

Homework 2

ISyE 6420

Spring 2021

1. 2-D Density Tasks. If

$$f(x, y) = \begin{cases} \frac{1}{4}xy(x+y)\exp\{-x-y\}, & 0 \leq x < \infty, 0 \leq y < \infty \\ 0, & \text{else} \end{cases}$$

Find

- (a) marginal distribution $f_X(x)$,
- (b) conditional distribution $f(y|x)$, and
- (c) expectation $\mathbb{E}[X]$.

2. Jeremy and Variance from Single Observation. Jeremy believes that his IQ test scores have normal distribution with mean 110 and unknown variance σ^2 . He takes a test and scores $X = 98$.

(a) If the prior on σ^2 is inverse gamma $\mathcal{IG}(\alpha, \beta)$ and the observation X is normal $\mathcal{N}(\mu, \sigma^2)$ with μ known, what is the posterior?

(b) Find a Bayes estimator (posterior mean) of σ^2 and its standard deviation in Jeremy's model if the prior on σ^2 is an inverse gamma $\mathcal{IG}(3, 100)$.

Hint. Random variable Y is said to have an inverse gamma $\mathcal{IG}(\alpha, \beta)$ distribution if its density is given by

$$f(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)y^{\alpha+1}} \exp\left\{-\frac{\beta}{y}\right\}, \alpha, \beta > 0.$$

The mean of Y is $\mathbb{E}[Y] = \frac{\beta}{\alpha-1}$, $\alpha > 1$ and the variance is $Var(Y) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$, $\alpha > 2$.

3. Exponential Lifetimes. A lifetime X (in years) of a particular device is modeled by an exponential distribution with unknown rate parameter θ . The lifetimes of $X_1 = 5$, $X_2 = 6$, and $X_3 = 4$ are observed. Assume that an expert familiar with this type of devices suggested that θ has exponential distribution with mean 3.

- (a) For the prior suggested by the expert, find the posterior.
- (b) What are the mean and variance of the posterior.

4. Cell Clusters in 3D Petri Dishes. The number of cell clusters in a 3D Petri dish has a Poisson distribution with mean $\lambda = 5$. Find the percentage of Petri dishes that have

(a) 0 clusters, (b) at least one cluster, (c) more than 8 clusters, and (d) between 4 and 6 clusters inclusive.