# Homework 5

**ISyE 6420**
Spring 2021

**1. Bipetal Diameter and Age of Fetus.** A study by Hadlock et al (1982) found a strong correlation between fetal bipetal diameter (BPD) and menstrual age of the fetus in weeks (MA). The BPD is one of the basic biometric parameters used to assess fetal size. It is important for judging the weight and growth patterns of the fetus. Hadlock et al provided BPDs of 533 fetuses with ma ranging from 12 to 40 weeks. The data are provided in `hadlock.dat|xlsx`. The first column is MA and the second column is BPD.

Fit the model

$$\text{BPD} = \beta_0 + \beta_1\sqrt{\text{MA}} + \epsilon,$$

where $\epsilon$ is normal $\mathcal{N}(0, \sigma^2)$.

(a) What are Bayes estimators of $\beta_0, \beta_1$ and $\sigma^2$? Use standard noninformative priors. What is $R^2$?

(b) Test $H_0 : \beta_1 \leq 8/3$ versus the alternative $H_1 : \beta_1 > 8/3$. Report their posterior probabilities.

(c) Find 95% credible set for population intercept $\beta_0$.

(d) What BPD is predicted when MA = 10.5? Find 95% credible set for the mean response and for the individual response (prediction).

**2. Binary Regression and IOP.** Laser refractive surgery often decreases Intraocular Pressure (IOP) and may lead to hypotony (clinically significant low IOP that may lead to corneal decompensation, accelerated cataract formation, maculopathy, and discomfort).

An investigator wished to determine whether the post-operative IOP in patients after laser refractive surgery was related to the residual thickness of the cornea. In a sample of 140 patients who had undergone laser surgery, post-operative IOP and the thickness of the cornea were measured. Data set is provided in startup file `iop2.dat|mat` which consists of two columns, (1) indicator of low IOP (IOP $< 10$) and (2) central corneal thickness (in micrometers)

(a) Fit the logistic regression with cornea thickness as the predictor of incidence of low IOP. Use standard noninformative priors.

(b) For a person who had a refractive surgery with residual thickness of cornea of 420 micrometers, what is the risk of a low IOP.

(c) Compare deviances for two links: `logit` (as in (a)), and `probit`. Which link provides better fit?

**3. Assessing the Volume in Shortleaf Pine.** *Pinus echinata* (shortleaf pine) forests provided innumerable railroad ties for our nation's expanding railroad network in the late 19th and early 20th century. The wood is now used for general construction, exterior and interior finishing, and pulpwood.

Volume is the most widely used measure of wood quantity and is often estimated in standing trees for the assessment of economic value or commercial utilization potential. Volume is usually estimated from such measurements as diameter and merchantable height. The proposed equation for volume is

$$V = \alpha_0 D^{\alpha_1} H^{\alpha_2} \eta,$$

where $D$ is the diameter at breast height (1.3 m) and $H$ is the merchantable height. Parameters $\alpha_0, \alpha_1$ and $\alpha_2$ depend on the tree species while $\eta$ is multiplicative error with lognormal distribution with parameters $\mu = 0$ and $\sigma^2$.

Bruce and Schumacher (1935) provided data on 70 shortleaf pine trees consisting of $D$ (in), $H$ (ft), and $V$ (cubic ft). The dataset is in file `shortleaf0.dat`.

(a)Set the model as

$$\log V = \gamma_0 + \gamma_1 \log D + \gamma_2 \log H,$$

and estimate parameters $\gamma_0, \gamma_1$ and $\gamma_2$.

(b)For $D = 15$ in and $H = 85$ ft estimate the mean volume and find 95% credible set.

**Hint.** Be careful in (b), the model is for $\log V$ and the estimate and CS is needed for $V$.