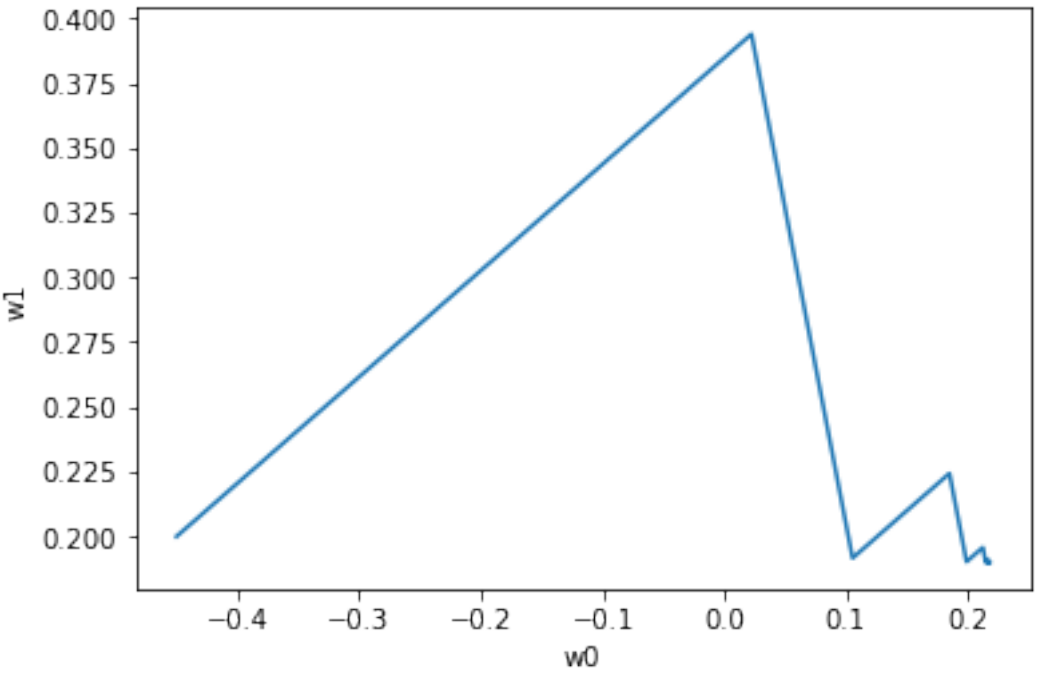


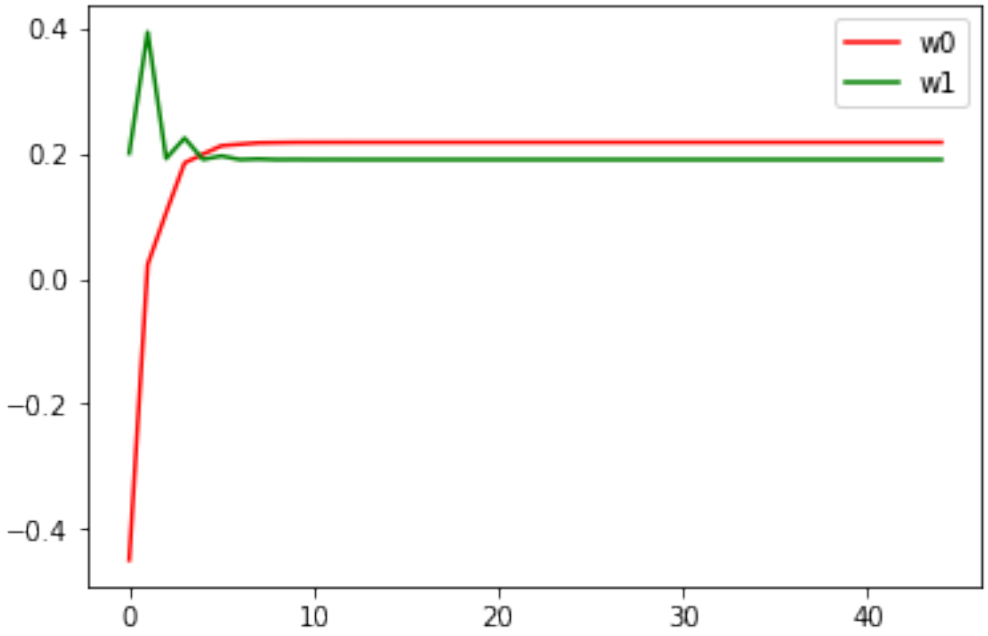


1.2 (b) Line Search

In [3]:
```
H = X.dot(X.T)

def calc_optimal_step_size(w, gt, dt):
    return dt.T.dot(gt)/(dt.T.dot(H)).dot(dt)

w_gradient_descent_array = []
w_tmp = W1
g1 = func_g(W1, X, Y)
step_size = calc_optimal_step_size(w_tmp, g1, g1)
for i in range(0, 100):
    w_gradient_descent_array.append(w_tmp)
    gt = func_g(w_tmp, X, Y)
    step_size = calc_optimal_step_size(w_tmp, gt, gt)
    w_tmp = w_tmp - step_size * gt
W = np.array(w_gradient_descent_array).reshape(100, 2)

plt.plot(W[:, 0], W[:, 1])
plt.xlabel('w0')
plt.ylabel("w1")
plt.show()

plt.plot(range(0, W.shape[0]), W[:, 0], color='r', label='w0')
plt.plot(range(0, W.shape[0]), W[:, 1], color='g', label='w1')
plt.legend()
plt.show()
```

```
/usr/local/lib/python2.7/site-packages/ipykernel_launcher.py:4: RuntimeWarning: invalid value en
  after removing the cwd from sys.path.
```

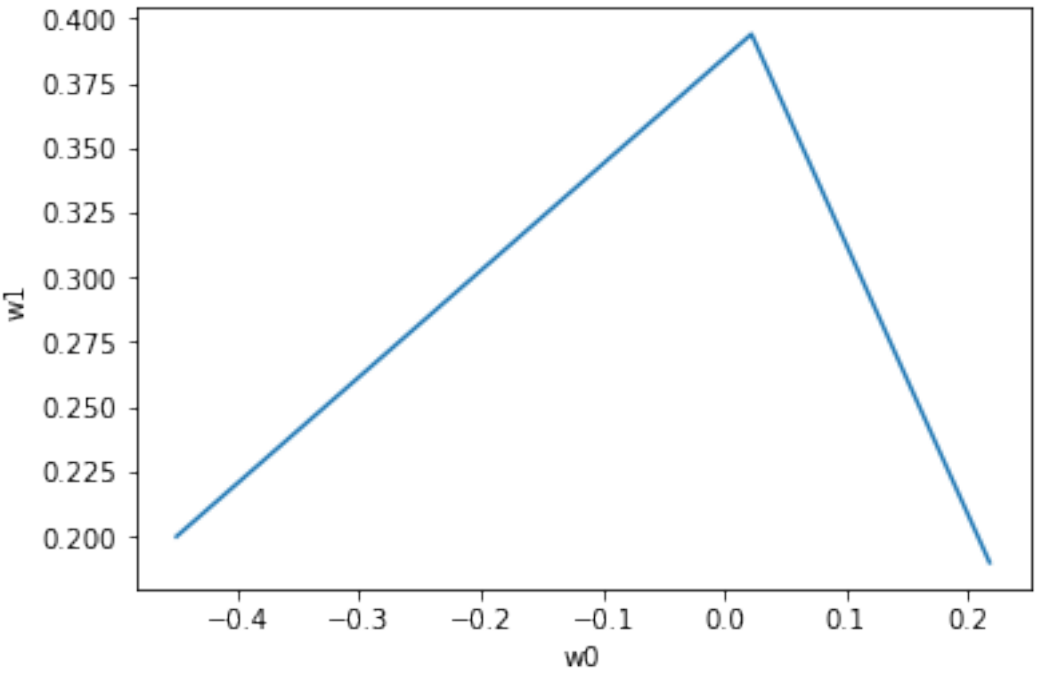

0.400
0.375
0.350
0.325
0.300
0.275
0.250
0.225
0.200
w1
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
w0

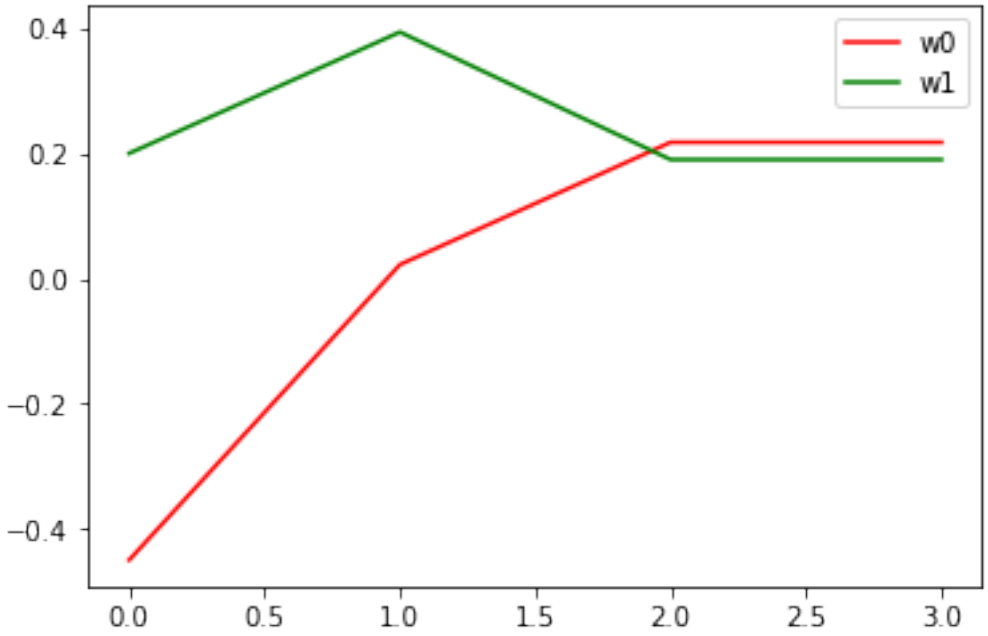

0.4
0.2
0.0
−0.2
−0.4
0
10
20
30
40
w0
w1

1.3 (c) Conjugate Gradient

In [14]:
```
def calc_beta(g_new, g_old):
    return g_new.T.dot(g_new)/g_old.T.dot(g_old)
w_gradient_descent_array = []
w_tmp = W1
g1 = func_g(W1, X, Y)
d1 = -g1
gt_new = g1
dt = d1
step_size = calc_optimal_step_size(w_tmp, g1, d1)
for i in range(0, 100):
    w_gradient_descent_array.append(w_tmp)
    step_size = calc_optimal_step_size(w_tmp, gt_new, dt)
    w_tmp = w_tmp - step_size * dt
    gt_old = gt_new
    gt_new = func_g(w_tmp, X, Y)
    beta = calc_beta(gt_new, gt_old)
    dt = gt_new - beta * dt
W = np.array(w_gradient_descent_array).reshape(100, 2)

plt.plot(W[:, 0], W[:, 1])
plt.xlabel('w0')
plt.ylabel("w1")
plt.show()

plt.plot(range(0, W.shape[0]), W[:, 0], color='r', label='w0')
plt.plot(range(0, W.shape[0]), W[:, 1], color='g', label='w1')
plt.legend()
plt.show()
```

```
/usr/local/lib/python2.7/site-packages/ipykernel_launcher.py:4: RuntimeWarning: invalid value en
  after removing the cwd from sys.path.
```


0.400
0.375
0.350
0.325
0.300
0.275
0.250
0.225
0.200
w1
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
w0


0.4
0.2
0.0
−0.2
−0.4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
w0
w1

In [ ]: