Homework 3

ISyE 6420
Spring 2021

1. Neuron Fires. The data set `neuronfires.dat|mat|xlsx` comes from the Lab of Dr. Steve Potter at BME, Georgia Tech. It consists of 989 firing times of a cell culture of neurons. The recorded firing times are time instances when a neuron sent a signal to another linked neuron (a spike). The cells, from the cortex of an embryonic rat brain, were cultured for 18 days on multielectrode arrays. The measurements were taken while the culture was stimulated at the rate of 1 Hz. It was postulated that firing times form a Poisson process; thus the interspike intervals should have an exponential distribution.

(a) Calculate the interspike intervals $T_i, i = 1, \ldots, 988$ and check the histogram for T. Discuss its resemblance to the exponential density and find the MLE for exponential rate parameter λ.

(b) Given the exponential model for T_i's, find the posterior distribution of rate parameter λ when the prior for λ is gamma $\mathcal{G}a(18, 20)$. What is the Bayes estimator for λ (posterior mean)? Find the posterior variance of λ.

(c) If the model for T_i's is parametrized by a scale parameter μ $(= 1/\lambda)$, find the posterior mean if the prior on μ is inverse-gamma $\mathcal{IG}(18, 20)$. What is the posterior variance of μ?

(d) From the invariance principle of MLEs it follows that $\hat{\mu}_{MLE} = 1/\hat{\lambda}_{MLE} = \bar{T}$. Is it true that $\hat{\mu}_B = 1/\hat{\lambda}_B$ for Bayes estimators $\hat{\lambda}_B$ and $\hat{\mu}_B$ found in (b) and (c)?

2. Improvement of Surgical Procedure. In a disease in which the postoperative mortality is usually 10%, a surgeon devises a new surgical technique. He tries the technique on 15 patients and has no fatalities.

(a) What is the probability of the surgeon having no fatalities in treating 15 patients if the postoperative mortality rate is $\theta = 0.1$?

(b) The surgeon claims that his new surgical technique significantly improves the survival rate. Conduct the classical (frequentist) test

$$H_0 : \theta = 0.1 \quad v.s. \quad H_1 : \theta < 0.1,$$

and report p-value.

(c) If the prior on θ was beta $\mathcal{B}e(1, 9)$, what is the posterior probability of H_0 in testing

$$H_0 : \theta \geq 0.1 \quad v.s. \quad H_1 : \theta < 0.1.$$

What is the Bayes factor B_{01}?

(d) A Bayesian wants to test a precise null hypothesis

$$H_0 : \theta = 0.1 \quad v.s. \quad H_1 : \theta \neq 0.1,$$

and adopts prior

$$\pi(\theta) = 0.5 \cdot \mathbf{1}(\theta = 0.1) + 0.5 \cdot \mathcal{B}e(1, 9).$$

What is the posterior probability of H_0? What is the Bayes factor B_{01}? (Note from the form of prior that prior probabilities of the hypotheses are equal, $\pi_0 = \pi_1 = 1/2$).