Exercise H 9.1

a). Let assume $x^{(d)+}$ is a positive sample,

and $x^{(d)-}$ is a negative sample.

Then the Euclidean distance between $x^{(d)+}$ and $x^{(d)-}$

will be. $d(x^{(d)+}, x^{(d)-}) = \frac{\underline{w}}{\|\underline{w}\|} \cdot (x^{(d)+} - x^{(d)-})$.

$x^{(d)+}$ is a positive sample, so

$$\underline{w}^T \cdot \underline{x}^{(d)+} + b = 1, \quad \underline{w}^T \cdot \underline{x}^{(d)+} = 1 - b$$

$x^{(d)-}$ is a negative sample, so

$$\underline{w}^T \cdot x^{(d)-} + b = -1, \quad w^T \cdot x^{(d)-} = -1 - b.$$

Therefore,
$$d(x^{(d)+}, x^{(d)-}) = \frac{\underline{w}}{\|\underline{w}\|} \cdot (x^{(d)+} - x^{(d)-})$$
$$= \frac{1}{\|\underline{w}\|} (\underline{w} x^{(d)+} - \underline{w} x^{(d)-})$$
$$= \frac{1}{\|\underline{w}\|} (1 - b - (-1 - b))$$
$$= \frac{2}{\|\underline{w}\|} = \frac{1 \cdot 2}{\|\underline{w}\|}$$

The number 2 is constant, so our target is to maximize

$\frac{1}{\|\underline{w}\|}$, therefore $d(\underline{x}^{(d)}, \underline{w}, b) \geq \frac{1}{\|\underline{w}\|}$

Exercise H 9.1

b). constraints : $y_T^{(\alpha)}(\underline{w}^T \cdot \underline{x}^{(\alpha)} + b) - 1 = 0$

target : $\frac{1}{2}\|w\|^2$

$$L(\underline{w}, b, \lambda_\alpha) = \frac{1}{2}\|w\|^2 - \sum_{\alpha=1}^{P} \lambda_\alpha \cdot \{ y_T^{(\alpha)}(\underline{w}^T \cdot \underline{x}^{(\alpha)} + b) - 1 \}$$

$$\frac{\partial L}{\partial w} = \underline{w} - \sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{x}^{(\alpha)} = 0$$

$$\underline{w} = \sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{x}^{(\alpha)}$$

$$\frac{\partial L}{\partial b} = - \sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} = 0$$

$$L(\underline{w}, b, \lambda_\alpha) = \frac{1}{2}\|w\|^2 - \sum_{\alpha=1}^{P} \lambda_\alpha \cdot \{ y_T^{(\alpha)}(\underline{w}^T \cdot \underline{x}^{(\alpha)} + b) - 1 \}$$

$$= \frac{1}{2}\left(\sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{x}^{(\alpha)}\right) \cdot \left(\sum_{\beta=1}^{P} \lambda_\beta \cdot y_T^{(\beta)} \cdot \underline{x}^{(\beta)}\right) - \sum_{\alpha=1}^{P} \left(\lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{w}^T \cdot \underline{x}^{(\alpha)} + \lambda_\alpha \cdot y_T^{(\alpha)} \cdot b - \lambda_\alpha\right)$$

$$= \frac{1}{2}\left(\sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{x}^{(\alpha)}\right) \cdot \left(\sum_{\beta=1}^{P} \lambda_\beta \cdot y_T^{(\beta)} \cdot \underline{x}^{(\beta)}\right) - \left(\sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{x}^{(\alpha)}\right) \cdot \left(\sum_{\beta=1}^{P} \lambda_\beta \cdot y_T^{(\beta)} \cdot \underline{x}^{(\beta)}\right) - \sum_{\alpha=1}^{P} \lambda_\alpha y_T^{(\alpha)} \cdot b + \sum_{\alpha=1}^{P} \lambda_\alpha$$

$$= \sum_{\alpha=1}^{P} \lambda_\alpha - \frac{1}{2}\left(\sum_{\alpha=1}^{P} \lambda_\alpha \cdot y_T^{(\alpha)} \cdot \underline{x}^{(\alpha)}\right) \cdot \left(\sum_{\beta=1}^{P} \lambda_\beta \, y_T^{(\beta)} \cdot \underline{x}^{(\beta)}\right)$$

$$= \sum_{\alpha=1}^{P} \lambda_\alpha - \frac{1}{2} \sum_{\alpha=1}^{P} \sum_{\beta=1}^{P} \lambda_\alpha \cdot \lambda_\beta \cdot y_T^{(\alpha)} \cdot y_T^{(\beta)} \cdot \underline{x}^{(\alpha)} \cdot \underline{x}^{(\beta)}$$