Class : Discrete Mathematics
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Assignment II. Relations.

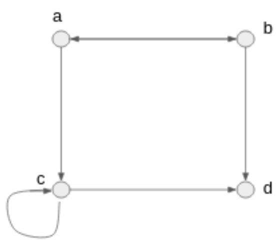
Important Instructions to Remember

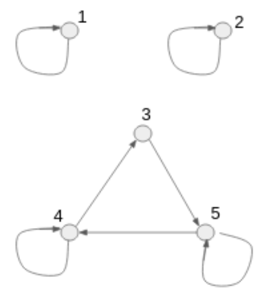
These are some of the instructions to upload each assignment. It is mandatory to read them before doing the homework.

1. Do not forget the header. It must have the student's name, code, class group and assignment name. Homework without a student's name will NOT be graded.
2. Each exercise must have its number and its complete statement. You must indicate if you are doing additional problems.
3. Do not forget to check the file before uploading it to see if it is working or not. It is the student's responsibility to upload the files to the virtual platform.
4. Be careful if the assignment is handwritten, it must have clear handwriting and be scanned properly.

Let's do it! 👍

1. **(4 points)** Write the matrix representation of each relation:
   a. Let R be a relation over $\{1, 2, 3, 4\}$ defined as $(x, y) \in R$ if and only if $x^2 \geq y$.
   b. Let R be a relation from the set X of planets to $\mathbb{Z}$ defined as $(x, y) \in R$ if and only if x is at the position y with respect to the Sun. For example, the Earth is located in position 3.
2. **(4 points)** Draw the graph of the following relations:
   a. The relation $R = \{(1, 2); (2, 1); (3, 3); (1, 1); (2, 2)\}$ over $X = \{1, 2, 3\}$.
   b. The relation $R = \{(1, 2); (2, 3); (3, 4); (4, 1)\}$ over $X = \{1, 2, 3, 4\}$.
3. **(4 points)** Write the relation set of each graph.

a. 

b. 

4. **(4 points)** A computer program consists of five modules: $M_1, M_2, ..., M_5$. A relation R on the set of modules is defined by the rule: $M_i R M_j$ if M_i is in the calling sequence of M_j. The relation matrix for R is shown below:

$$\begin{array}{c c} & \begin{array}{ccccc} M_1 & M_2 & M_3 & M_4 & M_5 \end{array} \\ \begin{array}{c} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{array} & \begin{bmatrix} T & F & T & T & F \\ F & T & T & F & F \\ F & F & T & F & F \\ F & F & T & T & F \\ F & F & T & T & T \end{bmatrix} \end{array}$$

   a. Verify that R is reflexive, antisymmetric and transitive.
   b. Which module is the main program?

5. **(4 points)** Let $A = \{1, 2, 3\}$. Each of the following subsets of $A \times A$ defines a relation on A. Is each relation reflexive, symmetric, skewsymmetric and/or transitive? Justify your answered.

   a. $C = \{(1, 2); (2, 3); (3, 1)\}$.
   b. $D = \{(1, 1); (1, 2); (2, 2); (2, 3); (3, 3); (1, 3)\}$.

6. **(5 points)** Determine if each relation is and equivalence relation over $\{1, 2, 3, 4, 5\}$, and it is an equivalence relation list the equivalence classes.

   a. $A = \{(1, 1); (2, 2); (3, 3); (4, 4); (5, 5); (1, 3); (3, 1); (3, 4); (4, 3)\}$.
   b. $B = \{(x, z) : 1 \leq x \leq 5 \text{ y } 1 \leq z \leq 5\}$.

7. **(5 points)** Let A be a set and think of $\subseteq$ as defining a relation on $\mathcal{P}(A)$, i.e.,

$$R = \{(X, Y) : X \subseteq Y\}.$$

   Is it an equivalence relation or an order relation? Justify your answer.

8. **(5 points)** Let $X = \{\text{San Francisco, Pittsburg, Chicago, San Diego, Filadelphia, Los Ángeles}\}$ be a set, and let R be a relation over X such that xRy if and only if x and y are in the same stated.

a. Is R an equivalence relation? Justify you answer.

b. Write the quotient set given by the relation.

9. **(5 points)** List all order pairs, and give an example of an equivalence relation over $A = \{1, 2, 3, 4, 5, 6\}$ which has a four size quotient set ($|A/\sim| = 4$).

10. **(10 points)** Let $A = \{a, b, c\}$. Then $\tau_A = \{A/\sim_1, A/\sim_2, A/\sim_3\}$ be the set of partitions of the set A, where

- $A/\sim_1 = \{\{a\}, \{b\}, \{c\}\}$,
- $A/\sim_2 = \{\{b, c\}, \{a\}\}$,
- $A/\sim_3 = \{\{a, b, c\}\}$.

Also, we see that $E(A) = \{R_1, R_2, R_3, R_4\}$ be the set of equivalence relations on A, where these are defined by the sets

- $R_1 = \{(a, a); (b, b); (c, c); (a, b); (b, a)\}$,
- $R_2 = \{(a, a); (b, b); (c, c)\}$,
- $R_3 = \{(a, a); (b, b); (c, c); (b, c); (c, b)\}$,
- $R_4 = \{(a, a)\,; (b, b)\,; (c, c)\,; (a, c)\,; (c, a)\}$.

Now, with the last information follow step by step the following points:

i. Write in enumerated form the set $\mathcal{P}(A)$.

ii. Verify if the set τ_A has all possible partitions of A, if it has not complete the set.

iii. Based on $(ii.)$ verify if the set $E(A)$ is the set of all possible equivalence relations of the set A, if it is not complete the set.

iv. Finally, write from the sets τ_A and $E(A)$ the correspondence between their elements.

Math Student's Problems:

1. ¿What is wrong in the following argument? ¿It does prove that any relation R over X, which is symmetric and transitive, then it is reflexive?

Let $x \in X$. Since R is symmetric $(x, y), (y, x) \in R$, and by transitive property $(x, x) \in R$, so R is reflexive.

2. Which of the following families of subsets of $\mathbb{R}$ are partitions of $[0, \infty)$?

a. $H = \{[n-1, n)\}_{n\in\mathbb{N}}$.

b. $G = \{[x-1, x)\}_{x\in[0,\infty)}$.

c. $F = \{\{x\}\}_{x\in[0,\infty)}$.

d. $I = \{[n-1, n+1)\}_{n\in\mathbb{N}}$.

e. $J = \{[0, n)\}_{n\in\mathbb{N}}$.

f. $K = \{[2^{n-1}-1, 2^n-1)\}_{n\in\mathbb{N}}$.

"For those of you who don't know about maths is difficult to understand that they are beautiful, the most beautiful of nature. If you want to learn about nature, and appreciate it, it is necessary to understand its speaking language."

Richard Feynman.