Class : Discrete Mathematics
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Assignment I. Set Problems.

Important Instructions to Remember

These are some of the instructions to upload each assignment. It is mandatory to read them before doing the homework.

1. Do not forget the header. It must have the student's name, code, class group and assignment name. Homework without a student's name will NOT be graded.
2. Each exercise must have its number and its complete statement. You must indicate if you are doing additional problems.
3. Do not forget to check the file before uploading it to see if it is working or not. It is the student's responsibility to upload the files to the virtual platform.
4. Be careful if the assignment is handwritten, it must have clear handwriting and be scanned properly.

Let's do it! 👍

1. **(4 points)** Consider a group of 191 students from which 10 take French, Business, and Music; 36 take French and Business; 20 take French and Music; 18 take Business and Music; 65 take French; 76 take Business, and 63 take Music.
   a. How many students take French and Music classes but not Business class?
   b. How many students take Business class but neither French nor Music classes?
   c. How many students take Music or French class or both, but not Business class?
   d. How many students take none of the three classes?
2. **(4 points)** In a group of students, each takes math or computer science or both. One-fifth of those who take math also take computer science and one-eighth of those who take computer science are also in math course. Are more than one-third of the students taking the math course?

3. **(4 points)** Let $A = \{a, b, c, d\}$ be a set. List all elements of $\mathcal{P}(A)$. Which and how many are the subsets X of the set A such that $X \neq A$?

4. **(4 points)** Write a pseudo code which represents and print each set of the following set operations $X \cup Y$, and $X \cap Y$.

5. **(4 points)** Set a universal set and write a pseudo code which represents the complement set of a given set X, and $X \times Y$ of two given sets. In each case print the initial set and the outcome set.

6. **(5 points)** Which of the following statements are true and which are false? Please justify your answer.

   a. $\{x\} \subseteq \{x\}$
   b. $\{x\} \in \{x, \{x\}\}$
   c. $\{1, 2, 2, 3\} = \{1, 2, 3\}$
   d. $\{x : x^2 + x = 2\} = \{1, 2\}$
   e. $3 \in (3, 7]$
   f. $[5, 7] \subseteq (4, \infty)$

7. **(5 points)** Let X be a set. If $|X| = 10$.

   a. What is the cardinal of $\mathcal{P}(X)$?
   b. How many subsets $Y \subseteq A$ such that $Y \neq A$ are there?

8. **(5 points)** Which of the following are true and which are false? Please justify your answer.

   a. $\phi \subseteq G$ for all sets G.
   b. $\phi \in G$ for all sets G.
   c. $\phi \in \mathcal{P}(G)$ for all sets G.
   d. $\{\{\phi\}\} \subseteq \mathcal{P}(\phi)$.
   e. $\{\phi\} \subseteq \{\{\phi, \{\phi\}, \{\{\phi\}\}\}\}$.
   f. $\mathcal{P}(\{\phi\}) = \{\phi, \{\phi\}\}$.

9. **(5 points)** What is the relation between A and B such that each statement is true?

   a. $A \cap B = A$.
   b. $A \cup B = A$.
   c. $\overline{A \cap B} = B$.
   d. $X \times Y = Y \times X$.

10. **(10 points)** Let U be a set and $\{A_n : n \in \mathbb{N}\}$ be a set of subsets of U (Family of sets). Define

$$B = \bigcap_{n=0}^{\infty}(\bigcup_{k=0}^{\infty} A_{n+k}),$$

$$C = \bigcup_{n=0}^{\infty}(\bigcap_{k=0}^{\infty} A_{n+k}).$$

Prove:

a. $\bigcup_{n=0}^{\infty}(\bigcap_{k=0}^{\infty}\overline{A_{n+k}}) = \overline{\bigcap_{n=0}^{\infty}(\bigcup_{k=0}^{\infty}A_{n+k})}$.

b. If $A_1 \subset A_2 \subset ...$ or $... \subset A_2 \subset A_1$, then $B = C$.

Hit: Read the text in the virtual classroom which talks about *algunas generalizaciones del álgebra de conjuntos.*

Math Student's Problems:

1. Let A and B be sets. Suppose that $A \subseteq B$. Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

2. Give a proof or a counterexample of each of the following statements. Let X, Y and Z be subsets of a universal set U, and suppose that the Cartesian product universal set is $U \times U$.

   a, $\overline{X \cap Y} \subseteq X$ for all sets.

   b. $\overline{X \times Y} = \overline{X} \times \overline{Y}$ for all sets.

   c. $X \times \phi = \phi$ for all set.

3. Prove or give a counterexample for each of the following statements.

   a. Let A and B be sets. Then $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

   b. Let A and B be sets. Then $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.

> "If people do not believe that the mathematics are simple, it is only because they do not realize how complicated life is. "
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> John Von Neumann.