MCMC_HW2

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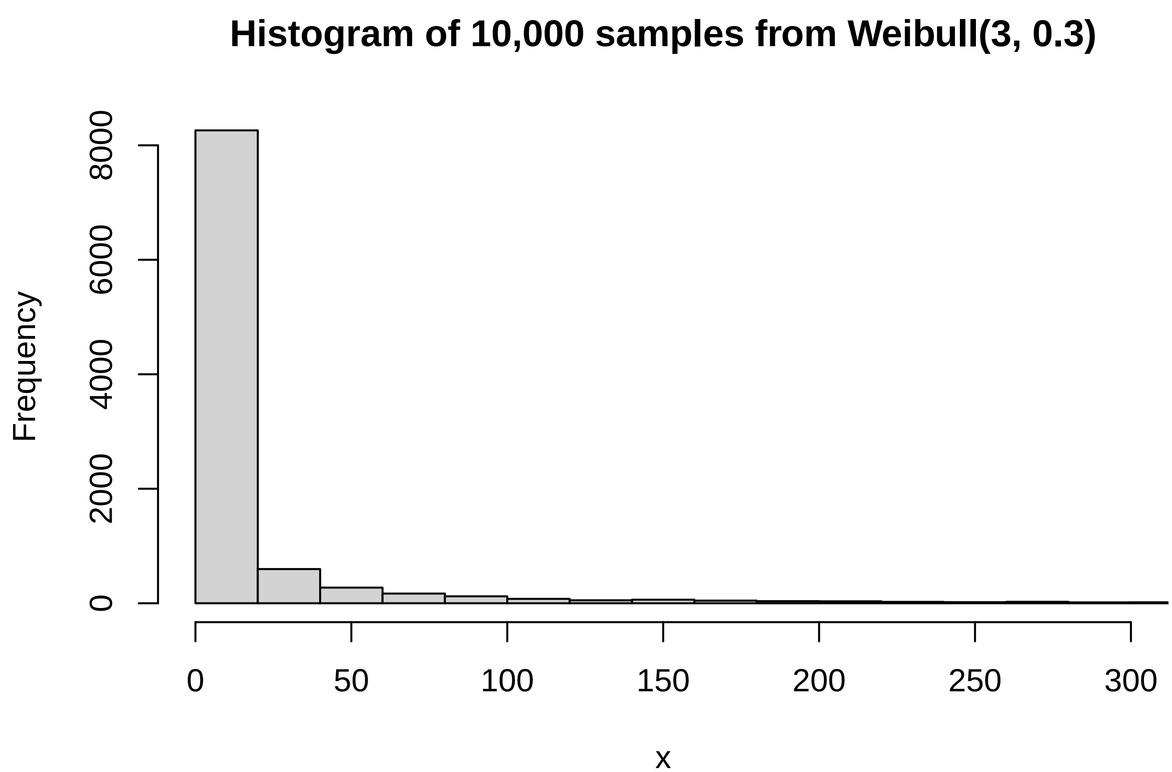
[HW 1] Generate 10,000 samples from Weibull(3, 0.3) and draw the histogram.

Let $X \sim Weibull(\lambda, k)$ and $u \sim Unif(0, 1)$.
Then by using inverse-cdf method, we can get

$$x = \lambda[-log(1-u)]^{\frac{1}{k}}$$

And using this fact, we can get the result below.

```
lambda = 3 ; k = 0.3
u = runif(10000)
x = lambda*(-log(1-u))^(1/k)
hist(x, nclass=500, xlim= c(0, 300),main="Histogram of 10,000 samples from Weibull(3, 0.3)")
```



[HW 2] Polar Methods for Generating Normal random variables.

Let X and Y follows standard normal distribution. Let r and θ denote the polar coordinate of the vector (X, Y).

$$r^2 = X^2 + Y^2 \quad tan\theta = \frac{Y}{X}$$

Since X and Y are independent, the joint density of X and Y is

$$f(x, y) = \frac{1}{2\pi} e^{-\frac{(x^2+y^2)}{2}}$$

Using polar method, X and Y can be expressed as $rcos\theta, rsin\theta$ respectively.

$$x^2 + y^2 = r^2 cos^2\theta + r^2 sin^2\theta = r^2$$

Therefore, $f(x, y)$ is transformed to $f(r, \theta)$

$$f(r, \theta) = \frac{1}{2\pi} \times \frac{1}{2} e^{-\frac{r^2}{2}}$$

And we can think that $\frac{1}{2\pi}$ as the distribution of θ and it follows $Unif(0, 2\pi)$ and $r^2 \sim exp(\frac{1}{2})$

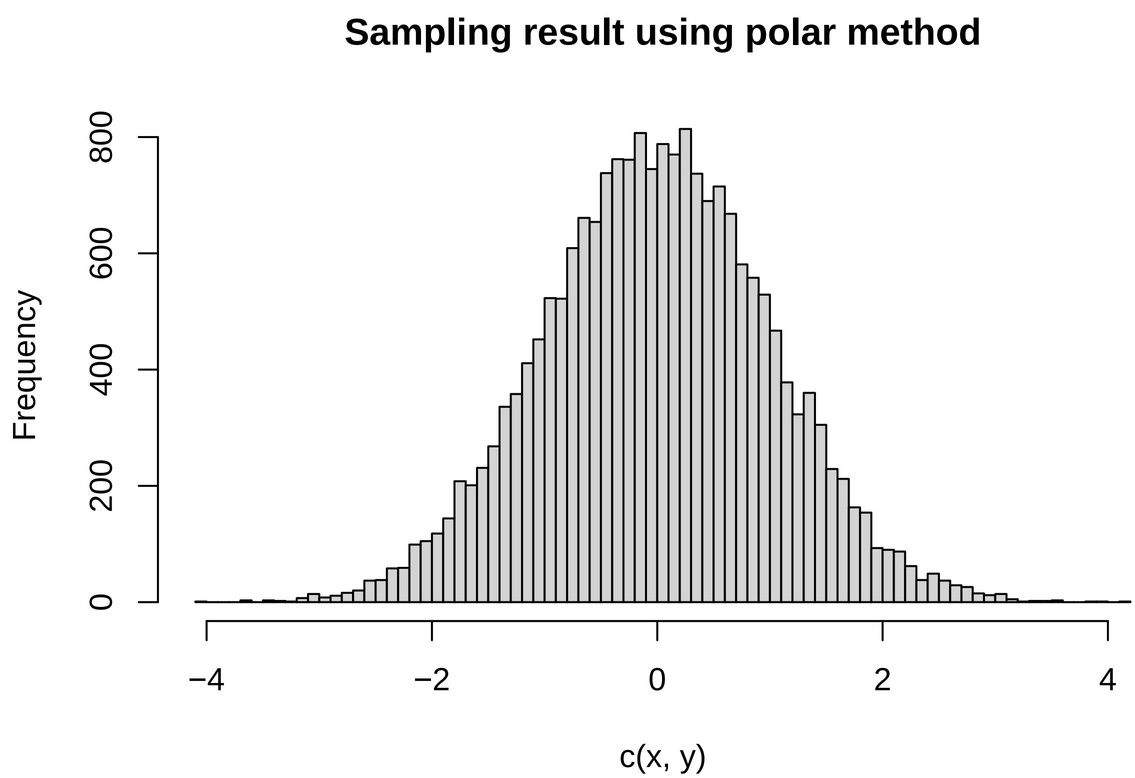
By inverse-CDF method, let $u \sim Unif(0, 1)$. Then we can the result below.

$$\theta = 2\pi u, \quad r = \sqrt{-2logu}$$

$$x = \sqrt{-2logu_1} cos2\pi u_2$$
$$y = \sqrt{-2logu_1} sin2\pi u_2$$

```r
u1 = runif(10000)
u2 = runif(10000)
r = sqrt(-2*log(u1))
theta = 2*pi*u2
x = r*cos(theta)
y = r*sin(theta)

# histogram
hist(c(x,y), nclass=100, main="Sampling result using polar method")
```



[**HW 3**]

(1) Find a density for that ARMS is applicable

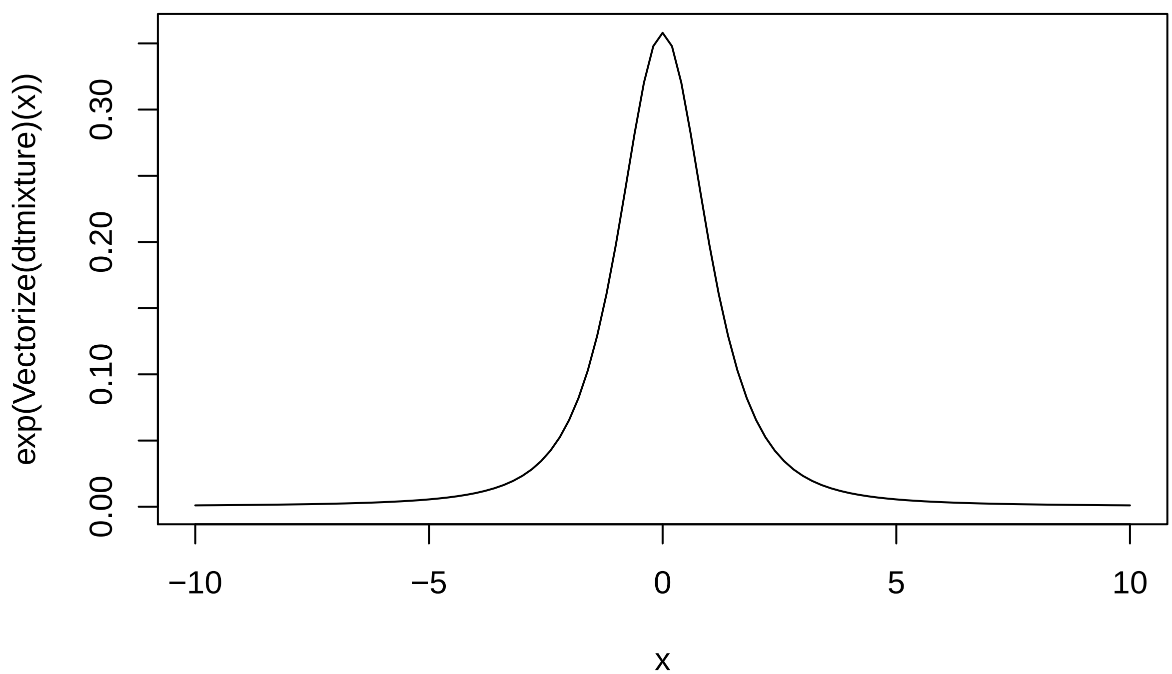
I will use t-mixture distribution in this section.

$$x \sim 0.3t(1) + 0.7t(4)$$

(2) Apply arms command in armspp package

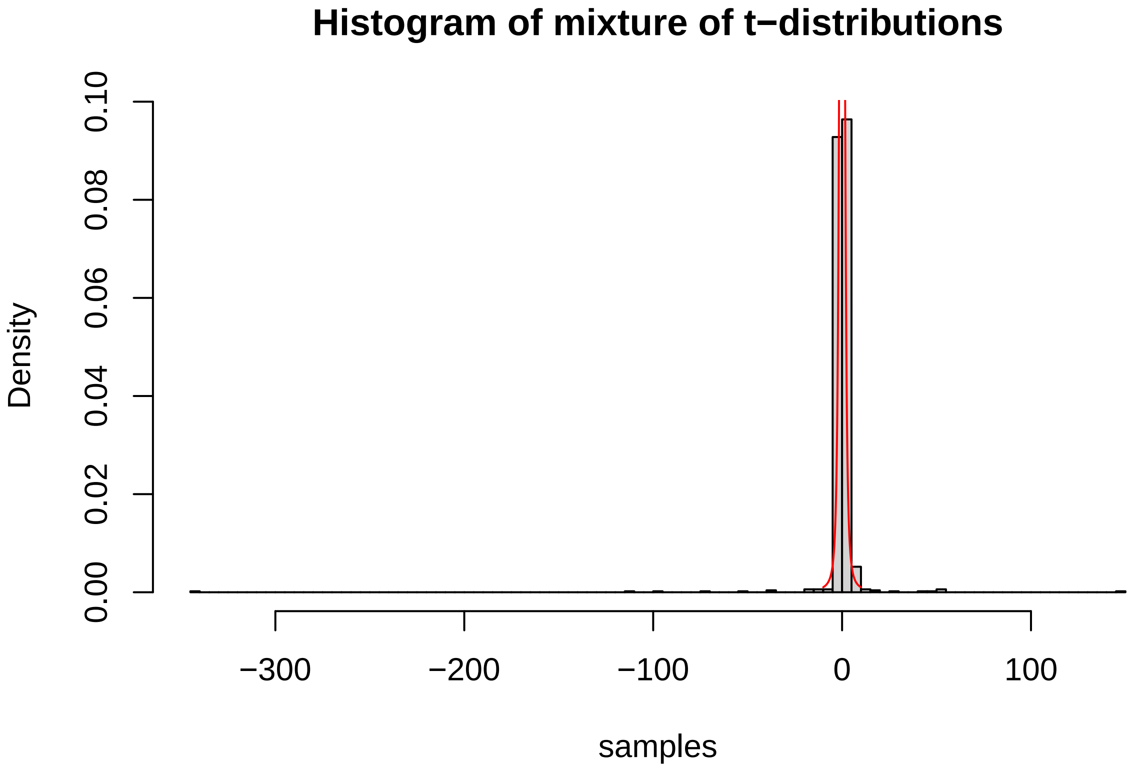
```
library(armspp)

# mixture of t distributions
dtmixture <- function(x) {
    parts <- log(c(0.3, 0.7)) + dt(x, df=c(1,4), log=TRUE)
    log(sum(exp(parts - max(parts)))) + max(parts)
}
curve(exp(Vectorize(dtmixture)(x)), -10, 10)
```



(3) Generate results

```
samples <- arms(1000, dtmixture, -1000, 1000)
hist(samples, freq=FALSE, nclass = 100, main="Histogram of mixture of t-distributions")
curve(exp(Vectorize(dtmixture)(x)),-10,10,col="red", add=TRUE)
```


Histogram of mixture of t−distributions
Density
0.00
0.02
0.04
0.06
0.08
0.10
−300
−200
−100
0
100
samples