Class : Discrete Mathematics
Code : 23012
Author : Alejandra Torres Manotas
Email : alejandra.torresm@konradlorenz.edu.co

Assignment III. Counting principles, permutations and combinations.

Important Instructions to Remember

These are some of the instructions to upload each assignment. It is mandatory to read them before doing the homework.

1. Do not forget the header. It must have the student's name, code, class group and assignment name. Homework without a student's name will NOT be graded.
2. Each exercise must have its number and its complete statement. You must indicate if you are doing additional problems.
3. Do not forget to check the file before uploading it to see if it is working or not. It is the student's responsibility to upload the files to the virtual platform.
4. Be careful if the assignment is handwritten, it must have clear handwriting and be scanned properly.

Let's do it! 👍

1. **(4 points)** The Braille system for representing characters was developed in the early 9th century by Louis Braille. Special characters for the blind consist of raised dots. The positions for the dots are selected in two vertical columns of three dots each. There must be at least one raised dot. How many different Braille characters can there be?
2. **(4 points)** Answer the following questions:
   a. How many ways can five people's birthdays be different?
   b. How many possibilities are there for the birthday months of five people?
3. **(4 points)** How many possible telephone numbers are there when there are seven digits, the first two of which are between 2 and 9 inclusive, the third digit between 1 and 9 inclusive, and each of the remaining may be between 0 and 9 inclusive?
4. **(4 points)** How many integers between 1 and 10^4 contain exactly one 8 and one 9?

5. **(4 points)** There are 10 copies of a book, and 10 books with a unique copy of each one. In how many ways can be selected 10 books?

6. **(5 points)** In how many ways can 10 people arrange themselves

   a. In a row of 7 chairs?

   b. In a circle of 10 chairs?

7. **(5 points)** In how many ways can the digits $0, 1, 2, 3, 4, 5, 6, 7, 8,$ and 9 be arranged so that

   a. The digits 0 and 1 are adjacent?

   b. The digits $0, 1, 2,$ and 3 are adjacent.

8. **(5 points)** Suppose there are 15 red balls and 5 white balls. Assume that the balls are distinguishable and that a sample of 5 balls is to be selected.

   a. How many samples of 5 balls are there?

   b. How many samples contain at least 4 red balls?

9. **(5 points)** Consider the set $S = \{a, e, i, b, c, d, f, g, h, m, n, p\}$ of letters.

   a. How many 5-letter words containing 2 different vowels and 3 different consonants?

   b. How many 5-letter words have which begin with a and contain b?

10. **(10 points)** Choose two of the five solutions, and determine whether or not the following each one is correct.

   a. There are $C(4, 2)C(50, 3)$ 5-card hands with at least 2 aces.

   b. There are $C(52, 5) - C(36, 5)$ 5-card hands with at least one of each of the honor cards (ace, king, queen, and jack).

   c. There are $C(39, 5)$ 5-card hands that contain only spades, hearts, and diamonds. For any choice of 3 suits, there are $C(39, 5)$ 5-card hands that contain cards from at most these 3 suits. Thus, there are $C(4, 3)$ $C(39, 5)$ 5-card hands that contain cards from at most three suits.

   d. There are $C(4, 2)(48)(44)(40)$ 5-card hands that contain exactly one pair of kings and no other matching cards because we can choose the 2 kings in $C(4, 2)$ ways, remove all kings from the deck, choose one card in 48 ways, remove all cards of that kind, choose another card from the remaining 44 cards in 44 ways, remove all cards of that kind, and, finally, choose a last card from the remaining cards in 40 ways.

   e. A team can either win, lose, or tie each game it plays. Thus, there are $C(20, 7)2^{13}$ different team records with exactly 7 wins in a 20-game season since we can choose the games for wins in $C(20, 7)$ ways and then there are 2 possibilities (lose or tie) in the other 13 games. Therefore, there are $C(20, 7)3^{13}$ team records with at least 7 wins.

Math Student's Problems: They replace the first point.

1. Let $S_{n,k}$ be the number of ways in which is possible doing a partition of a set with n elements into k nonempty subsets. The subsets order is not taking account. (The number $S_{n,k}$ is known as the Stirling number of second type.)

   a. Prove that $S_{n,k} = 0$ when $k < n$.

   b. Prove that $S_{n,n} = 1$ for all $n \geq 1$.

   c. Prove that $S_{n,1} = 1$ for all $n \geq 1$.

   d. Prove that $S_{3,2} = 3$.

   e. Prove that $S_{4,2} = 7$.

   f. Prove that $S_{n,2} = 2^{n-1} - 1$ for all $n \geq 2$.

   g. Prove that $S_{n,n-1} = C(n, 2)$ for all $n \geq 2$.

2. Prove by mathematical induction that one-half of the 6^n outcomes of rolling n distinguishable dice have an even sum.

> "Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."
>
> David Hilbert.